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Article

Generalized Class of Mean Estimators with Known Measures for Outliers Treatment

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Abstract: In estimation theory, the researchers have put their efforts to develop some estimators of population mean which may give more precise results when adopting ordinary least squares (OLS) method or robust regression techniques for estimating regression coefficients. But when the correlation is negative and the outliers are presented, the results can be distorted and the OLS-type estimators may give misleading estimates or highly biased estimates. Hence, this paper mainly focuses on such issues through the use of non-conventional measures of dispersion and a robust estimation method. Precisely, we have proposed generalized estimators by using the ancillary information of non-conventional measures of dispersion (Gini's mean difference, Downton's method and probability-weighted moment) using ordinary least squares and then finally adopting the Huber M-estimation technique on the suggested estimators. The proposed estimators are investigated in the presence of outliers in both situations of negative and positive correlation between study and auxiliary variables. Theoretical comparisons and real data application are provided to show the strength of the proposed generalized estimators. It is found that the proposed generalized Huber-M-type estimators are more efficient than the suggested generalized estimators under the OLS estimation method considered in this study. The new proposed estimators will be useful in the future for data analysis and making decisions.

Keywords: Product estimators; ratio estimators; regression estimators; ordinary least square; Huber M; mean squared error; efficiency
MSC: 62D05; 62G35

1 Introduction

For obtaining proficient estimators in sampling theory, a multiplicity of techniques has been used and the commonly one is the simple random sampling without replacement (SRSWOR) to obtain an estimator for the population mean, when auxiliary information is not available. But when auxiliary information is available and even has a relationship with study variable, there are lots of methods by which this auxiliary information can be incorporated viz., ratio, product, difference and regression, etc. Utilizing this auxiliary information for parameters will increase the estimation efficiency. The utilization of auxiliary information has been made in a number of ways for achieving the improved estimates of population parameters. Some latest uses of auxiliary information are provided in [1–4]. As data collected from different fields, which is the basis for statistical inference, most of the time, the data will not be symmetrical and may contain outliers. The latter can distort results since the classical methods are sensitive to outliers [5]. However, [6], and [7–9] have recommended different estimators that adopted different robust regression techniques when the correlation is positive. For more details of robust regression methods for obtaining mean estimation of sensitive variables by using auxiliary information, see [10–12]. In this study, we focus on a more generalized form of estimators when outliers are presented. On how to deal with that situation, we first proposed generalized estimators utilizing the auxiliary information of non-conventional measures of scattering using OLS and then finally adopting the Huber M-estimation technique on the suggested estimators, in the presence of outliers. Then, we adopted the Huber M-estimation instead of ordinary least square on the recommended generalized estimators in order to get valid findings so that our inference will be valuable for future analysis or application. Hence, the importance of our present paper is that this work uses the robust (Huber M) estimation method and non-conventional measures of dispersion, which can curb the influence of outliers in the estimation of population mean.

The rest of the paper is organized as follows. In Section 2 shows the generalized estimator, outliers present, negative correlation exist and the adaptation of the OLS method with the expressions of Bias and the mean squared error (MSE) derived up to the second degree of approximation. The generalized estimators based on adopting Huber M estimation instead of OLS and their bias and MSE equations are proposed in Section 3. Efficiency comparisons between the proposed and existing estimators are considered in Section 4. The results of the numerical examples are reported in Section 5. Discussion is devoted to Section 6, and the paper is concluded in the last section.

2 Proposed Generalized Estimators Using OLS

Let $S = (S_1, S_2, \ldots, S_M)$ be a finite population of size M units. Let U and V be the response and ancillary variables, respectively. Let m be the sample size m ($m < M$) drawn using SRSWOR to estimate $\bar{U} = (1/M)\sum_{i=1}^{M} u_i$. Based on the m observations, let $(\bar{u}, \bar{v})$ be the sample means which are unbiased estimators of the population means $(\bar{U}, \bar{V})$. The usual ratio and product estimators for $\bar{U}$ are, respectively, $\bar{U}_R = \bar{u}\left(\frac{\bar{V}}{\bar{v}}\right)$ and $\bar{U}_P = \bar{u}\left(\frac{\bar{v}}{\bar{V}}\right)$, where $\bar{u} = (1/m)\sum_{i=1}^{m} u_i$ and $\bar{v} = (1/m)\sum_{i=1}^{m} v_i$. When $\rho_z C_u/C_v > 1/2$, the ratio estimator is proficient and when $\rho_z C_u/C_v < -1/2$, the product method is proficient ([13]). Here, C_u, C_v and ρ_z are the coefficients of variation of v and u, and the correlation coefficient between v and u, respectively. Hence,

$$C_v = \frac{S_v}{\bar{V}},\ S_v^2 = \frac{\sum_{i=1}^{M}\left(v_i - \bar{V}\right)^2}{M-1},\ C_u = \frac{S_u}{\bar{U}},\ S_u^2 = \frac{\sum_{i=1}^{M}\left(u_i - \bar{U}\right)^2}{M-1},\ \rho_z = \frac{S_{xy}}{S_x S_y},\ S_{uv} = \frac{\sum_{i=1}^{M}\left(v_i - \bar{V}\right)\left(u_i - \bar{U}\right)}{M-1}.$$

Reference [14] proposed ratio estimators of the mean based on the simple random sampling (SRS) method as $\hat{\mu}_{USRSQ_1} = \frac{\bar{U}_{SRS}(\mu_V + Q_1)}{\bar{V}_{SRS} + Q_1}$ and $\hat{\mu}_{USRSQ_3} = \frac{\bar{U}_{SRS}(\mu_V + Q_3)}{\bar{V}_{SRS} + Q_3}$, where $\bar{V}_{SRS}$ and $\bar{U}_{SRS}$ are the sample means of the variable of interest and the auxiliary variable, Q_1 and Q_3 represent the first and third quartiles, respectively, of the auxiliary variable V. Also, [15] introduced ratio estimators of the population mean using extreme ranked set sampling. Later, [16] investigated some ratio estimators of population mean using auxiliary information based on simple random sampling and the median ranked set sampling methods. Reference [17] investigated some ratio estimators of the population mean with missing values using the ranked set sampling method. The dual to ratio estimator is introduced firstly by Srivenkataramna [18], dual to ratio product estimator is discussed by Bandyopadhyay [19] and ratio cum product estimators are due to the valuable efforts of [20] and [21]. The efforts on ratio, dual to ratio and dual to product estimators for estimation population mean using OLS method are due to [22] and [23]. Reference [24] used the dual auxiliary information to develop a new optimal estimator. For another method using some statistical tests to construct an estimator for the finite population mean, see [25].

