*Computer Systems Science & Engineering*
DOI: 10.32604/csse.2023.029255
***Article***

# The Laplacian Energy of Hesitancy Fuzzy Graphs in Decision-Making Problems

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Received: 28 February 2022; Accepted: 14 April 2022

**Abstract:** Decision-making (DM) is a process in which several persons concurrently engage, examine the problems, evaluate potential alternatives, and select an appropriate option to the problem. Technique for determining order preference by similarity to the ideal solution (TOPSIS) is an established DM process. The objective of this report happens to broaden the approach of TOPSIS to solve the DM issues designed with Hesitancy fuzzy data, in which evaluation evidence given by the experts on possible solutions is presents as Hesitancy fuzzy decision matrices, each of which is defined by Hesitancy fuzzy numbers. Findings: we represent analytical results, such as designing a satellite communication network and assessing reservoir operation methods, to demonstrate that our suggested thoughts may be used in DM. Aim: We studied a new testing method for the artificial communication system to give proof of the future construction of satellite earth stations. We aim to identify the best one from the different testing places. We are also finding the best operation schemes in the reservoir. In this article, we present the concepts of Laplacian energy (LE) in Hesitancy fuzzy graphs (HFGs), the weight function of LE of HFGs, and the TOPSIS method technique is used to produce the hesitancy fuzzy weighted-average (HFWA). Also, consider practical examples to illustrate the applicability of the finest design of satellite communication systems and also evaluation of reservoir schemes.

**Keywords:** Hesitancy fuzzy graphs (HFGs); laplacian energy; satellite communication system; reservoir operation schemes; decision-making

## 1 Introduction

The concept of "fuzzy sets (FSs)" was proposed by Zadeh (1965) [1]. He has introduced some basic definitions, algebraic operations, and properties of union, intersection, and complementation on FSs. Furthermore, the union and intersection of FSs were shown. Several fundamental graph notions such as bridges and trees are described as fuzzy analog, and many of their characteristics are introduced by Rosenfeld (1975) [2]. Certain fundamental properties of fuzzy relations are also discussed. Kaufmann [3] introduced the idea of fuzzy graphs (FGs), based on Zadeh's fuzzy relations [4]. The operations of Cartesian product and composition on fuzzy sub-graphs of graphs $G_1$ and $G_2$ were defined by Mordeson et al. [5]. Mordeson et al. (2000) [6] have introduced the concept of FGs and fuzzy hypergraphs. Parvathi et al. (2006) [7], established the definition of an intuitionistic fuzzy graph (IFG).

Balakrishnan (2004) [8], proposed the energy of a simple graph G as the summation of the actual values of Graph's Eigen values. Based on their energy, other types of graphs are classified, notably hyper-energetic, hypo-energetic, and equi-energetic, and additional information about them may be found here [9–12] Anjali et al. (2013) [13], developed the notion of Energy of FG. The FGs adjacency matrix is defined and the two boundaries of the energy of FG are derived. Praba et al. [14] introduced the idea of energy of an IFG in 2014. They expanded the idea of FG's energy to the energy of an IFG and also defined lower limit and upper limit on the energy of an IFG. In 2005, Gutman et al. [15,16] was defined and investigated the concept of Laplacian Energy (LE) of a graph. And they also established new properties of LE. In 2014, Sharbaf et al. [17] have introduced the idea of LE of a FG and also found boundaries of LE of a FG. The adjacency matrix of an IFG was established by Basha et al. [18], and the LE of an IFG is defined in terms of its adjacency matrix. They have also established the lower limit and upper limit for an IFG's energy, which are also calculated and validated with appropriate IFG's. Xu et al. [19] were presented the concept of an interval-valued intuitionistic fuzzy multiple attribute decision making problems with preference information on schemes and incomplete weight information.

In 2010, Torra [20] pioneered the notion of hesitant fuzzy sets (HFSs). He demonstrated that when applied to the envelope of HFSs, and the operations are compatible with those of intuitionistic fuzzy sets and also demonstrated some fundamental operations. A long-standing research challenge, coping with uncertainty in real world issues has led to the development of diverse methods and theories. Fuzzy Graphs (FGs) and their developments, like interval-valued FGs and intuitionistic fuzzy graphs (IFGs), have given a diverse collection of tools for dealing with uncertainty in various sorts of issues. Recently, a newer expansion of IFGs known as hesitancy fuzzy graphs (HFGs) was developed to cope with hesitant situations that have not been adequately addressed by past tools. Many researchers are suggested several expansions, various sorts of operators to calculate with such forms of information, and finally certain applications have been developed as a result of HFGs attracting their attention so rapidly.

Pathinathan et al. [21] developed a graphs design named as HFGs and explained several fundamental notions about it. Despite the fact that proposed the notion of HFG, they do not attach hesitant fuzzy elements (HFEs) to the network's nodes and paths. They employed IF-values instead of HFEs, and these IF-values are defined as three that included the membership, nonmembership and hesitancy degrees of nodes and paths. The concept has a design that would be same as to neutrosophic graphs [22–24] in certain ways. Ramesh et al. [25] were introduced the concept of group DM of selecting partners based on signless Laplacian energy (SLE) of an IFG. They developed the concept of the MATLAB program for computing the SLE of an intuitionistic fuzzy matrix, the weight characteristic of SLEs of an IFG, and the intuitionistic fuzzy weighted averaging (IFWA) using the TOPSIS technique. The rationality of certain group DM on the LE of an IFG has been shown by Basha et al. [26]. They are given numerical examples, such as the selection of an Alliance partner for a software firm. Alghamdi et al. [27] were suggested and presented TOPSIS techniques for multi-various methods of DM with several examples. Naz et al. [28] defined energy, Laplacian energy, and signless Laplacian energy in single-valued neutrosophic graphs, and detailed some of their features and developed relationships between them. They also had given a realistic example.

In this article, we present the concepts of LE of HFGs, the weight function of LE of HFGs, and the TOPSIS method technique is used to produce the hesitancy fuzzy weighted average. And also the real-time examples are illustrated to the applicability of the finest design of satellite communication systems and also evaluation of reservoir schemes. Satellite communication has a broad coverage area for communication, is not geographically limited, and is less vulnerable to the effects of natural disasters. In various application domains, including remote places, islands, mountains, traveling aviation, and sea ships, there is a distinct benefit of satellite communication. As a result, satellite communication not only efficiently substitutes the lack of other communication methods but also plays an important function as the principal way of communication in the mass media, particularly the military. For these purposes, we are implementing the TOPSIS model to find the finest design of the satellite communication system for the growth of society. The reservoir was constructed as a joint effort between irrigation and the NHPC (National Hydroelectric Power Corporation). The reservoir was built for a variety of different reasons, including power generation, irrigation, and the overall water supply for farming, business sectors, surrounding householders (residents), and the environment. Here, we are implementing the TOPSIS technique to find the finest reservoir that will fulfill the above aspects.

The following is how the article is designed. A brief literature review is presented in Section 2. The preliminaries for HFG, the algorithm, and the flow chart for finding HFPRs are presented in Section 3. Section 4 provides an application of HFG's Laplacian energy in decision-making for the construction of a satellite communication system and reservoir operation schemes evaluation. Finally, the article ends with the conclusion.

## 2 Literature Review

In the research literature, there have been several useful studies undertaken for the design of a satellite communication system and the evaluation of reservoir operation schemes.

