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Decision-Making Problems under the Environment of m-Polar Diophantine Neutrosophic N-Soft Set

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ABSTRACT

Fuzzy models are present everywhere from natural to artificial structures, embodying the dynamic processes in physical, biological, and social systems. As real-life problems are often uncertain on account of inconsistent and indeterminate information, it seems very demanding for an expert to solve those problems using a fuzzy model. In this regard, we develop a hybrid new model m-polar Diophantine neutrosophic N-soft set which is based on neutrosophic set and soft set. Additionally, we define several different sorts of compliments on the proposed set. A proposed set is a generalized form of fuzzy, soft, Pythagorean fuzzy, Pythagorean fuzzy soft, and Pythagorean fuzzy N-soft sets. In this manner, m-polar Diophantine neutrosophic N-soft set is more proficient, a versatile model to oversee vulnerabilities as it likewise survives the downsides of existing models which are to be summed up. Furthermore, we give the application of the proposed set in multi-attribute decision-making problems by defining a new choice-value function.

KEYWORDS

Neutrosophic set; soft set; N-soft set; m-polar diophantine neutrosophic N-soft set; decision making

1 Introduction

The idea of a set and set theory are incredible assets in arithmetic. Shockingly, a non-condition basic set theory for example that the component can either have a place in a set or not, is frequently not appropriate in genuine a daily existence where numerous unclear terms as "enormous benefit," "high pressing factor," "moderate temperature," "dependable instruments," "safe conditions," and so forth are broadly utilized. Tragically, such loose depictions cannot be sufficiently taken care of by ordinary mathematical tools.

In fuzzy theory, a recently characterized model by and large beats the downsides of recently characterized models. Because of uncertainty and weaknesses issues in numerous days by day life issues, routine math is not continuously accessible. To manage such issues, different methods such as the theory of possibility, rough set assumption, and fuzzy set theory has been considered as elective models and to keep away from weaknesses too. Inopportunely, the greater part of the options such as science have their own disadvantages and downsides. For example, a large portion of the words like expert, amazing, best, significant is most certainly not quantifiable and uncertain. The rules for words like superb, best, famous, and so forth, hesitate from individual to person.

To deal with such sort of equivocal and unsure data, Zadeh [1] investigated the idea of fuzzy set which is mapping from a universal set X to [0, 1]. Atanassov [2] proposed the fortuitous of intuitionistic fuzzy sets as an expansion of fuzzy sets by presenting the idea of membership and non-membership grades. Molodtsov [3] began the thought of soft set as a mathematical model to oversee vulnerabilities. The chance of soft set has another objective for the researchers due to them utilizes in a wide range of exuberant issues.

Ali et al. [4] presented some new operations on soft set theory. They introduced the ideas of expanded and restricted union and intersections in detail. In [5–7], Yager introduced and investigated several relations on Pythagorean fuzzy set. Peng et al. [8] discussed certain results on Pythagorean fuzzy sets and also defined the Pythagorean fuzzy number in. Peng et al. [9] set up some Pythagorean fuzzy data measures and their applications. Peng et al. [10] proposed some new approaches to manage single-regarded neutrosophic MADM reliant on MABAC, TOPSIS, and new closeness measure with score function. In [11–21], many decision-making problems and algebraic structures are discussed over different fuzzy environments.

Smarandache's introduced neutrosophic set and then proposed many operation on it [22–24]. Neutrosophic set (NS) based on three parameters namely, membership, indeterminacy, and non-membership. Wang et al. [25] propsed the concept of single valued neutrosophic sets. Deli et al. [26] introduced the idea of bipolar neutrosophic set and their applications in multi-criteria decision making problems in. Fatimah et al. [22] introduced the notion of an N-soft set which is an extension of a soft set. Many problems related to decision-making are discussed by using different kind of environments in [27–30].

There are many problems regarding decision-making that need to improveby investigating a new set or model. In this regard, we develop a proposed setthat provides a more batter approximation than existing sets. The proposed work is arranged as follows. In Section 2, some preliminary concepts are given to understand the proposed work. In Section 3, we define the notion of m-polar Diophantine neutrosophic N-soft set and then define some operations on it. In Section 4, we discuss different types of compliments on the proposed set. We give the comparison table and application in multi-attribute decision-making problems in Section 5.

2 Preliminaries

In this section, we give some preliminary concepts related to previous existing sets.

Definition 2.1. [1] A fuzzy set is a mapping μ from universal set X to [0, 1] such that, $\mu: X \to [0,1]$. The fuzzy set can be written in the form of

$$F_X = \{(x, \mu_F(x)) : x \in X\}.$$

Definition 2.2. [2] An intuitionistic fuzzy set on universal set X is defined as

$$\mathcal{I}_X = \{(x, \mu_I(x), \nu_I(x)) : x \in I\},$$

where $\mu_I: X \rightarrow [0,1]$ and $\nu_I: X \rightarrow [0,1]$ are the membership and non-membership functions, respectively.

Definition 2.3. [31] The m-polar fuzzy set on a universal set M is a mapping $\mu: M \rightarrow [0,1]^m$ and $m(X)$ is the collection of all m-polar fuzzy set on M.

Definition 2.4. [3] The soft set is defined by the set valued mapping $\varphi: \mathcal{I} \rightarrow 2^T$, where $\mathcal{I}$ denotes the set of parameters and 2^T is the power set of T. The soft set can be written as,

$$\varphi_{\mathcal{I}} = (\varphi, \mathcal{I}) = \left\{(t, \varphi(t)) : t \in \mathcal{I}, \varphi(t) \in 2^T\right\}.$$

Definition 2.5. [32,33] A fuzzy soft set is defined as

$$\Gamma_S = \{(t, \gamma_S(t)) : t \in T, \gamma_S \in \mathcal{F}(L)\},$$

where $\gamma_S: T \rightarrow \mathcal{F}(L)$ and $\mathcal{F}(L)$ is the collection of all fuzzy sets on L and T is the set of parameters with $S \subseteq T$.

Definition 2.6. [34] Let X be the crisp set. Intuitionistic fuzzy soft set (IFSS) is interpreted by multi-valued mapping $\psi: B \rightarrow IF^X$, where IF^X represents the collection of all IF-subsets defined over crisp set X (where $B \subseteq X$). Thus the IFSS can be expressed as

$$\mho_B = \left\{(e, \psi_B(e)) : e \in X, \psi_B \in IF^X\right\}.$$

Definition 2.7. [5] Let X be the crisp set. A *Pythagorean fuzzy set* (PFS) can be expressed as

$$P = \left\{< \rho, \mu_P(\rho), \nu_P(\rho) >: 0 \leq \mu_P^2(\rho) + \nu_P^2(\rho) \leq 1, \rho \in X\right\},$$

where $\mu_P: X \rightarrow [0,1]$ and $\nu_P: X \rightarrow [0,1]$ with the condition that $0 \leq \mu_P^2(\rho) + \nu_P^2(\rho) \leq 1$, is known as the degrees of membership and non-membership of $\rho \in X$ to the set P.

Definition 2.8. [35] The score function and accuracy function of Pythagorean fuzzy number $\alpha = (\mu_\alpha, \nu_\alpha)$ over X is defined as, $S(\gamma) = \mu_\gamma^2 - \nu_\gamma^2$ and $Q(\gamma) = \mu_\gamma^2 + \nu_\gamma^2$, with S–$1 \leq \gamma \leq 1$ and $0 \leq Q(\gamma) \leq 1$.

