

The *Arcsine-X* Family of Distributions with Applications to Financial Sciences

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Abstract: The heavy-tailed distributions are very useful and play a major role in actuary and financial management problems. Actuaries are often searching for such distributions to provide the best fit to financial and economic data sets. In the current study, a prominent method to generate new distributions useful for modeling heavy-tailed data is considered. The proposed family is introduced using trigonometric function and can be named as the *Arcsine-X* family of distributions. For the purposes of the demonstration, a specific sub-model of the proposed family, called the *Arcsine-Weibull* distribution is considered. The maximum likelihood estimation method is adopted for estimating the parameters of the *Arcsine-X* distributions. The resultant estimators are evaluated in a detailed Monte Carlo simulation study. To illustrate the *Arcsine-Weibull* two insurance data sets are analyzed. Comparison of the *Arcsine-Weibull* model is done with the well-known two parameters and four parameters competitors. The competitive models including the Weibull, Lomax, Burr-XII and beta Weibull models. Different goodness of fit measures are taken into account to determine the usefulness of the *Arcsine-Weibull* and other considered models. Data analysis shows that the *Arcsine-Weibull* distribution works much better than competing models in financial data analysis.

Keywords: Weibull distribution; *arcsine* function; financial sciences; maximum likelihood estimation; Monte Carlo simulation; comparative study

1 Introduction

One of the most important functions in actuarial and financial science is to have an accurate prediction of financial losses that occur with a significant heavy monetary value. Underestimation in such losses leads to serious operational risks such as underestimating premium, collapse bankruptcy, etc. Actuaries often seek to propose flexible heavy-tailed distributions to avoid such situations and to provide accurate predictions of financial science losses.

Actuarial and financial data sets are generally positive [1], right-handed, unimodal-shaped, and heavy tails [2]. Right skewness can be better modeled by the skewed distributions [3]. Therefore, new different unimodal positively skewed parametric distributions have been introduced to model such data; see [4–8]. The new distributions have been introduced in many different ways such as (i) transformation of



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variables; see [9,10], (ii) composition of two or more distributions; see [11,12], (iii) mixture distributions; see [13,14], and (iv) compounding of distributions; see [15,16]. For detailed information on proposing new methods, we refer to [17].

The introduction of new distributions using the methods mentioned above leads to a heavy-tailed distribution. Unfortunately, however, the above methods have certain limitations. For example, (i) the number of parameters is increased, causing difficulty in estimating the parameters of the model, (ii) the introduction of additional parameter(s) to the model brings more flexibility, however, usually such practices causing re-parameterization problems, (iii) using the method of composition and mixture distributions, the form of probability density function (pdf) becomes complicated, leading to computational problems, and (iv) some new proposed distributions using above methods do not have closed-form of cumulative distribution function (cdf) which makes the manual computation of statistical properties more difficult. The introduction of additional parameters to the existing models brings more flexibility which is a desirable feature. Unfortunately, it makes the inferences more complicated.

In the light of the above discussion, researchers are encouraged to introduce new distribution or distribution family with a simple pdf expression and a closed form of cdf. Therefore, in this article, an effort has been made to propose a new distributions family to avoid the issues discussed above and to provide the best fit to financial data sets. The proposed family is introduced using trigonometric function and can be called arcsine- X (with the short form of 'AS- X '). The proposed family can be found as a special model of the arcsine exponentiated- X family; see [18]. The AS- X family has a closed cdf form and a simple pdf expression. A random variable X is said to have the AS- X family, if its cdf is

$$G(x; \xi) = \frac{2}{\pi} \arcsine(F(x; \xi)), x \in R, \quad (1)$$

where $F(x; \xi)$ is cdf of the baseline random variable may depend on the parameter vector $\xi \in R$. The pdf corresponding to Eq. (1) is given by

$$g(x; \xi) = \frac{2}{\pi} \frac{f(x; \xi)}{\sqrt{1 - F(x; \xi)^2}}, x \in R. \quad (2)$$

The sf (survival function) and hrf (hazard rate function) of the AS- X distributions are

$$S(x; \xi) = 1 - \frac{2}{\pi} \arcsine(F(x; \xi)), x \in R,$$

and

$$h(x; \xi) = \frac{2f(x; \xi)}{\pi - 2\arcsine(F(x; \xi)) \left(\sqrt{1 - F(x; \xi)^2} \right)}, x \in R.$$

This paper is arranged as: A special case of the AS- X family is presented in Section 2. Mathematical properties of the AS- X distributions are presented in Section 3. The parameters of *Arcsine-Weibull* distribution are estimated in Section 4. In the same section, the Monte Carlo simulation study is provided. Real-life applications are analyzed in Section 5. Here, the proposed model is compared to the competitive model under various discriminatory measures and goodness of fit criteria. The last section concludes the article.

