

A Coordinated Search Algorithm for a Lost Target on the Plane

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Abstract: Concepts in search theory have developed since World War II. The study of search plans has found considerable interest among searchers due to its wide applications in our life. Searching for lost targets either located or moved is often a time-critical issue, especially when the target is very important. In many commercial and scientific missions at sea, it is of crucial importance to find lost targets underwater. We illustrate a technique known as coordinated search, that completely characterizes the search for a randomly located target on a plane. The idea is to avoid wasting time looking for a missing target. Two searchers or robots start from the center of a circle to search out a lost target, the first searcher looks for the target on the right side of the circular area, and the second one looks for it on the left side. The time taken to detect the target is obtained by assuming the target's position has a symmetric distribution. The procedures to facilitate the detection of the target are presented as an algorithm and as a flow-chart. An application demonstrates the applicability of this search technique and the associated decrease in search cost. Its effectiveness is illustrated by numerical results, which indicates considerable promise.

Keywords: Expected value; sensors; lost target; coordinated search; search algorithm; lost ship

1 Introduction

Search theory rests on the oldest area of operations research. The initial steps made by Koopman in the Anti-submarine Warfare Operations Research Group of the U.S. Navy during World War II for submarine detection are still widely used for this purpose. The theory has also been used by the Navy to search for objects such as the H-bomb lost in the ocean near Palomares, Spain, in 1966, the submarine Scorpion lost in 1968, and numerous lesser-known objects. The U.S. Coast Guard uses the search theory to plan some of its more complicated search and rescue efforts.

To mention some important models of search plans, we should begin with the linear search strategy, which has many life and mission applications, such as searching for a damaged unit in a large linear system (electrical power lines, telephone lines, and gas support lines), whether a linear system is independent or intersecting (see El-Rayes et al. [1], Mohamed [2], Mohamed et al. [3–5], Balkhi [6]).



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The coordinated search technique has been studied in the case of a linear search. Mohamed et al. [7–9] discussed the coordinated search technique for a target on two intersecting lines, where the target has a symmetric and asymmetric distribution (See Mohamed et al. [10], Afifi et al. [11]). More advanced work applied the coordinated search technique to a moving target on one line as well as on many independent lines. The coordinated search technique in the plane was discussed by Mohamed et al. [12]. El-Hadidy et al. [13,14] proposed a model in 3-D space to find a randomly located target by one or two searchers. Caraballo et al. [15] considered a model in search theory to find a randomly located target in 3-D space. Kassem et al. [16] studied a more sophisticated search model in 3-D space to find a randomly located target. More recently, Teamah et al. [17] obtained an optimal discrete search for randomly moving COVID-19 between several cells in the human body using a monitoring system. Afifi et al. [18] studied the existence of the multi-generalized linear search problem to detect a randomly located target in one of several real lines.

We present a coordinated algorithm, with the important advantage that it avoids wasting time searching for the target. The mission is carried out by two searchers starting from the origin of a known circular area on the sea surface. The first searcher looks for the target on the right side of the circular area, and the second one looks for it on the left side. It is important to note that neither searcher returns to the origin. We use modern means of communication and sign language to save much effort and time.

2 Problem Formulations

On May 19, 2020, maritime authorities announced that they had lost contact with a small ship on its way to the Socotra Archipelago port in the Indian Ocean, carrying two families in addition to the ship's crew. They suddenly received a call from the missing ship, which enabled them to determine its coordinates. Navy ships were then sent out to search for the missing ship. We present a coordinated algorithm to solve this kind of problem, with the advantage of a detection time saving element, as the probability distribution function of the ship's location is known to the searchers (sensors).

2.1 Search Framework

Search space: Circular two-dimensional area.

Target: The target is a lost ship randomly located on the sea surface.

Means of search: The search is performed by two searchers on a circular section. They start looking together from the point (0,0) (the center of the circular area), where the region is divided into right and left parts. We also divide the area into concentric circles, as in Fig. 1.

2.2 The Searching Technique

In this model, the search for the missing ship is carried out according to coordinated movement between the two searchers. After each move, both send signals to a marine ship's signal reception center by radio telex. There are two types of detection:

1. Perfect detection: One of the searchers detects the lost ship in the specified search section and sends a positive sign.
2. False detection: Neither searcher detects the lost ship in the specified search section, and one or both sends a positive sign.

