

# A Markov Model for Subway Composite Energy Prediction

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**Abstract:** Electric vehicles such as trains must match their electric power supply and demand, such as by using a composite energy storage system composed of lithium batteries and supercapacitors. In this paper, a predictive control strategy based on a Markov model is proposed for a composite energy storage system in an urban rail train. The model predicts the state of the train and a dynamic programming algorithm is employed to solve the optimization problem in a forecast time domain. Real-time online control of power allocation in the composite energy storage system can be achieved. Using standard train operating conditions for simulation, we found that the proposed control strategy achieves a suitable match between power supply and demand when the train is running. Compared with traditional predictive control systems, energy efficiency 10.5% higher. This system provides good stability and robustness, satisfactory speed tracking performance and control comfort, and significant suppression of disturbances, making it feasible for practical applications.

**Keywords:** Markov model; predictive control; composite energy storage; urban rail train

## 1 Introduction

Urbanization reflects the level of development of a civilized society. Social and economic development can lead to the expansion of urban scale which, in turn, will result in gradual increases in the urban population, the scale of population movement and the frequency of population mobility [1,2]. The infrastructure of expanding cities, especially rail transportation facilities, often fails to meet the requirements of urban development. For instance, urban traffic has become increasingly congested, which seriously hinders the long-term development of cities and socioeconomic [3,4]. Since urban rail transit has the advantages of high capacity and speed and high transportation and energy efficiency, the Chinese government vigorously promotes its construction. Urban rail transit stations are characterized by their short spacing along railway lines, which requires trains to start and stop frequently. At present, urban trains in China mainly use mechanical braking or regenerative-energy braking. The heat generated by mechanical or regenerative braking can cause substantial energy consumption and waste. Also,



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mechanical braking wears brake shoes, necessitating frequent replacement and maintenance. In subway tunnels, the heat generated by braking increases the air temperature, which must be mitigated using air conditioning and ventilation facilities, thus increasing the operating costs of subways [5,6].

Generally, energy storage systems can be added to urban rail trains to solve these energy issues. A vehicle energy storage system can store regenerative braking energy. Then, when the train is accelerating, the energy can be released to power the train [7,8]. Consequently, this can reduce braking energy losses and achieve energy savings and environmental protection. It can also decrease fluctuations in the train's traction power and achieve the purpose of "peak shaving and valley filling". Thus, it can lower the train's maximum demand on the power supply system. After the energy storage system is installed on the train, the train can completely charge the on-board energy storage system in a catenary area, which then powers the train so that it can smoothly pass through a contactless area. Therefore, the installation of train energy storage systems is of great significance to the development of rail transportation.

Aiming to resolve the issues caused by the separate use of batteries and supercapacitors, this paper adopts a composite energy storage system comprised of these two types of power supplies. This not only improves efficiency but also extends range. Also, the efficiency of a composite energy storage system mainly depends on its topology and its power distribution under operating conditions [9]. Under the coupling situations, it is necessary to select a reasonable energy control strategy to distribute system power and demanded power.

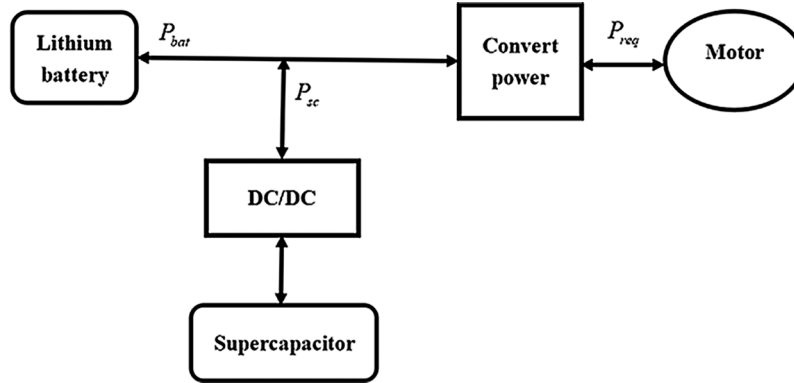
While there are many studies on the application of composite energy storage systems to electric vehicles, there are few on urban rail trains. There has been extensive research on the energy distribution strategies of composite energy electric vehicles [10,11], especially in the areas of rule-based and optimization-based control strategies. Although rule-based control strategies are relatively simple and easy to implement, they cannot achieve optimal control. However, global optimization based on dynamic programming is accomplished based on static analysis to attain optimal power allocation under specific cycle conditions [12–14]. It can usually be used as a reference for evaluating other energy allocation strategies, but the calculations required are complex and difficult to apply to vehicles. Urban rail trains generally use supercapacitors for energy control and utilization; i.e., the time-phase control strategies used in urban rail ground hybrid energy storage devices are based on the train's operating state. Also, the energy management strategy is based on equivalent hydrogen consumption, and the indirect current control strategy is from the vehicle supercapacitor energy storage system. In principle, composite energy storage is not implemented for energy storage and predictive control methods are not involved in energy control strategy design [15–19].

