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Power Quality Assessment Based on Rough AHP and Extension Analysis

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ABSTRACT

Due to the increasing power consumption of whole society and widely using of new non-linear and asymmetric electrical equipment, power quality assessment problem in the new period has attracted more and more attention. The mathematical essence of comprehensive assessment of power quality is a multi-attribute optimal decision-making problem. In order to solve the key problem of determining the indicator weight in the process of power quality assessment, a rough analytic hierarchy process (AHP) is proposed to aggregate the judgment opinions of multiple experts and eliminate the subjective effects of single expert judgment. Based on the advantage of extension analysis for solving the incompatibility problem, extension analysis method is adopted to assess the power quality. The assessment grades of both total power quality and each assessment indicator are obtained by correlation function. Through a case of 110 kV bus of a converting station in a wind farm of China, the feasibility and effectiveness of the propose method are demonstrated. The result shows that the proposed method can determine the overall power quality of power grid, as well as compare the differences among the performance of assessment indicators and provide the basis for further improving of power quality.

KEYWORDS

Power quality; comprehensive assessment; decision making; rough sets; analytic hierarchy process; extension analysis

Nomenclature:

I_j:	One assessment indicator, $j = 1, 2, \ldots, m$
D_l:	One quality grade, $l = 1, 2, \ldots, h$
U:	A domain which is actually a nonempty finite set of objects
Y:	Any object in U
S_i:	Any partition in U, $1 \leq i \leq n$
$U/R(Y)$:	The partition of the indistinct relationship $R(Y)$ in U
$\overline{AS}(S_i)$:	Upper approximation set of S_i in U
$\underline{AS}(S_i)$:	Lower approximation set of S_i in U
$RN(S_i)$:	Rough boundary interval of S_i in U
$\underline{L}(S_i)$:	Rough lower limit of $RN(S_i)$

$\overline{L}(S_i)$:	Rough upper limit of $RN(S_i)$
t:	The number of expert, $t = 1, 2, \ldots, p$
Φ^t:	Pairwise comparison matrix given by expert t
ϕ_{kj}^t:	One element in Φ^t, $k = 1, 2, \ldots, m$ and $j = 1, 2, \ldots, m$
CR^t:	Consistency ratio of Φ^t
CI^t:	Consistency index of Φ^t
$\lambda_{\max}^t$:	Largest eigenvalue of Φ^t
RI:	Random index
Φ:	Group decision matrix
ϕ_{kj}:	One element in Φ
$RN(\phi_{kj}^t)$:	Rough boundary interval of $\phi_{kj}^t \in \phi_{kj}$
$\phi_{kj}^{t,-}$:	Rough lower limit of ϕ_{kj}^t in set ϕ_{kj}
$\phi_{kj}^{t,+}$:	Rough upper limit of ϕ_{kj}^t in set ϕ_{kj}
$RN(\phi_{kj})$:	Rough boundary interval of ϕ_{kj}
$ARN(\phi_{kj})$:	Average form of $RN(\phi_{kj})$
ϕ_{kj}^-:	Is the rough lower limit of set ϕ_{kj}
ϕ_{kj}^+:	Is the rough upper limit of set ϕ_{kj}
EA:	Rough judgment matrix
EA^-:	Rough lower limit matrix
EA^+:	Rough upper limit matrix
VA^-:	Eigenvector corresponding to the maximum eigenvalue of EA^-
VA^+:	Eigenvector corresponding to the maximum eigenvalue of EA^+
va_j^-:	Value of VA^- on dimension j
va_j^+:	Value of VA^+ on dimension j
ω_j:	Weight value of indicator I_j
ω:	Weight vector
R_0:	Matter-element to be assessed
Y_0:	Unknown assessment grade of R_0
I:	Indicator set which represents the characteristics of R_0, $I = \{I_1, I_2, \ldots, I_m\}$
v_0:	Indicator value vector, $v_0 = [v_1, v_2, \ldots, v_m]$
v_j:	Indicator value on indicator I_j
R^l:	Classical domain
Y^l:	lth assessment grade of R_0
$[a_{lj}, b_{lj}]$:	Indicator value interval corresponding to Y^l on indicator I_j
R:	Joint domain
Y:	Set of all assessment grades of R_0
$[a_j, b_j]$:	Indicator value interval corresponding to all assessment grades on index I_j
$x = [a, b]$:	A bounded interval
$|x| = |b - a|$:	Module of bounded interval $x = [a, b]$
$\rho(x_0, x)$:	Distance from point x_0 to interval $x = [a, b]$

$K(x)$:   Correlation function

$K_l(v_j)$:   Correlation function which describes the quantitative relationship from indicator value v_j of matter-element R_0 to be assessed to two intervals $[a_{lj}, b_{lj}]$ and $[a_j, b_j]$

K_l: Comprehensive correlation function

1 Introduction

Scientific and reasonable comprehensive assessment of power quality of smart grid is the basis for restraining and urging power companies and power users to jointly maintain the power quality environment of public power grid, and is also the main basis for measuring power quality and formulating electricity price, which has profound theoretical and practical significance [1,2]. At present, the comprehensive judgment methods of power quality are mainly based on fuzzy mathematics [3], probability statistics and vector algebra method [4,5], evaluation method based on grey theory [6,7], evaluation method based on subjective and objective weights [8–10], grey cloud evidential reasoning [11], variable fuzzy set [12] and random forest [13].

In the process of evaluation, the above methods are uncertain to a certain extent or too subjective. For example, in reference [3], the concept of interval average distribution density is used to establish a fuzzy model for power quality evaluation, but a single subjective weighting method is used to determine the weight value, which has a great impact on the accuracy of power quality assessment results; according to the main characteristics of each single power quality index in reference [5], vector algebra method is used to normalize and quantify them effectively. However, selecting different reference values will produce different evaluation results, which is not conducive to the objective evaluation of power quality. The power quality assessment method based on genetic projection pursuit can extract the characteristics of high-dimensional nonlinear evaluation indexes by automatic search, which overcomes the defects of human subjective factors in traditional evaluation methods. However, due to the concealment in its own evaluation process, it increases the uncertainty of evaluation results.