Amanor et al. [29] proposed the design specifications and related design requirements of the physical layer suited for the moveable platforms. The systems engineering (SE) technique to physical layer building is utilized to contribute further sageness into the proposed design. David et al. obtained an inter-satellite connection length of one kilometer at a BER for an eight-level PPM scheme. Amanor et al. [30] used multi-objective optimization to find physical layer design factors that enhance the VLC receiver's signal-to-noise ratio (SNR). The SNR at the VLC receiver is increased. They applied the Non-dominated Sorting Genetic Algorithm 2 (NSGA-II) in MATLAB to find the Pareto front of 2 conflicting objective functions, and TOPSIS was used to find the finest outcome. The mathematical technique of the inter-satellite connection was built to investigate the feasibility of employing LEDs for well-founded data transfer in the existence of steady solar background light and the ISL's performance is measured in terms of BER and feasible data rates for 4 main intensity modulation and direct detection (IM/DD) techniques [31]. Radhakrishnan et al. [32] described several studies being undertaken in the small satellite community for establishing inter-satellite communications using the Open System Interconnection (OSI) approach. They gave a detailed list of design criteria that might be used to achieve inter-satellite communications for multiple small satellite missions.

BashaS [33] were using the TOPSIS technique to locate the outlines of reservoir activity to the representations of the specified aspects to rank the finest ones. BashaS [33] provided specifications for the IIFWA operator's interpretation and algorithm procedures, and then the concept was applied to reservoir operation and found to be feasible. The given analysis is expanded to different sorts of data forms and combinations with other operators.

In particular, Satellite communication has a broad coverage area for communication, is not geographically limited, and is less vulnerable to the effects of natural disasters. In various application domains, including remote places, islands, mountains, traveling aviation, and sea ships, there is a distinct benefit of satellite communication. As a result, satellite communication not only efficiently substitutes the lack of other communication methods but also plays an important function as the principal way of communication in mass media, particularly the military. For these purposes, we are implementing the TOPSIS model to find the finest design of the satellite communication system for the growth of society.

The reservoir was constructed as a joint effort between irrigation and the NHPC (National Hydroelectric Power Corporation). The reservoir was built for a variety of different reasons, including power generation, irrigation, and the overall water supply for farming, business sectors, surrounding householders (residents), and the environment. Here, we are implementing the TOPSIS technique to find the finest reservoir that will fulfill the above all aspects.

## 3 Preliminaries

**Definition 3.1.** An $HG=(V, E, \mu, \gamma, \beta)$ denote an HFG of the type where

- $V=\{v_1, v_2, \ldots, v_r\}$ such that $\mu_1:V\rightarrow[0, 1]$, $\gamma_1:V\rightarrow[0, 1]$ and $\beta_1:V\rightarrow[0, 1]$ are denotes the degree of membership, non-membership and hesitancy of the element respectively and $\mu_1(v_i)+\gamma_1(v_i)+\beta_1(v_i)=1$ for every $v_i\in V$, where $\beta_1(v_i)=1-[\mu_1(v_i)+\gamma_1(v_i)]$
- $E\subseteq V\times V$ where $\mu_2:V\times V\rightarrow[0, 1]$, $\gamma_2:V\times V\rightarrow[0, 1]$ and $\beta_2:V\times V\rightarrow[0, 1]$ are such that,

$$\mu_2(v_i,\ v_j)\leq\min[\mu_1(v_i),\ \mu_1(v_j)]$$

$$\gamma_2(v_i,\ v_j)\leq\max[\gamma_1(v_i),\ \gamma_1(v_j)]$$

$$\beta_2(v_i,\ v_j)\leq\min[\beta_1(v_i),\ \beta_1(v_j)]$$

$$0\leq\mu_2(v_i,\ v_j)+\gamma_2(v_i,\ v_j)+\beta_2(v_i,\ v_j)\leq 1,\quad\forall\ (v_i,\ v_j)\in E,$$

**Definition 3.2**. An $HFG's$ energy $HG=(V, E, \mu, \gamma, \beta)$ is indicated as

$$E(HG)=\left(\sum\nolimits^{r}\underset{\xi_i\epsilon\ S}{i=1}\ |\xi_i|,\ \sum\nolimits^{r}\underset{\delta_i\epsilon\ S}{i=1}\ |\delta_i|,\ \sum\nolimits^{r}\underset{\lambda_i\epsilon\ S}{i=1}\ |\lambda_i|\right)\tag{i}$$

where $\xi_i$, $\delta_i$, $\lambda_i$ is the Eigen roots hesitancy fuzzy adjacency matrix

**Definition 3.3**. If $HG=(V, E, \mu, \gamma, \beta)$ be an HFG, then the Laplacian matrix of an HFG is described as

$$L(HG)=D(HG)-\ A(HG),$$

where $A(HG)$ be an adjacency-matrix and $D(HG)$ be a degree-matrix of a $HFG$.

**Definition 3.4**. If $HG=(V, E, \mu, \gamma, \beta)$ be an HFG, then the Laplacian energy (LE) of an HFG is indicated as

$$LE(HG)=\left(\sum_{i=1}^{r}|\xi_i|,\ \sum_{i=1}^{r}|\varrho_i|,\ \sum_{i=1}^{r}|\eta_i|\right)\tag{ii}$$

where

$$\xi_i=\lambda_i-\frac{2\sum_{1\leq i\leq j\leq r}\mu(u_iv_j)}{n},$$

$$\varrho_i = \vartheta_i - \frac{2\sum_{1 \le i \le j \le r} \gamma(u_i v_j)}{n},$$

$$\eta_i = \kappa_i - \frac{2\sum_{1 \le i \le j \le r} \beta(u_i v_j)}{n}.$$

**Definition 3.5.** Weights of a wide range of Laplacian energy is calculated using the formula below (similar to the Bayesian formula)

$$W_r = ((W_\mu)_r,\ (W_\gamma)_r,\ (W_\beta)_r) = \left( \frac{LE((D_\mu)_r)}{\sum_{i=1}^{s} LE((D_\mu)_i)},\ \frac{LE((D_\gamma)_r)}{\sum_{i=1}^{s} LE((D_\gamma)_i)},\ \frac{LE((D_\beta)_r)}{\sum_{i=1}^{s} LE((D_\beta)_i)} \right), \quad r = 1,\ 2,\ \ldots,\ s, \cdot \qquad \text{(iii)}$$

### *3.1 TOPSIS Method for Finding Hesitancy Fuzzy Preference Relationship (HFPR)*

The compensating glomeration technique examines a predefined set of decisions by recognizing loads for each criterion, standardizing assessments of each rule, and calculating the geometric separation between each other option and the exact different alternative, which is the acceptable rating in all respects.

### *3.2 Computational Procedure for TOPSIS Method Problem*

We consider the $p$ choices $X_1, X_2, \ldots, X_p$. Each choice of $X_i$ is involved with $q$ rules $a_1, a_2, \ldots, a_q$ which are communicated through positive numbers $a_{ij}$. Measures $a_1, a_2, \ldots, a_k$ are an advantage (monotonically increasing incline), while rules $a_{k+1}, a_{k+2}, \ldots, a_q$ is a disadvantage (monotonically decreasing incline). Weights $w_r$ are assigned to the measurements $x_r$ so that $\sum_{r=1}^{n} w_r = 1$.

### *3.3 Initial Table and Decision Matrix*

Other alternatives, rules, and their weights are listed in the table for better comprehension. (see Tab. 1).