Real-time optimization based on model predictive control does not require prior knowledge of the future driving characteristics of the vehicle and is not limited by specific cycle conditions. The computation task is small and easy to implement on the basis of a guaranteed sub-optimal power distribution [20]. This paper investigates a composite energy storage system for an urban rail train consisting of a lithium battery-supercapacitor. The main objective is to achieve the lowest energy consumption during train operation. The composite energy control is based on Markov model predictive control (MPC) with consideration of the advantages of lithium batteries and supercapacitors in further optimizing this system.

## 2 Establishment of a Composite Energy Storage Structure for Urban Rail Trains

As shown in Fig. 1, based on the operating features of urban rail trains and the characteristics of lithium battery-supercapacitors, a composite energy storage system was constructed [21–23]. The supercapacitor is connected in series with a bidirectional DC/DC converter and then coupled in parallel with a lithium battery. Finally, a composite power system is formed by a parallel connection to a DC bus to drive a motor. While the urban rail train is accelerating or at a constant speed, the lithium batteries can provide stable output power,

while the high-specific-power quality of the supercapacitor provides transient power. During deceleration and braking of the train, the supercapacitor is used to recover energy. Thus, its instantaneous high-current charging characteristics can be used to recover regenerative braking energy and protect the lithium battery from instantaneous high-current shocks.



**Figure 1:** Structural diagram of an urban rail transit vehicle-mounted composite energy storage system

According to the architecture of the composite energy storage system shown in Fig. 1, the working models can be divided into four categories: (1) When the train is running at low speed or cruising at a uniform speed, the lithium battery runs in a separate driving mode. (2) When the train starts, accelerates and climbs, its supercapacitors alone will provide power. (3) When the train is accelerating or climbing for a long time and the charge-state of the supercapacitor decreases to its lower limit and cannot maintain the power demand, a co-driving mode (lithium battery and supercapacitor) is adopted. (4) When the train is in braking deceleration or going downhill it adopts braking regeneration mode, in which the supercapacitor can quickly recover energy [24–26]. Comprehensive analysis of the power features (lithium batteries, supercapacitors, DC/DC converter, motor controllers) and the train's operating characteristics allows optimal power distribution to be attained, as shown in Eq. (1).

$$\begin{cases} P_{req} = (P_{bat} + \eta_{dc}P_{sc})/\eta_c \\ P_{bat} = k_1P_{req} \\ P_{sc} = k_2P_{req} \\ k_1 + k_2\eta_{dc} = \eta_c \\ P_{req} \leq P_{bat} + P_{sc} \end{cases} \quad (1)$$

where  $P_{req}$  is the demand power of the whole vehicle;  $P_{bat}$  is the output power of the lithium battery;  $P_{sc}$  is the output power of the supercapacitor group;  $k_1$  represents the power distribution coefficient of the lithium battery;  $k_2$  represents the supercapacitor group's power distribution coefficient;  $\eta_{dc}$  is the DC-DC conversion efficiency; and  $\eta_c$  gives the motor controller conversion effectiveness.

### 3 Construction of an Energy Simulation Model

#### 3.1 Building of an Urban Rail Train Power Model

To build a model, the whole urban rail train can be treated as a particle point and longitudinal dynamics can be applied to attain the demand dynamics model:

$$P_{req} = Fv = \begin{cases} P_t = Ma(t)v(t) = M\dot{v}(t)v(t) \\ P_r = [Mg(w_0(t) + g[s(t)])]v(t) \end{cases} \quad (2)$$

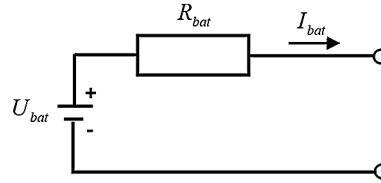
where  $F$  is the whole-vehicle traction force,  $v(t)$  is the train's running speed,  $M$  represents the train's overall quality,  $a(t)$  is equal to  $\dot{v}(t)$ , which is the real-time acceleration of the running train,  $P_t$  is the train's running power;  $P_r$  is the train's resistance power;  $w_0(t)$  is the basic resistance of the train; and  $g[s(t)]$  is the additional resistance of the train.