The combination of rough sets theory [14–16] and analytic hierarchy process (AHP) [17] can overcome the shortcomings of classical AHP and has been applied in many practical applications. Lee et al. [18] used rough set theory and group AHP for the evaluation of new service concepts and considerable results have been achieved. Zhang [19] studied rough set and AHP which was significant for the risk assessment of maintenance. Sun et al. [20] evaluated the performance of combined cooling heating and power (CCHP) system based on interval rough number AHP. Peng et al. [21] carried out the transformer state assessment based on rough sets theory and AHP. Wan et al. [22] adopted rough set and fuzzy AHP to determine customer demand weight. Extension analysis can solve the problem of incompatibility of various features of things [23–29]. Liu et al. [23] built the matter element extension model for risk analysis of buried crude oil pipeline. Zhai et al. [24] combined set pair analysis and extension theory coupling model and applied them for river health evaluation. Zhou et al. [25] proposed an improved extension AHP to evaluate the power failure. Jiang et al. [26] put forward a risk assessment approach of highway tunnel construction based on analytic hierarchy process and extension model.

In view of the shortcomings of the above methods, rough sets theory is introduced to improve the classical AHP when determining the weight of indicators in the power quality assessment indicator system and a rough AHP is proposed to aggregate the weight information of multiple experts, so as to eliminate the subjective influence of a single expert. Based on the characteristic of extension analysis

for solving the problem of incompatibility of various features of things, extension analysis method is adopted to assess the power quality.

2 Framework

2.1 Power Quality Assessment Indicator System

In this paper, six power quality standards of national grid are used as the basis of comprehensive assessment of power quality, and each power quality standard is taken as assessment indicator. The six assessment indicators of power quality are frequency deviation, total harmonic distortion rate of voltage, voltage fluctuation, voltage flicker, voltage deviation and voltage three-phase unbalance degree, which are expressed by I_1-I_6, respectively. They are divided into several quality grades which are expressed by D_1-D_h, respectively. The power quality assessment indicator system is shown in Fig. 1.
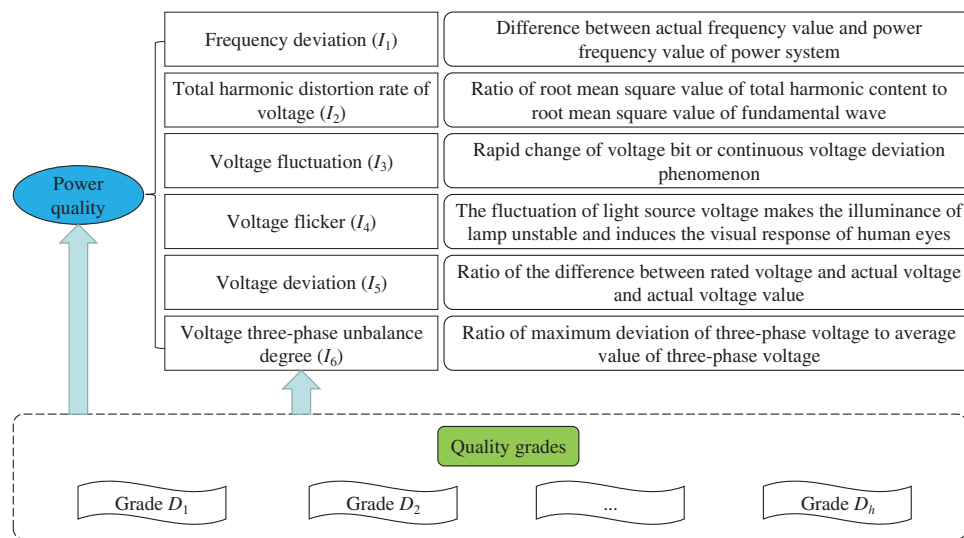

Figure 1: The power quality assessment indicator system

2.2 Research Framework

The overall research framework is shown in Fig. 2, which include three parts: the construction of assessment indicator system, the determination of assessment indicator weight and the assessment of power quality.

The construction of assessment indicator system is given in the above section. The other parts are explained in Fig. 2. The method and theoretical innovations are mainly elaborated as follows:

(1) In the determination of assessment indicator weight, multi-expert comprehensive judgment is adopted for eliminating the adverse effects of single expert judgment, rough sets theory is introduced to aggregate the judgment opinions of all experts, and the weights of the indicators is calculated by AHP.

(2) In the assessment of power quality, the classical domain matter-element, joint domain matter-element and matter-element to be assessed are obtained by extension analysis theory. Then based on correlation function both the assessment grade of total power quality and each assessment indicator can be obtained. According to the assessment result, the key factors affecting power quality will be analyzed.
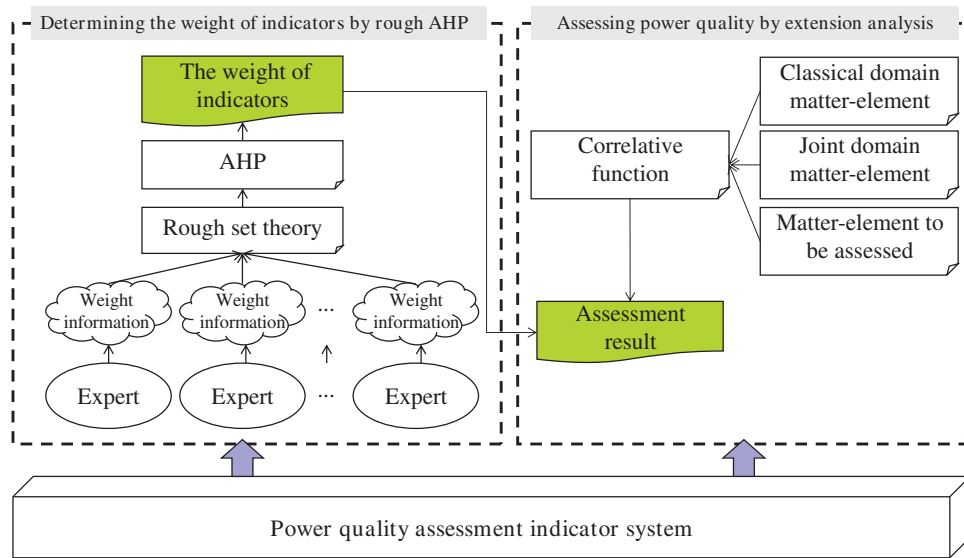
Figure 2: The research framework

3 Method

3.1 Determining the Weight of Indicator by Rough AHP

AHP method [17] was put forward by Thomas L. Saaty. This method can not only make clear the hierarchical structure of the components of complex problem, but also verify the consistency of the results. Therefore, it has been widely applied in the weighting of multi-attribute decision-making problem [30,31]. Experts inevitably have subjectivity in judging the relative importance of indicators. To a certain extent, the adverse effects of single expert judgment can be eliminated by using multiple experts for comprehensive judgment. The judgment of relative importance between indicators by multiple experts is obviously indistinguishable when aggregating the judgment opinions of all experts. Instead of a membership function, the concept of rough boundary interval in rough sets theory [14–16] can represent the indistinguishability as a set boundary area. It can better aggregate the judgment opinions of all experts. Accordingly, a rough AHP method is designed to determine the weight of indicators in power quality assessment.