2 The Arcsine-Weibull Distribution

In this section, we introduce the AS-Weibull (AS-W) distribution. Consider the Weibull random variable say X , with cdf $F(x; \xi) = 1 - e^{-\gamma x^\alpha}, x \geq 0, \alpha, \gamma > 0$, and pdf $f(x; \xi) = \alpha \gamma x^{\alpha-1} e^{-\gamma x^\alpha}$, where $\xi = (\alpha, \gamma)$. Then, the cdf of the AS-W random variable is

$$G(x) = \frac{2}{\pi} \arcsine(1 - e^{-\gamma x^\alpha}), x \geq 0, \alpha, \gamma > 0. \tag{3}$$

The density function and corresponding to Eq. (3) is given by

$$g(x) = \frac{2}{\pi} \frac{\alpha \gamma x^{\alpha-1} e^{-\gamma x^\alpha}}{\sqrt{1 - (1 - e^{-\gamma x^\alpha})^2}}, x > 0. \tag{4}$$

The density plots of the AS-W distribution are sketched in Fig. 1.

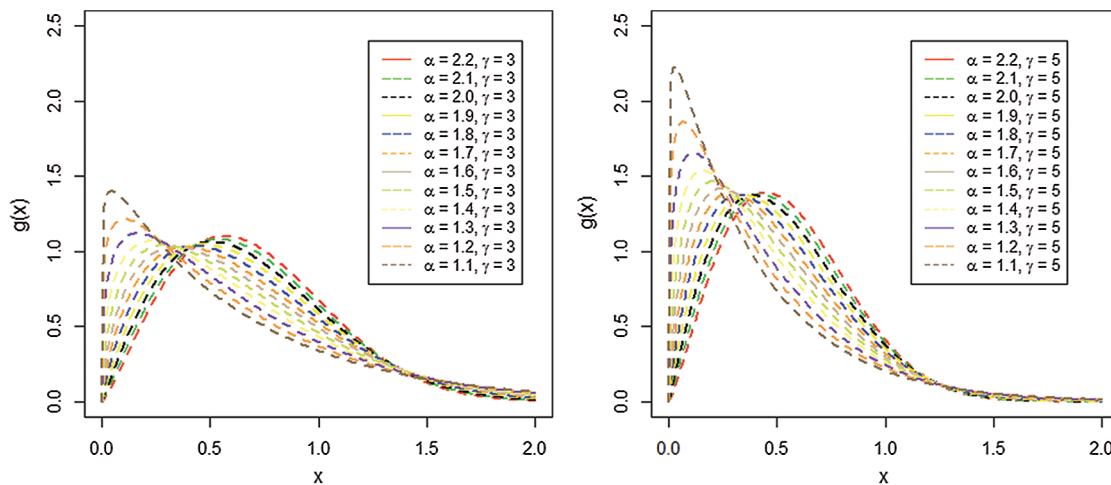


Figure 1: Plots for the pdf of the AS-W distribution for selected values of the model parameters

3 Mathematical Properties

In this section, some statistical properties such as the quantile function and moments are derived. The quantile function is very useful in generating random number for the simulation study, where the first four moments offers the basic characteristics of the AS-X distributions.

3.1 Quantile Function

Let X be the AS-X distributed random variable with pdf given by Eq. (2), the quantile function of X , say $Q(u)$, is defined by

$$x = Q(u) = G^{-1}(u) = F^{-1}\left(\sin\left(\frac{\pi}{2}u\right)\right), \tag{5}$$

where, $u \in (0, 1)$. The expression provided in Eq. (5) can be to generate random number form the AS-X distributions.