Let (X, Y) be independent random variables representing the position of the target, with cumulative distribution function (CDF) $F(x, y)$ and probability density function (PDF) $f(x, y)$.

Searchers s_1 and s_2 start searching together for the lost ship (target) from the origin $(0, 0)$ with equal speeds $v_1 = v_2 = 1$, and they search with a regular speed on the sectors and their tracks. Searcher s_1

searches on the right side, and searcher s_2 on the left side. Searcher s_1 goes through a +ve part y-axis with a distance r_1 (radius of the first circle) and starts searching sector L_1 . After searching sector L_1 , the searcher reaches the y-axis again but does not return to the origin point and moves toward sector L_2 with distance $r_2 - r_1$. When it completes the search of sector L_2 and does not find the target, it continues searching sector L_3 , and so on. Simultaneously, searcher s_2 searches the opposite area on the left hand-side, goes through a -ve part y-axis with distance r_1 , and starts searching sector M_1 . After searching this sector and not finding the target, it moves to sector M_2 with distance $r_2 - r_1$. When it completes searching sector M_2 and does not find the target, it continues searching sector M_3 , and so on. Searchers s_1 and s_2 search sectors L_1 and M_1 at the same time. If one of them detects the lost target, it will send a positive sign and the search will end. If both send negative signs, then the two searchers complete their search and move to the next sectors, L_2 and M_2 , and so on, until one of the searchers detects the lost ship. The searchers do not return to the origin so as to save search time and reduce the expected time to detect the target.

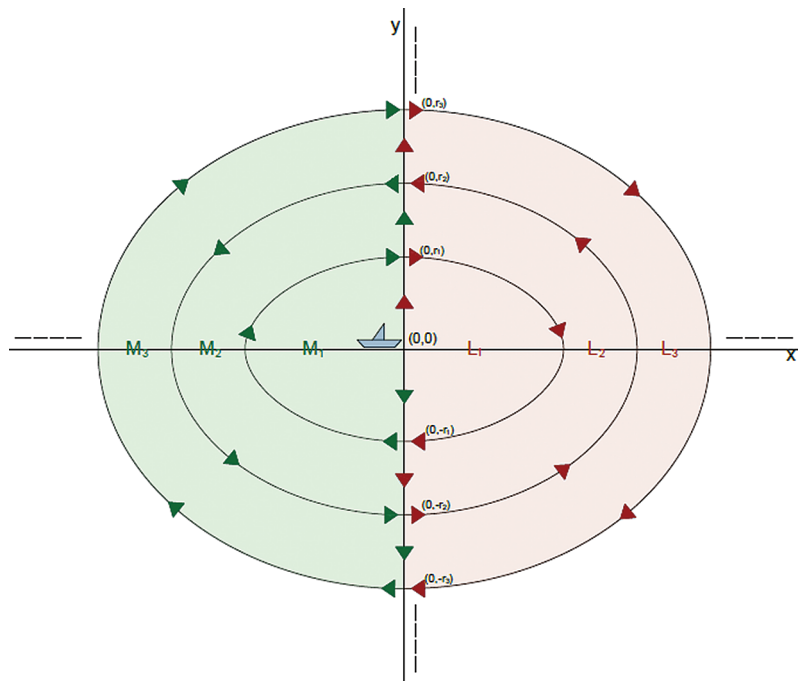


Figure 1: The search path

Any track has width $r_i - r_{i-1}$. The searcher moves on the y-axis +ve and -ve sides until it detects the target. We aim to calculate the expected length of time to detect the target and search the optimal search plan.

Searchers s_1 and s_2 follow a coordinated search path to find the target. Let E_i and F_i be the search paths of s_1 and s_2 , respectively, $i = 1, 2, \dots, n$, and let L_i and M_i be their respective search sectors. Let t_1 and t_2 be the respective times taken by s_1 and s_2 to search each sector in paths E_i and F_i , respectively.

The search proceeds as follows.