Definition 2.9. [36] The ranking function of Pythagorean fuzzy number $\gamma = (\mu_\gamma, \nu_\gamma)$ over X is defined as

$$R(\gamma) = \frac{1}{2} + r_\gamma \left(\frac{1}{2} - \frac{2\theta_\gamma}{\pi}\right),$$

where $r_\gamma = \sqrt{\mu_\gamma^2 + \nu_\gamma^2}$ is called commitment strength and θ_γ is the angle between r_γ and μ_γ. The direction of commitment d_γ is $d_\gamma = 1 - \dfrac{2\theta_\gamma}{\pi}$, where $\mu_\gamma = r_\gamma cos\theta_\gamma$, $\nu_\gamma = r_\gamma sin\theta_\gamma$.

Definition 2.10. [37] Let X be a non-empty universal set, S be the set of attributes, and $Y \subseteq S$. Let $D = \{0, 1, 2, \ldots, N-1\}$ be set of grading. The triple (F_p, Y, N) is said to be a Pythagorean fuzzy N-soft set on X, if F_p is a mapping $F_p: Y \rightarrow 2^{X \times D} \times PFN$, in which $F: Y \rightarrow 2^{X \times D}$, and $P: Y \rightarrow PFN$, where PFN is Pythagorean fuzzy number. That is $\mu: Y \rightarrow [0,1]$ and $\nu: Y \rightarrow [0,1]$ such that

$$0 \leq \mu_y^2(x) + \nu_y^2(x) \leq 1, \quad \forall y \in Y, \quad \forall x \in X.$$

Hence,

$$\left(F_p, Y, N\right) = \left(\left(x, d_y\right), \left(\mu_y\left(x\right), \nu_y\left(x\right)\right)\right), \quad d_y \in D.$$

Definition 2.11. [38] A neutrosophic fuzzy set (NS), S over the universal set X is defined as $S = \{(\psi, \mu_S(\psi), \lambda_S(\psi), \nu_S(\psi))\}$,

where mappings μ_S, λ_S, ν_S stand for degree of truth, degree of indeterminacy, degree of falsity. $\mu_S, \lambda_S, \nu_S \in [0,1]$, with $0 \leqslant \mu_S + \lambda_S + \nu_S \leqslant 3$.

3 m-Polar Diophantine Neutrosophic N-Soft Set

Definition 3.1. Let $\mathfrak{L} = \{0, 1, 2, \ldots, \mathrm{N}-1\}$ be the set of grades where $\mathrm{N} \in \{2, 3, 4, \ldots\}$. If X is a non-emty set and E is the family of attributes. Let A be a non-empty subset of E. A *m-poler Diophantine neutrosophic N-soft* (MPDNNS) set on X is denoted as ($\mho$, A, m, N) or $\mho_A^{(m,N)}$, where $\mho\colon A \to \mathbf{P}\left(PF^X \times \mathfrak{L}\right)$ is a mapping (where PF^X is the aggregate of all Diophantine neutrosophic subsets over X). That is

$$(\mho, A, m, N) = \left\{\left(e, \left\{\frac{\langle \rho, \mathfrak{l}_e(\rho)\rangle}{(\mu_1(\rho), \mu_2(\rho), \ldots, \mu_m(\rho); \lambda_1(\rho), \lambda_2(\rho), \ldots, \lambda_m(\rho); \nu_1(\rho), \nu_2(\rho), \ldots, \nu_m(\rho)}\right\}\right) \right.$$
$$\left. | e \in A, \rho \in X, \mathfrak{l}_e(\rho) \in \mathfrak{L}\right\},$$

where $\mu_e\colon X \to [0,1]^m$, $\nu_e\colon X \to [0,1]^m$, and $\lambda_e\colon X \to [0,1]^m$ are mappings along with the property,

$$0 \le \sum_{i=1}^{m} \mu_i^m(\rho) + \sum_{i=1}^{m} \nu_i^m(\rho) + \sum_{i=1}^{m} \lambda_i^m(\rho) \le 3m.$$

In particular, $\mu_i(\rho)$ represents the truth-membership, $\nu_i(\rho)$ denotes degree of falsity-membership, $\lambda_i(\rho)$ is the degree of indeterminacy and $\mathfrak{l}_e(\rho)$ denotes the grading value of the element $\rho \in X$ corresponding to the attribute $e \in A$ to the set ($\mho$, A, m, N). If we write $a_{ij} = \mu_{ei}(\rho_j)$, $b_{ij} = \nu_{e_i}\left(\rho_j\right)$, $d_{ij} = \lambda_{ei}\ (\rho_j)$, and $c_{ij} = \mathfrak{l}_{e_j}\left(\rho_j\right)$ where i runs from 1 to m and j runs from 1 to n then the MPDNNS set $\mho_A^{(m,N)}$ may be represented in tabular form as

$\mho_A^{(m,N)}$	e_1	e_2	$\cdots$	e_m
ρ_1	$\langle c_{11}, (a_{11}, a_{21}, \cdots, a_{m1};$	$\langle c_{12}, (a_{11}, a_{21}, \cdots, a_{m1};$		$\langle c_{1m}, (a_{11}, a_{21}, \cdots, a_{m1};$
	$d_{11}, d_{21}, \cdots, d_{m1};$	$d_{11}, d_{21}, \cdots, d_{m1};$	$\cdots$	$d_{11}, d_{21}, \cdots, d_{m1};$
	$b_{11}, b_{21}, \cdots, b_{m1})\rangle$	$b_{11}, b_{21}, \cdots, b_{m1})\rangle$	$\cdots$	$b_{11}, b_{21}, \cdots, b_{m1})\rangle$
ρ_2	$\langle c_{21}, (a_{12}, a_{22}, \cdots, a_{m2};$	$\langle c_{22}, (a_{12}, a_{22}, \cdots, a_{m2};$		$\langle c_{2m}, (a_{12}, a_{22}, \cdots, a_{m2};$
	$d_{12}, d_{22}, \cdots, d_{m2};$	$b_{12}, b_{22}, \cdots, b_{m2};$	$\cdots$	$b_{12}, b_{22}, \cdots, b_{m2};$
	$b_{12}, b_{22}, \cdots, b_{m2})\rangle$	$b_{12}, b_{22}, \cdots, b_{m2})\rangle$	$\cdots$	$b_{12}, b_{22}, \cdots, b_{m2})\rangle$
$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$
ρ_n	$\langle c_{n1}, (a_{1n}, a_{2n}, \cdots, a_{mn};$	$\langle c_{n2}, (a_{1n}, a_{2n}, \cdots, a_{mn};$		$\langle c_{nm}, (a_{1n}, a_{2n}, \cdots, a_{mn};$
	$d_{1n}, d_{2n}, \cdots, d_{mn};$	$d_{1n}, d_{2n}, \cdots, d_{mm};$	$\cdots$	$d_{1n}, d_{2n}, \cdots, d_{mn};$
	$b_{1n}, b_{2n}, \cdots, b_{mn})\rangle$	$b_{1n}, b_{2n}, \cdots, b_{mm})\rangle$	$\cdots$	$b_{1n}, b_{2n}, \cdots, b_{mn})\rangle$

and in matrix form as

$$(\mho, A, m, N) = \left[\langle c_{ij}, (a_{ij}, d_{ij}, b_{ij})\rangle\right]_{n\times m}$$