3.1.1 Basic Concepts of Rough Sets

Here, U is used to represent a domain which is actually a nonempty finite set of objects. Y is any object in U.

Concept 1. Upper and lower approximation sets [14–16].

In U, all objects are divided into n partitions: $S_1, S_2, \ldots, S_n$. If these n partition has the order of $S_1 < S_2 < \ldots < S_n$, the upper and lower approximation sets of any partition S_i ($1 \leq i \leq n$) can be defined as follows:

$$\overline{AS}(S_i) = \{Y \in K | K \subseteq U / R(Y) \wedge K \geq S_i\}$$
$$\underline{AS}(S_i) = \{Y \in K | K \subseteq U / R(Y) \wedge K \leq S_i\}$$
(1)

where $U/R(Y)$ represents the partition of the indistinct relationship $R(Y)$ in U.

Concept 2. Rough boundary interval [14–16].

According to the above definition, any ambiguous partition S_i in U can be represented by its rough boundary interval $RN(S_i)$. $RN(S_i)$ consists of its rough lower limit $\underline{L}(S_i)$ and rough upper limit $\overline{L}(S_i)$ which are defined as follows:

$$\underline{L}(S_i) = \frac{\sum\limits_{Y \in \underline{AS}(S_i)} R(Y)}{\underline{N}(S_i)} \tag{2}$$

$$\overline{L}(S_i) = \frac{\sum\limits_{Y \in \overline{AS}(S_i)} R(Y)}{\overline{N}(S_i)} \tag{3}$$

where $\overline{N}(S_i)$ is the number of objects in the upper approximation set of S_i and $\underline{N}(S_i)$ is the number of objects in the upper approximation set of S_i.

As can be seen, an ambiguous partition in the domain can be represented by a rough boundary interval containing a rough lower limit and a rough upper limit as follows:

$$RN(S_i) = [\underline{L}(S_i), \overline{L}(S_i)] \tag{4}$$

3.1.2 Rough AHP

In the proposed rough AHP, the concept of rough boundary interval in rough sets theory is introduced in classical AHP to aggregate the judgment opinions of multiple experts. The detailed steps are as follows:

Step 1. Construct the comparison matrix.

According to the indicator system shown in Fig. 1, multiple experts compare the importance of each indicator under the constraint of the overall objective. Here, power quality is the overall objective. Let the overall objective be the criterion, and the indicators controlled by it is $I_1 \sim I_m (m = 6)$. The relative importance of indicator with regard to the overall objective is the weight of indicator. The weight is determined by pairwise comparison of 1–9 scales shown in Table 1.

Table 1: Pairwise comparison of 1–9 scales

Scale	Meaning
1	The two factors have the same importance.
3	The former factor is slightly more important than the latter factor.
5	The former factor is obviously more important than the latter factor.
7	The former factor is intensively more important than the latter factor.
9	The former factor is more important than the latter factor in the extreme.
2, 4, 6, 8	The intermediate value of the above adjacent judgments.
Reciprocal	If the scale of importance of factor 1 to factor 2 is χ_{12}, then the scale of importance of factor 2 to factor 1 is $1/\chi_{12}$.

Assuming that there are p experts, each expert carries out the pairwise comparison of the indicators by 1–9 scales. Then p pairwise comparison matrices are obtained. The pairwise comparison matrix given by expert t ($t = 1, 2, \ldots, p$) is defined as follows:

$$\Phi^t = \begin{bmatrix} 1 & \phi_{12}^t & \cdots & \phi_{1m}^t \\ \phi_{21}^t & 1 & \cdots & \phi_{2m}^t \\ \vdots & \vdots & & \vdots \\ \phi_{m1}^t & \phi_{m2}^t & \cdots & 1 \end{bmatrix} \tag{5}$$

Step 2. Consistency test.

For pairwise comparison matrix Φ^t, its consistency ratio is defined as follows:

$$CR^t = \frac{CI^t}{RI} \tag{6}$$

where $CI^t = \dfrac{\lambda_{max}^t - m}{m - 1}$ is the consistency index of Φ^t, here λ_{max}^t is the largest eigenvalue of Φ^t and m is the dimensionality of Φ^t; RI is the random index which is only related to the matrix dimensionality and can be obtained according to Table 2.

Table 2: The average random consistency

Matrix dimensionality	RI value
1	0
2	0
3	0.52
4	0.89
5	1.12
6	1.26
7	1.36
8	1.41
9	1.46

When $CR^t < 0.1$, the consistency of pairwise comparison matrix Φ^t is acceptable; when $CR^t = 0.1$ or $CR^t > 0.1$, the consistency of pairwise comparison matrix Φ^t is not acceptable and the matrix should be modified by the corresponding expert.

Step 3. Judgment aggregation by rough boundary interval.

Based on pairwise comparison matrices $\Phi^1, \Phi^2, \ldots, \Phi^p$ given by p experts, the group decision matrix is constructed as follows:

$$\Phi = \begin{bmatrix} 1 & \phi_{12} & \cdots & \phi_{1m} \\ \phi_{21} & 1 & \cdots & \phi_{2m} \\ \vdots & \vdots & & \vdots \\ \phi_{m1} & \phi_{m2} & \cdots & 1 \end{bmatrix} \tag{7}$$

where $\phi_{kj} = \{\phi_{kj}^1, \phi_{kj}^2, \ldots, \phi_{kj}^p\} (k, j = 1, 2, \ldots, m$ and $k \neq j)$.

The rough boundary interval of $\phi_{kj}^t \in \phi_{kj}$ is obtained as follows.

$$RN(\phi_{kj}^t) = [\phi_{kj}^{t,-}, \phi_{kj}^{t,+}] \tag{8}$$

where $\phi_{kj}^{t,-}$ is the rough lower limit of ϕ_{kj}^t in set ϕ_{kj} and $\phi_{kj}^{t,+}$ is the rough upper limit of ϕ_{kj}^t in set ϕ_{kj}.