3.2 Moments

Moments play an essential role in statistical and financial analysis, especially in the applications. It helps to capture the essential features and characteristics of the distribution. The r^{th} moment of the AS- X family of distributions is derived as

$$\mu_r' = \int_{-\infty}^{\infty} x^r g(x) dx. \quad (6)$$

Using Eq. (2) in Eq. (6), we get

$$\mu_r' = \frac{2}{\pi} \int_{-\infty}^{\infty} x^r \frac{f(x; \xi)}{\sqrt{1 - F(x; \xi)^2}} dx. \quad (7)$$

Using the binomial expansion, we have

$$\frac{1}{\sqrt{1 - x^2}} = \sum_{n=0}^{\infty} \frac{1 \times 3 \times 5 \times \dots \times (2n - 1)}{n! 2^n} x^{2n}. \quad (8)$$

Let $x = F(x; \xi)$, in Eq. (8), we get

$$\frac{1}{\sqrt{1 - F(x; \xi)^2}} = \sum_{n=0}^{\infty} \frac{1 \times 3 \times 5 \times \dots \times (2n - 1)}{n! 2^n} F(x; \xi)^{2n}. \quad (9)$$

Using Eq. (9) in Eq. (7), we get

$$\mu_r' = \frac{1}{\pi} \sum_{n=0}^{\infty} \frac{1 \times 3 \times 5 \times \dots \times (2n - 1)}{n! 2^{n-1}} \eta_{r, 2n}, \quad (10)$$

Where $\eta_{r, 2n} = \int_{-\infty}^{\infty} x^r f(x; \xi) F(x; \xi)^{2n} dx$.

Furthermore, a general expression for moment generating function (mgf) of the AS- X family is given by

$$M_X(t) = \frac{1}{\pi} \sum_{r, n=0}^{\infty} \frac{1 \times 3 \times 5 \times \dots \times (2n - 1)}{r! n! 2^{n-1}} t^r \eta_{r, 2n}.$$

4 Maximum Likelihood Estimation and Monte Carlo Simulation

This section provides the maximum likelihood estimation and a brief Monte Carlo simulation study to test the performance of the estimators

4.1 Maximum Likelihood Estimation

This subsection deals with the MLEs (maximum likelihood estimators) of the parameters of AS- X distributions. Let X_1, X_2, \dots, X_n be a simple random sample from AS- X family with observed values x_1, x_2, \dots, x_n . The log-likelihood function of this sample is

$$\ell = n \log\left(\frac{2}{\pi}\right) + \sum_{i=1}^n \log f(x_i; \xi) - \frac{1}{2} \log\left(1 - F(x_i; \xi)^2\right). \tag{11}$$

Obtaining the partial derivative of Eq. (11), we get

$$\frac{\partial \ell}{\partial \xi} = \sum_{i=1}^n \frac{\partial f(x_i; \xi) / \partial \xi}{f(x_i; \xi)} + \sum_{i=1}^n \frac{F(x_i; \xi)}{[1 - F(x_i; \xi)^2]} \partial F(x_i; \xi) / \partial \xi. \tag{12}$$

Setting $\frac{\partial \ell}{\partial \xi}$ equal to zero and solving numerically, yields the maximum likelihood estimator of ξ . Using Eq. (12), we can obtain the maximum likelihood estimator(s) of the model parameter(s) for any sub-case of the proposed family with cdf and pdf given by $F(x_i; \xi)$ and $f(x_i; \xi)$, respectively.

4.2 Monte Carlo Simulation Study

In this sub-section, we evaluate the MLEs of the AS-W distribution. A numerical evaluation is done to examine the performance of MLEs of the AS-W model via the `optim()` R-function with the argument `method = "L-BFGS-B"`. The evaluation was performed based on the biases and the empirical mean square errors (MSEs) using the R package. The numerical steps are as follows:

- i) A random sample X_1, X_2, \dots, X_n of sizes; $n = 25, 50 \dots 500$ are considered; these random samples are generated from the AS-W distribution by using the inversion method.
- ii) The MLEs of AS-W model are obtained.
- iii) 500 repetitions are made to calculate the biases and the MSE of these estimators.
- iv) Formulas used for calculating bias and MSE are given by
- v) Step (iv) is also repeated for the other parameter γ .
- vi) $Bias(\hat{\alpha}) = \frac{1}{500} \sum_{i=1}^{500} (\hat{\alpha} - \alpha)$ and $MSE(\hat{\alpha}) = \frac{1}{500} \sum_{i=1}^{500} (\hat{\alpha} - \alpha)^2$, respectively.

The simulation results are provided in [Tabs. 1-4](#). From the simulation study ([Tabs. 1-4](#)), we can detect that the estimates are relatively stable and are close to the actual value of the parameters as the sample sizes increases. We can also observe that as the sample size increase the Biases and MSEs decay to zero.

Table 1: Simulation results of the AS-W model.