The energy consumed during train running is:

$$E = \int_{t_1}^{t_2} P_{req} dt \quad (3)$$

### 3.2 Building of a Lithium Battery Model

During the charging and discharging processes, complex chemical reactions are generated inside the lithium battery, which can cause it to exhibit a high degree of nonlinearity and strong coupling, thus making the accurate modelling and control of lithium batteries challenging [27–29]. In this work, a simplified Rint model is used, which can mimic the internal resistive information. The model equates the Li-ion battery model circuit with an ideal voltage source and a series of resistors, as shown in Fig. 2.



**Figure 2:** Equivalent circuit of lithium battery model

From the KVL law of Kirchhoff, the circuit can be analyzed to acquire the load power of the lithium battery during operation:

$$P_{bat} = (U_{bat} - I_{bat}R_{bat})I_{bat} \quad (4)$$

where  $P_{bat}$  is the lithium battery's power (positive values indicate the discharge state, negative values represent the charge state),  $U_{bat}$  is the battery open-circuit voltage,  $I_{bat}$  is the battery current, and  $R_{bat}$  is the battery equivalent internal resistance. The current flowing through the lithium battery can be calculated by Eq. (4), and can be expressed as:

$$I_{bat} = \frac{U_{bat} - \sqrt{U_{bat}^2 - 4P_{bat}R_{bat}}}{2R_{bat}} \quad (5)$$

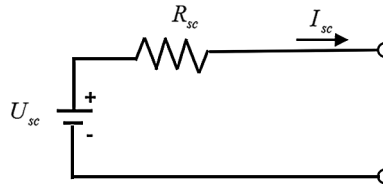
So, the state of charge (SOC) of the lithium battery can be attained as:

$$SOC_{bat} = SOC_{bat0} - \frac{1}{C_{bat}} \int I_{bat} dt \quad (6)$$

where  $SOC_{bat0}$  is the initial state of charge of the lithium battery, and  $C_{bat}$  is the ampere capacity of the battery (A·h).

### 3.3 Establishment of a Supercapacitor Group Model

The working principles of supercapacitors and lithium batteries are different; e.g., supercapacitors do not involve complex chemical reactions during the operating process. Currently, supercapacitor models are generally categorized into classical models, trapezoidal models, three-branch models, and impedance-based models according to their electrical characteristics. In this paper, the supercapacitor group model applies the classic RC circuit model, as shown in Fig. 3.



**Figure 3:** Supercapacitor model equivalent circuit

From the Kirchhoff principle [30–32], the load power of the supercapacitor in the composite energy storage system can be expressed as:

$$P_{sc} = (U_{sc} - I_{sc}R_{sc})I_{sc} \quad (7)$$

where  $P_{sc}$  is the supercapacitor's power (positive values indicate the discharge state and negative values indicate charge),  $U_{sc}$  represents the supercapacitor's open-circuit voltage,  $I_{sc}$  is the supercapacitor current, and  $R_{sc}$  is the supercapacitor's equivalent internal resistance.

With Eq. (5), the supercapacitor current can be calculated as:

$$I_{sc} = \frac{U_{sc} - \sqrt{U_{sc}^2 - 4P_{sc}R_{sc}}}{2R_{sc}} \quad (8)$$

Thus, the SOC of the supercapacitor can be obtained as:

$$SOC_{sc} = SOC_{sc0} - \frac{1}{C_{sc}U_{sc}} \int I_{sc} dt \quad (9)$$

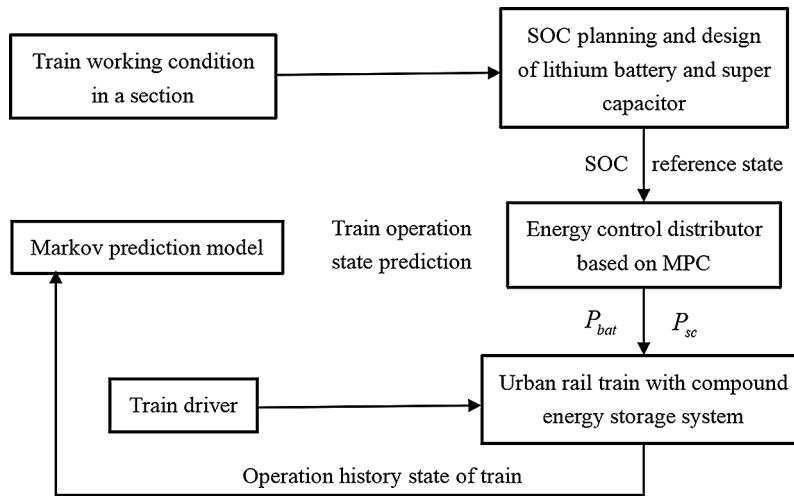
where  $SOC_{sc0}$  is the initial state of charge of the supercapacitor and  $C_{sc}$  is its capacity (F).

## 4 Energy-Predictive Control Strategy Based on a Markov Model

In the MPC algorithm, a model describing the dynamic performance of an object is required, whose job is to forecast the future dynamics of the system. For instance, the output at time  $k + 1$  can be predicted based on the state of system  $k$  and the control input at time  $k$ . The input at time  $k$  can be employed to regulate the output of the system at time  $k + 1$ , making it as close as possible to the expected value at time  $k + 1$ . Using MPC for the composite energy predictive control, it is compulsory to attain a local optimal solution from the optimization cost function in the control time-domain of the system, and then perform roll-forward optimization, thus enhancing the global control performance of the entire control system. However, the entire vehicle control process does not consider the global feature information of the disturbance quantity, which causes certain limitations in the predictive energy control strategy.

Based on the above reasons, it can be concluded that the state of charge trajectories of the battery and supercapacitor can directly affect the final control effect of the predicted-energy control system [33–35]. Therefore, the charging states of the lithium battery and supercapacitor can be elected as the state

variables of the system. Meanwhile, dynamic programming of the charging-state reference curve should be established, and an MPC energy-control distributor needs to be designed based on the Markov prediction model. Next, a control strategy for the composite energy storage system is created, which can implement train operating state monitoring and predictive control. Meanwhile, the traffic information is merged into the energy control system to establish an energy prediction control strategy with “traffic awareness”, as shown in Fig. 4.



**Figure 4:** Traffic-aware energy predictive control strategy

#### 4.1 Establishment of a Reference Target

To achieve optimal control of the energy in the composite energy storage system, it is necessary to optimize the power distribution between the lithium battery and the supercapacitor. In a specified operating interval under the conditions of known train speed, acceleration and running resistance, the total power required for train operation can be solved with Eqs. (1)–(3), while the operating energy consumed in the interval can be computed. To better optimize and control the energy consumption during train operation, this paper determines the energy consumption ratio (ERR), as shown in Eq. (10). Using the dynamic programming algorithm to attain global optimal power allocation control of the composite energy storage system, the desired reference trajectory of the lithium battery and supercapacitor SOC can be acquired [36].

$$ECR = \frac{1.1 \times 10^7 E}{L} \quad (10)$$

where  $E$  is the energy consumption of the train during travel, including those of both the lithium battery and supercapacitor, and  $L$  is the travel distance.

Based on Eqs. (1) and (2), the full demand power of the train can be calculated. Also, if the power demand of the train at any moment is established, it is essential to solve the power of the lithium battery or supercapacitor at any time. In addition, the power distribution factor, all power, and the conversion efficiency of each controller should be determined. Accordingly, when the power factor is assigned and any value of the required power of the system is attained, another demanded power can be found. Here, we investigate the power distribution factor and the required power of the supercapacitor, which are applied as control variables to develop a predictive control design based on a Markov model.

$$u = \{P_{bat}(t), k_1, k_2\} \quad (11)$$

In the process of designing the model predictive controller, there is a certain functional relationship between the required power and the charging states of the lithium battery and supercapacitor, and there are mutual constraints within a certain range. As a result, the charging state of the lithium battery  $SOC_{bat}$  and supercapacitor  $SOC_{sc}$  are chosen as state variables for global optimization of the system, giving:

$$x = \{SOC_{bat}(t), SOC_{sc}(t)\} \quad (12)$$

From our analysis, the power solution and predictive control of the composite energy storage system can be approximated as a nonlinear and time-discrete system, with the following formulas obtained:

$$\begin{cases} x(t+1) = f(x(t), u(t)) \\ x(0) = x_0 \end{cases} \quad (13)$$

where  $x(t)$  is the state of the system at time  $t$ ;  $x(t+1)$  is the state of the system at time  $t+1$ ; and  $f$  is the system state transfer function from the lithium battery and supercapacitor model.