$$= \begin{pmatrix} \langle c_{11}, (a_{11}, a_{21}, \cdots, a_{m1}; & \langle c_{12}, (a_{11}, a_{21}, \cdots, a_{m1}; & & \langle c_{1m}, (a_{11}, a_{21}, \cdots, a_{m1}; \\ d_{11}, d_{21}, \cdots, d_{m1}; & d_{11}, d_{21}, \cdots, d_{m1}; & \cdots & d_{11}, d_{21}, \cdots, d_{m1}; \\ b_{11}, b_{21}, \cdots, b_{m1})\rangle & b_{11}, b_{21}, \cdots, b_{m1})\rangle & \cdots & b_{11}, b_{21}, \cdots, b_{m1})\rangle \\ \langle c_{21}, (a_{12}, a_{22}, \cdots, a_{m2}; & \langle c_{22}, (a_{12}, a_{22}, \cdots, a_{m2}; & & \langle c_{2m}, (a_{12}, a_{22}, \cdots, a_{m2}; \\ d_{12}, d_{22}, \cdots, d_{m2}; & d_{12}, d_{22}, \cdots, d_{m2}: & \cdots & d_{12}, d_{22}, \cdots, d_{m2}; \\ b_{12}, b_{22}, \cdots, b_{m2})\rangle & b_{12}, b_{22}, \cdots, b_{m2})\rangle & \cdots & b_{12}, b_{22}, \cdots, b_{m2})\rangle \\ \vdots & \vdots & \ddots & \vdots \\ \langle c_{n1}, (a_{1n}, a_{2n}, \cdots, a_{mn}; & \langle c_{n2}, (a_{1n}, a_{2n}, \cdots, a_{mn}; & & \langle c_{nm}, (a_{1n}, a_{2n}, \cdots, a_{mn}; \\ d_{1n}, d_{2n}, \cdots, d_{mn}; & d_{1n}, d_{2n}, \cdots, d_{mm}; & \cdots & d_{1n}, d_{2n}, \cdots, d_{mn}; \\ b_{1n}, b_{2n}, \cdots, b_{mn})\rangle & b_{1n}, b_{2n}, \cdots, b_{mm})\rangle & \cdots & b_{1n}, b_{2n}, \cdots, b_{mn})\rangle \end{pmatrix}$$

This matrix is called *m-Polar Diophantine neutrosophic N-Soft matrix* or shortly MPDNNS matrix.

Note: We use the expression $x^n + y^n = z^n$ in Definition 3.1, which is similar to Diophantine equation. Thats why we call m-polar Diophantine neutrosophic N-soft set instead of m-polar neutrosophic N-soft set.

Definition 3.2. An MPDNNS set $\mho_A^{(m,N)}$ over X is known as *null MPDNNS set*, symbolized as $\mho_{\phi;}^{(m,0)}$ and defined as

$$\mho_{\phi}^{(m,0)} = \left\{\left(e, \left\{\frac{\langle \rho, \mathfrak{l}_{\phi}(\rho)\rangle}{(\mu_{\phi_1}(\rho), \mu_{\phi_2}(\rho), \cdots, \mu_{\phi_m}(\rho); \lambda_{\phi_1}(\rho), \lambda_{\phi_2}(\rho), \cdots, \lambda_{\phi_m}(\rho); \nu_{\phi_1}(\rho), \nu_{\phi_2}(\rho), \cdots, \nu_{\phi_m}(\rho)}\right\}\right)\right.$$
$$\left.|e \in A, \rho \in X, \mathfrak{l}_{\phi}(\rho) \in \mathfrak{L}\right\},$$

where, $\mu_{\phi_i}(\rho) = 0$, $\lambda_{\phi_i}(\rho) = 1$, $\nu_{\phi_i}(\rho) = 1$, $1 \le i \le m$, and $\mathfrak{l}_{\phi}(\rho) = 0$.

Definition 3.3. An MPDNNS set $\mho_A^{(m,N)}$ over X is known as *absolute MPDNNS set*, symbolized as $\mho_E^{(m,N-1)}$ and defined as

$$\mho_E^{(m,N-1)} = \left\{\left(e, \left\{\frac{\langle \rho, \mathfrak{l}_E(\rho)\rangle}{\left(\mu_{E_1}(\rho), \mu_{E_2}(\rho), \cdots, \mu_{E_m}(\rho); \nu_{E_1}(\rho)\right), \nu_{E_2}(\rho), \cdots, \nu_{E_m}(\rho)}\right\}\right) :\right.$$
$$\left. e \in A, \rho \in X, \mathfrak{l}_E(\rho) \in \mathfrak{L}\right\},$$

where $\mu_{E_i}(\rho) = 1$, $\lambda_{E_i}(\rho) = 0$, $\nu_{E_i}(\rho) = 0$, $1 \le i \le m$, and $\mathfrak{l}_{\phi}(\rho) = N - 1$.

Example 3.1. Let $X = \{\rho_1, \rho_2, \rho_3, \rho_4, \rho_5, \rho_6, \rho_7\}$, $E = \{e_1, e_2, e_3\}$ and $\mathfrak{L} = \{0, 1, 2, \ldots, 5\}$. Suppose that $A = \{e_1, e_3\}$. Then

$$\begin{aligned}\mho_A^{(3,6)} = \{&(e_1, \{\langle\rho_1, 3, (.3, .4, .5; .5, .4, .3; .1, .6, .4)\rangle, \quad \langle\rho_4, 4, (.4, .3, .1; .4, .4, .2; .3, .4, .5)\rangle,\\ &\langle\rho_7, 5, (.6, .2, .55; .3, .1, .2; .4, .3, .5)\rangle\}), \quad (e_3, \{\langle\rho_2, 2, (.2, .1, .7; .6, .6, .5; .8, .4, .6)\rangle,\\ &\langle\rho_3, 1, (.5, .6, .7; .7, .5, .4; .4, .3, .1)\rangle, \quad \langle\rho_5, 2, (.4, .6, .2; .7, .3, .5; .7, .5, .3)\rangle\})\}\end{aligned}$$

is a 3PDN6S set over X. The tabular representation of $\mho_A^{(3,6)}$ is

$\mho_A^{(3,6)}$	e_1	e_2	e_3
ρ_1	$\langle 3, (.3, .4, .5; .5, .4, .3; .1, .6, .4)\rangle$	$\langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1)\rangle$	$\langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1)\rangle$
ρ_2	$\langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1)\rangle$	$\langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1)\rangle$	$\langle 2, (.2, .1, .7; .6, .6, .5; .8, .4, .6)\rangle$
ρ_3	$\langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1)\rangle$	$\langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1)\rangle$	$\langle 1, (.5, .6, .7; .7, .5, .4; .4, .3, .1)\rangle$
ρ_4	$\langle 4, (.4, .3, .1; .4, .4, .2; .3, .4, 0.5)\rangle$	$\langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1)\rangle$	$\langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1)\rangle$
ρ_5	$\langle 0, (0, 0, 0; 1, 1, 1)\rangle$	$\langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1)\rangle$	$\langle 2, (.4, .6, .2; .7, .3, .5; .7, .5, .3)\rangle$
ρ_6	$\langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1)\rangle$	$\langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1)\rangle$	$\langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1)\rangle$
ρ_7	$\langle 5, (.6, .2, .55; .3, .1, .2; .4, .3, 0.5)\rangle$	$\langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1)\rangle$	$\langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1)\rangle$

The corresponding 3PDN6S matrix is $(\mho, A, 3, 6) = [a_{ij}, d_{ij}, b_{ij}]_{7\times 3}$

$$= \begin{pmatrix} \langle 3, (.3, .4, .5; .5, .4, .3; .1, .6, .4)\rangle & \langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1)\rangle & \langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1)\rangle \\ \langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1)\rangle & \langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1)\rangle & \langle 2, (.2, .1, .7; .6, .6, .5; .8, .4, .6)\rangle \\ \langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1)\rangle & \langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1)\rangle & \langle 1, (.5, .6, .7; .7, .5, .4; .4, .3, .1)\rangle \\ \langle 4, (.4, .3, .1; .4, .4, .2; .3, .4, 0.5)\rangle & \langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1)\rangle & \langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1)\rangle \\ \langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1)\rangle & \langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1)\rangle & \langle 2, (.4, .6, .2; .7, .3, .5; .7, .5, .3)\rangle \\ \langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1)\rangle & \langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1)\rangle & \langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1)\rangle \\ \langle 5, (.6, .2, .55; .3, .1, .2; .4, .3, .5)\rangle & \langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1)\rangle & \langle 0, (0, 0, 0; 1, 1, 1; 1, 1, 1)\rangle \end{pmatrix}$$