Reference [13] and [26–32], and ultimately, suggested generalized estimator using ancillary information for estimating the population parameters such as the mean in SRSWOR. Motivated by their works, our proposed estimators are given as

$$\bar{u}_{si} = [\bar{u} + k\lambda(\bar{v} - \bar{V})]\left[\frac{\bar{v}A + B}{\bar{V}A + B}\right]^{k\delta}, \tag{1}$$

where $k = \begin{cases} 1 & \text{for product estimator} \\ -1 & \text{for ratio estimator} \end{cases}$, A is a reasonably selected constant, δ is unknown constant and B is also a suitably chosen constant, where $G(v) = (4/M-1)\sum_{i=1}^{M}[(2i-M-1)/2M]V_{(i)}$, $D(v) = \left[2\sqrt{\bar{\lambda}}/M(M-1)\right]\sum_{i=1}^{M}\left(i - \frac{M+1}{2}\right)V_{(i)}$, and $S_{pw}(v) = \left(\sqrt{\bar{\lambda}}/M^2\right)\sum_{i=1}^{M}(2i-M-1)V_{(i)}$, the Gini's mean difference, Downton's method, probability weighted moments, respectively, or their functions. It is assumed that the population mean $\bar{V}$ of the auxiliary variable v is known. The $\lambda = \frac{S_{uv}}{S_v^2}$ is obtained by the OLS method. To determine the MSEs together with the bias, the proposed generalized estimators using OLS, where the members of this generalized class of estimator are given in Tab. 1, we let

$$\eta_0 = \frac{\bar{u} - \bar{U}}{\bar{U}},\ \eta_1 = \frac{\bar{v} - \bar{V}}{\bar{V}},\ E\left(\eta_0^2\right) = \frac{1-t}{m}C_u^2,\ E\left(\eta_1^2\right) = \frac{1-t}{m}C_v^2,\ E(\eta_0\eta_1) = \frac{1-t}{m}\rho_z C_u C_v,\ t = \frac{m}{M} \tag{2}$$

Eqs. (1) and (2) can be transformed as

$$\bar{u}_{si} = [\bar{U}(1+\eta_0) + \bar{V}k\lambda\eta_1][1+\theta_i\eta_1]^{k\delta},\ \theta_i = \frac{A\bar{V}}{A\bar{V} + B_i}. \tag{3}$$

Using Taylor expansion of order 2 of $[1+\theta_1\eta_1]^{k\delta}$ for Eq. (3) we have

$$\bar{u}_{si} \cong \bar{U}(1 + \eta_0 + k\lambda W\eta_1)\left(1 + k\delta\theta_i\eta_1 + \frac{k\delta(k\delta-1)}{2!}\theta_i^2\eta_1^2 + \ldots\right). \tag{4}$$

Therefore, the bias of the estimator is

$$B(\bar{u}_{si}) = E(\bar{u}_{si} - \bar{U}) = \frac{1-t}{m}\bar{U}\left\{\left(\frac{k\delta(k\delta-1)}{2}\theta_i^2 + k^2\delta\theta_i\lambda W\right)C_v^2 + k\delta\theta_i\rho_z C_u C_v\right\}. \tag{5}$$

The MSE of the proposed estimator in (1) can be obtained by using the Taylor series approximation as:

$$\begin{aligned} MSE(\bar{u}_{si}) &= E(\bar{u}_{si} - \bar{U})^2 \\ &= E\{\bar{U}(\eta_0 + k(\delta\theta_i + \lambda W)\eta_1)\}^2 \\ &= \frac{1-t}{m}\bar{U}\left\{C_u^2 + 2k(\delta\theta_i + \lambda W)\rho_z C_u C_v + k^2(\delta\theta_i + \lambda W)^2 C_v^2\right\}W \cdot \\ &= \frac{\bar{V}}{\bar{U}} = \frac{1}{R}. \end{aligned} \tag{6}$$

Table 1: A few members from the suggested class based on product and ratio estimators under OLS

A	B	λ	δ	Product Estimators $k = 1$	Ratio Estimators $k = -1$
1	$G(v)$	λ	1	$\bar{u}_{sp1} = Ж\frac{\bar{v}+G(v)}{\bar{V}+G(v)}$ Abid et al. [33]	$\bar{u}_{sr1} = \acute{K}\frac{\bar{V}+G(v)}{\bar{v}+G(v)}$ Abid et al. [33]
1	$D(v)$	λ	1	$\bar{u}_{sp2} = Ж\frac{\bar{v}+D(v)}{\bar{V}+D(v)}$ Abid et al. [33]	$\bar{u}_{sr2} = \acute{K}\frac{\bar{V}+D(v)}{\bar{v}+D(v)}$ Abid et al. [33]
1	$S_{pw}(v)$	λ	1	$\bar{u}_{sp3} = Ж\frac{\bar{v}+S_{pw}(v)}{\bar{V}+S_{pw}(v)}$ Abid et al. [33]	$\bar{u}_{sr3} = \acute{K}\frac{\bar{V}+S_{pw}(v)}{\bar{v}+S_{pw}(v)}$ Abid et al. [33]
1	ψ_1	λ	1	$\bar{u}_{sp4} = Ж\frac{\bar{v}+\psi_1}{\bar{V}+\psi_1}$ Subzar et al. [32]	$\bar{u}_{sr4} = \acute{K}\frac{\bar{V}+\psi_1}{\bar{v}+\psi_1}$ Subzar et al. [32]
1	ψ_2	λ	1	$\bar{u}_{sp5} = Ж\frac{\bar{v}+\psi_2}{\bar{V}+\psi_2}$ Subzar et al. [32]	$\bar{u}_{sr5} = \acute{K}\frac{\bar{V}+\psi_2}{\bar{v}+\psi_2}$ Subzar et al. [32]
1	ψ_3	λ	1	$\bar{u}_{sp6} = Ж\frac{\bar{v}+\psi_3}{\bar{V}+\psi_3}$ Subzar et al. [32]	$\bar{u}_{sr6} = \acute{K}\frac{\bar{V}+\psi_3}{\bar{v}+\psi_3}$ Subzar et al. [32]
ρ_z	ψ_1	λ	1	$\bar{u}_{sp7} = Ж\frac{\rho_z\bar{v}+\psi_1}{\rho_z\bar{V}+\psi_1}$ Subzar et al. [32]	$\bar{u}_{sr7} = \acute{K}\frac{\bar{V}\rho_z+\psi_1}{\bar{v}\rho_z+\psi_1}$ Subzar et al. [32]
ρ_z	ψ_2	λ	1	$\bar{u}_{sp8} = Ж\frac{\rho_z\bar{v}+\psi_2}{\rho_z\bar{V}+\psi_2}$ Subzar et al. [32]	$\bar{u}_{sr8} = \acute{K}\frac{\bar{V}\rho_z+\psi_2}{\bar{v}\rho_z+\psi_2}$ Subzar et al. [32]
ρ_z	ψ_3	λ	1	$\bar{u}_{sp9} = Ж\frac{\rho_z\bar{v}+\psi_3}{\rho_z\bar{V}+\psi_3}$ Subzar et al. [32]	$\bar{u}_{sr9} = \acute{K}\frac{\bar{V}\rho_z+\psi_3}{\bar{v}\rho_z+\psi_3}$ Subzar et al. [32]
ρ_z	$G(v)$	λ	1	$\bar{u}_{sp10} = Ж\frac{\rho_z\bar{v}+G(v)}{\rho_z\bar{V}+G(v)}$ Subzar et al. [32]	$\bar{u}_{sr10} = \acute{K}\frac{\bar{V}\rho_z+G(v)}{\bar{v}\rho_z+G(v)}$ Subzar et al. [32]