Step 1: The two searchers move from $(0,0)$ to detect the target. Searcher s_1 follows search path e_1 as follows: Searcher s_1 goes to $(0, r_1)$ with distance $|r_1|$ through a +ve part in y-axis. If the target is not found, it completely searches sector L_1 and continues tracking until the point $(0, -r_1)$. At the same time, searcher s_2 goes to $(0, -r_1)$ with distance $|r_1|$ through a -ve part in x-axis. If the target is not found, it completely searches sector M_1 and continues tracking until the point $(0, r_1)$. At this time, the two searchers send signals

to a marine ship's signal reception center. If one of them sends a positive signal, then the search ends; however, if the two searchers send a negative signal, the searchers will move on to the following step.

Step 2: Searcher s_1 completes the search for the lost target and moves to the point $(0, r_2)$ with a distance $|r_2 - r_1|$. The first searcher follows the search path e_2 as follows: searcher s_1 goes to $(0, r_2)$ with distance $|r_2 - r_1|$ through a +ve part on the y-axis. If the target is not found, it completely searches sector L_2 and keeps tracking until the point $(0, -r_2)$. At the same time, searcher s_2 goes to $(0, -r_2)$ with distance $|r_2 - r_1|$ through a -ve part on the x-axis. If the target is not found, it completely searches sector M_2 and keeps tracking until the point $(0, r_2)$. At this time, the two searchers send signals to a marine ship's signal reception center. If one of them sends a positive signal, then the search ends; however, if the two searchers send a negative signal, the searchers will move on to the following step.

Step 3: Searcher s_1 completes the search for the lost target and moves to the point $(0, r_3)$ with a distance $|r_3 - r_2|$. The first searcher follows the search path e_3 as follows: searcher s_1 goes to $(0, r_3)$ with distance $|r_3 - r_2|$ through a +ve part on the y-axis. If the target is not found, it completely searches sector L_2 and keeps tracking until the point $(0, -r_2)$. At the same time, searcher s_2 goes to $(0, -r_3)$ with distance $|r_3 - r_2|$ through the -ve part on the x-axis. If the target is not found, it completely searches sector M_3 and keeps tracking until the point $(0, r_3)$. At this time, the two searchers send signals to a marine ship's signal reception center. If one of them sends a positive signal, then the search ends; however, if the two searchers send a negative signal, the searchers will move on to the next sectors, and so on, until one of the two searchers detects the lost ship.

The searchers s_1 and s_2 follow the search path. Let E_i be the path for the first searcher, where $i \geq 0$, t_1 is the time for the first searcher, L_i is the sector which s_1 searches, $i = 1, 2, \dots, n$, and F_i the path of the second searcher, where $i \geq 0$, M_i is the sector which s_2 searches, $i = 1, 2, \dots, n$, and the two searchers move from $(0, 0)$ to detect the target. The first search path e_1 of s_1 is as follows: searcher s_1 goes to $(0, r_1)$ with distance $|r_1|$ through a +ve part on the y-axis. If the target is not found, it completely searches sector L_1 and keeps tracking until the point $(0, -r_1)$. The time taken in this step is $t_1 = |r_1| + \frac{\pi}{w_1}$, and the searcher moves to the point $(0, r_2)$ with distance $|r_2 - r_1|$. It takes time $t_1 = |r_2 - r_1| + \frac{\pi}{w_2}$ if it does not find the target and moves on to sector L_2 while continuing to track. If the target is not found, it moves toward $(0, r_3)$ to search sector L_3 and keeps tracking until the point $(0, -r_3)$ with distance $|r_3 - r_2|$; it takes time $t_1 = |r_3 - r_2| + \frac{\pi}{w_3}$, and so on. The number of circles is $i = 1, 2, \dots$. These circles are divided into two equal two sectors, L_i and M_i .

Searcher s_2 goes to $(0, -r_1)$ with distance $|r_1|$ through a -ve part on the x-axis. If the target is not found, it completely searches sector M_1 and continues tracking until the point $(0, r_1)$ with time $t_2 = |r_1| + \frac{\pi}{w_1}$, then moves to the point $(0, r_2)$ with distance $|r_2 - r_1|$ and takes time $t_2 = |r_2 - r_1| + \frac{\pi}{w_2}$. If the target is not found, it searches sector M_2 until the point $(0, r_2)$. If the target is not found, it moves toward $(0, -r_3)$, searches sector M_3 , and keeps tracking until the point $(0, r_3)$ with distance $|r_3 - r_2|$, and so on.