Therefore, the rough boundary interval of ϕ_{kj} can be represented as follows:

$$RN(\phi_{kj}) = \{RN(\phi_{kj}^1), RN(\phi_{kj}^2), \ldots, RN(\phi_{kj}^p)\} \tag{9}$$

Based on the operational rule of rough boundary interval, the average form of $RN(\phi_{kj})$ is obtained as follows:

$$ARN(\phi_{kj}) = [\phi_{kj}^-, \phi_{kj}^+] = \left[\frac{\sum\limits_{t=1}^{p} \phi_{kj}^{t,-}}{p}, \frac{\sum\limits_{t=1}^{p} \phi_{kj}^{t,+}}{p} \right] \tag{10}$$

where ϕ_{kj}^- is the rough lower limit of set ϕ_{kj} and ϕ_{kj}^+ is the rough upper limit of set ϕ_{kj}.

The rough judgment matrix is constructed as follows:

$$EA = \begin{bmatrix} 1 & ARN(\phi_{12}) & \cdots & ARN(\phi_{1m}) \\ ARN(\phi_{21}) & 1 & \cdots & ARN(\phi_{2m}) \\ \vdots & \vdots & & \vdots \\ ARN(\phi_{m1}) & ARN(\phi_{m2}) & \cdots & 1 \end{bmatrix} \tag{11}$$

Step 4. Divide rough judgment matrix and solve the eigenvectors.

Rough judgment matrix EA is divided into EA^- and EA^+. Here, EA^- is the rough lower limit matrix and EA^+ is the rough upper limit matrix. EA^- and EA^+ are expressed as follows:

$$EA^- = \begin{bmatrix} 1 & \phi_{12}^- & \cdots & \phi_{1m}^- \\ \phi_{21}^- & 1 & \cdots & \phi_{2m}^- \\ \vdots & \vdots & & \vdots \\ \phi_{m1}^- & \phi_{m2}^- & \cdots & 1 \end{bmatrix} \tag{12}$$

$$EA^+ = \begin{bmatrix} 1 & \phi_{12}^+ & \cdots & \phi_{1m}^+ \\ \phi_{21}^+ & 1 & \cdots & \phi_{2m}^+ \\ \vdots & \vdots & & \vdots \\ \phi_{m1}^+ & \phi_{m2}^+ & \cdots & 1 \end{bmatrix} \tag{13}$$

The eigenvectors corresponding to the maximum eigenvalues of EA^- and EA^+ are obtained respectively as follows:

$$VA^- = [va_1^-, va_2^-, \cdots, va_m^-]^{\mathrm{T}} \tag{14}$$

$$VA^+ = [va_1^+, va_2^+, \cdots, va_m^+]^{\mathrm{T}} \tag{15}$$

where va_j^- are the value of VA^- on dimension j and va_j^+ are the value of VA^+ on dimension j, here $j = 1, 2, \ldots, m$.

Step 5. Weight value calculation.

The weight value of indicator I_j is calculated as follows:

$$\omega_j = \frac{|va_j^-| + |va_j^+|}{2} \tag{16}$$

Lastly the weight vector is obtained as follows:

$$\omega = [\omega_1, \omega_2, \ldots, \omega_m]^{\mathrm{T}} \tag{17}$$

3.2 Assessing Power Quality by Extension Analysis

Extension analysis theory can classify the assessment indicators of power quality from the dynamic and transformation perspective and quantitatively express the quality grade of the indicators by the value of correlation function. The calculation of correlation function is based on the actual measured objective data.

Concept 3. Matter-element [27–29].

According to the extension analysis theory, the matter-element to be assessed is expressed by R_0. R_0 is defined as an ordered triples: $R_0 = (Y_0, I, v_0)$. Here Y_0 represents the unknown assessment grade of R_0, $I = \{I_1, I_2, \ldots, I_m\}$ is the indicator set which represents the characteristics of R_0 and $v_0 = [v_1, v_2, \ldots, v_m]$ is the corresponding indicator value vector. Therefore R_0 can be expressed as follows:

$$R_0 = \begin{bmatrix} Y_0 & I_1 & v_1 \\ & I_2 & v_2 \\ & \vdots & \vdots \\ & I_m & v_m \end{bmatrix} \tag{18}$$

Concept 4. Classical domain [27–29].

The classical domain refers to the range of values specified by the corresponding characteristics of each assessment grade of the object to be assessed. The classical domain matrix can be expressed as follows:

$$R^l = \begin{bmatrix} Y^l & I_1 & [a_{l1}, b_{l1}] \\ & I_2 & [a_{l2}, b_{l2}] \\ & \vdots & \vdots \\ & I_m & [a_{lm}, b_{lm}] \end{bmatrix} \tag{19}$$

where Y^l ($l = 1, 2, \ldots, h$) is the lth assessment grade of R_0 and $[a_{lj}, b_{lj}]$ is the indicator value interval corresponding to Y^l on indicator I_j.

Concept 5. Joint domain [27–29].

The joint domain refers to the range of values specified by all assessment grades of the object to be assessed. The joint domain matrix can be expressed as follows:

$$R = \begin{bmatrix} Y & I_1 & [a_1, b_1] \\ & I_2 & [a_2, b_2] \\ & \vdots & \vdots \\ & I_m & [a_m, b_m] \end{bmatrix} \tag{20}$$

where Y is the set of all assessment grades of R_0 and $[a_j, b_j]$ is the indicator value interval corresponding to all assessment grades on index I_j.

Concept 6. Correlation function [27–29].

Correlation function describes the mapping of elements in matter element to real axis, which is specifically expressed as the quantitative relationship between v_j and two interval $[a_{lj}, b_{lj}]$ and

$[a_j, b_j]$. Through the correlation function, we can describe quantitatively and objectively the degree and its change of an element belonging to a certain property. For the elements in the same domain, we can also divide the elements into different grades according to the value of the correlation function value.