**Table 1:** The TOPSIS method initial table

| Criteria | $a_1$ cr.1 | $a_2$ cr.2 | $a_3$ cr.3 | … … | $a_q$ cr.q |
|---|---|---|---|---|---|
| Weights | $w_1$ | $w_2$ | $w_3$ | … | $w_q$ |
| $X_1$ | $a_{11}$ | $a_{12}$ | $a_{13}$ | … | $a_{1q}$ |
| $X_2$ | $a_{21}$ | $a_{22}$ | $a_{23}$ | … | $a_{2q}$ |
| … | … | … | … | … | … |
| $X_p$ | $a_{p1}$ | $a_{p2}$ | $a_{p3}$ | … | $a_{pq}$ |

For HFGs, the cumulative grid may be obtained by hesitancy fuzzy weighted averaging (HFWA), as shown below.

$$HFWA(R_{ij}^{(1)}, R_{ij}^{(2)}, \ldots, R_{ij}^{(n)}) = (1 - \underset{i=1}{\overset{s}{\pi}} (1 - (\mu_{jk}^{(i)})^{w_i}), \underset{i=1}{\overset{s}{\pi}} (\gamma_{jk}^{(i)})^{w_i}, \underset{i=1}{\overset{s}{\pi}} (\beta_{jk}^{(i)})^{w_i}) \tag{iv}$$

where $w_i$ be the weight function, $R_{ij}^{(1)}, R_{ij}^{(2)}, \ldots, R_{ij}^{(n)}$ be an individual HFPR, $\mu_{jk}$ be the membership element, $\gamma_{jk}$ be the nonmembership element, and $\beta_{jk}$ be the hesitant of the element.

### *3.4 Algorithm*

The procedure for determining the most significant satellite communication network and reservoir operation strategy is given as

**Input:** $Z = \{z_1, z_2, \ldots, z_n\}$ is the set of schemes, $e = \{e_1, e_2, \ldots, e_n\}$ is the expert set, and build of HFPR for every expert $R_k = (r_{ij}^{(k)})_{n\times n}$.

**Outcome:** The process of selecting the best scheme.

**Step 1.** Start

**Step 2.** Compute the Laplacian energy of every HFG $C_k$, $k = 1, 2, \ldots, n$.

$$LE = \left| \lambda_i - \frac{2\sum_{1\le i\le j\le n} \mu(u_i u_j)}{n} \right| \tag{v}$$

**Step 3.** Compute the weight for experts on the basis of Laplacian energy of HFGs using Eq. (3)

**Step 4.** Compute the HFWA by utilizing by using Eq. (4)

**Step 5.** Using the HFWA operator; calculate a collective HFE $p_i (i = 1, 2, \ldots, n)$ of the testing venue $C_i$ over all other testing locations $R = (r_{ij})_{n\times n}$.

**Step 6.** Calculate their out-degrees by utilizing

$$out - d(v_k)\ (k = 1,\ 2,\ \ldots,\ n).$$

**Step 7.** Compute the ranking of the factors of $v_k$ ($k = 1, 2, \ldots, n$) on the basis of membership degrees of $out - d(v_k)$.

**Step 8.** Identify the best test venue in the ranking of the factors $v_k$.

**Step 9.** Stop.

### *3.5 Flow Chart*

Fig. 1 shows a GDM method by using TOPSIS technique.

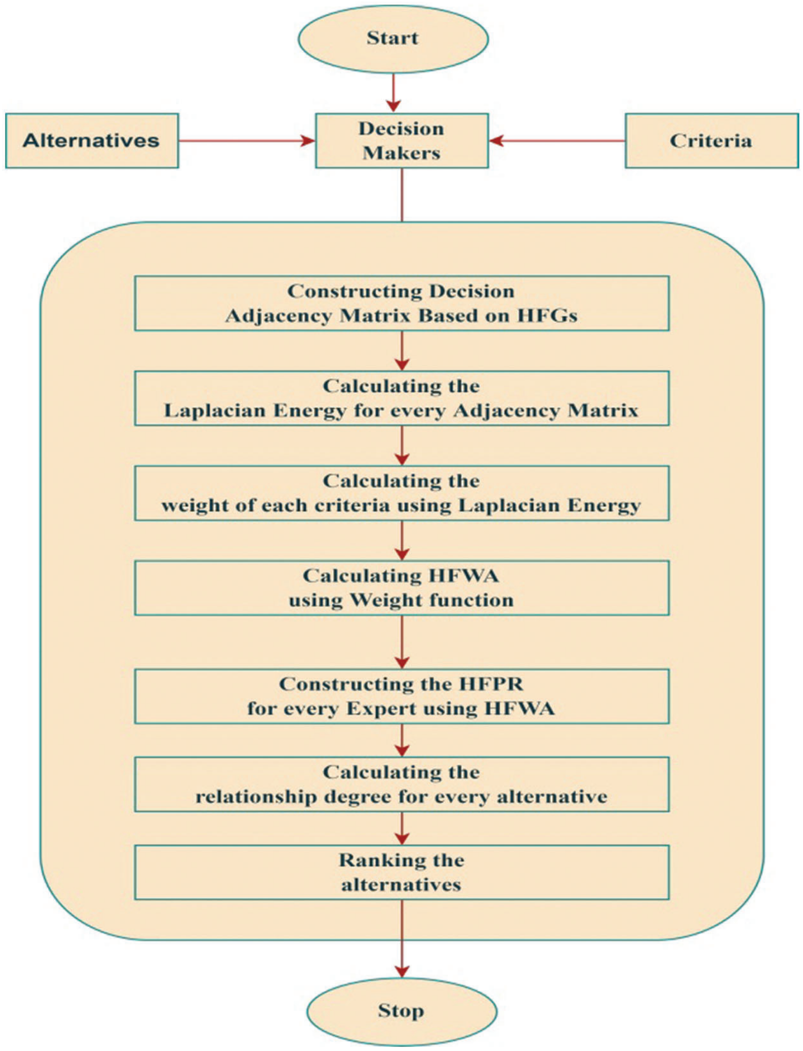


**Figure 1:** Shows a GDM method by using TOPSIS technique

## 4 Application of HFG's Laplacian Energy in Decision-Making

Group decision-making is a popular method in human activities which identifies the best option from a certain set of finite alternatives utilizing the information provided by a group of decision-makers or experts for review. Group decision-making performs a major essential part in dealing with decision-making problems as society develops quickly.

This section contains, to demonstrate our proposed ideas of the HFGT in decision-making, we utilise two practical examples: the design of a satellite communication system and the evaluation of reservoir operation methods.