Definition 3.4. Let $\mho_A^{(m,N_1)}$ and $\mho_B^{(m,N_2)}$ be MPDNNS sets over X. Then $\mho_A^{(m,N_1)}$ is *MPDNNS subset* of $\mho_B^{(m,N_2)}$, i.e., $\mho_A^{(m,N_1)} \mathrel{\tilde{\subseteq}} \mho_B^{(m,N_2)}$, if

- $A \subseteq B$,
- $N_1 \leq N_2$,
- $\mu_{i,A}(\rho) \leq \mu_{i,B}(\rho)$, $1 \leq i \leq m$,
- $\lambda_{i,A}(\rho) \geq \lambda_{i,B}(\rho)$, $1 \leq i \leq m$,
- $\nu_{i,A}(\rho) \geq \nu_{i,B}(\rho)$, $1 \leq i \leq m$, and
- $\mathfrak{l}_A(\rho) \leq \mathfrak{l}_B(\rho)$.

It is worth mentioning that $\mho_A^{(m,N_1)} \mathrel{\tilde{\subseteq}} \mho_B^{(m,N_2)}$ it is not necessary each element of $\mho_A^{(m,N_1)}$ is also in ${\mho_B}^{(m,N_2)}$.

Definition 3.5. Let $\mho_A^{(m,N_1)}$ and $\mho_B^{(m,N_2)}$ be MPDNNS sets over X. Then $\mho_A^{(m,N_1)}$ is *MPDNNS equal* of $\mho_B^{(m,N_2)}$ i.e., $\mho_A^{(m,N_1)} \cong \mho_B^{(m,N_2)}$, if

- $A = B$,
- $N_1 = N_2$,
- $\mu_{i,A}(\rho) = \mu_{i,B}(\rho),\ 1 \leq i \leq m$,
- $\lambda_{i,A}(\rho) = \lambda_{i,B}(\rho),\ 1 \leq i \leq m$,
- $\nu_{i,A}(\rho) = \nu_{i,B}(\rho),\ 1 \leq i \leq m$, and
- $\mathfrak{l}_A(\rho) = \mathfrak{l}_B(\rho)$.

Proposition 3.1. If $\mho_A^{(m,N)}$ is any MPDNNS set over X, then

(*i*) $\mho_\phi^{(m,0)} \tilde{\subseteq} \mho_A^{(m,N)}$,

(*ii*) $\mho_A^{(m,N)} \tilde{\subseteq} \mho_E^{(m,N-1)}$.

Proof.

(i) The truth-membership, degree of indeterminacy and degree of falsity always fall in [0, 1] according to Definition 3.1 of MPDNNS set. So, $0 \leq \mu_{i,A}(\rho),\ \nu_{i,A}(\rho) \leq 1,\ \lambda_{i,A}(\rho) \leq 1$, and grading value $0 \leq \mathfrak{l}_A(\rho),\ \forall \rho \in X$ and $1 \leq i \leq m$.

Thus, it follows from Definitions 3.4 and 3.2 of null MPDNNS set $\mho_\phi^{(m,0)}$. Hence, $\mho_\phi^{(m,0)} \tilde{\subseteq} \mho_A^{(m,N)}$.

(ii) Clearly, $\mu_{i,A}(\rho) \leq 1$, $\nu_{i,A}(\rho) \geq 0$ and $\nu_{i,A}(\rho) \geq 0$ and grading value $\mathfrak{l}_A(\rho) \leq N-1$ for all $\rho \in X$ and $1 \leq i \leq m$. Thus, it follows from Definitions 3.4 and 3.3 of absolute MPDNNS set. Hence, $\mho_A^{(m,N)} \tilde{\subseteq} \mho_E^{(m,N-1)}$.

Proposition 3.2. If $\mho_A^{(m,N)}$, $\mho_B^{(m,N)}$ and $\mho_C^{(m,N)}$ are MPDNNS sets over X, then

(*i*) $\mho_A^{(m,N)} \tilde{\subseteq} \mho_A^{(m,N)}$.

(*ii*) $\mho_A^{(m,N)} \tilde{\subseteq} \mho_B^{(m,N)}$ and $\mho_B^{(m,N)} \tilde{\subseteq} \mho_C^{(m,N)} \Rightarrow \mho_A^{(m,N)} \tilde{\subseteq} \mho_C^{(m,N)}$.

Definition 3.6. Let $\mho_A{}^{(m,N_1)}$ and $\mho_B{}^{(m,N_2)}$ be MPDNNS sets over X. Then their *extended union* is defined as $\mho_C^{(m,N)^*} = \mho_A^{(m,N_1)} \tilde{\cup}_E \mho_B^{(m,N_2)}$,

$$\mho_C^{(m,N)^*} = \left\{\left(e, \left\{\frac{\langle \rho, \mathfrak{l}_C(\rho)\rangle}{(\mu_{1,C}(\rho), \mu_{2,C}(\rho), \cdots, \mu_{m,C}(\rho); \lambda_{1,C}(\rho), \lambda_{2,C}(\rho) \cdots, \lambda_{m,C}(\rho); \nu_{1,C}(\rho), \nu_{2,C}(\rho) \cdots, \nu_{m,C}(\rho)}\right\}\right) \middle| e \in C, \rho \in X, \mathfrak{l}_C(\rho) \in \mathfrak{L}\right\}$$

where,

- $C = A \cup B$,
- $(m,\ N)^* = (m,\ \max\{N_1,\ N_2\})$,

- $\mu_C(\rho) = \max\{\mu_{i,A}(\rho),\ \mu_{i,B}(\rho)\},\ 1\ \le i \le m,$
- $\lambda_C(\rho) = \min\{\lambda_{i,A}(\rho),\ \lambda_{i,B}(\rho)\}\ 1\ \le i \le m,$
- $\nu_C(\rho) = \min\{\nu_{i,A}(\rho),\ \nu_{i,B}(\rho)\}\ 1\ \le i \le m,$ and
- $\mathfrak{l}_C(\rho) = \max\{\mathfrak{l}_A(\rho),\ \mathfrak{l}_B(\rho)\}\ \forall\ e\ \in C.$

Definition 3.7. Let $\mho_A^{(m,N_1)}$ and $\mho_B^{(m,N_2)}$ be MPDNNS sets over X. Then their *restricted union* is defined as $\mho_D^{(m,N)^\diamond} = \mho_A^{(m,N_1)} \widetilde{\cup}_R \mho_B^{(m,N_2)}$,

$$\mho_D^{(m,N)^\diamond} = \left\{ \left(e, \left\{ \frac{\langle \rho, \mathfrak{l}_D(\rho) \rangle}{(\mu_{1,D}(\rho), \mu_{2,D}(\rho), \cdots, \mu_{m,D}(\rho); \lambda_{1,D}(\rho), \lambda_{2,D}(\rho) \cdots, \lambda_{m,D}(\rho); \nu_{1,D}(\rho), \nu_{2,D}(\rho) \cdots, \nu_{m,D}(\rho)} \right\} \right) \right.$$

$$\left. | e \in D, \rho \in X, \mathfrak{l}_D(\rho) \in \mathfrak{L} \right\}$$

where,

- $D = A \cap B,$
- $(m,N)^\diamond = (m,\ \max\{N_1,\ N_2\}),$
- $\mu_D(\rho) = \max\{\mu_{i,A}(\rho),\ \mu_{i,B}(\rho)\},\ 1\ \le i \le m,$
- $\lambda_D(\rho) = \min\{\lambda_{i,A}(\rho),\ \lambda_{i,B}(\rho)\}\ 1\ \le i \le m,$
- $\nu_D(\rho) = \min\{\nu_{i,A}(\rho),\ \nu_{i,B}(\rho)\}\ 1\ \le i \le m,$ and
- $\mathfrak{l}_D(\rho) = \max\{\mathfrak{l}_A(\rho),\ \mathfrak{l}_B(\rho)\}\ \forall\ e\ \in D.$