In this study, it is assumed that the global optimization objective function of urban rail trains running in a certain section is:

$$J = \int_0^t ECR[x(t), u(t), t] dt \quad (14)$$

Specifically, during the operation of urban rail trains, the performance of the composite energy storage system is affected by the performance of the lithium batteries and supercapacitors. Under certain constraints, it is necessary to ensure that the power required by the vehicle reaches the ideal state in order to ensure stable, safe and energy-efficient train operation. The constraints can be summarized in Eq. (15):

$$\begin{cases} SOC_{bat,min} \leq SOC_{bat}(t) \leq SOC_{bat,max} \\ SOC_{sc,min} \leq SOC_{sc}(t) \leq SOC_{sc,max} \\ I_{bat,min} \leq I_{bat}(t) \leq I_{bat,max} \\ I_{sc,min} \leq I_{sc}(t) \leq I_{sc,max} \\ P_{bat,min} \leq P_{bat}(t) \leq P_{bat,max} \\ P_{sc,min} \leq P_{sc}(t) \leq P_{sc,max} \end{cases} \quad (15)$$

In summary, the energy control problem related to the dynamic planning of a composite energy storage system for urban rail transit can be investigated through the following steps:

(1) The system can be divided into several phases and the state variables can be discretized within the allowable range of variation.

(2) The required power and power allocation factor can be computed in the system interval to determine the objective function.

(3) In the process of going from the initial state  $x(0)$  to the final state  $x(t)$ , the following can be solved in reverse: the optimal control amount  $u(t)$  for each phase, the dispersion points of each state variable, and the minimum cost function to the final state.

(4) Based on the initial value of the state variable, the optimal control sequence can be sought in the forward direction for the entire-cycle condition.

#### 4.2 Building of a Markov-Based Predictive Model

When the train runs at certain intervals, it is crucial to construct a predictive model for each stage of the travel process using information such as the current vehicle speed and acceleration to predict the speed and acceleration within a finite period. This prediction can be applied to forecast the operational state of the train in the time domain and calculate the power demand of the train for use in the later energy optimization control problem [37–39]. The control effect is strongly related to the control accuracy of operating speed prediction, while the train running state is also influenced by the external environment and the driver's state, which are unknown. Therefore, it can be regarded as a Markov process, and it can be considered that the future running state of the train is independent of historical data and only depends on the current operating state.

The train speed and acceleration state are adopted in the Markov model to forecast the train's running state. By selecting the specified operating interval collection data as the observation sample, the nearest-neighbour method can be selected to discretize the sample vehicle speed and acceleration information into a limited number of series.

$$\begin{cases} v \in \{v_1, v_2, \dots, v_l\} \\ a \in \{a_1, a_2, \dots, a_s\} \end{cases} \quad (16)$$

The collected train speed and acceleration samples are analyzed and summarized at a certain interval between the two stations, and the maximum likelihood estimation method can be adopted for the control system. The probability that the acceleration corresponding to the discrete velocity point  $v_n (1 \leq n \leq l)$  can be transferred from  $a_i$  to  $a_j$  can be calculated as:

$$\begin{cases} p_{ni,j} = \frac{m_{ni,j}}{m_{nj}} \\ m_{nj} = \sum_{j=1}^s m_{ni,j} \end{cases} \quad (17)$$

where  $m_{ni,j}$  is the number of shifts in acceleration from  $a_i$  to  $a_j$ , which is required to understand the discrete velocity point  $v_n$ ; and  $m_{nj}$  is the sum of the number times acceleration  $a_i$  is transferred when the discrete velocity point  $v_n$  is attained.