### *4.1 The Construction of a Satellite Communication System*

Communication is intimately linked to the growth of society. Satellite communication [18], in particular, has a broad coverage area for communication, is not geographically limited, and is less vulnerable to the effects of natural disasters. In various application domains, including remote places, islands, mountains, travelling aviation's and sea ships, there is a distinct benefit of satellite communication. As a result, satellite communication not only efficiently substitutes the lack of other communication methods but also plays an important function as the principal way of communication in mass media, particularly the military. The ability to quantify the communication service's quality is essential in military warfare. Suppose the joint division for communication is planning to develop a satellite communication system in India. A new testing method must thus be studied for the synthetic communication system so as to give proof of the future construction of satellite earth stations. There are six different testing places $C_i$ ($i = 1, 2, 3, 4, 5, 6$) to select from (ISRO $C_1$, SDSC $C_2$, VSSC $C_3$, TERLS $C_4$, URSC $C_5$, NRSC $C_6$), as per the expeditions. Therefore, due to limited resources, finances, and other considerations, only the best of these would be chosen. A decision-making committee of six experts $e_k$, $k = 1, 2, \ldots, 6$ provides the judgements with six distinct HFPRs [28] $R_k = (r_{ij}^{(k)})_{6\times 6}$, $k = 1, 2, \ldots, 6$ as follows:

$$R_1 = \begin{bmatrix} (0,0,0) & (0.5,0.3,0.1) & (0,0,0) & (0.4,0.4,0.2) & (0,0,0) & (0.4,0.2,0.2) \\ (0.5,0.3,0.1) & (0,0,0) & (0.3,0.2,0.1) & (0,0,0) & (0.2,0.4,0.1) & (0.4,0.4,0.1) \\ (0,0,0) & (0.3,0.2,0.1) & (0,0,0) & (0.3,0.2,0.2) & (0.2,0.2,0.5) & (0.3,0.1,0.4) \\ (0.4,0.4,0.2) & (0,0,0) & (0.3,0.2,0.2) & (0,0,0) & (0.2,0.4,0.2) & (0,0,0) \\ (0,0,0) & (0.2,0.4,0.1) & (0.2,0.2,0.5) & (0.2,0.4,0.2) & (0,0,0) & (0.2,0.2,0.4) \\ (0.4,0.2,0.2) & (0.4,0.4,0.1) & (0.3,0.1,0.4) & (0,0,0) & (0.2,0.2,0.4) & (0,0,0) \end{bmatrix}$$

$$R_2 = \begin{bmatrix} (0,0,0) & (0.1,0.4,0.1) & (0,0,0) & (0.2,0.4,0.4) & (0,0,0) & (0.2,0.3,0.4) \\ (0.1,0.4,0.1) & (0,0,0) & (0.1,0.4,0.3) & (0,0,0) & (0.1,0.4,0.2) & (0.1,0.4,0.4) \\ (0,0,0) & (0.1,0.4,0.3) & (0,0,0) & (0.2,0.4,0.3) & (0.5,0.2,0.2) & (0.4,0.1,0.3) \\ (0.2,0.4,0.4) & (0,0,0) & (0.2,0.4,0.3) & (0,0,0) & (0.2,0.4,0.2) & (0,0,0) \\ (0,0,0) & (0.1,0.4,0.2) & (0.5,0.2,0.2) & (0.2,0.4,0.2) & (0,0,0) & (0.4,0.2,0.2) \\ (0.2,0.3,0.4) & (0.1,0.4,0.4) & (0.4,0.1,0.3) & (0,0,0) & (0.4,0.2,0.2) & (0,0,0) \end{bmatrix}$$

$$R_3 = \begin{bmatrix} (0,0,0) & (0.3,0.5,0.1) & (0,0,0) & (0.2,0.5,0.2) & (0,0,0) & (0.2,0.5,0.2) \\ (0.3,0.5,0.1) & (0,0,0) & (0.2,0.5,0.1) & (0,0,0) & (0.2,0.5,0.1) & (0.2,0.5,0.1) \\ (0,0,0) & (0.2,0.5,0.1) & (0,0,0) & (0.2,0.3,0.4) & (0.2,0.3,0.2) & (0.2,0.3,0.4) \\ (0.2,0.5,0.2) & (0,0,0) & (0.2,0.3,0.4) & (0,0,0) & (0.2,0.6,0.2) & (0,0,0) \\ (0,0,0) & (0.2,0.5,0.1) & (0.2,0.3,0.2) & (0.2,0.6,0.2) & (0,0,0) & (0.2,0.6,0.2) \\ (0.2,0.5,0.2) & (0.2,0.5,0.1) & (0.2,0.3,0.4) & (0,0,0) & (0.2,0.6,0.2) & (0,0,0) \end{bmatrix}$$

$$R_4 = \begin{bmatrix} (0,0,0) & (0.1,0.5,0.2) & (0,0,0) & (0.1,0.2,0.2) & (0,0,0) & (0.1,0.8,0.1) \\ (0.1,0.5,0.2) & (0,0,0) & (0.3,0.4,0.2) & (0,0,0) & (0.1,0.6,0.2) & (0.1,0.5,0.1) \\ (0,0,0) & (0.3,0.4,0.2) & (0,0,0) & (0.3,0.3,0.2) & (0.1,0.6,0.3) & (0.1,0.3,0.1) \\ (0.1,0.2,0.2) & (0,0,0) & (0.3,0.3,0.2) & (0,0,0) & (0.1,0.6,0.2) & (0,0,0) \\ (0,0,0) & (0.1,0.6,0.2) & (0.1,0.6,0.3) & (0.1,0.6,0.2) & (0,0,0) & (0.1,0.6,0.1) \\ (0.1,0.8,0.1) & (0.1,0.5,0.1) & (0.1,0.3,0.1) & (0,0,0) & (0.1,0.6,0.1) & (0,0,0) \end{bmatrix}$$

$$R_5 = \begin{bmatrix} (0,0,0) & (0.2,0.5,0.1) & (0,0,0) & (0.2,0.2,0.1) & (0,0,0) & (0.1,0.2,0.1) \\ (0.2,0.5,0.1) & (0,0,0) & (0.2,0.5,0.3) & (0,0,0) & (0.2,0.5,0.1) & (0.1,0.8,0.1) \\ (0,0,0) & (0.2,0.5,0.3) & (0,0,0) & (0.2,0.3,0.3) & (0.3,0.6,0.1) & (0.1,0.3,0.1) \\ (0.2,0.2,0.1) & (0,0,0) & (0.2,0.3,0.3) & (0,0,0) & (0.2,0.1,0.1) & (0,0,0) \\ (0,0,0) & (0.2,0.5,0.1) & (0.3,0.6,0.1) & (0.2,0.1,0.1) & (0,0,0) & (0.1,0.8,0.1) \\ (0.1,0.2,0.1) & (0.1,0.8,0.1) & (0.1,0.3,0.1) & (0,0,0) & (0.1,0.8,0.1) & (0,0,0) \end{bmatrix}$$

$$R_6 = \begin{bmatrix} (0,\ 0,\ 0) & (0.2,\ 0.3,\ 0.2) & (0,\ 0,\ 0) & (0.1,\ 0.2,\ 0.2) & (0,\ 0,\ 0) & (0.7,\ 0.1,\ 0.1) \\ (0.2,\ 0.3,\ 0.2) & (0,\ 0,\ 0) & (0.2,\ 0.3,\ 0.3) & (0,\ 0,\ 0) & (0.2,\ 0.3,\ 0.3) & (0.2,\ 0.3,\ 0.1) \\ (0,\ 0,\ 0) & (0.2,\ 0.3,\ 0.3) & (0,\ 0,\ 0) & (0.1,\ 0.3,\ 0.3) & (0.4,\ 0.2,\ 0.3) & (0.4,\ 0.2,\ 0.1) \\ (0.1,\ 0.2,\ 0.2) & (0,\ 0,\ 0) & (0.1,\ 0.3,\ 0.3) & (0,\ 0,\ 0) & (0.1,\ 0.1,\ 0.3) & (0,\ 0,\ 0) \\ (0,\ 0,\ 0) & (0.2,\ 0.3,\ 0.3) & (0.4,\ 0.2,\ 0.3) & (0.1,\ 0.1,\ 0.3) & (0,\ 0,\ 0) & (0.6,\ 0.1,\ 0.1) \\ (0.7,\ 0.1,\ 0.1) & (0.2,\ 0.3,\ 0.1) & (0.4,\ 0.2,\ 0.1) & (0,\ 0,\ 0) & (0.6,\ 0.1,\ 0.1) & (0,\ 0,\ 0) \end{bmatrix}$$

Fig. 2 shows, the HFGs $C_k$ corresponding to HFPRs in matrices $R_k$ $k = 1, 2, 3, 4, 5, 6$.