(Continued)

Table 1 (continued).

A	B	λ	δ	Product Estimators $k=1$	Ratio Estimators $k=-1$
ρ_z	$D(v)$	λ	1	$\bar{u}_{sp11} = Ж\dfrac{\rho_z\bar{v}+D(v)}{\rho_z\bar{V}+D(v)}$ Subzar et al. [32]	$\bar{u}_{sr11} = \acute{K}\dfrac{\bar{V}\rho_z+D(v)}{\bar{v}\rho_z+D(v)}$ Subzar et al. [32]
ρ_z	$S_{pw}(v)$	λ	1	$\bar{u}_{sp12} = Ж\dfrac{\rho_z\bar{v}+S_{pw}(v)}{\rho_z\bar{V}+S_{pw}(v)}$ Subzar et al. [32]	$\bar{u}_{sr12} = \acute{K}\dfrac{\bar{V}\rho_z+S_{pw}(v)}{\bar{v}\rho_z+S_{pw}(v)}$ Subzar et al. [32]
$G(v)$	ρ_z	λ	1	$\bar{u}_{sp13} = Ж\dfrac{G(v)\bar{v}+\rho_z}{G(v)\bar{V}+\rho_z}$	$\bar{u}_{sr13} = \acute{K}\dfrac{G(v)\bar{V}+\rho_z}{G(v)\bar{v}+\rho_z}$
A	B	λ	0	$\bar{u}_{sp14} = Ж$ Regression Estimator	$\bar{u}_{sr14} = \bar{u}-\lambda(\bar{v}-\bar{V})$ Regression Estimator
1	$G(v)$	λ	Δ	$\bar{u}_{sp15} = \left(Ж\dfrac{\bar{v}+G(v)}{\bar{V}+G(v)}\right)^{\Delta}$	$\bar{u}_{sr15} = \acute{K}\left(\dfrac{\bar{V}+G(v)}{\bar{v}+G(v)}\right)^{-\Delta}$
1	$D(v)$	λ	Δ	$\bar{u}_{sp16} = \left(Ж\dfrac{\bar{v}+D(v)}{\bar{V}+D(v)}\right)^{\Delta}$	$\bar{u}_{sr16} = \acute{K}\left(\dfrac{\bar{V}+D(v)}{\bar{v}+D(v)}\right)^{-\Delta}$
1	$S_{pw}(v)$	λ	Δ	$\bar{u}_{sp17} = \left(Ж\dfrac{\bar{v}+S_{pw}(v)}{\bar{V}+S_{pw}(v)}\right)^{\Delta}$	$\bar{u}_{sr15} = \acute{K}\left(\dfrac{\bar{V}+S_{pw}(v)}{\bar{v}+S_{pw}(v)}\right)^{-\Delta}$

Note: $\psi_1 = (Md+G(v))$, $\psi_2 = (Md+D(v))$, $\psi_3 = (Md+S_{pw}(v))$, $Ж = \bar{u}+\lambda(\bar{v}-\bar{V})$, $\Delta = \rho_z\left(\frac{C_u}{C_v}\right)$, $\acute{K} = \bar{u}-\lambda(\bar{v}-\bar{V})$

3 Proposed Class of Estimators using Huber M-Estimation

The main issue on which we focus in the present study is the proposition of a generalized class of ratio and product estimators that are suitable for data with the existence of outliers. To deal with this situation, we have adopted the Huber M-estimation technique to the developed generalized class of estimators, displayed in (1), to obtain valid results while estimating parameters in that situation, i.e.,

$$\bar{u}_{pi} = [\bar{u}+k\lambda_{Huber\,M}(\bar{v}-\bar{V})]\left(\frac{\bar{v}A+B}{\bar{V}A+B}\right)^{k\delta}. \tag{7}$$

In adopting the Huber M-estimates, the outlier's negative effect is reduced and valid results are obtained; hence, valid inferences will be drawn from the results. The compromise between h^2 and $|h|$ is the function $\rho_z(h)$ which is used in Huber M-estimator; h is the error term in regression model $u = c+dv+h$, c being the constant of the model. The function $\rho_z(h)$ has the form

$$\rho_z(h) = \begin{cases} h^2 & -l \le h \le l \\ 2l|h|-l^2 & h<-l \ \ or \ \ l<h \end{cases} \tag{8}$$

where l is a tuning constant that controls the robustness of the estimator and the value of regression coefficient $\lambda_{Huber\,M}$ is obtained by minimizing

$$\sum_{i=1}^{m} \rho_z(u_i - c - dv_i). \tag{9}$$

with respect to c and d. To determine the MSE together with the bias of the developed generalized estimators using Huber M-estimation, we use Eq. (2) and transform it into Eq. (6) to obtain

$$\bar{u}_{pi} = (\bar{U}(1+\eta_0) + \bar{V}k\lambda_{Huber\,M}\eta_1)(1+\theta_i\eta_1)^{k\delta},\ \theta_i = \frac{A\bar{V}}{A\bar{V}+B_i}. \tag{10}$$