3 Expected Time to Detect Target

Theorem 1: The expected value of the time to detect the target is

$$\begin{aligned}
 E(t_\phi) &= 2 \left[\left[\left(|r_1| + \frac{\pi}{w_1} \right) \int_0^{\frac{\pi}{2}} \int_0^{r_1} g_1(r, \theta) r dr d\theta \right] + \left[\left(|r_2 - r_1| + \frac{\pi}{w_2} \right) \int_0^{\frac{\pi}{2}} \int_{r_1}^{r_2} g_2(r, \theta) r dr d\theta \right] \right. \\
 &+ \left[\left(|r_3 - r_2| + \frac{\pi}{w_3} \right) \int_0^{\frac{\pi}{2}} \int_{r_2}^{r_3} g_3(r, \theta) r dr d\theta \right] + \dots + \left[\left(|r_n - r_{n-1}| + \frac{\pi}{w_n} \right) \int_0^{\frac{\pi}{2}} \int_{r_{n-1}}^{r_n} g_n(r, \theta) r dr d\theta \right] \\
 &= 2 \left[\left[\left(|r_i - r_{i-1}| + \frac{\pi}{w_i} \right) \int_0^{\frac{\pi}{2}} \int_{r_{i-1}}^{r_i} g_i(r, \theta) r dr d\theta \right] \right]
 \end{aligned} \tag{1}$$

where $i = 1, 2, 3, \dots, n, r_0 = 0$.

Proof:

1) For s_1 :

If the target is in sector L1, then $t_1 = |r_1| + \frac{\pi}{w_1}$.

If the target is in sector L2, then $t_1 = |r_2 - r_1| + \frac{\pi}{w_2}$.

If the target is in sector L3, then $t_1 = |r_3 - r_2| + \frac{\pi}{w_3}$, and so on.

2) For s_2 :

If the target is in sector M1, then $t_2 = |r_1| + \frac{\pi}{w_1}$.

If the target is in sector M2, then $t_2 = |r_2 - r_1| + \frac{\pi}{w_2}$.

If the target is in sector M3, then $t_2 = |r_3 - r_2| + \frac{\pi}{w_3}$, and so on.