Set 1: $\alpha = 0.7$ and $\gamma = 1$				
n	Parameters	MLEs	MSEs	Biases
25	$\hat{\alpha}$	0.7416	0.0160	0.0416
	$\hat{\gamma}$	1.0817	0.1043	0.0817
50	$\hat{\alpha}$	0.7253	0.0071	0.0253
	$\hat{\gamma}$	1.0306	0.0302	0.0306
75	$\hat{\alpha}$	0.7202	0.0060	0.0202
	$\hat{\gamma}$	1.0139	0.0191	0.0139
100	$\hat{\alpha}$	0.7119	0.0052	0.0119
	$\hat{\gamma}$	1.0116	0.0144	0.0116

(Continued)

Table 1 (continued).

Set 1: $\alpha = 0.7$ and $\gamma = 1$				
150	$\hat{\alpha}$	0.7108	0.0041	0.0059
	$\hat{\gamma}$	1.0082	0.0108	0.0043
200	$\hat{\alpha}$	0.7062	0.0031	0.0062
	$\hat{\gamma}$	1.0027	0.0059	0.0027
250	$\hat{\alpha}$	0.7057	0.0023	0.0057
	$\hat{\gamma}$	1.0090	0.0040	0.0090
300	$\hat{\alpha}$	0.7060	0.0018	0.0060
	$\hat{\gamma}$	1.0029	0.0030	0.0029
350	$\hat{\alpha}$	0.7034	0.0015	0.0034
	$\hat{\gamma}$	1.0049	0.0025	0.0049
400	$\hat{\alpha}$	0.7043	0.0010	0.0043
	$\hat{\gamma}$	1.0099	0.0021	0.0099
450	$\hat{\alpha}$	0.7037	0.0008	0.0037
	$\hat{\gamma}$	1.0029	0.0016	0.0029
500	$\hat{\alpha}$	0.7030	0.0002	0.0030
	$\hat{\gamma}$	1.0026	0.0014	0.0026

Table 2: Simulation results of the AS-W model.

Set 1: $\alpha = 1.8$ and $\gamma = 1$				
n	Parameters	MLEs	MSEs	Biases
25	$\hat{\alpha}$	1.9061	0.1220	0.1061
	$\hat{\gamma}$	1.0677	0.0914	0.0677
50	$\hat{\alpha}$	1.8606	0.0483	0.0606
	$\hat{\gamma}$	1.0284	0.0351	0.0284
75	$\hat{\alpha}$	1.8325	0.0278	0.0325
	$\hat{\gamma}$	1.0285	0.0231	0.0285
100	$\hat{\alpha}$	1.8304	0.0197	0.0304
	$\hat{\gamma}$	1.0273	0.0153	0.0273
150	$\hat{\alpha}$	1.8140	0.0139	0.0140
	$\hat{\gamma}$	1.0180	0.0107	0.0180
200	$\hat{\alpha}$	1.8128	0.0105	0.0128
	$\hat{\gamma}$	1.0125	0.0080	0.0125
250	$\hat{\alpha}$	1.8127	0.0085	0.0127
	$\hat{\gamma}$	1.0100	0.0060	0.0100

Table 2 (continued).

Set 1: $\alpha = 1.8$ and $\gamma = 1$				
300	$\hat{\alpha}$	1.8066	0.0061	0.0066
	$\hat{\gamma}$	1.0021	0.0042	0.0021
350	$\hat{\alpha}$	1.8043	0.0059	0.0043
	$\hat{\gamma}$	1.0027	0.0037	0.0027
400	$\hat{\alpha}$	1.8077	0.0044	0.0077
	$\hat{\gamma}$	1.0086	0.0037	0.0086
450	$\hat{\alpha}$	1.8044	0.0040	0.0043
	$\hat{\gamma}$	1.0063	0.0034	0.0063
500	$\hat{\alpha}$	1.8071	0.0039	0.0071
	$\hat{\gamma}$	1.0067	0.0028	0.0067

Table 3: Simulation results of the AS-W model.

Set 1: $\alpha = 1.2$ and $\gamma = 0.7$				
n	Parameters	MLEs	MSEs	Biases
25	$\hat{\alpha}$	1.2755	0.0475	0.0755
	$\hat{\gamma}$	0.7566	0.0453	0.0566
50	$\hat{\alpha}$	1.2427	0.0206	0.0427
	$\hat{\gamma}$	0.7219	0.0149	0.0219
75	$\hat{\alpha}$	1.2287	0.0129	0.0287
	$\hat{\gamma}$	0.7146	0.0104	0.0146
100	$\hat{\alpha}$	1.2176	0.0094	0.0176
	$\hat{\gamma}$	0.7008	0.0066	0.0008
150	$\hat{\alpha}$	1.2114	0.0063	0.0114
	$\hat{\gamma}$	0.7104	0.0045	0.0104
200	$\hat{\alpha}$	1.2089	0.0040	0.0089
	$\hat{\gamma}$	0.6997	0.0036	0.0002
250	$\hat{\alpha}$	1.2114	0.0037	0.0114
	$\hat{\gamma}$	0.7036	0.0027	0.0036
300	$\hat{\alpha}$	1.2047	0.0028	0.0047
	$\hat{\gamma}$	0.7037	0.0024	0.0037
350	$\hat{\alpha}$	1.2062	0.0023	0.0062
	$\hat{\gamma}$	0.7011	0.0019	0.0011

(Continued)

Table 3 (continued).