Then, using the Taylor expansion of order 2 of $[1+\theta_1\eta_1]^{k\delta}$ in (7) we determine

$$\bar{u}_{pi} \cong \bar{U}(1+\eta_0+k\lambda_{Huber\,M}W\eta_1)\left(1+k\delta\theta_i\eta_1+\frac{k\delta(k\delta-1)}{2!}\theta_i^2\eta_1^2+\ldots\right). \tag{11}$$

Hence, the bias of the estimator is

$$B(\bar{u}_{pi}) = E(\bar{u}_{pi}-\bar{U}) = \frac{1-t}{m}\bar{U}\left\{\left(\frac{k\delta(k\delta-1)}{2}\theta_i^2 + k^2\delta\theta_i\lambda_{Huber\,M}W\right)C_v^2 + k\delta\theta_i\rho_z C_u C_v\right\}, \tag{12}$$

and the MSE of (7) can be obtained based on the Taylor series approximation as

$$MSE(\bar{u}_{pi}) = \frac{1-t}{m}\bar{U}\left\{C_u^2 + 2k(\delta\theta_i+\lambda_{Huber\,M}W)\rho_z C_u C_v + k^2(\delta\theta_i+\lambda_{Huber\,M}W)^2 C_v^2\right\} \tag{13}$$

Substituting the different values of A, B, δ and k results in some class members of this family of estimators. Also, the use of the robust measure (non-parametric) of the regression coefficient and the different non-conventional measures of dispersion helps in producing estimators that are not really affected by outliers and these estimators are mentioned in Tab. 2, that may be used when a set of data contains outliers.

Table 2: Some class members of product and ratio estimators

A	B	$\lambda_{Huber\,M}$	δ	Product Estimators $k=1$	Ratio Estimators $k=-1$
1	$G(v)$	$\lambda_{Huber\,M}$	1	$\bar{u}_{pp1} = \Psi_{Huber\,M}\frac{\bar{v}+G(v)}{\bar{V}+G(v)}$	$\bar{u}_{pr1} = \hbar_{Huber\,M}\frac{\bar{V}+G(v)}{\bar{v}+G(v)}$
1	$D(v)$	$\lambda_{Huber\,M}$	1	$\bar{u}_{pp2} = \Psi_{Huber\,M}\frac{\bar{v}+D(v)}{\bar{V}+D(v)}$	$\bar{u}_{pr2} = \hbar_{Huber\,M}\frac{\bar{V}+D(v)}{\bar{v}+D(v)}$
1	$S_{pw}(v)$	$\lambda_{Huber\,M}$	1	$\bar{u}_{pp3} = \Psi_{Huber\,M}\frac{\bar{v}+S_{pw}(v)}{\bar{V}+S_{pw}(v)}$	$\bar{u}_{pr3} = \hbar_{Huber\,M}\frac{\bar{V}+S_{pw}(v)}{\bar{v}+S_{pw}(v)}$
1	ψ_1	$\lambda_{Huber\,M}$	1	$\bar{u}_{pp4} = \Psi_{Huber\,M}\frac{\bar{v}+\psi_1}{\bar{V}+\psi_1}$	$\bar{u}_{pr4} = \hbar_{Huber\,M}\frac{\bar{V}+\psi_1}{\bar{v}+\psi_1}$
1	ψ_2	$\lambda_{Huber\,M}$	1	$\bar{u}_{pp5} = \Psi_{Huber\,M}\frac{\bar{v}+\psi_2}{\bar{V}+\psi_2}$	$\bar{u}_{pr5} = \hbar_{Huber\,M}\frac{\bar{V}+\psi_2}{\bar{v}+\psi_2}$
1	ψ_3	$\lambda_{Huber\,M}$	1	$\bar{u}_{pp6} = \Psi_{Huber\,M}\frac{\bar{v}+\psi_3}{\bar{V}+\psi_3}$	$\bar{u}_{pr6} = \hbar_{Huber\,M}\frac{\bar{V}+\psi_3}{\bar{v}+\psi_3}$

(Continued)

Table 2 (continued).