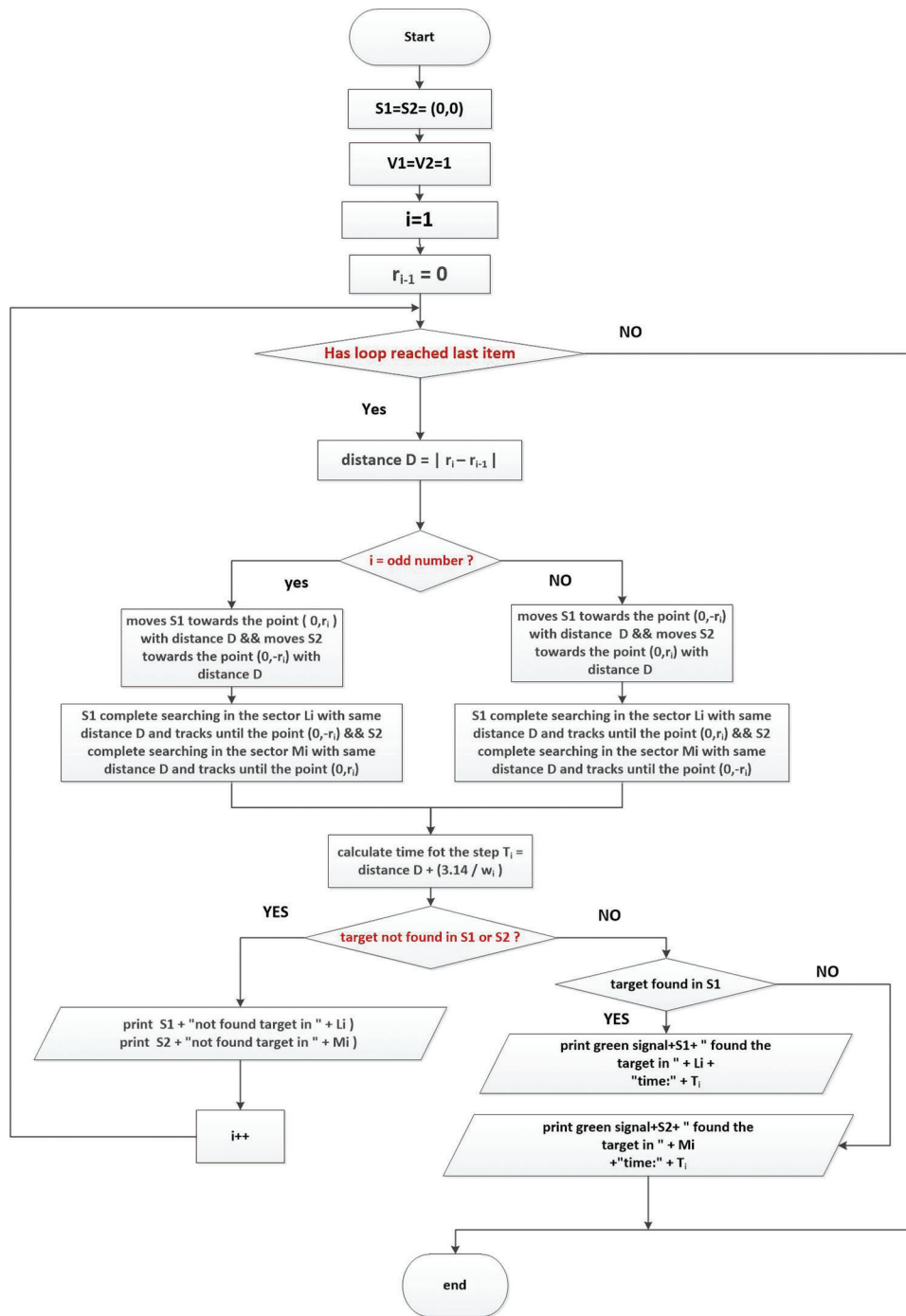
Then

$$\begin{aligned}
 E(t_\phi) &= \left[\left(|r_1| + \frac{\pi}{w_1} \right) \int_0^{\frac{\pi}{2}} \int_0^{r_1} g_1(r, \theta) r dr d\theta \right] + \left[\left(|r_2 - r_1| + \frac{\pi}{w_2} \right) \int_0^{\frac{\pi}{2}} \int_{r_1}^{r_2} g_2(r, \theta) r dr d\theta \right] + \dots \\
 &\left[\left(|r_3 - r_2| + \frac{\pi}{w_3} \right) \int_0^{\frac{\pi}{2}} \int_{r_2}^{r_3} g_3(r, \theta) r dr d\theta \right] + \dots + \left[\left(|r_n - r_{n-1}| + \frac{\pi}{w_n} \right) \int_0^{\frac{\pi}{2}} \int_{r_{n-1}}^{r_n} g_n(r, \theta) r dr d\theta \right] + \\
 &\left[\left(|r_1| + \frac{\pi}{w_1} \right) \int_0^{\frac{\pi}{2}} \int_0^{r_1} g_1(r, \theta) r dr d\theta \right] + \left[\left(|r_2 - r_1| + \frac{\pi}{w_2} \right) \int_0^{\frac{\pi}{2}} \int_{r_1}^{r_2} g_2(r, \theta) r dr d\theta \right] + \\
 &\left[\left(|r_3 - r_2| + \frac{\pi}{w_3} \right) \int_0^{\frac{\pi}{2}} \int_{r_2}^{r_3} g_3(r, \theta) r dr d\theta \right] + \dots + \left[\left(|r_n - r_{n-1}| + \frac{\pi}{w_n} \right) \int_0^{\frac{\pi}{2}} \int_{r_{n-1}}^{r_n} g_n(r, \theta) r dr d\theta \right] \\
 &= 2 \left[\left[\left(|r_i - r_{i-1}| + \frac{\pi}{w_i} \right) \int_0^{\frac{\pi}{2}} \int_{r_{i-1}}^{r_i} g_i(r, \theta) r dr d\theta \right] \right]
 \end{aligned}$$

4 Algorithm and Flowchart

The calculation of the expected value of the time to detect the lost target is described in the following algorithm and flowchart.