The module of bounded interval $x = [a, b]$ is defined as follows:

$$|x| = |b - a| \tag{21}$$

The distance from point x_0 to interval $x = [a, b]$ is defined as follows:

$$\rho(x_0, x) = \left| x_0 - \frac{1}{2}(a + b) \right| - \frac{1}{2}(b - a) \tag{22}$$

Assuming that there are two intervals $x^1 = [a^1, b^1]$ and $x^2 = [a^2, b^2]$ and they satisfy that $x^1 \subset x^2$, the correlation function is defined as follows:

$$K(x) = \begin{cases} -\dfrac{\rho(x, x^1)}{|x^1|}, & x \in x^1 \\ \dfrac{\rho(x, x^1)}{\rho(x, x^2) - \rho(x, x^1)}, & x \notin x^1 \end{cases} \tag{23}$$

The properties of correlation function $K(x)$ are as follows:

(1) When $K(x) > 1$, it indicates that the assessment object exceeds the upper limit required.
(2) When $0 \leq K(x) \leq 1$, it indicates that the assessment object meets the requirement. The larger its value, the closer it is to the upper limit required.
(3) When $-1 \leq K(x) < 0$, it indicates that the assessment object does not meet the requirement, but has the condition that can be converted to meet the requirement.
(4) When $K(x) < -1$, it indicates that the assessment object does not meet the requirement, and does not have the condition to convert to meet the requirement.

Here, let $x = v_j$, $x^1 = [a_{lj}, b_{lj}]$ and $x^2 = [a_j, b_j]$, correlation function $K_l(v_j)$ describes the quantitative relationship from indicator value v_j of matter-element R_0 to be assessed to two intervals $[a_{lj}, b_{lj}]$ and $[a_j, b_j]$. It represents the membership degree of matter-element R_0 to be assessed with respect to assessment grade Y^l on indicator I_j and is defined as follows:

$$K_l(v_j) = \begin{cases} -\dfrac{\rho(v_j, [a_{lj}, b_{lj}])}{|[a_{lj}, b_{lj}]|}, & v_j \in [a_{lj}, b_{lj}] \\ \dfrac{\rho(v_j, [a_{lj}, b_{lj}])}{\rho(v_j, [a_j, b_j]) - \rho(v_j, [a_{lj}, b_{lj}])}, & v_j \notin [a_{lj}, b_{lj}] \end{cases} \tag{24}$$

Based on weight vector $\omega = [\omega_1, \omega_2, \ldots, \omega_m]^T$ obtained by Eqs. (16) and (17), the comprehensive correlation function value is obtained as follows:

$$K_l = \sum_{j=1}^{m} \omega_j K_l(v_j) \tag{25}$$

If $K_g = \max_{l=1,2,\ldots,h} \{K_l\}$, here $g = 1, 2, \ldots, h$, the assessment grade of the power quality to be assessed is D_g.

4 Case Study

With the rapid development of economy and the gradual improvement of people's living standards, the power consumption of the whole society is increasing, and various new non-linear asymmetric electrical equipment have been widely used. A converting station in a wind farm of China is in line with these characteristics and is very representative in China. Therefore its 110 kV bus can be taken as research object to illustrate the feasibility of the proposed method. Its power quality is comprehensively assessed by the proposed method as follows. The power quality is divided into four quality grades: high quality, good, medium and qualified. They are expressed by D_1–D_4, respectively.

The grading limits and measured data of power quality indicators are shown in Table 3.

Table 3: Grading limits and measured data of power quality indicators

Indicator	D_1	D_2	D_3	D_4	Measured data
I_1	[0, 0.05]	[0.05, 0.1]	[0.1, 0.15]	[0.15, 0.2]	0.04
I_2 (%)	[0, 0.5]	[0.5, 1]	[1, 1.5]	[1.5, 2]	0.82
I_3 (%)	[0, 0.5]	[0.5, 1]	[1, 1.5]	[1.5, 2]	1.05
I_4	[0, 0.2]	[0.2, 0.5]	[0.5, 0.8]	[0.8, 1]	0.34
I_5 (%)	[0, 2]	[2, 5]	[5, 8]	[8, 10]	1.66
I_6 (%)	[0, 0.5]	[0.5, 1]	[1, 1.5]	[1.5, 2]	1.07

Firstly the weight of indicator is determined by the proposed rough AHP as follows. Assuming that there are three experts, each expert carries out the pairwise comparison of the indicators according to 1–9 scales in Table 1, then three pairwise comparison matrices are obtained as shown in Tables 4–6.

Table 4: Pairwise comparison matrix Φ^1 of the indicators given by Expert 1

	I_1	I_2	I_3	I_4	I_5	I_6
I_1	1	5	2	1/2	7	4
I_2	1/5	1	1/3	1/8	2	2
I_3	1/2	3	1	1/4	4	3
I_4	2	8	4	1	8	6
I_5	1/7	1/2	1/4	1/8	1	1/2
I_6	1/4	1/2	1/3	1/6	2	1

Table 5: Pairwise comparison matrix Φ^2 of the indicators given by Expert 2

	I_1	I_2	I_3	I_4	I_5	I_6
I_1	1	2	7	2	1	3
I_2	1/2	1	3	2	1/2	2
I_3	1/7	1/3	1	1/3	1/8	1/3

(Continued)

Table 5 (continued)

	I_1	I_2	I_3	I_4	I_5	I_6
I_4	1/2	1/2	3	1	1/3	2
I_5	1	2	8	3	1	3
I_6	1/3	1/2	3	1/2	1/3	1

Table 6: Pairwise comparison matrix Φ^3 of the indicators given by Expert 3

	I_1	I_2	I_3	I_4	I_5	I_6
I_1	1	4	5	2	3	1/2
I_2	1/4	1	2	1/3	1/2	1/7
I_3	1/5	1/2	1	1/3	1/2	1/9
I_4	1/2	3	3	1	2	1/5
I_5	1/3	2	2	1/2	1	1/6
I_6	2	7	9	5	6	1

The consistency index and consistency ratio of the three pairwise comparison matrices is shown in Table 7.

Table 7: Consistency index and consistency ratio

	CI	CR
Φ^1	0.0320	0.0254
Φ^2	0.0204	0.0162
Φ^3	0.0201	0.0159

As can be seen, the consistency ratios of the three pairwise comparison matrices all have values less than 0.1, so the consistency of each pairwise comparison matrix is acceptable. Then according to Eq. (7) the group decision matrix is constructed as shown in Table 8.