Example 3.2. Let $X = \{\rho_i\colon\ i = 1,\ 2, \ldots,\ 8\}$, $E = \{e_i\colon\ i = 1,\ 2,\ 3\}$ and $\mathfrak{L} = \{0,\ 1,\ 2, \ldots,\ 12\}$. Assume that $A = \{e_1, e_3\} \subseteq E$ and $B = \{e_2, e_3\} \subseteq E$. Then

$$\begin{aligned}\mho_A^{(3,13)} = \{&(e_1, \{\langle \rho_2, 11, (.1, .9, .7; .5, .3.2; .4, .6, .8)\rangle, \langle \rho_5, 9, (.2, .1, .2; .3, .4, .5; .6, .7, .8)\rangle,\\ &\langle \rho_7, 8, (.3, .4, .5; .6, .7, .6; .5, .4, .3)\rangle\}), (e_3, \{\langle \rho_3, 3, (.4, .3, .4; .5, .6, .8; .7, .6, .5)\rangle,\\ &\langle \rho_4, 6, (.5, .4, .5; .6, .7, .7; .6, .4, .2)\rangle, \langle \rho_6, 5, (.6, .5, .6; .7, .8, .9; .8, .7, .6)\rangle\})\}\end{aligned}$$

is a 3PDN13S set over X. The corresponding $\mho_A^{(3,13)}$ matrix is

$$\mho_A^{(3,13)} = \left[\left\langle c_i, \left(a_{ij}; d_{ij}; b_{ij}\right)\right\rangle\right]_{8\times 3}$$

$$= \begin{pmatrix} \langle 0,(0,0,0;1,1,1;1,1,1)\rangle & \langle 0,(0,0,0;1,1,1;1,1,1)\rangle & \langle 0,(0,0,0;1,1,1;1,1,1)\rangle \\ \langle 11,(.1,.9,.7;.5,.3,0.2;.4,.6,.8)\rangle & \langle 0,(0,0,0;1,1,1;1,1,1)\rangle & \langle 0,(0,0,0;1,1,1;1,1,1)\rangle \\ \langle 0,(0,0,0;1,1,1;1,1,1)\rangle & \langle 0,(0,0,0;1,1,1;1,1,1)\rangle & \langle 3,(.4,.3,.4;.5,.6,.8;.7,.6,.5)\rangle \\ \langle 0,(0,0,0;1,1,1;1,1,1)\rangle & \langle 0,(0,0,0;1,1,1;1,1,1)\rangle & \langle 6,(.5,.4,.5;.6,.7,.7;.6,.4,.2)\rangle \\ \langle 9,(.2,.1,.2;.3,.4.5;.6,.7,.8)\rangle & \langle 0,(0,0,0;1,1,1;1,1,1)\rangle & \langle 0,(0,0,0;1,1,1;1,1,1)\rangle \\ \langle 0,(0,0,0;1,1,1;1,1,1)\rangle & \langle 0,(0,0,0;1,1,1;1,1,1)\rangle & \langle 5,(.6,.5,.6;.7,.8.9;.8,.7,.6)\rangle \\ \langle 8,(.3,.4,.5;.6,.7,.6;.5,.4,.3)\rangle & \langle 0,(0,0,0;1,1,1;1,1,1)\rangle & \langle 0,(0,0,0;1,1,1;1,1,1)\rangle \\ \langle 0,(0,0,0;1,1,1;1,1,1)\rangle & \langle 0,(0,0,0;1,1,1;1,1,1)\rangle & \langle 0,(0,0,0;1,1,1;1,1,1)\rangle \end{pmatrix} \tag{1}$$

$$\begin{aligned} \mho_B^{(3,10)} = \{(e_2, \{\langle \rho_1, 9, (.7,.6,.4,:.2,.1,.4;.3,.5,.7)\rangle, \langle \rho_4, 6, (.8,.6,.5;.6,.4,.7;.5.8,.9)\rangle,\\ \langle \rho_8, 8, (.1,.3,.4;.8,.7,.4;.7,.3,.1)\rangle\}), (e_3, \{\langle \rho_2, 1, (.2,.1,.3;.6,.7,.9;.7,.3,.6)\rangle,\\ \langle \rho_5, 5, (.1,.2,.4;.8,.7,.1;.8,.3,.7)\rangle, \langle \rho_7, 3, (.2,.3,.6;.9,.8,.7;.8,.7,.5)\rangle\})\} \end{aligned}$$

is a 3PDF10S set over X. The corresponding $\mho_B^{(3,10)}$ matrix is

$$\mho_B^{(3,10)} = [\langle c_i, (a_{ij}; b_{ij})\rangle]_{8\times 3}$$

$$\mho_B^{(3,10)} = \begin{pmatrix} \langle 0,(0,0,0;1,1,1;1,1,1)\rangle & \langle 9,(.7,.6,.4,:.2,.1,.4;.3,.5,.7)\rangle & \langle 0,(0,0,0;1,1,1;1,1,1)\rangle \\ \langle 0,(0,0,0;1,1,1;1,1,1)\rangle & \langle 0,(0,0,0;1,1,1;1,1,1)\rangle & \langle 1,(.2,.1,.3;.6,.7,.9;.7,.3,.6)\rangle \\ \langle 0,(0,0,0;1,1,1;1,1,1)\rangle & \langle 0,(0,0,0;1,1,1;1,1,1)\rangle & \langle 0,(0,0,0;1,1,1;1,1,1)\rangle \\ \langle 0,(0,0,0;1,1,1;1,1,1)\rangle & \langle 6,(.8,.6,.5;.6,.4,.7;.5,.8,.9)\rangle & \langle 0,(0,0,0;1,1,1;1,1,1)\rangle \\ \langle 0,(0,0,0;1,1,1;1,1,1)\rangle & \langle 0,(0,0,0;1,1,1;1,1,1)\rangle & \langle 5,(.1,.2,.4;.8,.7,.1;.8,.3,.7)\rangle \\ \langle 0,(0,0,0;1,1,1;1,1,1)\rangle & \langle 0,(0,0,0;1,1,1;1,1,1)\rangle & \langle 0,(0,0,0;1,1,1;1,1,1)\rangle \\ \langle 0,(0,0,0;1,1,1;1,1,1)\rangle & \langle 0,(0,0,0;1,1,1;1,1,1)\rangle & \langle 3,(.2,.3,.6;.9,.8,.7;.8,.7,.5)\rangle \\ \langle 0,(0,0,0;1,1,1;1,1,1)\rangle & \langle 8,(.1,.3,.4;.8,.7,.4;.7,.3,.1)\rangle & \langle 0,(0,0,0;1,1,1;1,1,1)\rangle \end{pmatrix} \tag{2}$$

Extended union of $\mho_A^{(3,13)}$ and $\mho_B^{(3,10)}$ is given below in Eq. (3):