Set 1: $\alpha = 1.2$ and $\gamma = 0.7$				
400	$\hat{\alpha}$	1.2015	0.0020	0.0015
	$\hat{\gamma}$	0.7017	0.0016	0.0017
450	$\hat{\alpha}$	1.2038	0.0018	0.0038
	$\hat{\gamma}$	0.7025	0.0013	0.0025
500	$\hat{\alpha}$	1.2028	0.0017	0.0018
	$\hat{\gamma}$	0.7007	0.0013	0.0007

Table 4: Simulation results of the AS-W model.

Set 1: $\alpha = 0.8$ and $\gamma = 1.2$				
n	Parameters	MLEs	MSEs	Biases
25	$\hat{\alpha}$	0.8416	0.0195	0.0416
	$\hat{\gamma}$	1.3111	0.1676	0.1111
50	$\hat{\alpha}$	0.8287	0.0085	0.0287
	$\hat{\gamma}$	1.2601	0.0584	0.0601
75	$\hat{\alpha}$	0.8150	0.0053	0.0150
	$\hat{\gamma}$	1.2328	0.0327	0.0328
100	$\hat{\alpha}$	0.8152	0.0042	0.0152
	$\hat{\gamma}$	1.2268	0.0245	0.0268
150	$\hat{\alpha}$	0.8059	0.0022	0.0059
	$\hat{\gamma}$	1.2174	0.0129	0.0174
200	$\hat{\alpha}$	0.8067	0.0016	0.0067
	$\hat{\gamma}$	1.2137	0.0102	0.0137
250	$\hat{\alpha}$	0.8031	0.0011	0.0031
	$\hat{\gamma}$	1.2051	0.0075	0.0051
300	$\hat{\alpha}$	0.8051	0.0010	0.0051
	$\hat{\gamma}$	1.2091	0.0072	0.0091
350	$\hat{\alpha}$	0.8064	0.0009	0.0064
	$\hat{\gamma}$	1.2085	0.0047	0.0085
400	$\hat{\alpha}$	0.8038	0.0008	0.0038
	$\hat{\gamma}$	1.2063	0.0041	0.0063
450	$\hat{\alpha}$	0.8032	0.0006	0.0032
	$\hat{\gamma}$	1.2063	0.0039	0.0063
500	$\hat{\alpha}$	0.8016	0.0005	0.0016
	$\hat{\gamma}$	1.2055	0.0031	0.0055

5 Applications to Financial Sciences

In this section, we consider two real life applications from financial sciences. For these data sets, the AS-W distribution is compared with the other well-known distributions such as two-parameter Weibull, two-parameter Lomax distribution, three-parameter Weibull-Loss (W-Loss) model, four-parameter Burr-XII (B-XII), and a four-parameter beta Weibull (BW) distribution. The cdfs of these models are:

- Weibull distribution

$$G(x; \alpha, \gamma) = 1 - e^{-\gamma x^\alpha}, \quad x \geq 0, \alpha, \gamma > 0.$$

- Lomax distribution

$$G(x; \alpha, \gamma) = 1 - (1 + \gamma x)^{-\alpha}, \quad x \geq 0, \alpha, \gamma > 0.$$

- B-XII distribution

$$G(x; \alpha, \gamma) = 1 - (1 + x^\gamma)^{-\alpha}, \quad x \geq 0, \alpha, \gamma > 0.$$

- BW distribution

$$G(x; a, b\alpha, \gamma) = I_{(1-e^{-\gamma x^\alpha})}(a, b), \quad x \geq 0, a, b, \alpha, \gamma > 0.$$

- W-Loss distribution

$$G(x; \alpha, \gamma, \sigma) = 1 - \frac{\sigma e^{-\gamma x^\alpha}}{\sigma + \gamma x^\alpha}, \quad x \geq 0, \alpha, \gamma, \sigma > 0.$$

To determine the best fitting capabilities of the applied distributions, we look at certain analytical measures. In this regard, we consider two measures of discrimination such as the Akaike information criterion (AIC) and Bayesian information criterion (BIC). In addition to the measures of discrimination, we also consider other goodness of fit measures such as Anderson Darling (AD) test statistic, Cramer Von-Messes (CM) test statistic and Kolmogorov-Smirnov (KS) test statistics with associating p-values.