Table 8: Group decision matrix Φ

	I_1	I_2	I_3	I_4	I_5	I_6
I_1	{1, 1, 1}	{5, 2, 4}	{2, 7, 5}	{1/2, 2, 2}	{7, 1, 3}	{4, 3, 1/2}
I_2	{1/5, 1/2, 1/4}	{1, 1, 1}	{1/3, 3, 2}	{1/8, 2, 1/3}	{2, 1/2, 1/2}	{2, 2, 1/7}
I_3	{1/2, 1/7, 1/5}	{3, 1/3, 1/2}	{1, 1, 1}	{1/4, 1/3, 1/3}	{4, 1/8, 1/2}	{3, 1/3, 1/9}
I_4	{2, 1/2, 1/2}	{8, 1/2, 3}	{4, 3, 3}	{1, 1, 1}	{8, 1/3, 2}	{6, 2, 1/5}
I_5	{1/7, 1, 1/3}	{1/2, 2, 2}	{1/4, 8, 2}	{1/8, 3, 1/2}	{1, 1, 1}	{1/2, 3, 1/6}
I_6	{1/4, 1/3, 2}	{1/2, 1/2, 7}	{1/3, 3, 9}	{1/6, 1/2, 5}	{2, 1/3, 6}	{1, 1, 1}

Next $\phi_{12} = \{5, 2, 4\}$ is taken as an example to show the calculation process of rough boundary interval based on Eqs. (1)–(4). In $\phi_{12} = \{5, 2, 4\}$, the lower and upper approximation sets of $\phi_{12}^1 = 5$ are $\{5, 2, 4\}$ and $\{5\}$. Therefore, the rough lower limit of ϕ_{12}^1 in set ϕ_{12} is $\phi_{12}^{1,-} = \dfrac{5+2+4}{3} = 3.6667$, while the rough upper limit of ϕ_{12}^1 in set ϕ_{12} is $\phi_{12}^{1,+} = \dfrac{5}{1} = 5$. As a result, the rough boundary interval of ϕ_{12}^1 is obtained as $RN(\phi_{12}^1) = [3.6667, 5]$. Similarly, the rough boundary intervals of $\phi_{12}^2 = 2$ and $\phi_{12}^3 = 4$ are obtained as $RN(\phi_{12}^2) = [2, 3.6667]$ and $RN(\phi_{12}^3) = [3, 4.5]$. According to Eq. (10), the rough boundary intervals of ϕ_{12} is obtained as $ARN(\phi_{12}) = [2.8889, 4.3889]$.

Then the rough judgment matrix is constructed as shown in Table 9.

Table 9: Rough judgment matrix EA

	I_1	I_2	I_3	I_4	I_5	I_6
I_1	[1, 1]	[2.8889, 4.3889]	[3.3889, 5.8889]	[1.1667, 1.8333]	[2.2222, 5.2222]	[1.5833, 3.3333]
I_2	[0.2472, 0.3972]	[1, 1]	[1.0926, 2.4259]	[0.3912, 1.3287]	[0.6667, 1.3333]	[0.9683, 1.7937]
I_3	[0.1984, 0.3770]	[0.6759, 2.0093]	[1, 1]	[0.2870, 0.3241]	[0.6597, 2.5972]	[0.4938, 1.9383]
I_4	[0.6667, 1.3333]	[2.0278, 5.7778]	[3.1111, 3.5556]	[1, 1]	[1.6481, 5.4815]	[1.3444, 4.2444]
I_5	[0.2910, 0.7196]	[1.1667, 1.8333]	[1.5972, 5.4722]	[0.5486, 1.9861]	[1, 1]	[0.5741, 1.9907]
I_6	[0.4676, 1.3426]	[1.2222, 4.1111]	[2.0370, 6.3704]	[0.7963, 3.2130]	[1.4259, 4.2593]	[1, 1]

Rough judgment matrix EA is then divided into rough lower limit matrix EA^- and rough upper limit matrix EA^+ as shown in Tables 10 and 11.

Table 10: Rough lower limit matrix EA^-

	I_1	I_2	I_3	I_4	I_5	I_6
I_1	1.0000	2.8889	3.3889	1.1667	2.2222	1.5833
I_2	0.2472	1.0000	1.0926	0.3912	0.6667	0.9683
I_3	0.1984	0.6759	1.0000	0.2870	0.6597	0.4938
I_4	0.6667	2.0278	3.1111	1.0000	1.6481	1.3444
I_5	0.2910	1.1667	1.5972	0.5486	1.0000	0.5741
I_6	0.4676	1.2222	2.0370	0.7963	1.4259	1.0000

Table 11: Rough upper limit matrix EA^+

	I_1	I_2	I_3	I_4	I_5	I_6
I_1	1.0000	4.3889	5.8889	1.8333	5.2222	3.3333
I_2	0.3972	1.0000	2.4259	1.3287	1.3333	1.7937
I_3	0.3770	2.0093	1.0000	0.3241	2.5972	1.9383
I_4	1.3333	5.7778	3.5556	1.0000	5.4815	4.2444
I_5	0.7196	1.8333	5.4722	1.9861	1.0000	1.9907
I_6	1.3426	4.1111	6.3704	3.2130	4.2593	1.0000

By Matlab, the eigenvectors corresponding to the maximum eigenvalues of EA^- and EA^+ are obtained respectively as:

$EA^- = [0.6592, 0.2361, 0.1725, 0.5173, 0.2676, 0.3753]^\mathrm{T}$;

$EA^+ = [0.5274, 0.2218, 0.2110, 0.5332, 0.3221, 0.4901]^\mathrm{T}$.

According to Eqs. (16) and (17), the weight vector is obtained as $\omega = [0.2617, 0.1010, 0.0846, 0.2317, 0.1301, 0.1909]^\mathrm{T}$.

Based on weight vector ω and the grading limits and measured data shown in Table 3, the correlation function values are obtained as shown in Table 12 according to Eq. (24).

Table 12: Correlation function value

	D_1	D_2	D_3	D_4
I_1	0.2000	−0.2000	−0.6000	−0.7333
I_2	−0.2807	0.3600	−0.1800	−0.4533
I_3	−0.3667	−0.0500	0.1000	−0.3214
I_4	−0.2917	0.4667	−0.3200	−0.5750
I_5	0.1700	−0.1700	−0.6680	−0.7925
I_6	−0.3800	−0.0700	0.1400	−0.3162

At last, the comprehensive correlation function values are obtained according to Eq. (25) as: $K_1 = -0.1250$, $K_2 = 0.0524$, $K_3 = -0.3011$ and $K_4 = -0.5616$. Because $K_2 = \max_{l=1,2,3,4}\{K_l\}$, the power quality to be assessed belongs to assessment grade D_2: good.