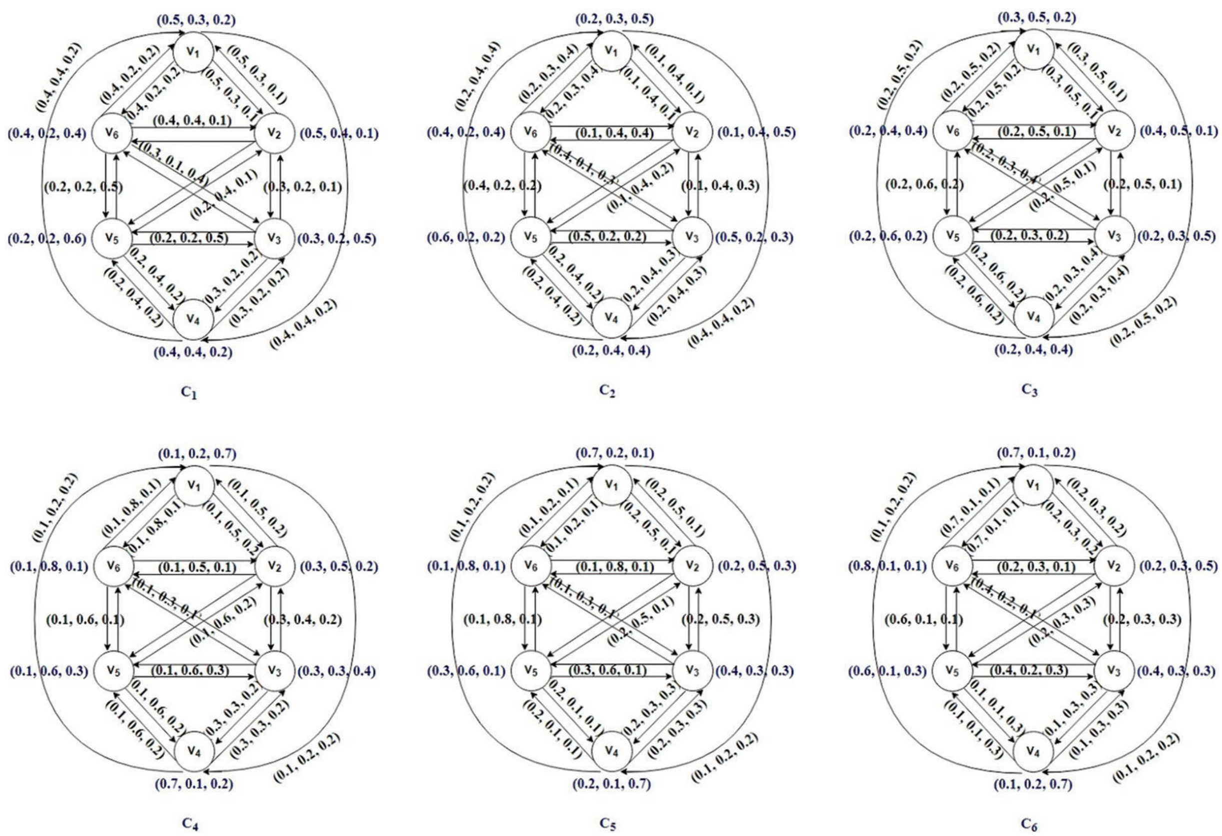


**Figure 2:** Hesitancy fuzzy graphs

By substituting Eigen roots in Eq. (2) and calculating we have the Laplacian Energy of $R_1$ is $LE(R_1) = [3.2077, \quad 3.1997, \quad 2.9361]$

In the same way, each HFG's Laplacian energy is calculated as follows:

$LE\ (R_2) = (3.1997,\ \ 3.3454,\ 2.8298)$, $\quad LE\ (R_3) = (2.0692,\ \ 4.6135,\ 2.4233)$,

$LE\ (R_4) = (1.5401,\ \ 5.7351,\ 1.9156)$, $\quad LE\ (R_5) = (1.9156,\ \ 6.3398,\ 1.9156)$, and

$LE\ (R_6) = (3.0027,\ \ 2.4496,\ 2.5983)$.

The calculation of every expert's weight by using Eq. (3) we get,

$$w_1 = [0.2148, \quad 0.1246, \quad 0.2008] \quad w_2 = [0.2142, \quad 0.1303, \quad 0.1936]$$

$$w_3 = [0.1385, \quad 0.1796, \quad 0.1658] \quad w_4 = [0.1031, \quad 0.2233, \quad 0.1310]$$

$$w_5 = [0.1283, \quad 0.2468, \quad 0.1310] \quad w_6 = [0.2011, \quad 0.0954, \quad 0.1777]$$

Hence, the six experts weight vector $e_k$ $k = 1, 2, \ldots, 6$ is calculated as:

$$w = [(0.2148, \quad 0.1246, \quad 0.2008), (0.2142, \quad 0.1303, \quad 0.1936), (0.1385, \quad 0.1796, \quad 0.1658), (0.1031, \quad 0.2233, \quad 0.1310), (0.1283, \quad 0.2468, \quad 0.1310), (0.2011, \quad 0.0954, \quad 0.1777)]$$

Using the HFWA operator, we calculate a collective HFE $p_i (i = 1, 2, 3, 4, 5, 6)$ of the testing venue $C_i$ over all other testing locations:

$$R = \begin{bmatrix} (0,\ 0,\ 0) & (0.2630,\ 0.4340,\ 0.1239) & (0,\ 0,\ 0) & (0.2205,\ 0.2813,\ 0.2089) & (0,\ 0,\ 0) & (0.3655,\ 0.3171,\ 0.1687) \\ (0.2630,\ 0.4340,\ 0.1239) & (0,\ 0,\ 0) & (0.2137,\ 0.3926,\ 0.1902) & (0,\ 0,\ 0) & (0.1695,\ 0.4686,\ 0.1523) & (0.2074,\ 0.5052,\ 0.1308) \\ (0,\ 0,\ 0) & (0.2137,\ 0.3926,\ 0.1902) & (0,\ 0,\ 0) & (0.2149,\ 0.2961,\ 0.2751) & (0.3207,\ 0.3605,\ 0.2488) & (0.2911,\ 0.2181,\ 0.2057) \\ (0.2205,\ 0.2813,\ 0.2089) & (0,\ 0,\ 0) & (0.2149,\ 0.2961,\ 0.2751) & (0,\ 0,\ 0) & (0.1708,\ 0.2931,\ 0.1963) & (0,\ 0,\ 0) \\ (0,\ 0,\ 0) & (0.1695,\ 0.4686,\ 0.1523) & (0.3207,\ 0.3605,\ 0.2488) & (0.1708,\ 0.2931,\ 0.1963) & (0,\ 0,\ 0) & (0.3276,\ 0.4103,\ 0.1695) \\ (0.3655,\ 0.3171,\ 0.1687) & (0.2074,\ 0.5052,\ 0.1308) & (0.2911,\ 0.2181,\ 0.2057) & (0,\ 0,\ 0) & (0.3276,\ 0.4103,\ 0.1695) & (0,\ 0,\ 0) \end{bmatrix}$$

Fig. 3 shows a directed network related to a collective HFPR.