A	B	$\lambda_{Huber\,M}$	δ	Product Estimators $k = 1$	Ratio Estimators $k = -1$
ρ_z	ψ_1	$\lambda_{Huber\,M}$	1	$\bar{u}_{pp7} = \Psi_{Huber\,M}\dfrac{\rho_z\bar{v}+\psi_1}{\rho_z\bar{V}+\psi_1}$	$\bar{u}_{pr7} = \hbar_{Huber\,M}\dfrac{\bar{V}\rho_z+\psi_1}{\bar{v}\rho_z+\psi_1}$
ρ_z	ψ_2	$\lambda_{Huber\,M}$	1	$\bar{u}_{pp8} = \Psi_{Huber\,M}\dfrac{\rho_z\bar{v}+\psi_2}{\rho_z\bar{V}+\psi_2}$	$\bar{u}_{pr8} = \hbar_{Huber\,M}\dfrac{\bar{V}\rho_z+\psi_2}{\bar{v}\rho_z+\psi_2}$
ρ_z	ψ_3	$\lambda_{Huber\,M}$	1	$\bar{u}_{pp9} = \Psi_{Huber\,M}\dfrac{\rho_z\bar{v}+\psi_3}{\rho_z\bar{V}+\psi_3}$	$\bar{u}_{pr9} = \hbar_{Huber\,M}\dfrac{\bar{V}\rho_z+\psi_3}{\bar{v}\rho_z+\psi_3}$
ρ_z	$G(v)$	$\lambda_{Huber\,M}$	1	$\bar{u}_{pp10} = \Psi_{Huber\,M}\dfrac{\rho_z\bar{v}+G(v)}{\rho_z\bar{V}+G(v)}$	$\bar{u}_{pr10} = \hbar_{Huber\,M}\dfrac{\bar{V}\rho_z+G(v)}{\bar{v}\rho_z+G(v)}$
ρ_z	$D(v)$	$\lambda_{Huber\,M}$	1	$\bar{u}_{pp11} = \Psi_{Huber\,M}\dfrac{\rho_z\bar{v}+D(v)}{\rho_z\bar{V}+D(v)}$	$\bar{u}_{pr11} = \hbar_{Huber\,M}\dfrac{\bar{V}\rho_z+D(v)}{\bar{v}\rho_z+D(v)}$
ρ_z	$S_{pw}(v)$	$\lambda_{Huber\,M}$	1	$\bar{u}_{pp12} = \Psi_{Huber\,M}\dfrac{\rho_z\bar{v}+S_{pw}(v)}{\rho_z\bar{V}+S_{pw}(v)}$	$\bar{u}_{pr12} = \hbar_{Huber\,M}\dfrac{\bar{V}\rho_z+S_{pw}(v)}{\bar{v}\rho_z+S_{pw}(v)}$
$G(v)$	ρ_z	$\lambda_{Huber\,M}$	1	$\bar{u}_{pp13} = \Psi_{Huber\,M}\dfrac{G(v)\bar{v}+\rho_z}{G(v)\bar{V}+\rho_z}$	$\bar{u}_{pr13} = \hbar_{Huber\,M}\dfrac{G(v)\bar{V}+\rho_z}{G(v)\bar{v}+\rho_z}$
A	B	$\lambda_{Huber\,M}$	0	$\bar{u}_{pp14} = \Psi_{Huber\,M}$ Huber-M regression estimator	$\bar{u}_{pr14} = \hbar_{Huber\,M}$ Huber-M regression estimator
1	$G(v)$	$\lambda_{Huber\,M}$	Δ	$\bar{u}_{pp15} = \Psi_{Huber\,M}\left(\dfrac{\bar{v}+G(v)}{\bar{V}+G(v)}\right)^{\Delta}$	$\bar{u}_{pr15} = \hbar_{Huber\,M}\left(\dfrac{\bar{V}+G(v)}{\bar{v}+G(v)}\right)^{-\Delta}$
1	$D(v)$	$\lambda_{Huber\,M}$	Δ	$\bar{u}_{pp16} = \Psi_{Huber\,M}\left(\dfrac{\bar{v}+D(v)}{\bar{V}+D(v)}\right)^{\Delta}$	$\bar{u}_{pr16} = \hbar_{Huber\,M}\left(\dfrac{\bar{V}+D(v)}{\bar{v}+D(v)}\right)^{-\Delta}$
1	$S_{pw}(v)$	$\lambda_{Huber\,M}$	Δ	$\bar{u}_{pp17} = \Psi_{Huber\,M}\left(\dfrac{\bar{v}+S_{pw}(v)}{\bar{V}+S_{pw}(v)}\right)^{\Delta}$	$\bar{u}_{pr15} = \hbar_{Huber\,M}\left(\dfrac{\bar{V}+S_{pw}(v)}{\bar{v}+S_{pw}(v)}\right)^{-\Delta}$

Note: $\Delta = \rho_z\left(\frac{C_u}{C_v}\right)$, $\Psi_{Huber\,M} = \bar{u} + \lambda_{Huber\,M}(\bar{v} - \bar{V})$, $\hbar_{Huber\,M} = \bar{u} - \lambda_{Huber\,M}(\bar{v} - \bar{V})$.

4 Comparison of Efficiencies

The efficiencies of the generalized estimators using ancillary information when OLS is adopted are compared with the generalized estimator using the same ancillary information but with Huber M-estimation. For $\bar{u}_{pi}$ to be more efficient than $\bar{u}_{si}$, we have

$$MSE(\bar{u}_{pi}) \le MSE(\bar{u}_{si})$$

$$\Rightarrow\ 2k(\delta\theta_i + \lambda_{Huber\,M}W)\rho_z C_u C_v + k^2(\delta\theta_i + \lambda_{Huber\,M}W)^2 C_v^2 < 2k(\delta\theta_i + \lambda W)\rho_z C_u C_v + k^2(\delta\theta_i + \lambda W)^2 C_v^2$$

$$\Rightarrow\ 2\rho_z C_u C_v k[\delta\theta_i + \lambda_{Huber\,M}W - \delta\theta_i - \lambda W] + C_v^2 k^2\left[(\delta\theta_i + \lambda_{Huber\,M}W)^2 - (\delta\theta_i + \lambda W)^2\right] < 0$$

$$\Rightarrow\ 2\rho_z C_u C_v W k[\lambda_{Huber\,M} - \lambda] + C_v^2 k^2[(\delta\theta_i + \lambda_{Huber\,M}W) - (\delta\theta_i + \lambda W)][(\delta\theta_i + \lambda_{Huber\,M}W + \delta\theta_i + \lambda W)] < 0$$

$$\Rightarrow\ 2W\delta[\lambda_{Huber\,M} - \lambda] + k[(\lambda_{Huber\,M} - \lambda)W][2\delta\theta_i + W(\lambda_{Huber\,M} + \lambda)] < 0$$

$$\Rightarrow \quad W[\lambda_{Huber\,M} - \lambda][2\delta + k(2\delta\theta_i + W(\lambda_{Huber\,M} + \lambda))] < 0$$

$$\Rightarrow \quad W[\lambda_{Huber\,M} - \lambda][2\delta(1 + k\theta_i) + Wk(\lambda_{Huber\,M} + \lambda)] < 0$$

$$\Rightarrow \quad [\lambda_{Huber\,M} - \lambda][2\delta(1 + k\theta_i) + Wk(\lambda_{Huber\,M} + \lambda)] < 0.$$

Since, $W > 0$, either $\lambda_{Huber\,M} - \lambda < 0$ and $2\delta(1 + k\theta_i) + Wk(\lambda_{Huber\,M} + \lambda) < 0$. This implies that

$$\Rightarrow \quad \lambda_{Huber\,M} < \lambda \text{ and } 2\delta(1 + k\theta_i) > -kW(\lambda_{Huber\,M} + \lambda) < 0 \tag{14}$$

$$\text{Or } \lambda_{Huber\,M} > \lambda \text{ and } 2\delta(1 + k\theta_i) < -Wk(\lambda_{Huber\,M} + \lambda) < 0. \tag{15}$$

When the conditions given in (14) or (15) are satisfied, a proposed class of estimators in which Huber-M is adopted is more proficient than the generalized estimators in which OLS is taken.

5 Application and Numerical Illustration

In this section, we consider three real data populations and their descriptive statistics are summarized in Tab. 3. The first population (Pop.) is taken from [34]. The second population data is taken from the book entitled "Advanced Sampling Theory with Applications" by Singh [35], p. 147, Example (3.2.2.1). This second data is collected from a little town in the USA in which Psychologist want to estimate, in average, the sleep duration (in minutes) during the night for people of 50 years old and more. It is realized that there are 30 people living in the town matured 50 and over. Rather than asking everyone, the clinician chooses a SRSWOR sample of six people of this age gathering and records the data. The third population data set is taken from Myers, [36] in which the study is conducted on transistor gain between emitter and collector in an integrated circuit device (hFC), where emitter drive-in time (in minutes) is denoted by v and gain or hFC is denoted by u.