A model with smallest analytical measures values is considered a good candidate model among the distributions taken for analyzing data sets. By looking at these analytical tools, we have noticed that the AS-W distribution provides the best fit compare to other distributions because the it has the smallest values of the considered measures.

5.1 The Vehicle Insurance Loss Data

This data set representing vehicle insurance losses taken from [19]. We fitted the AS-W and other distributions to this data set. The parameter values are reported in [Tab. 5](#), and the analytical measures are presented in [Tab. 6](#). From the results obtained in [Tab. 6](#), we can see that for the AS-W distribution, the values of AIC=428.018, BIC=430.956, CM=0.024, AD=0.152, KS=0.103 and p-value is 0.846 against the model (W-Loss) with the second smallest values are AIC=430.856, BIC=433.965, CM=0.031, AD=0.208, KS=0.128 and p-value is 0.665.

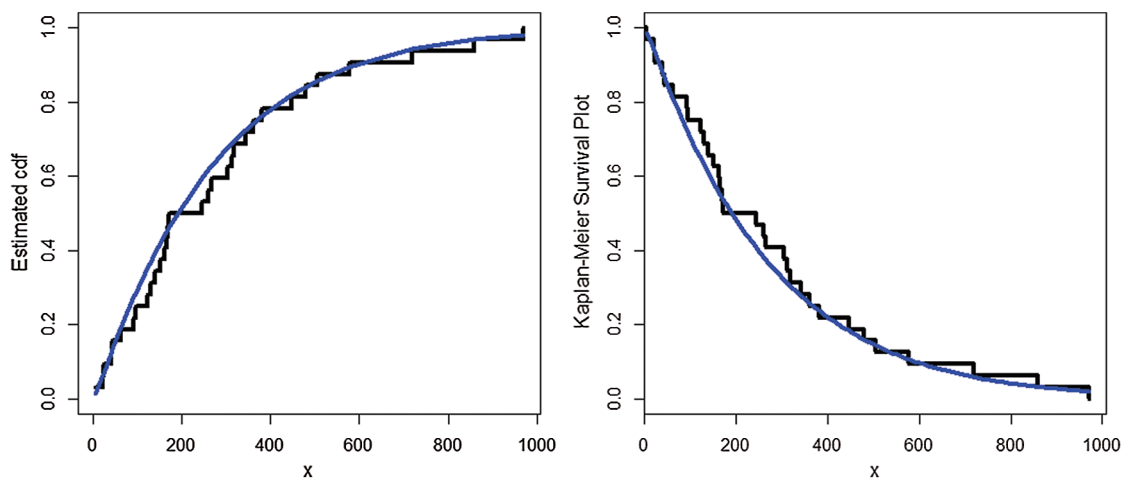
The estimated cdf and Kaplan-Meier survival plots of the AS-W distribution are presented in [Fig. 2](#). The PP (probability-probability) and the QQ (quantile-quantile) plots of the AS-W distribution are provided in [Fig. 3](#).

Table 5: MLEs with standard errors (in parentheses) of the competing models for data 1

Models	$\hat{\alpha}$	$\hat{\gamma}$	\hat{a}	\hat{b}	$\hat{\sigma}$
AS-W	1.124 (0.0707)	0.003 (0.0013)	–	–	
Weibull	1.019 (0.9445)	0.003 (1.6540)	–	–	
Lomax	0.495 (0.4334)	30.008 (9.618)	–	–	
B-XII	0.049 (0.1134)	4.427 (2.2671)	–	–	
BW	0.031 (0.1032)	2.097 (1.9431)	1.230 (0.6309)	1.409 (1.4097)	
W-Loss	1.137 (0.1078)	0.075 (0.0106)			1.185 (0.0385)

Table 6: Analytical measures of the proposed and other competing models for data 1

Models	AIC	BIC	CM	AD	KS	p-value
AS-W	428.018	430.956	0.024	0.152	0.103	0.846
Weibull	432.353	439.256	0.054	0.447	0.185	0.597
Lomax	460.191	463.021	0.083	0.520	0.207	0.108
B-XII	503.477	506.383	0.228	1.362	0.416	0.208
BW	463.873	473.467	0.138	0.643	0.324	0.158
W-Loss	430.856	433.965	0.031	0.208	0.128	0.665