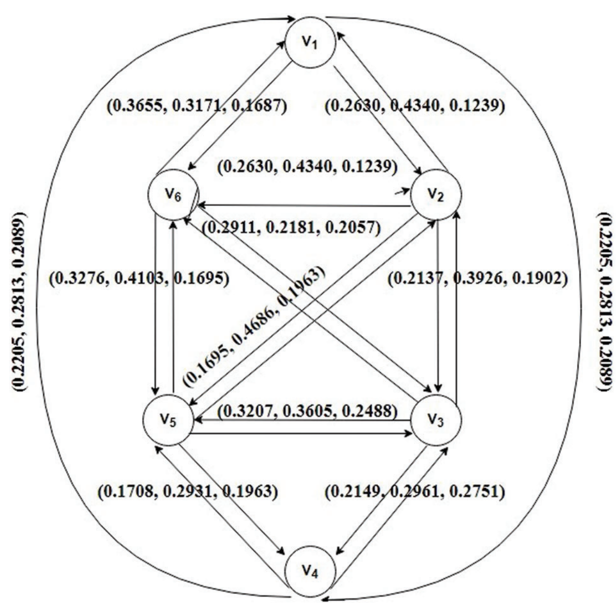


**Figure 3:** Hesitancy fuzzy graph

In the partially directed network, calculate the out-degrees $out-d(v_k)$ ( $k = 1, 2, 3, 4, 5, 6$) from all criteria as below:

$out - d(v_1) = \langle 0.8490, \ \ 1.0324, \ \ 0.5015 \rangle, \ \ out - d(v_2) = \langle 1.0536, \ \ 1.7039, \ \ 0.5972 \rangle,$

$out - d(v_3) = \langle 1.0404, \ \ 1.2673, \ \ 0.9198 \rangle, \quad out - d(v_4) = \langle 0.6062, \ \ 0.8705, \ \ 0.6803 \rangle,$

$out - d(v_5) = \langle 0.9886, \ \ 1.5325, \ \ 0.7669 \rangle, \quad out - d(v_6) = \langle 1.1916, \ \ 1.4507, \ \ 0.6747 \rangle.$

The ranking of the factors $v_k$ ($k = 1, 2, 3, 4, 5, 6$) are calculated based upon the membership degrees of $out-d(v_k)$ ($k = 1, 2, 3, 4, 5, 6$), as follows:

$v_6 > v_2 > v_3 > v_5 > v_1 > v_4$

Hence, the $v_6$ test venue is the best of the six test venues.

### *4.2 Reservoir Operation Schemes Evaluation*

This section focuses on assessing reservoir operation schemes. The Indirasagar reservoir is a multipurpose major project constructed in the Narmada River basin. The Indira Sagar Dam has been built to supply water to surrounding communities in Bhandara, and was known as the Gosikhurd Project. The reservoir was constructed as a joint effort between irrigation and the NHPC (National Hydroelectric Power Corporation). The reservoir was built for a variety of different reasons, including power generation, irrigation, and the overall water supply for farming, business sectors, surrounding householders (residents), and the environment. Six reservoirs operating scheme, and are proposed due to various needs for the division of the volume of water. $v_1$: The maximum production of plants, sufficient water supply in the Narmada River basin, the economy, and the society has the lowest and the highest supplies;

$v_2$: The maximum production of plants, sufficient water supply in the Narmada basin, lower and greater social and economic supplies, and lower ecosystem supply;

$v_3$: The maximum production of plants, adequate water supply utilized in the Narmada basin, lower and higher social and economic supplies, total ecosystem and environmental supply, with 90 percent of this supply being distributed to low-water flushing sands;

$v_4$: The maximum production of plants, adequate water supply utilized in the Narmada basin, lower and higher social and economic supplies, total ecosystem and environmental supply, with 60 percent of this supply being distributed to low-water flushing sands;

$v_5$: The maximum production of plants, adequate water supply utilized in the Narmada basin, lower and higher social and economic supplies, total ecosystem, and environmental supply, with 40 percent of this supply being distributed to low-water flushing sands;

$v_6$: The maximum production of plants, adequate water supply utilized in the Narmada basin, lower and higher social and economic supplies, overall supply to the environment and ecosystem and flooding times.

Example2:

To choose the best scheme, six experts $e_k$, ($k = 1, 2, 3, 4, 5, 6$) are invited by the government to assess the six plans. On the basis of their experience, the experts compare every combination, and provide the following HFPRs individual judgments ($k = 1, 2, 3, 4, 5, 6$) [29].

$$R_1 = \begin{bmatrix} (0,0,0) & (0.2,0.4,0.4) & (0.2,0.3,0.4) & (0,0,0) & (0.1,0.4,0.4) & (0.2,0.3,0.2) \\ (0.2,0.4,0.4) & (0,0,0) & (0.3,0.2,0.3) & (0.3,0.2,0.2) & (0,0,0) & (0.2,0.4,0.2) \\ (0.2,0.3,0.4) & (0.3,0.2,0.3) & (0,0,0) & (0.4,0.2,0.2) & (0.2,0.2,0.3) & (0.3,0.4,0.2) \\ (0,0,0) & (0.3,0.2,0.2) & (0.4,0.2,0.2) & (0,0,0) & (0.2,0.2,0.2) & (0.4,0.4,0.1) \\ (0.1,0.4,0.4) & (0,0,0) & (0.2,0.2,0.3) & (0.2,0.2,0.2) & (0,0,0) & (0.2,0.4,0.2) \\ (0.2,0.3,0.2) & (0.2,0.4,0.2) & (0.3,0.4,0.2) & (0.4,0.4,0.1) & (0.2,0.4,0.2) & (0,0,0) \end{bmatrix}$$

$$R_2 = \begin{bmatrix} (0,0,0) & (0.2,0.4,0.4) & (0.2,0.3,0.4) & (0,0,0) & (0.2,0.6,0.2) & (0.2,0.5,0.2) \\ (0.2,0.4,0.4) & (0,0,0) & (0.2,0.4,0.4) & (0.2,0.4,0.2) & (0,0,0) & (0.2,0.5,0.2) \\ (0.2,0.3,0.4) & (0.2,0.4,0.4) & (0,0,0) & (0.4,0.2,0.2) & (0.2,0.6,0.2) & (0.3,0.5,0.2) \\ (0,0,0) & (0.2,0.4,0.2) & (0.4,0.2,0.2) & (0,0,0) & (0.2,0.6,0.2) & (0.3,0.5,0.2) \\ (0.2,0.6,0.2) & (0,0,0) & (0.2,0.6,0.2) & (0.2,0.6,0.2) & (0,0,0) & (0.2,0.6,0.2) \\ (0.2,0.5,0.2) & (0.2,0.5,0.2) & (0.3,0.5,0.2) & (0.3,0.5,0.2) & (0.2,0.6,0.2) & (0,0,0) \end{bmatrix}$$

$$R_3 = \begin{bmatrix} (0,0,0) & (0.3,0.3,0.3) & (0,0.4,0.3) & (0,0,0) & (0.5,0.3,0.1) & (0.5,0.2,0.2) \\ (0.3,0.3,0.3) & (0,0,0) & (0,0.6,0.4) & (0.3,0.4,0.1) & (0,0,0) & (0.3,0.3,0.2) \\ (0,0.4,0.3) & (0,0.6,0.4) & (0,0,0) & (0,0.6,0.1) & (0,0.3,0.1) & (0,0.6,0.2) \\ (0,0,0) & (0.3,0.4,0.1) & (0,0.6,0.1) & (0,0,0) & (0.5,0.4,0.1) & (0.5,0.3,0.1) \\ (0.5,0.3,0.1) & (0,0,0) & (0,0.3,0.1) & (0.5,0.4,0.1) & (0,0,0) & (0.6,0.3,0.1) \\ (0.5,0.2,0.2) & (0.3,0.3,0.2) & (0,0.6,0.2) & (0.5,0.3,0.1) & (0.6,0.3,0.1) & (0,0,0) \end{bmatrix}$$