Table 3: Data sets descriptive

Parameter	Pop. 1	Pop. 2	Pop. 3	Parameter	Pop. 1	Pop. 2	Pop. 3
M	20	30	70	C_v	0.3943	0.13711	14.73
m	8	6	14	ρ_z	−0.9199	−0.8552	0.611
$\bar{U}$	19.55	384.2	1251.8	Md	13.55	55.27	1269.5
$\bar{V}$	18.8	67.267	248.21	$G(v)$	5.104	60.208	659.79
S_u	6.9441	59.402	226.1	$D(v)$	4.789	59.087	563.74
S_{uv}	−47.352	−472.607	5053.116	$S_{pw}(v)$	5.3122	60.907	690.94
C_u	0.3552	0.15588	18.06	λ	−0.8617	−5.5446	3.7786
S_v	7.4128	9.2324	36.56	$\lambda_{Huber\,M}$	−0.4917	−2.2267	2.2058

We applied to these data different class members of estimators using both proposed methods with the same auxiliary information; OLS and Huber M-estimation technique. The bias, mean squared error and percent relative efficiency (PRE) of some product types estimators for populations 1, 2 and 3 are given in Tabs. 4–6, respectively. The Tabs. 7–9 present the values of bias, MSE and PRE of some ratio types estimators for the populations 1, 2 and 3, respectively.

Table 4: MSE's and bias of some member type products from the classes for population 1

Estimators	Bias (OLS)	MSE (OLS)	Estimators	Bias (Huber M)	MSE (Huber M)	PRE
$\bar{u}_{sp1}$	−2.3771	1.6106	$\bar{u}_{pp1}$	−1.8668	0.7114	226.42
$\bar{u}_{sp2}$	−2.4088	1.5775	$\bar{u}_{pp2}$	−1.8917	0.6918	228.02
$\bar{u}_{sp3}$	−2.3566	1.6323	$\bar{u}_{pp3}$	−1.8506	0.7242	225.39
$\bar{u}_{sp4}$	−1.5171	2.6620	$\bar{u}_{pp1}$	−1.1914	1.3935	191.03
$\bar{u}_{sp5}$	−1.5300	2.6441	$\bar{u}_{pp5}$	−1.2015	1.3811	191.45
$\bar{u}_{sp6}$	−1.5087	2.6737	$\bar{u}_{pp6}$	−1.1848	1.4016	190.76
$\bar{u}_{sp7}$	38.4376	377.0613	$\bar{u}_{pp1}$	30.1856	358.6372	105.14
$\bar{u}_{sp8}$	50.0254	604.8798	$\bar{u}_{pp8}$	39.2856	581.4802	104.02
$\bar{u}_{sp9}$	33.3341	293.7308	$\bar{u}_{pp1}$	26.1777	277.4980	105.85
$\bar{u}_{sp10}$	−4.2879	0.3314	$\bar{u}_{pp10}$	−3.3674	0.2526	131.19
$\bar{u}_{sp11}$	−4.1799	0.3649	$\bar{u}_{pp11}$	−3.2826	0.2397	152.22
$\bar{u}_{sp12}$	−4.3625	0.3111	$\bar{u}_{pp12}$	−3.4259	0.2643	117.71
$\bar{u}_{sp13}$	−3.0517	0.9925	$\bar{u}_{pp13}$	−2.3965	0.3829	259.20
$\bar{u}_{sp14}$	0.0000	0.0222	$\bar{u}_{pp14}$	0.0000	**0.0117**	189.34
$\bar{u}_{sp15}$	2.8738	9.9511	$\bar{u}_{pp15}$	2.4017	7.1854	138.49
$\bar{u}_{sp16}$	2.8738	10.0245	$\bar{u}_{pp16}$	2.4453	7.2475	138.32
$\bar{u}_{sp17}$	2.8738	9.9038	$\bar{u}_{pp17}$	2.3736	7.1453	138.60

Table 5: MSE's and bias of some member type products from the classes for population 2

C	Bias (OLS)	MSE (OLS)	Estimators	Bias (Huber M)	MSE (Huber M)	PRE
$\bar{u}_{sp1}$	−5.9238	13.5806	$\bar{u}_{pp1}$	−4.1528	6.0296	225.23
$\bar{u}_{sp2}$	−5.9763	13.5042	$\bar{u}_{pp2}$	−4.1896	5.9846	225.65
$\bar{u}_{sp3}$	−5.8915	13.6278	$\bar{u}_{pp3}$	−4.1302	6.0574	224.98
$\bar{u}_{sp4}$	−4.1322	16.3379	$\bar{u}_{pp1}$	−2.8968	7.7157	211.75
$\bar{u}_{sp5}$	−4.1577	16.2966	$\bar{u}_{pp5}$	−2.9147	7.6896	211.93
$\bar{u}_{sp6}$	−4.1164	16.3635	$\bar{u}_{pp6}$	−2.8858	7.7318	211.64
$\bar{u}_{sp7}$	11.1437	51.7997	$\bar{u}_{pp1}$	7.8122	34.0438	152.16
$\bar{u}_{sp8}$	11.3635	52.4662	$\bar{u}_{pp8}$	7.9663	34.5788	151.73
$\bar{u}_{sp9}$	11.0109	51.3992	$\bar{u}_{pp1}$	7.7191	33.7227	152.42
$\bar{u}_{sp10}$	240.8527	3164.9230	$\bar{u}_{pp10}$	168.8476	3009.8200	105.15
$\bar{u}_{sp11}$	413.8980	8705.1820	$\bar{u}_{pp11}$	290.1594	8446.6120	103.06
$\bar{u}_{sp12}$	191.0471	2079.1650	$\bar{u}_{pp12}$	133.9318	1953.8410	106.41

(Continued)

Table 5 (continued).