-
- Main algorithm
 - initialize searchers S_1 and S_2 to start point (0,0);
 - Velocities $V_1 = V_2 = 1$;
 - $i = 1$;
 - $r_{i-1} = 0$;
 - for (i to k)
 - distance $D = |r_i - r_{i-1}|$;
 - if (i = odd number) then
 - move S1 toward point (0, r_i) with distance D && move S2 toward point (0,- r_i) with distance D ;
 - S1 completes searching in sector L_i with same distance D and tracks until point (0,- r_i) && S2 completes searching in sector M_i with same distance D and tracks until point (0, r_i);
 - else
 - move S1 toward point (0,- r_i) with distance D && move S2 toward point (0, r_i) with distance D ;
 - S1 completes searching in sector L_i with same distance D and tracks until point (0, r_i) && S2 completes searching in sector M_i with same distance D and tracks until point (0,- r_i);
 - end if
 - calculate time for step $T_i = \text{distance } D + (3.14/w_i)$
 - if (target not found in S1 || S2) then
 - print S1 + “not found target in” + L_i)
 - print S2 + “not found target in” + M_i)
 - else
 - if (target found in S1) then
 - print green signal+S1+ “found target in” + L_i + “time:” + T_i
 - else
 - print green signal+S2+ “found target in” + M_i + “time:” + T_i
 - exit a for loop
 - end if
 - End for
 - End Main
-



5 Application

Searchers s_1 and s_2 have a mission to search for a lost ship on the sea surface. The ship's location follows a standard bivariate normal distribution for two independent random variables x, y , $f(x, y) = \frac{1}{2\pi} e^{-\frac{x^2+y^2}{2}}$. Changing to polar coordinates $x = r \cos \theta$, $y = r \sin \theta$, the Jacobian matrix is defined as

$$J(u, v) = \det \begin{pmatrix} \frac{\partial x}{\partial r} & \frac{\partial x}{\partial \theta} \\ \frac{\partial y}{\partial r} & \frac{\partial y}{\partial \theta} \end{pmatrix} = J(u, v) = \det \begin{pmatrix} \cos \theta & -r \sin \theta \\ \sin \theta & r \cos \theta \end{pmatrix} = r \cos^2 \theta + r \sin^2 \theta = r(\cos^2 \theta + \sin^2 \theta) = r.$$

So, $g(r, \theta) = \frac{1}{2\pi} e^{-\frac{r^2}{2}}$, and by substituting in Eq. (1), the expected value of the time to detect the target will be

$$E(t_\phi) = 2[|r_i - r_{i-1}| + \frac{\pi}{w_i}] \int_0^{\frac{\pi}{2}} \int_{r_{i-1}}^{r_i} \frac{1}{2\pi} r e^{-\frac{r^2}{2}} r dr d\theta = \int_0^{\frac{\pi}{2}} \int_{r_{i-1}}^{r_i} r^2 e^{-\frac{r^2}{2}} dr d\theta = \frac{\pi}{2} \int_{r_{i-1}}^{r_i} r^2 e^{-\frac{r^2}{2}} dr$$

$$\text{The first search if } r: 0 \rightarrow r_1; \text{ then } E(t_\phi) = \frac{\pi}{2} \left[\frac{e^{-\frac{r_1^2}{2}} \left(\sqrt{\pi} e^{\frac{r_1^2}{2}} \operatorname{erf} \frac{r_1}{\sqrt{2}} - \sqrt{2} r_1 \right)}{\sqrt{2}} \right]$$

The Second search if $r: r_1 \rightarrow r_2$; then the expected value will be

$$= \frac{\pi}{2} \left[\frac{e^{-\frac{r_2^2}{2}} \left(\sqrt{\pi} e^{\frac{r_2^2}{2}} \operatorname{erf} \frac{r_2}{\sqrt{2}} - \sqrt{2} r_2 \right)}{\sqrt{2}} - \frac{e^{-\frac{r_1^2}{2}} \left(\sqrt{\pi} e^{\frac{r_1^2}{2}} \operatorname{erf} \frac{r_1}{\sqrt{2}} - \sqrt{2} r_1 \right)}{\sqrt{2}} \right]$$