$$R_4 = \begin{bmatrix} (0,0,0) & (0.3,0.3,0.3) & (0.3,0.5,0) & (0,0,0) & (0.1,0,0.6) & (0.2,0.2,0.6) \\ (0.3,0.3,0.3) & (0,0,0) & (0.4,0.6,0) & (0.1,0.3,0.3) & (0,0,0) & (0.2,0.2,0.3) \\ (0.3,0.5,0) & (0.4,0.6,0) & (0,0,0) & (0.1,0.6,0) & (0.1,0.6,0) & (0.2,0.6,0) \\ (0,0,0) & (0.1,0.3,0.3) & (0.1,0.6,0) & (0,0,0) & (0.1,0.4,0.5) & (0.1,0.4,0.5) \\ (0.1,0,0.6) & (0,0,0) & (0.1,0.6,0) & (0.1,0.4,0.5) & (0,0,0) & (0.1,0.3,0.1) \\ (0.2,0.2,0.6) & (0.2,0.2,0.3) & (0.2,0.6,0) & (0.1,0.4,0.5) & (0.1,0.3,0.1) & (0,0,0) \end{bmatrix}$$

$$R_5 = \begin{bmatrix} (0,0,0) & (0.2,0.2,0.1) & (0.2,0.1,0.5) & (0,0,0) & (0.2,0.1,0.5) & (0.2,0.3,0.2) \\ (0.2,0.2,0.1) & (0,0,0) & (0.4,0.2,0.1) & (0.3,0.2,0.1) & (0,0,0) & (0.5,0.2,0.1) \\ (0.2,0.1,0.5) & (0.4,0.2,0.1) & (0,0,0) & (0.3,0.1,0.5) & (0.4,0.1,0.5) & (0.4,0.3,0.2) \\ (0,0,0) & (0.3,0.2,0.1) & (0.3,0.1,0.5) & (0,0,0) & (0.3,0.1,0.5) & (0.3,0.1,0.2) \\ (0.2,0.1,0.5) & (0,0,0) & (0.4,0.1,0.5) & (0.3,0.1,0.5) & (0,0,0) & (0.5,0.3,0.2) \\ (0.2,0.3,0.2) & (0.5,0.2,0.1) & (0.4,0.3,0.2) & (0.3,0.1,0.2) & (0.5,0.3,0.2) & (0,0,0) \end{bmatrix}$$

$$R_6 = \begin{bmatrix} (0,0,0) & (0.1,0.2,0.2) & (0.5,0.1,0.2) & (0,0,0) & (0.5,0,0.2) & (0.2,0.3,0.2) \\ (0.1,0.2,0.2) & (0,0,0) & (0.1,0.2,0.4) & (0.1,0.2,0.4) & (0,0,0) & (0.1,0.3,0.5) \\ (0.5,0.1,0.2) & (0.1,0.2,0.4) & (0,0,0) & (0.5,0.1,0.4) & (0.5,0.1,0.4) & (0.2,0.3,0.4) \\ (0,0,0) & (0.1,0.2,0.4) & (0.5,0.1,0.4) & (0,0,0) & (0.5,0,0.4) & (0.2,0.3,0.4) \\ (0.5,0,0.2) & (0,0,0) & (0.5,0.1,0.4) & (0.5,0,0.4) & (0,0,0) & (0.2,0,0.5) \\ (0.2,0.3,0.2) & (0.1,0.3,0.5) & (0.2,0.3,0.4) & (0.2,0.3,0.4) & (0.2,0,0.5) & (0,0,0) \end{bmatrix}$$

The HFGs $C_t$ corresponding to HFPRs given in matrices $R_t$ $(t = 1, 2, 3, 4, 5, 6)$ are shown in Fig. 4

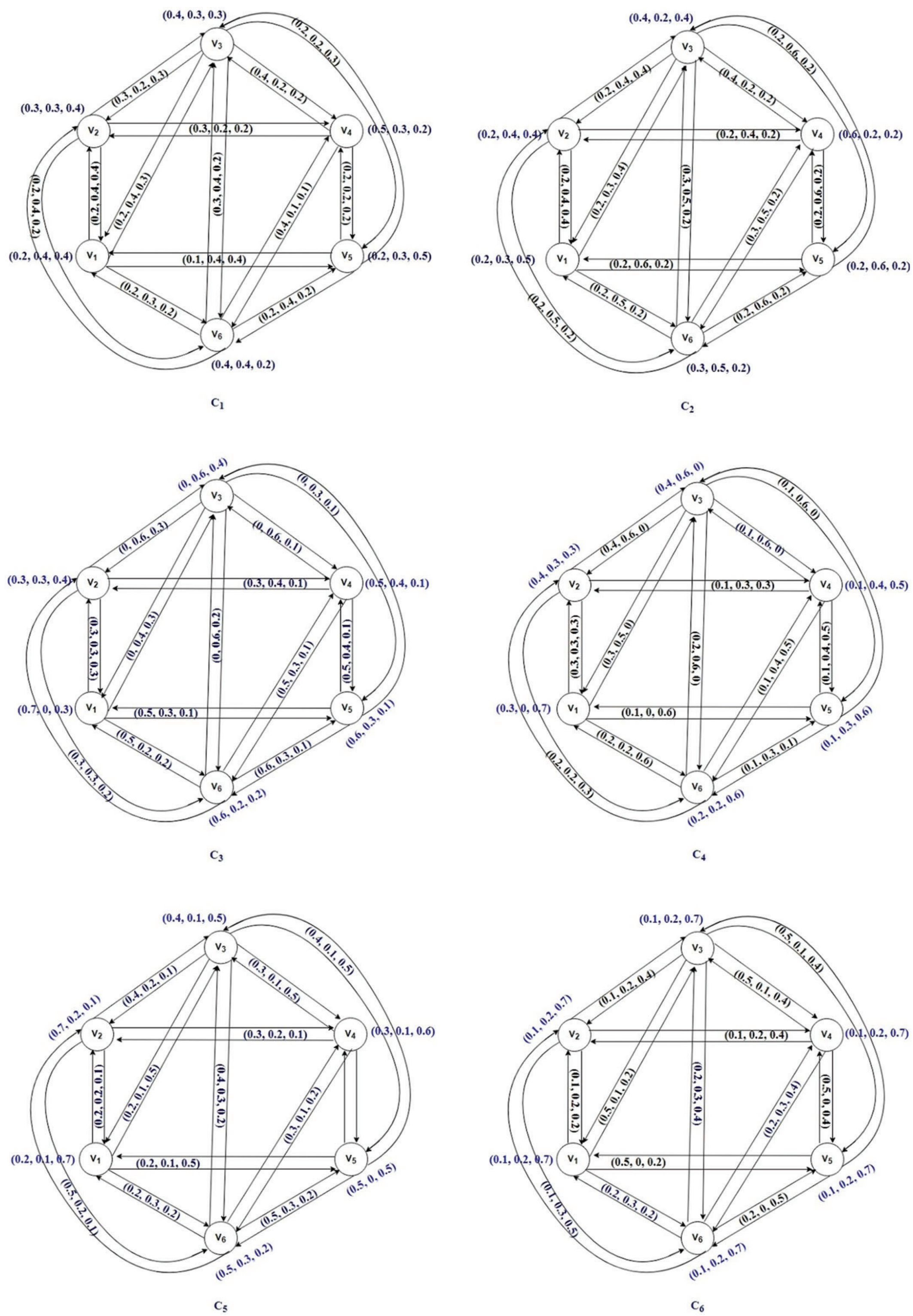


**Figure 4:** Hesitancy fuzzy graphs

By substituting Eigen roots in Eq. (2) and calculating we have the Laplacian Energy of $R_1$ is