As a result, a one-step state transition probability matrix  $P_n$  can be achieved that corresponds to the  $l$  vehicle speed discrete points at each discrete velocity point  $v_n$ . Thus, a total of  $l$  one-step transition probability matrix can be achieved to build a one-step Markov model that satisfies the system.

$$P_n = \begin{bmatrix} P_{n1,1} & P_{n1,2} & \cdots & P_{n1,s} \\ P_{n2,1} & P_{n2,2} & \cdots & P_{n2,s} \\ \vdots & \vdots & \ddots & \vdots \\ P_{ns,1} & P_{ns,2} & \cdots & P_{ns,s} \end{bmatrix} \quad (18)$$

From the current train running speed  $v(k)$ , acceleration  $a(k)$  and transition probability matrix  $P_n$ , the acceleration process with the highest probability can be employed to predict the acceleration  $a(k+1)$  of the train at the next moment. Meanwhile, the speed  $v(k)$  and acceleration  $a(k)$  of the train can be applied to calculate its speed  $v(k+1)$  at the next moment, which can then be predicted until the end of the time domain. Finally, all the train running speeds and accelerations in the predicted time domain can be acquired.

#### 4.3 Constraint Optimization Issues and Solutions

According to the prediction and control model of the train's composite energy storage system, it can be expected that the future prediction time-domain disturbance is known. Consequently, its control system can



only attain the local optimal solution in the prediction time-domain at each control moment. Regarding the condition of the known global disturbance, the global optimal power allocation control strategy can be realized via the dynamic programming algorithm within a certain operating interval, thus obtaining the reference trajectory of the lithium battery and the supercapacitor SOC under ideal cases.

To acquire the desired reference trajectory, it is essential to guarantee that the lithium battery and supercapacitor SOC operate within a certain range. The objective function of each period is constructed within the operating interval as shown in Eq. (19).

$$L[x(t), u(t)] = \min_{u(t)} \left\{ \int_0^t ECR[x(t), u(t), t] dt \right. \\ \left. + \tau_{bat}[SOC_{bat}(t) - SOC_{bate}(t)] \right. \\ \left. + \tau_{sc}[SOC_{sc}(t) - SOC_{sce}(t)] \right\} \quad (19)$$

where  $\tau_{bat}$  and  $\tau_{sc}$  correspond to the weighting coefficients of the lithium battery and supercapacitor SOC predictive control output and the desired state.  $SOC_{bate}(t)$  and  $SOC_{sce}(t)$  are the desired SOC of the lithium battery and supercapacitor, respectively.

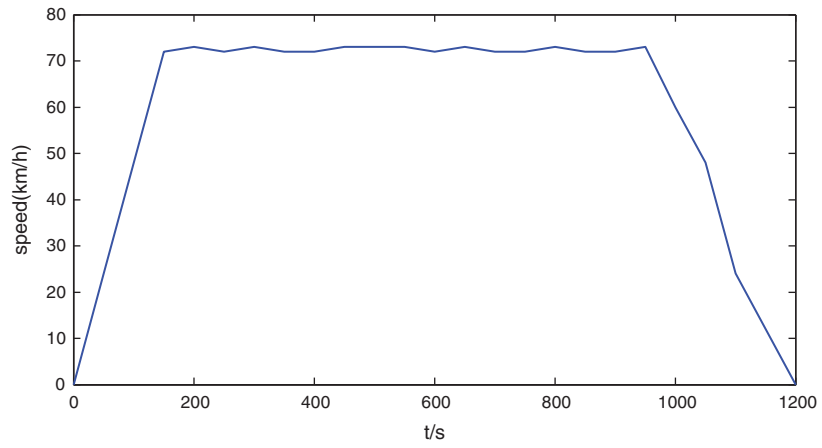
Considering the features of the energy storage system and train operation, and to optimize the performance of the control system, it is necessary to constrain each part of the composite system according to Eq. (15). In this system, the predicted time domain is assumed to be  $p$ , the control time domain is  $m$ , and  $m \leq p$ . The vector of the optimal control variable is acquired by the dynamic programming algorithm in the set predictive time domain  $[t, t + p]$ . Based on the specific optimization dynamic programming solving steps above, the optimal lithium battery distribution power sequence can be achieved.

If the optimal control variables attained in the first step can be applied to the composite system, the next cycle is started and the process can be repeated. Based on the dynamic programming solution, the composite optimization problem can be solved to accomplish optimal energy control and consumption for the system.