C	Bias (OLS)	MSE (OLS)	Estimators	Bias (Huber M)	MSE (Huber M)	PRE
$\bar{u}_{sp13}$	−11.2283	7.1426	$\bar{u}_{pp13}$	−7.8715	2.7633	258.48
$\bar{u}_{sp14}$	0.0000	0.3196	$\bar{u}_{pp14}$	0.0000	0.0289	1106.54
$\bar{u}_{sp15}$	7.3025	36.8603	$\bar{u}_{pp15}$	5.5805	22.3234	165.12
$\bar{u}_{sp16}$	7.3811	36.9896	$\bar{u}_{pp16}$	5.6439	22.4222	164.97
$\bar{u}_{sp17}$	7.2543	36.7810	$\bar{u}_{pp17}$	5.5417	22.2628	165.21

Table 6: MSE's and bias of some member type products from the classes for population 3

C	Bias (OLS)	MSE (OLS)	Estimators	Bias (Huber M)	MSE (Huber M)	PRE
$\bar{u}_{sp1}$	−88997.86	530757.70	$\bar{u}_{pp1}$	−70474.42	385872.40	137.547
$\bar{u}_{sp2}$	−99525.91	513770.30	$\bar{u}_{pp2}$	−78811.23	373267.40	137.641
$\bar{u}_{sp3}$	−86045.96	535602.30	$\bar{u}_{pp3}$	−68136.91	389488.20	137.514
$\bar{u}_{sp4}$	−37111.39	621117.30	$\bar{u}_{pp1}$	−29387.27	454633.30	136.619
$\bar{u}_{sp5}$	−38823.93	617958.80	$\bar{u}_{pp5}$	−30743.36	452187.70	136.660
$\bar{u}_{sp6}$	−36587.99	622085.00	$\bar{u}_{pp6}$	−28972.80	455383.10	136.607
$\bar{u}_{sp7}$	27775.66	749649.60	$\bar{u}_{pp1}$	21994.61	556155.30	134.791
$\bar{u}_{sp8}$	29362.17	753008.40	$\bar{u}_{pp8}$	23250.92	558853.70	134.742
$\bar{u}_{sp9}$	27297.32	748638.90	$\bar{u}_{pp1}$	21615.83	555343.70	134.806
$\bar{u}_{sp10}$	97169.20	906210.30	$\bar{u}_{pp10}$	76945.03	683829.70	132.520
$\bar{u}_{sp11}$	119817.80	961581.60	$\bar{u}_{pp11}$	94879.67	729773.20	131.764
$\bar{u}_{sp12}$	91556.54	892813.50	$\bar{u}_{pp12}$	72500.55	672769.40	132.707
$\bar{u}_{sp13}$	−325572.50	258658.60	$\bar{u}_{pp13}$	−257810.00	212251.60	121.864
$\bar{u}_{sp14}$	0.00	86714.26	$\bar{u}_{pp14}$	0.00	63933.60	135.632
$\bar{u}_{sp15}$	−60111.50	557794.30	$\bar{u}_{pp15}$	−44961.00	406163.10	137.333
$\bar{u}_{sp16}$	−70477.93	529514.60	$\bar{u}_{pp16}$	−51796.09	384946.00	137.556
$\bar{u}_{sp17}$	−57329.50	564985.00	$\bar{u}_{pp17}$	−43054.12	411603.50	137.264

Table 7: MSE's and bias of some type member ratios from the classes for population 1

C	Bias (OLS)	MSE (OLS)	Estimators	Bias (Huber M)	MSE (Huber M)	PRE
$\bar{u}_{sp1}$	−0.000054	1.355528	$\bar{u}_{pp1}$	−0.510384	0.565790	239.58
$\bar{u}_{sp2}$	−0.000054	1.385855	$\bar{u}_{pp2}$	−0.517200	0.582487	237.92
$\bar{u}_{sp3}$	−0.000053	1.336132	$\bar{u}_{pp3}$	−0.505977	0.555208	240.65
$\bar{u}_{sp4}$	−0.000034	0.687007	$\bar{u}_{pp1}$	−0.325739	0.266521	257.77

(Continued)

Table 7 (continued).

C	Bias (OLS)	MSE (OLS)	Estimators	Bias (Huber M)	MSE (Huber M)	PRE
$\bar{u}_{sp5}$	−0.000035	0.694834	$\bar{u}_{pp5}$	−0.328502	0.268823	258.47
$\bar{u}_{sp6}$	−0.000034	0.681941	$\bar{u}_{pp6}$	−0.323938	0.265057	257.28
$\bar{u}_{sp7}$	−0.000033	0.649607	$\bar{u}_{pp1}$	−0.312200	0.256197	253.56
$\bar{u}_{sp8}$	−0.000033	0.657102	$\bar{u}_{pp8}$	−0.314960	0.258173	254.52
$\bar{u}_{sp9}$	−0.000033	0.644760	$\bar{u}_{pp1}$	−0.310402	0.254945	252.90
$\bar{u}_{sp10}$	−0.000053	1.314725	$\bar{u}_{pp10}$	−0.501068	0.543618	241.85
$\bar{u}_{sp11}$	−0.000053	1.345963	$\bar{u}_{pp11}$	−0.508216	0.560562	240.11
$\bar{u}_{sp12}$	−0.000052	1.294792	$\bar{u}_{pp12}$	−0.496454	0.532913	242.97
$\bar{u}_{sp13}$	−0.000068	2.016678	$\bar{u}_{pp13}$	−0.642786	0.962165	209.60
$\bar{u}_{sp14}$	0.000000	0.022234	$\bar{u}_{pp14}$	0.000000	**0.011743**	190.24
$\bar{u}_{sp15}$	−0.077200	0.974100	$\bar{u}_{pp15}$	−0.492400	0.374700	259.99
$\bar{u}_{sp16}$	−0.079400	0.995600	$\bar{u}_{pp16}$	−0.500500	0.384300	259.07
$\bar{u}_{sp17}$	−0.075800	0.960400	$\bar{u}_{pp17}$	−0.487200	0.368600	260.53

Table 8: MSE’s and bias of some type member ratios from the classes for population 2