$\mho_C^{(3,13)} = \mho_A^{(3,13)} \widetilde{\cup}_E \mho_B^{(3,10)}$ where $C = \{e_1, e_2, e_3\}$

$$= \begin{pmatrix} \langle 0,(0,0,0;1,1,1;1,1,1)\rangle & \langle 9,(.7,.6,.4,:\ .2,.1,.4;.3,.5,.7)\rangle & \langle 0,(0,0,0;1,1,1;1,1,1)\rangle \\ \langle 11,(.1,.9,.7;.5,.3,.2;.4,.6,.8)\rangle & \langle 0,(0,0,0;1,1,1;1,1,1)\rangle & \langle 1,(.2,.1,.3;.6,.7,.9;.7,.3,.6)\rangle \\ \langle 0,(0,0,0;1,1,1;1,1,1)\rangle & \langle 0,(0,0,0;1,1,1;1,1,1)\rangle & \langle 3,(.4,.3,.4;.5,.6,.8;.7,.6,.5)\rangle \\ \langle 0,(0,0,0;1,1,1;1,1,1)\rangle & \langle 6,(.8,.6,.5;.6,.4,.7;.5.8,.9)\rangle & \langle 6,(.5,.4,.5;.6,.7,.7;.6,.4,.2)\rangle \\ \langle 9,(.2,.1,.2;.3,.4.5;.6,.7,.8)\rangle & \langle 0,(0,0,0;1,1,1;1,1,1)\rangle & \langle 5,(.1,.2,.4;8,.7,.1;.8,.3,.7)\rangle \\ \langle 0,(0,0,0;1,1,1;1,1,1)\rangle & \langle 0,(0,0,0;1,1,1;1,1,1)\rangle & \langle 5,(.6,.5,.6;.7,.8.9;.8,.7,.6)\rangle \\ \langle 8,(.3,.4,.5;.6,.7,.6;.5,.4,.3)\rangle & \langle 0,(0,0,0;1,1,1;1,1,1)\rangle & \langle 3,(.2,.3,.6;.9,.8,.7;.8,.7,.5)\rangle \\ \langle 0,(0,0,0;1,1,1;1,1,1)\rangle & \langle 8,(.1,.3,.4;.8,.7,.4;.7,.3,.1)\rangle & \langle 0,(0,0,0;1,1,1;1,1,1)\rangle \end{pmatrix} \tag{3}$$

Restricted union of $\mho_A^{(3,13)}$ and $\mho_B^{(3,10)}$ is given below in Eq. (4):

$\mho_D^{(3,13)} = \mho_A^{(3,13)} \widetilde{\cup}_R \mho_B^{(3,10)}$ where $D = \{e_3\}$

$$= \begin{pmatrix} \langle 0,(0,0,0;1,1,1;1,1,1)\rangle & \langle 0,(0,0,0;1,1,1;1,1,1)\rangle & \langle 0,(0,0,0;1,1,1;1,1,1)\rangle \\ \langle 0,(0,0,0;1,1,1;1,1,1)\rangle & \langle 0,(0,0,0;1,1,1;1,1,1)\rangle & \langle 1,(.2,.1,.3;.6,.7,.9;.7,.3,.6)\rangle \\ \langle 0,(0,0,0;1,1,1;1,1,1)\rangle & \langle 0,(0,0,0;1,1,1;1,1,1)\rangle & \langle 3,(.4,.3,.4;.5,.6,.8;.7,.6,.5)\rangle \\ \langle 0,(0,0,0;1,1,1;1,1,1)\rangle & \langle 0,(0,0,0;1,1,1;1,1,1)\rangle & \langle 6,(.5,.4,.5;.6,.7,.7;.6,.4,.2)\rangle \\ \langle 0,(0,0,0;1,1,1;1,1,1)\rangle & \langle 0,(0,0,0;1,1,1;1,1,1)\rangle & \langle 5,(.1,.2,.4;8,.7,.1;.8,.3,.7)\rangle \\ \langle 0,(0,0,0;1,1,1;1,1,1)\rangle & \langle 0,(0,0,0;1,1,1;1,1,1)\rangle & \langle 5,(.6,.5,.6;.7,.8.9;.8,.7,.6)\rangle \\ \langle 0,(0,0,0;1,1,1;1,1,1)\rangle & \langle 0,(0,0,0;1,1,1;1,1,1)\rangle & \langle 3,(.2,.3,.6;.9,.8,.7;.8,.7,.5\rangle \\ \langle 0,(0,0,0;1,1,1;1,1,1)\rangle & \langle 0,(0,0,0;1,1,1;1,1,1)\rangle & \langle 0,(0,0,0;1,1,1;1,1,1)\rangle \end{pmatrix} \tag{4}$$

Definition 3.8. Let $\mho_A^{(m,N_1)}$ and $\mho_B^{(m,N_2)}$ be MPDFNS set over X. Then their *extended intersection* is defined as

$\mho_C^{(m,N)*} = \mho_A^{(m,N_1)} \widetilde{\cap}_E \mho_B^{(m,N_2)}$,

$$\mho_C^{(m,N)*} = \left\{ \left(e, \left\{ \frac{\langle \rho, \mathfrak{l}_C(\rho)\rangle}{(\mu_{1,C}(\rho), \mu_{2,C}(\rho), \cdots, \mu_{m,C}(\rho); \nu_{1,C}(\rho), \nu_{2,C}(\rho) \cdots, \nu_{m,C}(\rho)} \right\} \right) : \right.$$
$$\left. e \in C, \rho \in X, \mathfrak{l}_C(\rho) \in \mathfrak{L} \right\}$$

where,

- $C = A \cup B$,
- $(m,\ N)^* = (m,\ \min\{N_1,\ N_2\})$,
- $\mu_C(\rho) = \min\{\mu_{i,A}(\rho),\ \mu_{i,B}(\rho)\},\ 1 \le i \le m$,
- $\nu_C(\rho) = \max\{\nu_{i,A}(\rho),\ \nu_{i,B}(\rho)\}\ 1 \le i \le m$, and
- $\mathfrak{l}_C(\rho) = \min\{\mathfrak{l}_A(\rho),\ \mathfrak{l}_B(\rho)\}\ \forall\, e \in C$.

Definition 3.9. Let $\mho_A^{(m,N_1)}$ and $\mho_B^{(m,N_2)}$ be MPDFNS set over X. Then their *restricted intersetion* is defined as

$$\mho_D^{(m,N)_\diamond} = \mho_A^{(m,N_1)} \widetilde{\cup}_R \mho_B^{(m,N_2)},$$

$$\mho_D^{(m,N)_\diamond} = \left\{\left(e, \left\{\frac{\langle \rho, \mathfrak{l}_D(\rho)\rangle}{(\mu_{1,D}(\rho), \mu_{2,D}(\rho), \cdots, \mu_{m,D}(\rho); \nu_{1,D}(\rho), \nu_{2,D}(\rho)\cdots, \nu_{m,D}(\rho)}\right\}\right) : e \in D, \rho \in X, \mathfrak{l}_D(\rho) \in \mathfrak{L}\right\}$$

where,

- $D = A \cap B$,
- $(m, N)^\diamond = (m, \min\{N_1, N_2\})$,
- $\mu_D(\rho) = \min\{\mu_{i,A}(\rho), \mu_{i,B}(\rho)\}$, $1 \le i \le m$,
- $\nu_D(\rho) = \max\{\nu_{i,A}(\rho), \nu_{i,B}(\rho)\}$ $1 \le i \le m$, and
- $\mathfrak{l}_D(\rho) = \min\{\mathfrak{l}_A(\rho), \mathfrak{l}_B(\rho)\}$ $\forall\, e \in D$.