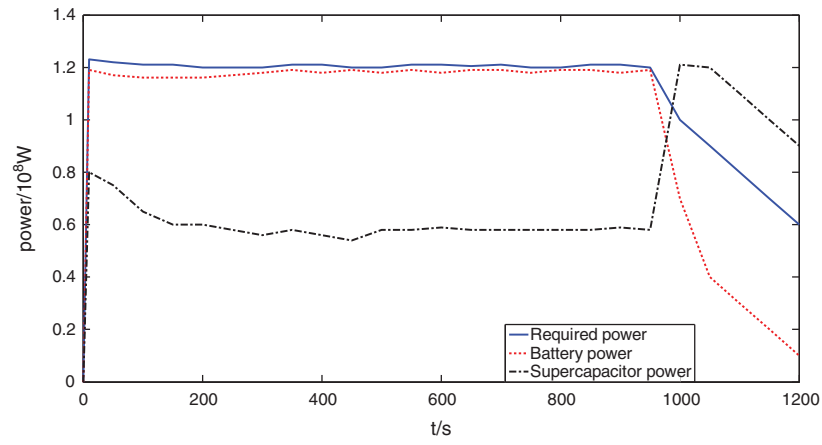
## 5 Simulation Experiment and Analysis

This paper used the 4M2T grouping method for a B-type urban rail train. The partial parameters are: whole-vehicle mass = 180 t, traction motor power = 180–300 kW, and DC-DC conversion efficiency = 0.987. The motor controller has a conversion efficiency of 0.975 and the power supply voltage is 1500 V DC. Also, the maximum running speed is 80 km/h, the maximum starting acceleration (0–35 km/h) is 0.95 m/s<sup>2</sup>, and the average acceleration (0–60 km/h) is  $\geq 0.5$  m/s<sup>2</sup>. Besides, the deceleration is 1.0 m/s<sup>2</sup>, the emergency braking deceleration is 1.2 m/s<sup>2</sup>, the DC-DC conversion efficiency is 0.987, and the motor controller conversion efficiency is 0.992. The entire vehicle's traction and resistance can be computed and the train's running speed and acceleration can be measured based on real-time acquisition and calculation. Assuming that the predicted time domain is 10 and the control time domain is 6, the composite energy storage system and predictive control energy controller were simulated in the MATLAB/Simulink environment to better confirm the optimization effect of the energy control system control strategy according to the predictive control model [40–43].

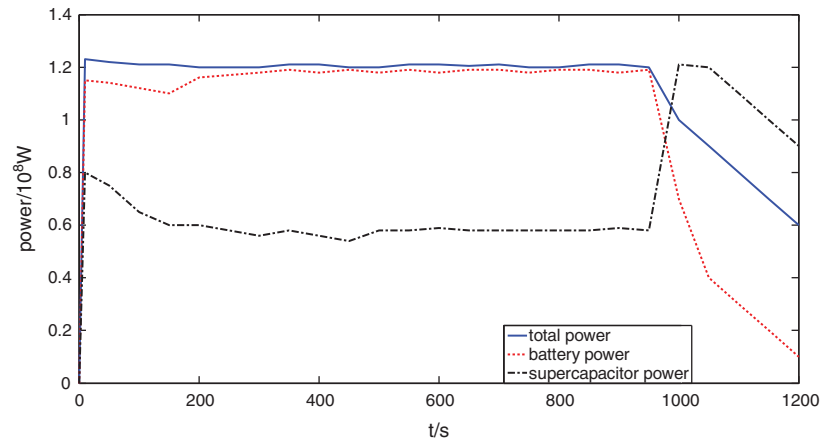
In the simulation, experiments were conducted in a part of the city with the upper and lower steep slopes of Line 2. The effects of the energy control strategy are compared and analyzed from the dynamic programming and model prediction control. The simulation findings are shown in Figs. 5–9.