To demonstrate the validity and applicability, the assessment result of the proposed method is compared with the assessment result of D-S evidence theory [32]. By treating every indicator as evidence, the identification framework is built in which the assessment grades (D_1–D_4) are elements. Based on the indicator measured data in Table 3 and the weight vector obtained above, weighted evidence synthesis is carried out. The comparison of the assessment results are shown in Fig. 3.

From Fig. 3, it can be seen that the assessment results of proposed method and D-S evidence theory are consistent. The power quality to be assessed belongs to assessment grade D_2: good, and the order of membership of the four assessment grades is as follows: D_2, D_1, D_3 and D_4. This shows that the proposed method in this paper is feasible. In conclusion, this proposed method is superior to

D-S evidence theory in two aspects: (1) By the proposed method, the assessment result can be analyzed based on correlation function shown in Table 12, and the grade of each indicator can be obtained which reflects the main problems existing in power grid. By D-S evidence theory, these cannot be achieved. (2) The calculation amount of evidence synthesis is very large by D-S evidence theory, especially when there are many indicators. By the proposed method, only correlation function and comprehensive correlation function need to be calculated according to Eqs. (24) and (25).
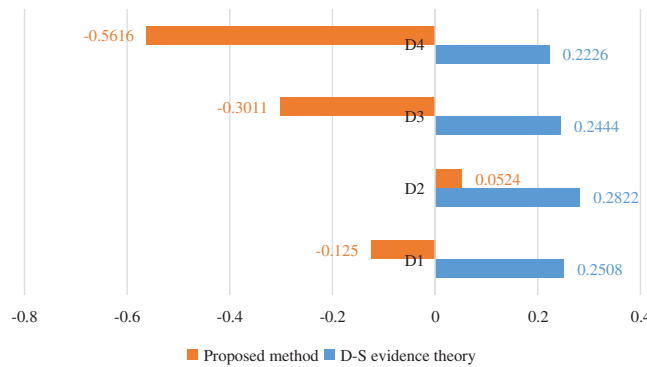
Figure 3: The comparison of the assessment results of proposed method and D-S evidence theory

According to the nature of correlation function, the assessment result is analyzed as follows. The grade of each indicator, which is shown in Fig. 4, can reflect the main problems existing in power grid.
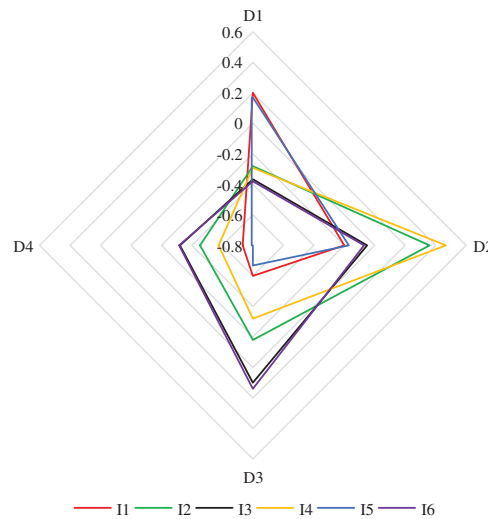

Figure 4: The grade of each indicator

As can be seen in Table 12, the grade of frequency deviation (I_1) is D_1, the grade of total harmonic distortion rate of voltage (I_2) is D_2, the grade of voltage fluctuation (I_3) is D_3, the grade of voltage flicker (I_4) is D_2, the grade of voltage deviation (I_5) is D_1 and the grade of voltage three-phase unbalance degree (I_6) is D_3. Compared with other indicators, voltage fluctuation (I_3) and voltage three-phase unbalance degree (I_6) have lower assessment grade. Therefore, voltage fluctuation and voltage three-phase unbalance degree are the key factors affecting power quality, and corresponding measures should be taken to restrain these two indicators.

5 Conclusions

In this paper, the method of combining extension analysis with rough AHP is used to assess power quality comprehensively. By rough AHP, the adverse effects of single expert judgment can be eliminated. Through the concept of matter-element and correlation function, the relationship between each assessment indicator of power quality and the assessment interval of each quality grade can be quantified. At last the case analysis and application show that the proposed method can determine the overall power quality of power grid, compare the differences among the performance of assessment indicators, find out the key factors affecting power quality, and provide the basis for further improving the power quality of power grid. However, rough AHP is a kind of subjective weighting method and may lead to the lack of effective weighting information. Synthesis of subjective and objective weights is a significant way to overcome this shortcoming. In addition, the definition of assessment grade standard, classical domain and joint domain needs to be further studied in extension analysis.

Conflicts of Interest: The authors declare that they have no conflicts of interest to report regarding the present study.

References

1. Hemapriya, C., Suganyadevi, M., Krishnakumar, C. (2020). Detection and classification of multi-complex power quality events in a smart grid using Hilbert–Huang transform and support vector machine. *Electrical Engineering, 102(3),* 1681–1706. DOI 10.1007/s00202-020-00987-8.

2. Reddy, D., Narayana, S., Ganesh, V. (2019). Towards an enhancement of power quality in the distribution system with the integration of BESS and FACTS device. *International Journal of Ambient Energy, 2,* 1–19. DOI 10.1080/01430750.2019.1636866.

3. Tan, J., Huang, S. (2006). Research on synthetic evaluation method of power quality based on fuzzy theory. *Relay, 34(3),* 55–59. DOI 10.3969/j.issn.1674-3415.2006.03.014.

4. Xiong, Y., Cheng, H., Wang, H., Xu, J. (2009). Synthetic evaluation of power quality based on improved AHP and probability statistics. *Power System Protection and Control, 37(13),* 48–52. DOI 10.3969/j.issn.1674-3415.2009.13.010.

5. Gao, W., Wang, H., Yao, C. (2009). Comprehensive evaluation of power quality based on probability and vector algebra. *Shanxi Electric Power, 30(6),* 65–67. DOI 10.3969/j.issn.1671-0320.2009.06.022.

6. Lei, G., Gu, W., Yuan, X. (2009). Application of gray theory in power quality comprehensive evaluation. *Electric Power Automation Equipment, 37(11),* 62–65. DOI 10.3969/j.issn.1006-6047.2009.11.014.

7. Zhou, H., Yang, H., Wu, C. (2012). A power quality comprehensive evaluation method based on grey clustering. *Power System Protection and Control, 40(15),* 70–75. DOI 10.3969/j.issn.1674-3415.2012.15.013.