Example 3.3. Consider $\mho_A^{(3,13)}$ and $\mho_B^{(3,10)}$ as given in Example 3.2 by Eqs. (1) and (2), respectively. Extended intersection of $\mho_A^{13}$ and $\mho_B^{10}$ is given below in Eq. (5):

$$\mho_C^{(3,10)} = \mho_A^{(3,13)} \widetilde{\cap}_E \mho_B^{(3,10)} \text{ where } C = \{e_1, e_2, e_3\}$$

$$\mho_C^{(3,10)} = \begin{pmatrix} \langle 0, (0,0,0;1,1,1;1,1,1)\rangle & \langle 0, (0,0,0;1,1,1;1,1,1)\rangle & \langle 0, (0,0,0;1,1,1;1,1,1)\rangle \\ \langle 0, (0,0,0;1,1,1;1,1,1)\rangle & \langle 0, (0,0,0;1,1,1;1,1,1)\rangle & \langle 0, (0,0,0;1,1,1;1,1,1)\rangle \\ \langle 0, (0,0,0;1,1,1;1,1,1)\rangle & \langle 0, (0,0,0;1,1,1;1,1,1)\rangle & \langle 0, (0,0,0;1,1,1;1,1,1)\rangle \\ \langle 0, (0,0,0;1,1,1;1,1,1)\rangle & \langle 0, (0,0,0;1,1,1;1,1,1)\rangle & \langle 0, (0,0,0;1,1,1;1,1,1)\rangle \\ \langle 0, (0,0,0;1,1,1;1,1,1)\rangle & \langle 0, (0,0,0;1,1,1;1,1,1)\rangle & \langle 0, (0,0,0;1,1,1;1,1,1)\rangle \\ \langle 0, (0,0,0;1,1,1;1,1,1)\rangle & \langle 0, (0,0,0;1,1,1;1,1,1)\rangle & \langle 0, (0,0,0;1,1,1;1,1,1)\rangle \\ \langle 0, (0,0,0;1,1,1;1,1,1)\rangle & \langle 0, (0,0,0;1,1,1;1,1,1)\rangle & \langle 0, (0,0,0;1,1,1;1,1,1)\rangle \\ \langle 0, (0,0,0;1,1,1;1,1,1)\rangle & \langle 0, (0,0,0;1,1,1;1,1,1)\rangle & \langle 0, (0,0,0;1,1,1;1,1,1)\rangle \end{pmatrix} \quad (5)$$

Restricted intersection of $\mho_A^{(3,13)}$ and $\mho_B^{(3,10)}$ is given below Eq. (6):

$$\mho_D^{(3,10)} = \mho_A^{(3,13)} \widetilde{\cap}_R \mho_B^{(3,10)} \text{ where } D = \{e_3\}$$

$$\mho_D^{(3,10)} = \begin{pmatrix} \langle 0,(0,0,0;1,1,1;1,1,1)\rangle & \langle 0,(0,0,0;1,1,1;1,1,1)\rangle & \langle 0,(0,0,0;1,1,1;1,1,1)\rangle \\ \langle 0,(0,0,0;1,1,1;1,1,1)\rangle & \langle 0,(0,0,0;1,1,1;1,1,1)\rangle & \langle 0,(0,0,0;1,1,1;1,1,1)\rangle \\ \langle 0,(0,0,0;1,1,1;1,1,1)\rangle & \langle 0,(0,0,0;1,1,1;1,1,1)\rangle & \langle 0,(0,0,0;1,1,1;1,1,1)\rangle \\ \langle 0,(0,0,0;1,1,1;1,1,1)\rangle & \langle 0,(0,0,0;1,1,1;1,1,1)\rangle & \langle 0,(0,0,0;1,1,1;1,1,1)\rangle \\ \langle 0,(0,0,0;1,1,1;1,1,1)\rangle & \langle 0,(0,0,0;1,1,1;1,1,1)\rangle & \langle 0,(0,0,0;1,1,1;1,1,1)\rangle \\ \langle 0,(0,0,0;1,1,1;1,1,1)\rangle & \langle 0,(0,0,0;1,1,1;1,1,1)\rangle & \langle 0,(0,0,0;1,1,1;1,1,1)\rangle \\ \langle 0,(0,0,0;1,1,1;1,1,1)\rangle & \langle 0,(0,0,0;1,1,1;1,1,1)\rangle & \langle 0,(0,0,0;1,1,1;1,1,1)\rangle \\ \langle 0,(0,0,0;1,1,1;1,1,1)\rangle & \langle 0,(0,0,0;1,1,1;1,1,1)\rangle & \langle 0,(0,0,0;1,1,1;1,1,1)\rangle \end{pmatrix} \tag{6}$$

Proposition 3.3. If $\mho_A^{(m,N_1)}$, $\mho_B^{(m,N_2)}$, $\mho_C^{(m,N_3)}$ are MPDNFNS set over X, then

(*i*) $\mho_A^{(m,N_1)} \widetilde{\cap}_E \mho_A^{(m,N_1)} = \mho_A^{(m,N_1)}$.

(*ii*) $\mho_A^{(m,N_1)} \widetilde{\cup}_E \mho_A^{(m,N_1)} = \mho_A^{(m,N_1)}$.

(*iii*) $\mho_A^{(m,N_1)} \widetilde{\cap}_E \mho_B^{(m,N_2)} = \mho_B^{(m,N_2)} \widetilde{\cap}_E \mho_A^{(m,N_1)}$.

(*iv*) $\mho_A^{(m,N_1)} \widetilde{\cup}_E \mho_B^{(m,N_2)} = \mho_B^{(m,N_2)} \widetilde{\cup}_E \mho_A^{(m,N_1)}$.

(*v*) $\mho_A^{(m,N_1)} \widetilde{\cap}_E \left(\mho_B^{(m,N_2)} \widetilde{\cap}_E \mho_C^{(m,N_3)}\right) = \left(\mho_A^{(m,N_1)} \widetilde{\cap}_E \mho_B^{(m,N_2)}\right) \widetilde{\cap} \mho_C^{(m,N_3)}$.

(*vi*) $\mho_A^{(m,N_1)} \widetilde{\cup}_E \left(\mho_B^{(m,N_2)} \widetilde{\cup}_E \mho_C\right) = \left(\mho_A^{(m,N_1)} \widetilde{\cup}_E \mho_B^{(m,N_2)}\right) \widetilde{\cup}_E \mho_C^{(m,N_3)}$.

(*vii*) $\mho_A^{(m,N_1)} \widetilde{\cap}_R \mho_A^{(m,N_1)} = \mho_A^{(m,N_1)}$.

(*viii*) $\mho_A^{(m,N_1)} \widetilde{\cup}_R \mho_A^{(m,N_1)} = \mho_A^{(m,N_1)}$.

(*ix*) $\mho_A^{(m,N_1)} \widetilde{\cap}_R \mho_B^{(m,N_2)} = \mho_B^{(m,N_2)} \widetilde{\cap}_R \mho_A^{(m,N_1)}$.

(*x*) $\mho_A^{(m,N_1)} \widetilde{\cup}_R \mho_B^{(m,N_2)} = \mho_B^{(m,N_2)} \widetilde{\cup}_R \mho_A^{(m,N_1)}$.

(*xi*) $\mho_A^{(m,N_1)} \widetilde{\cap}_R \left(\mho_B^{(m,N_2)} \widetilde{\cap}_R \mho_C^{(m,N_3)}\right) = \left(\mho_A^{(m,N_1)} \widetilde{\cap}_R \mho_B^{(m,N_2)}\right) \widetilde{\cap} \mho_C^{(m,N_3)}$.

(*xii*) $\mho_A^{(m,N_1)} \widetilde{\cup}_R \left(\mho_B^{(m,N_2)} \widetilde{\cup}_R \mho_C\right) = \left(\mho_A^{(m,N_1)} \widetilde{\cup}_R \mho_B^{(m,N_2)}\right) \widetilde{\cup}_R \mho_C^{(m,N_3)}$.

Proof. The proof is obvious from Definitions 3.4, 3.5, 3.6, 3.7, 3.8 and 3.9.

Proposition 3.4. If $\mho_A^{(m,N_1)}$ and $\mho_B^{(m,N_2)}$ are MPDNFNS sets over X, then

(*i*) $\left(\mho_A^{(m,N_1)} \widetilde{\cap}_E \mho_B^{(m,N_2)}\right) \widetilde{\subseteq} \mho_A^{(m,N_1)} \widetilde{\subseteq} \left(\mho_A^{(m,N_1)} \widetilde{\cup}_E \mho_B^{(m,N_2)}\right)$.