C	Bias (OLS)	MSE (OLS)	Estimators	Bias (Huber M)	MSE (Huber M)	PRE
$\bar{u}_{sp1}$	−0.0048	3.6054	$\bar{u}_{pp1}$	−1.7758	2.0236	178.17
$\bar{u}_{sp2}$	−0.0048	3.6340	$\bar{u}_{pp2}$	−1.7915	2.0208	179.83
$\bar{u}_{sp3}$	−0.0048	3.5879	$\bar{u}_{pp3}$	−1.7661	2.0255	177.14
$\bar{u}_{sp4}$	−0.0034	2.7823	$\bar{u}_{pp1}$	−1.2387	2.2718	122.47
$\bar{u}_{sp5}$	−0.0034	2.7920	$\bar{u}_{pp5}$	−1.2463	2.2662	123.20
$\bar{u}_{sp6}$	−0.0033	2.7764	$\bar{u}_{pp6}$	−1.2340	2.2753	122.02
$\bar{u}_{sp7}$	−0.0030	2.6390	$\bar{u}_{pp1}$	−1.1190	2.3673	111.48
$\bar{u}_{sp8}$	−0.0030	2.6473	$\bar{u}_{pp8}$	−1.1263	2.3610	112.13
$\bar{u}_{sp9}$	−0.0030	2.6339	$\bar{u}_{pp1}$	−1.1145	2.3711	111.08
$\bar{u}_{sp10}$	−0.0044	3.3767	$\bar{u}_{pp10}$	−1.6443	2.0572	164.14
$\bar{u}_{sp11}$	−0.0045	3.4032	$\bar{u}_{pp11}$	−1.6601	2.0522	165.83
$\bar{u}_{sp12}$	−0.0044	3.3605	$\bar{u}_{pp12}$	−1.6346	2.0603	163.10
$\bar{u}_{sp13}$	−0.0091	7.7629	$\bar{u}_{pp13}$	−3.3645	3.0123	257.71
$\bar{u}_{sp14}$	0.0000	0.3196	$\bar{u}_{pp14}$	0.0000	**0.0289**	1106.54
$\bar{u}_{sp15}$	−0.0229	3.3017	$\bar{u}_{pp15}$	−1.6173	2.0729	159.28
$\bar{u}_{sp16}$	−0.0233	3.3268	$\bar{u}_{pp16}$	−1.6331	2.0673	160.92
$\bar{u}_{sp17}$	−0.0226	3.2864	$\bar{u}_{pp17}$	−1.6077	2.0764	158.27

Table 9: MSE's and bias of some type member ratios from the classes for population 3

C	Bias (OLS)	MSE (OLS)	Estimators	Bias (Huber M)	MSE (Huber M)	PRE
$\bar{u}_{sp1}$	34754.07	279084.10	$\bar{u}_{pp1}$	16230.63	220918.80	126.329
$\bar{u}_{sp2}$	41013.26	287533.90	$\bar{u}_{pp2}$	20298.58	224986.00	127.801
$\bar{u}_{sp3}$	33080.66	276796.50	$\bar{u}_{pp3}$	15171.61	219860.00	125.897
$\bar{u}_{sp4}$	10544.88	244079.20	$\bar{u}_{pp1}$	2820.75	207512.40	117.622
$\bar{u}_{sp5}$	11167.77	245058.40	$\bar{u}_{pp5}$	3087.21	207778.80	117.942
$\bar{u}_{sp6}$	10356.90	243782.30	$\bar{u}_{pp6}$	2741.71	207433.40	117.523
$\bar{u}_{sp7}$	−4197.67	215835.90	$\bar{u}_{pp1}$	1583.38	206279.50	104.633
$\bar{u}_{sp8}$	−4341.94	215361.50	$\bar{u}_{pp8}$	1769.31	206465.50	104.309
$\bar{u}_{sp9}$	−4152.14	215980.90	$\bar{u}_{pp1}$	1529.35	206225.40	104.731
$\bar{u}_{sp10}$	−862.47	204732.60	$\bar{u}_{pp10}$	19361.70	224062.50	91.373
$\bar{u}_{sp11}$	4499.39	205382.10	$\bar{u}_{pp11}$	29437.49	234139.80	87.718
$\bar{u}_{sp12}$	−1866.06	204896.90	$\bar{u}_{pp12}$	17189.93	221890.40	92.341
$\bar{u}_{sp13}$	285027.40	578580.50	$\bar{u}_{pp13}$	217264.80	421936.90	137.125
$\bar{u}_{sp14}$	0.00	63933.60	$\bar{u}_{pp14}$	0.00	86714.26	73.729
$\bar{u}_{sp15}$	13326.80	266965.70	$\bar{u}_{pp15}$	−1823.70	215546.30	123.855
$\bar{u}_{sp16}$	15910.33	279679.70	$\bar{u}_{pp16}$	−2771.50	221197.60	126.439
$\bar{u}_{sp17}$	12656.01	263997.50	$\bar{u}_{pp17}$	−1619.37	214328.30	123.174

6 Discussion

From Tabs. 1 and 2, it can be seen that the generalized class members of estimators can deliver various kinds of product and ratio estimators utilizing different auxiliary information under the adoption of OLS and Huber-M methods, respectively. Tabs. 4–6 present the numerical delineation of the productivity of certain members from these generalized classes of estimators. From these tables, it is found that while utilizing the same auxiliary information in the case of OLS and Huber M-estimations through product method of estimation, Huber-M-type (robust) estimators provide more efficient results than the OLS-type estimators when outliers are presented in the data. It is also observed that the Huber-M product regression estimator $\bar{u}_{pp14}$ has the least MSE in all the populations under consideration. This is seconded by $\bar{u}_{pp2}$. Similarly, from Tabs. 7–9, it is found that while utilizing the same auxiliary information in case of OLS and Huber M-estimations through ratio method of estimation, Huber-M-type (robust) estimators still provide more efficient results than the OLS-type estimators when in the presence of outliers in the data. It is also observed that the Huber-M ratio regression estimator $\bar{u}_{pp14}$ has the smallest MSE in all the populations under investigation. In the present study, we have also shown that the Huber-M-type classes of estimators have higher efficiencies than the OLS-type estimators, mainly where there exists the influence of outliers in the data. One can also generate different ratio and product estimators from the generalized class of estimators by substituting different parameters of auxiliary variable when outliers are existing in the data.

7 Conclusion

Based on the above discussion and numerical study, we can conclude that adopting Huber M instead of OLS, especially when outliers are presented, has superiority in precision (see Tabs. 7–9). The main feature of adopting the Huber M-estimation method that it provides an estimator that is easy to compute in practice with more efficient results. Beside these facts, our new proposed estimators will be useful in future study for data analysis and making decisions. Thus, a valid inference could be drawn from accurate results for future study or application, and, hence, providing better alternative estimators in practical situations. The proposed generalized estimators in this paper can be modified using different robust regression techniques [37] under different sampling techniques such as [38], systematic, two-Phase, and may be based on ranked set sampling methods [39–45].

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