8. Li, N., He, Z. (2009). Power quality comprehensive evaluation combining subjective weight with objective weight. *Power System Technology, 33(6),* 55–61. DOI 10.13335/j.1000-3673.pst.2009.0344.

9. Liu, B., Li, Q., Dong, X. (2009). Comprehensive assessment study of the power quality based on the time-varying weight. *Power System Protection and Control, 37(14),* 6–9. DOI 10.3969/j.issn.1674-3415.2009.14.002.

10. Li, N., He, Z. (2009). Combinatorial weighting method for comprehensive evaluation of power quality. *Power System Protection and Control, 37(16),* 128–134. DOI 10.3969/j.issn.1674-3415.2009.16.031.

11. Gu, S., Shi, W., Lan, Y., Zhuo, J., Zhang, W. (2020). Power quality online assessment of all-electric ship based on grey cloud evidential reasoning. *Power System Protection and Control, 48(8),* 17–24. DOI 10.19783/j.cnki.pspc.190665.

12. Zhang, K., Zhu, W., Cheng, Z., Fan, Y. (2019). Power quality assessment method based on variable fuzzy set. *Journal of Anhui University (Natural Sciences), 43(5),* 60–67. DOI 10.3969/j.issn.1000-2162.2019.05.010.

13. Li, S., Chen, D., Qiu, Q., Shi, J., Xu, W. et al. (2019). Comprehensive assessment of power quality based on random forest. *Modern Electric Power, 36(2),* 81–87. DOI 10.3969/j.issn.1007-2322.2019.02.011.

14. Kang, Y., Wu, S., Li, Y., Liu, J., Chen, B. (2018). A variable precision grey-based multi-granulation rough set model and attribute reduction. *Knowledge-Based Systems, 148(5),* 131–145. DOI 10.1016/j.knosys.2018.02.033.

15. Mandal, P., Ranadive, A. (2018). Decision-theoretic rough sets under Pythagorean fuzzy information. *International Journal of Intelligent Systems, 33(4),* 818–835. DOI 10.1002/int.21969.

16. Sheeja, T., Kuriakose, A. (2018). A novel feature selection method using fuzzy rough sets. *Computers in Industry, 97,* 111–121. DOI 10.1016/j.compind.2018.01.014.

17. Bharathi, S. (2019). Forewarned is forearmed assessment of IoT information security risks using analytic hierarchy process. *Benchmarking, 26(8),* 2443–2467. DOI 10.1108/BIJ-08-2018-0264.

18. Lee, C., Lee, H., Seol, H., Park, Y. (2012). Evaluation of new service concepts using rough set theory and group analytic hierarchy process. *Expert Systems with Applications, 39(3),* 3404–3412. DOI 10.1016/j.eswa.2011.09.028.

19. Zhang, P. (2018). Research on aircraft maintenance risk based on rough set and analytic hierarchy process. *Aviation Maintenance & Engineering, 7,* 41–44. DOI 10.3969/j.issn.1672-0989.2018.07.017.

20. Sun, S., Chen, K., Li, Y., Li, C., Pan, M. et al. (2020). Comprehensive performance evaluation method for CCHP system based on interval rough number analytic hierarchy process. *Thermal Power Generation, 49(4),* 131–137. DOI 10.19666/j.rlfd.201909235.

21. Peng, D., Chen, Y., Fan, J., Qian, Y. (2019). Research on assessment of transformer state using analytic hierarchy process and rough set theory. *High Voltage Apparatus, 55(7),* 150–157. DOI 10.13296/j.1001-1609.hva.2019.07.022.

22. Wan, R., Yan, R. (2018). Method for determining customer demand weight based on rough set and FAHP. *Science and Technology Management Research, 38(4),* 204–208. DOI 10.3969/j.issn.1000-7695.2018.04.030.

23. Liu, J., Sun, D., Chao, H., Bai, H., Lei, Y. (2020). Risk analysis of buried crude oil pipeline based on PSR-matter element extension model. *Oil-Gasfield Surface Engineering, 39(3),* 90–94. DOI 10.3969/j.issn.1006-6896.2020.03.017.

24. Zhai, D., Xu, C., Wang, R., Xu, T. (2020). The application of set pair analysis and extension theory coupling model in river health evaluation. *China Rural Water and Hydropower, 7,* 65–70. DOI 10.3969/j.issn.1007-2284.2020.07.013.

25. Zhou, J., Luo, T., Li, Z., Chen, G., Yu, H. et al. (2019). Comprehensive evaluation of power failure based on improved extension analytic hierarchy process. *Power System Protection and Control, 47(3),* 31–38. DOI 10.7667/PSPC180151.

26. Jiang, A., Dong, Y., Zhang, X. (2019). Risk assessment of highway tunnel construction based on analytic hierarchy process and extension model. *Mathematics in Practice and Theory, 49(11),* 297–305. DOI 10.3969/j.issn.1000-0984.2019.034.

27. He, Y., Dai, A., Zhu, J. (2011). Risk assessment of urban network planning in China based on the matter-element model and extension analysis. *International Journal of Electrical Power & Energy Systems, 33(3),* 775–782. DOI 10.1016/j.ijepes.2010.12.037.

28. Meng, J., Xu, R., Li, D., Chen, X. (2018). Combining the matter-element model with the associated function of performance indices for automatic train operation algorithm. *IEEE Transactions on Intelligent Transportation Systems, 2018,* 1–11. DOI 10.1109/TITS.2018.2805917.

29. Zhang, X., Yue, J. (2017). Measurement model and its application of enterprise innovation capability based on matter element extension theory. *Procedia Engineering, 174,* 275–280. DOI 10.1016/j.proeng.2017.01.136.

30. Bakeshlou, E., Khamseh, A., Asl, M. (2014). Evaluating a green supplier selection problem using a hybrid MODM algorithm. *Journal of Intelligent Manufacturing, 28(4),* 1–15. DOI 10.1007/s10845-014-1028-y.

31. Kar, A. (2014). Revisiting the supplier selection problem: An integrated approach for group decision support. *Expert Systems with Applications, 41(6),* 2762–2771. DOI 10.1016/j.eswa.2013.10.009.

32. Li, L., Wang, H. (2018). A parts supplier selection framework of mechanical manufacturing enterprise based on D-S evidence theory. *Journal of Algorithms & Computational Technology, 12(4),* 333–341. DOI 10.1177/1748301818779044.