**Figure 2:** Estimated cdf and Kaplan-Meier survival plots of the AS-W distribution for the first data set

5.2 The Earthquake Insurance Data Set

In this sub-section, we consider second data set related to the earthquake insurance available at: <https://earthquake.usgs.gov/data/>. The data has already been studied by [19]. For this data set, we also compared the AS-W distribution to other distributions mentioned above. For the second data set, the MLEs of the competing models are provided in Tab. 7. The discrimination measures and goodness of fit measures are provided in Tab. 8. From the results obtained in Tab. 8, we can see that for the AS-W distribution, the

values of AIC = 27439.37, BIC = 27455.12, CM = 6.837, AD = 0.771, KS = 0.016 and p-value is 0.695 against the model (W-Loss) with the second smallest values are AIC = 27451.37, BIC=27461.12, CM = 7.185, AD = 0.813, KS = 0.019 and p-value is 0.598.

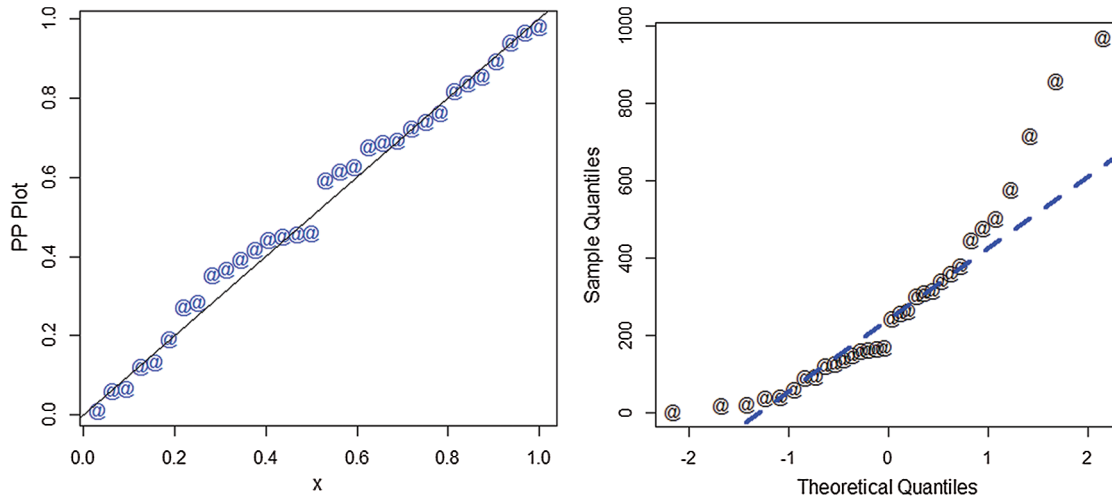


Figure 3: PP and QQ plots of the AS-W distribution for the first data set

Table 7: MLEs with standard errors (in parentheses) of the competing models for data 2

Models	$\hat{\alpha}$	$\hat{\gamma}$	\hat{a}	\hat{b}	$\hat{\sigma}$
AS-W	2.222 (0.0120)	1.268 (0.0111)			
Weibull	2.113 (0.0116)	0.711 (0.0056)			
Lomax	73.69 (5.2704)	75.956 (5.4564)			
B-XII	1.219 (0.0092)	2.967 (0.0185)			
BW	1.717 (0.0371)	1.256 (0.0877)	1.495 (0.0590)	0.807 (0.0679)	
W-Loss	2.046 (0.1069)	1.507 (0.0543)			1.487 (0.9759)

Table 8: Analytical measures of the proposed and other competing models for data 2

Models	AIC	BIC	CM	AD	KS	p-value
AS-W	27439.37	27455.12	6.837	0.771	0.016	0.695
Weibull	27461.75	27475.60	8.495	1.078	0.021	0.519
Lomax	40390.05	40404.19	10.062	5.139	0.235	0.478
B-XII	28662.28	28678.03	9.509	7.502	0.062	0.221
BW	27450.4	27456.88	8.012	1.017	0.019	0.578
W-Loss	27451.37	27461.12	7.185	0.813	0.019	0.598

For this data set, the estimated cdf and Kaplan-Meier survival plots of the AS-W distribution are presented in Fig. 4. The PP and the QQ plots of the AS-W distribution are provided in Fig. 5.