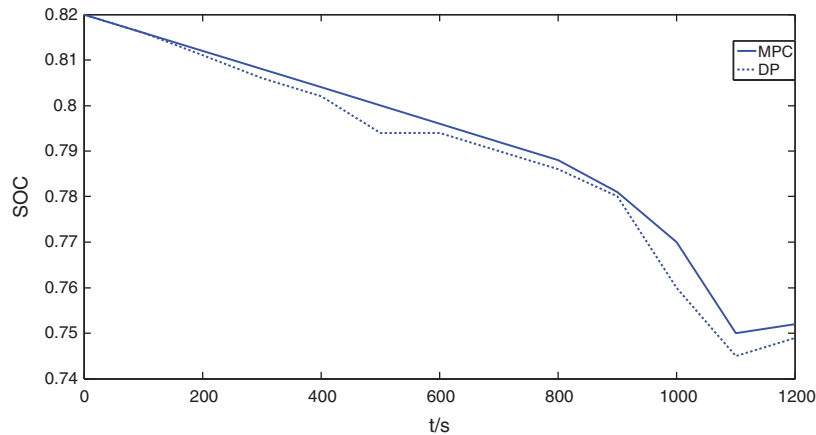
**Figure 5:** Train speed-time curve



**Figure 6:** Power distribution curve based on the dynamic programming control strategy



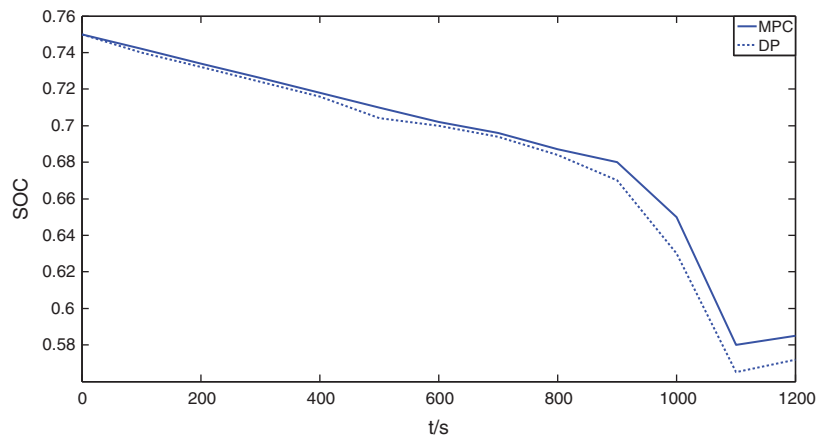
**Figure 7:** Power distribution curve under the model predictive control strategy



**Figure 8:** Lithium battery SOC variation with time under different control modes

Figs. 6 and 7 show the vehicle power distribution curves under the dynamic programming control strategy and the model prediction control strategy, respectively. The power demand curves represent the total power required by the train to operate within a certain interval. Also, it can be seen from the distribution power curves of the composite energy storage system under the two control distribution strategies that lithium batteries cannot absorb any energy during the regenerative braking process, while supercapacitors can. This protects the lithium battery from large currents and acts to “pad the valleys”. When the power demand of the whole vehicle is high, the supercapacitors provide the peak power, thus reducing the workload of the lithium battery and “clipping the peaks”.

Figs. 8 and 9 show the charging states of the lithium batteries and supercapacitors in different control modes. The SOC of the lithium battery under dynamic programming control is 0.82, while that under model predictive control is 0.75; hence, the mode predictive control strategy provides slightly poor control. Also, the total running time of the train in this section is 1200 s and the total time required for the offline model predictive control simulation is 15 s. This also reflects that the energy control strategy based on model predictive control can provide real-time performance.



**Figure 9:** Supercapacitor SOC under different control modes

From the above, the dynamic programming-based energy control allocation strategy is globally optimal according to the specific cycle conditions. Meanwhile, the results of the model predictive control strategy are similar, being suboptimal but easier to implement online.

## 6 Conclusions

A Markov model was proposed to predict the speed and acceleration states of a train in a future time domain, thereby precisely predicting the required power. The energy control strategy based on model predictive control can provide real-time performance, providing a solid foundation for the design of real-time energy control strategies.

Based on a Markov model of predictive control, a control strategy for predicting the energy of an urban rail composite energy storage system was designed. By selecting characteristic operation intervals for simulation testing, the effects of dynamic planning and model-based predictive control were compared. Dynamic programming is a globally optimal offline static method for analyzing future working conditions and highlights the effectiveness and real-time performance of model predictive energy control strategies.

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**Conflicts of Interest:** There are no conflicts of interest related to this paper, which was approved for publication by all authors. On behalf of my co-authors, I would like to declare that the work described is original research that has not been previously published in whole or in part.

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