$$= \frac{\pi}{2} \left[\frac{e^{-\frac{r_i^2}{2}} \left(\sqrt{\pi} e^{\frac{r_i^2}{2}} \operatorname{erf} \frac{r_i}{\sqrt{2}} - \sqrt{2} r_i \right) - e^{-\frac{r_{i-1}^2}{2}} \left(\sqrt{\pi} e^{\frac{r_{i-1}^2}{2}} \operatorname{erf} \frac{r_{i-1}}{\sqrt{2}} - \sqrt{2} r_{i-1} \right)}{\sqrt{2}} \right]$$

$$E(t_\phi) = 2 \cdot \frac{1}{2\pi} \cdot \frac{\pi}{2\sqrt{2}} \left[\left(|r_i - r_{i-1}| + \frac{\pi}{w_i} \right) \left(-r_i e^{-\frac{r^2}{2}} + \sqrt{\frac{\pi}{2}} \operatorname{erf} \frac{r_i}{\sqrt{2}} + r_{i-1} e^{-\frac{r_i^2}{2}} - \sqrt{\frac{\pi}{2}} \operatorname{erf} \frac{r_{i-1}}{\sqrt{2}} \right) \right],$$

$$E(t_\phi) = \frac{1}{2\sqrt{2}} \left[|r_i - r_{i-1}| + \frac{\pi}{w_i} \right] \left[-r_i e^{-\frac{r_i^2}{2}} + r_{i-1} e^{-\frac{r_{i-1}^2}{2}} + \sqrt{\frac{\pi}{2}} \left(\operatorname{erf} \frac{r_i}{\sqrt{2}} - \operatorname{erf} \frac{r_{i-1}}{\sqrt{2}} \right) \right]. \tag{2}$$

Special cases:

If $r_1 - r_0 = r \rightarrow (3)$, where $r_0=0$, then $r_2 - r_1 = r \rightarrow (4)$, and $r_2 = 2r$. Also, $r_3 - r_2 = r \Rightarrow r_3 = 3r$.

Hence,

$$E(t_\phi) = \frac{1}{2\sqrt{2}} \left[r + \frac{\pi}{w_i} \right] \left[-r_i e^{-\frac{r_i^2}{2}} + (r_i - r) e^{-\frac{(r_i - r)^2}{2}} + \sqrt{\frac{\pi}{2}} \left(\operatorname{erf} \frac{r_i}{\sqrt{2}} - \operatorname{erf} \frac{(r_i - r)}{\sqrt{2}} \right) \right],$$

$$E(t_\phi) = \frac{1}{2\sqrt{2}} \left[r + \frac{\pi}{w_i} \right] \left[-ir e^{-\frac{(ir)^2}{2}} + r(i - 1) e^{-\frac{(r(i - r))^2}{2}} + \sqrt{\frac{\pi}{2}} \left(\operatorname{erf} \frac{ir}{\sqrt{2}} - \operatorname{erf} \frac{r(i - 1)}{\sqrt{2}} \right) \right]. \tag{5}$$

By considering the values of i , r , and w_i in [Tab. 1](#) using the Mathematica program, and substituting in [Eq. \(5\)](#), we can obtain the different values of $E(t_\phi)$.

Table 1: Expected values of the time taken to detect the lost ship

i	r	w_i	$E(t_\phi)$
0	0	10	0
0	1	10	0.115721
0	2	10	0.978161
0	3	10	1.46451
0	4	10	1.91147
1	0	10	0
1	1	10	0.115721
1	2	10	0.757269
1	3	10	1.42546
1	4	10	1.90942
2	0	10	0
2	1	10	0.314292

(Continued)

Table 1 (continued).			
i	r	w_i	$E(t_\phi)$
2	2	10	1.68174
2	3	10	0.0430129
2	4	10	0.000121085
3	0	10	0
3	1	10	0.135183
3	2	10	0.442948
3	3	10	7.03006
3	4	10	0.00409326
4	0	10	0
4	1	10	0.0163944
4	2	10	0.0016467
4	3	10	0.117145
4	4	10	18.3028
0	0	20	0
0	1	20	0.101895
0	2	20	0.911795
0	3	20	1.35793
0	4	20	1.83993

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