$LE(C_1) = [3.1613, \quad 3.4370, \quad 3.1462]$

In the same way, Each HFG's Laplacian energy is calculated as follows:

$LE\ (C_2) = (2.6435, \quad 4.9522, \quad 3.3835), \qquad LE\ (C_3) = (4.8976, \quad 4.5470, \quad 2.8670),$

$LE\ (C_4) = (2.7466, \quad 5.0802, \quad 4.4557), \qquad LE\ (C_5) = (3.7042, \quad 2.2466, \quad 4.2608),$ and

$LE\ (C_6) = (4.2608, \ 2.7835, \ 4.3490).$

The calculation of every expert's weight is as follows:

$w_1 = [0.2148, \quad 0.1246, \quad 0.2008] \quad w_2 = [0.2142, \quad 0.1303, \quad 0.1936]$

$w_3 = [0.1385, \quad 0.1796, \quad 0.1658] \quad w_4 = [0.1031, \quad 0.2233, \quad 0.1310]$

$w_5 = [0.1283, \quad 0.2468, \quad 0.1310] \quad w_6 = [0.2011, \quad 0.0954, \quad 0.1777]$

Hence, the six experts' weight vector $e_k \ \ k = 1, 2, \ldots, 6$ is calculated as:

$$w = [(0.2148, \quad 0.1246, \quad 0.2008), (0.2142, \quad 0.1303, \quad 0.1936), (0.1385, \quad 0.1796, \quad 0.1658), (0.1031, \quad 0.2233, \quad 0.1310), (0.1283, \quad 0.2468, \quad 0.1310), (0.2011, \quad 0.0954, \quad 0.1777)]$$

Using the HFWA operator, we calculate a collective HFG $p_i (i = 1, 2, 3, 4, 5, 6)$ of the testing venue $C_i$ over all other testing locations (Fig. 5).

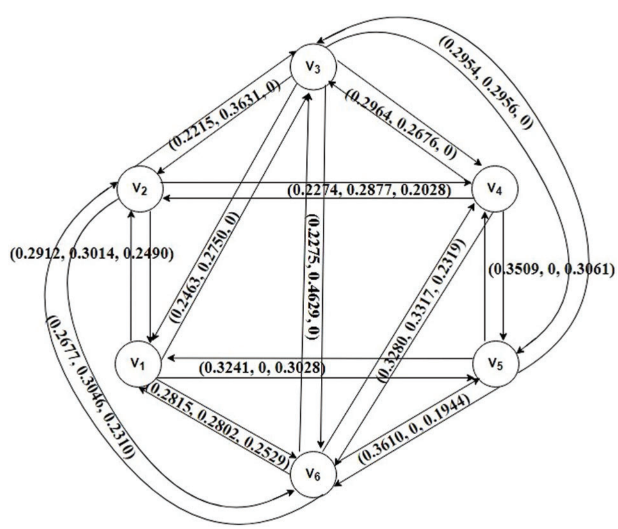


**Figure 5:** Hesitancy fuzzy graph

$$R = \begin{bmatrix} (0,\ 0,\ 0) & (0.2192,\ 0.3014,\ 0.2490) & (0.2463,\ 0.2750,\ 0) \\ (0.2192,\ 0.3014,\ 0.2490) & (0,\ 0,\ 0) & (0.2215,\ 0.3631,\ 0) \\ (0.2463,\ 0.2750,\ 0) & (0.2215,\ 0.3631,\ 0) & (0,\ 0,\ 0) \\ (0,\ 0,\ 0) & (0.2274,\ 0.2877,\ 0.2028) & (0.2964,\ 0.2676,\ 0) \\ (0.3241,\ 0,\ 0.3028) & (0,\ 0,\ 0) & (0.2594,\ 0.2956,\ 0) \\ (0.2815,\ 0.2802,\ 0.2529) & (0.2677,\ 0.3046,\ 0.2310) & (0.2275,\ 0.4629,\ 0) \end{bmatrix.$$

$$\begin{matrix} (0,\ 0,\ 0) & (0.3241,\ 0,\ 0.3028) & (0.2815,\ 0.2802,\ 0.2529) \\ (0.2274,\ 0.2877,\ 0.2028) & (0,\ 0,\ 0) & (0.2677,\ 0.3046,\ 0.2310) \\ (0.2964,\ 0.2676,\ 0) & (0.2594,\ 0.2956,\ 0) & (0.2275,\ 0.4629,\ 0) \\ (0,\ 0,\ 0) & (0.3509,\ 0,\ 0.3031) & (0.3280,\ 0.3317,\ 0.2319) \\ (0.3509,\ 0,\ 0.3031) & (0,\ 0,\ 0) & (0.3610,\ 0,\ 0.1944) \\ (0.3280,\ 0.3317,\ 0.2319) & (0.3610,\ 0,\ 0.1944) & (0,\ 0,\ 0) \end{matrix} \end{bmatrix}$$

In the partially directed network, calculate the out-degrees $out-d(v_k)$ ( $k = 1, 2, 3, 4, 5, 6$) from all criteria as below:

$$out - d(v_1) = \langle 1.0711,\ \ 0.8566,\ \ 0.8047 \rangle, \quad out - d(v_2) = \langle 0.9358,\ \ 1.2568,\ \ 0.6828 \rangle,$$

$$out - d(v_3) = \langle 1.2506,\ \ 1.6642,\ \ 0 \rangle, \quad out - d(v_4) = \langle 1.2027,\ \ 0.8870,\ \ 0.7378 \rangle,$$

$$out - d(v_5) = \langle 1.2954,\ \ 0.2956,\ \ 0.8003 \rangle, \quad out - d(v_6) = \langle 1.4657,\ \ 1.3794,\ \ 0.9102 \rangle.$$

The ranking of the factors $v_k$ ($k = 1, 2, 3, 4, 5, 6$) are calculated based upon the membership degrees of $out-d(v_k)$ ($k = 1, 2, 3, 4, 5, 6$), as follows:

$v_6 > v_5 > v_3 > v_4 > v_1 > v_2$

Hence, the $v_6$ scheme is the best scheme of the six schemes.

## 5 Conclusion

In this article, we present the concepts of LE in an HFG. An HFG can accurately characterize the ambiguity of all types of networks. We also discussed the topic of an HFG's LE and also its applicability in decision-making problems. These concepts are also presented through real-time examples like satellite communications and an evaluation of reservoir schemes. We intend to broaden our study to include *(1)* Signless Energy and LE of HFGS, as well as *(2)* Simplified an interval-valued HFG. We were extended the approach for order performance by a similarity to an ideal solution (TOPSIS) to the HFG in the procedures and utilized the extended TOPSIS to rank and pick the best alternative.

**Funding Statement:** The authors received no specific funding for this study.

**Conflicts of Interest:** The authors declare that they have no conflicts of interest to report regarding the present study.

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