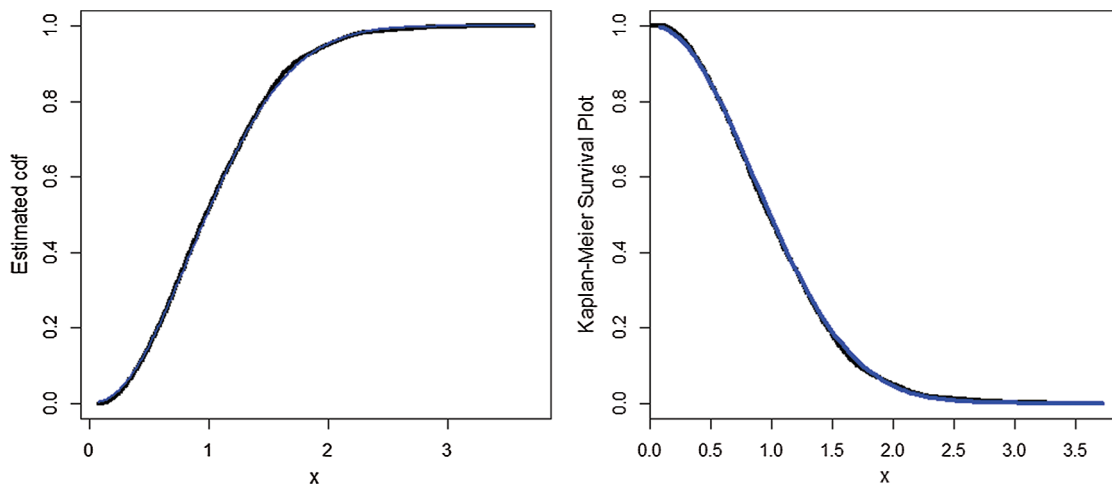


Figure 4: Estimated cdf and Kaplan-Meier survival plots of the AS-W distribution for the second data set

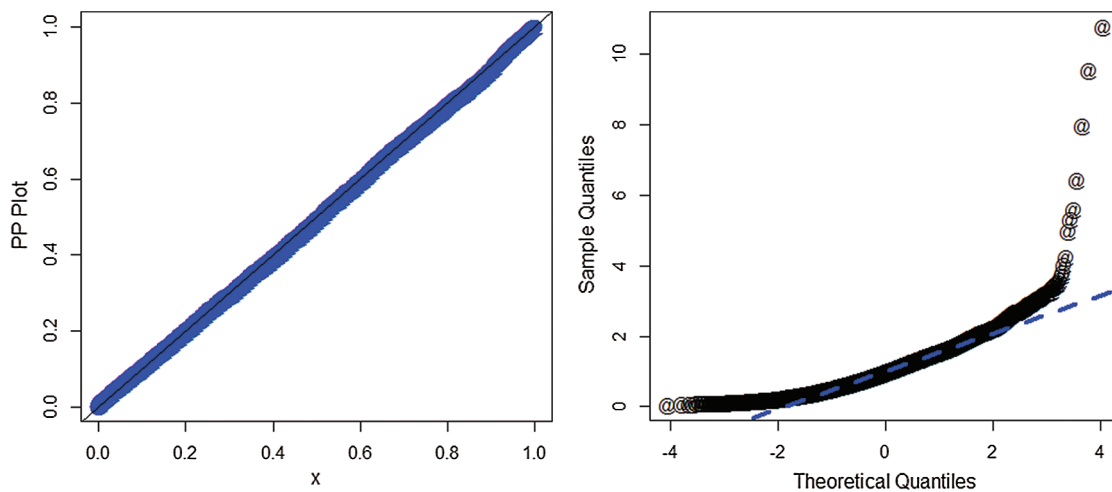


Figure 5: PP and QQ plots of the AS-W distribution for the second data set

6 Concluding Remarks

Statistical theory addresses the nature of uncertainty and provides a rational framework for dealing with financial decision-making problems. Financial data sets are mostly skewed to the right, and the skewed distributions are reasonable candidate models for such type of data. Keeping in view the importance of statistical distributions, an attempt has been made to propose a new distribution family using the trigonometric function. The proposed method is called the AS- X distributions family. A special model of the AS- X family offers best fitting in financial data analysis. The AS-W distribution is applied to the vehicle insurance losses and earth quick insurance data sets. Comparison of the AS-W model was made to some well-known competitors such as Weibull, Lomax, BX-II and beta-Weibull distributions. Comparisons were made based on two discriminatory measures (AIC and BIC) and three goodness of fit measures (CM, AD and KS test statistics with corresponding p-values). Using these measures, it is observed that that the AS-W model could be a suitable model for analyzing heavy-tailed financial data.

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