(*ii*) $\left(\mho_A^{(m,N_1)} \widetilde{\cap}_E \mho_B^{(m,N_2)}\right) \widetilde{\subseteq} \mho_B^{(m,N_2)} \widetilde{\subseteq} \left(\mho_A^{(m,N_1)} \widetilde{\cup}_E \mho_B^{(m,N_2)}\right)$.

(*iii*) $\left(\mho_A^{(m,N_1)} \widetilde{\cap}_R \mho_B^{(m,N_2)}\right) \widetilde{\subseteq} \mho_A^{(m,N_1)} \widetilde{\subseteq} \mho_A^{(m,N_1)} \widetilde{\cup}_E \mho_B^{(m,N_2)}\Big)$.

(*iv*) $\left(\mho_A^{(m,N_1)} \widetilde{\cap}_R \mho_B^{(m,N_2)}\right) \widetilde{\subseteq} \mho_B^{(m,N_2)} \widetilde{\subseteq} \left(\mho_A^{(m,N_1)} \widetilde{\cup}_E \mho_B^{(m,N_2)}\right)$.

Proof. Proof of this proposition is obvious from Definitions 3.4, 3.6, 3.8 and 3.9.

4 Complements of m-Polar Neutrosophic Diophantine N-Soft Set

Definition 4.1. Let $\mho_A^{(m,N)}$ be MPDNNS sets over X. Then their *weak complement* is represented as $\left(\mho_A^{(m,N)}\right)^{wc}$ and defined by

$$\left(\mho_A^{(m,N)}\right)^{wc} = \left\{\left(e, \left\{\frac{\left\langle \rho, \mathfrak{l}_A^{wc}(\rho)\right\rangle}{\left(\mu_{1,A}^{wc}(\rho), \mu_{2,A}^{wc}(\rho), \cdots, \mu_{m,A}^{wc}(\rho); \lambda_{1,A}^{wc}(\rho), \lambda_{2,A}^{wc}(\rho), \cdots, \lambda_{m,A}^{wc}(\rho); \nu_{1,A}^{wc}(\rho), \nu_{2,A}^{wc}(\rho) \cdots, \nu_{m,A}^{wc}(\rho)\right)}\right\}\right) \mid e \in A, \rho \in X, \mathfrak{l}_A^{wc}(\rho) \in \mathfrak{L}\right\}$$

where,

- $\mu_{i,A}^{wc}(\rho) = \nu_{i,A}(\rho),\ 1 \leq i \leq m,$
- $\lambda_{i,A}^{wc}(\rho) = 1 - \lambda_{i,A}(\rho),\ 1 \leq i \leq m,$
- $\nu_{i,A}^{wc}(\rho) = \mu_{i,A}(\rho),\ 1 \leq i \leq m,$ and
- $\mathfrak{l}_A^{wc}(\rho) \cap \mathfrak{l}_A(\rho) = \varnothing,\ \forall\ e \in A.$

Example 4.1. Consider $\mho_A^{(3,13)}$ and $\mho_B^{(3,10)}$ as given in Example 3.2 by Eqs. (1) and (2), respectively. Weak complement of $\mho_A^{(3,13)}$ and $\mho_B^{(3,10)}$ are given below in Eqs. (7) and (8):

$$\mho_A^{(3,13)wc} = \begin{pmatrix} \langle 0, (0,0,0;1,1,1;1,1,1)\rangle & \langle 0, (0,0,0;1,1,1;1,1,1)\rangle & \langle 0, (0,0,0;1,1,1;1,1,1)\rangle \\ \langle 10, (.4,.6,.8;.5,.7,0.8;.1,.9,.7)\rangle & \langle 0, (0,0,0;1,1,1;1,1,1)\rangle & \langle 0, (0,0,0;1,1,1;1,1,1)\rangle \\ \langle 0, (0,0,0;1,1,1;1,1,1)\rangle & \langle 0, (0,0,0;1,1,1;1,1,1)\rangle & \langle 5, (.7,.6,.5;.5,.4,.2;.4,.3,.4)\rangle \\ \langle 0, (0,0,0;1,1,1;1,1,1)\rangle & \langle 0, (0,0,0;1,1,1;1,1,1)\rangle & \langle 8, (.6,.4,.2;.4,.3,.3;.5,.4,.5)\rangle \\ \langle 8, (.6,.7,.8;.7,.6,.5;.2,.1,.2)\rangle & \langle 0, (0,0,0;1,1,1;1,1,1)\rangle & \langle 0, (0,0,0;1,1,1;1,1,1)\rangle \\ \langle 0, (0,0,0;1,1,1;1,1,1)\rangle & \langle 0, (0,0,0;1,1,1;1,1,1)\rangle & \langle 7, (.8,.7,.6;.3,.2.1;.6,.5,.6)\rangle \\ \langle 6, (.5,.4,.3;.4,.3,.4;.3,.4,.5)\rangle & \langle 0, (0,0,0;1,1,1;1,1,1)\rangle & \langle 0, (0,0,0;1,1,1;1,1,1)\rangle \\ \langle 0, (0,0,0;1,1,1;1,1,1)\rangle & \langle 0, (0,0,0;1,1,1;1,1,1)\rangle & \langle 0, (0,0,0;1,1,1;1,1,1)\rangle \end{pmatrix} \quad (7)$$

$$\mho_B^{(3,13)wc} = \begin{pmatrix} \langle 0,(0,0,0;1,1,1;1,1,1)\rangle & \langle 8,(.3,.5,.7;.8,.9,.6;.7,.6,.4)\rangle & \langle 0,(0,0,0;1,1,1;1,1,1)\rangle \\ \langle 0,(0,0,0;1,1,1;1,1,1)\rangle & \langle 0,(0,0,0;1,1,1;1,1,1)\rangle & \langle 7,(.7,.3,.6;.4,.3,.1;.2,.1,.3)\rangle \\ \langle 0,(0,0,0;1,1,1;1,1,1)\rangle & \langle 0,(0,0,0;1,1,1;1,1,1)\rangle & \langle 0,(0,0,0;1,1,1;1,1,1)\rangle \\ \langle 0,(0,0,0;1,1,1;1,1,1)\rangle & \langle 4,(.5,.8,.9;.4,.6,.3;.8,.6,.5)\rangle & \langle 0,(0,0,0;1,1,1;1,1,1)\rangle \\ \langle 0,(0,0,0;1,1,1;1,1,1)\rangle & \langle 0,(0,0,0;1,1,1;1,1,1)\rangle & \langle 6,(.8,.3,.7;.2,.3,.9;.1,.2,.4)\rangle \\ \langle 0,(0,0,0;1,1,1;1,1,1)\rangle & \langle 0,(0,0,0;1,1,1;1,1,1)\rangle & \langle 0,(0,0,0;1,1,1;1,1,1)\rangle \\ \langle 0,(0,0,0;1,1,1;1,1,1)\rangle & \langle 0,(0,0,0;1,1,1;1,1,1)\rangle & \langle 2,(.8,.7,.5;.1,.2,.3;.2,.3,.6)\rangle \\ \langle 0,(0,0,0;1,1,1;1,1,1)\rangle & \langle 6,(.7,.3,.1;.8,.7,.4;.1,.3,.4)\rangle & \langle 0,(0,0,0;1,1,1;1,1,1)\rangle \end{pmatrix} \tag{8}$$

Remark 4.1 Again taking week complement of $\left(\mho_A^{(3,13)}\right)^{wc}$ and $\left(\mho_B^{(3,10)}\right)^{wc}$ which given above in Eqs. (7), (8), respectively. There compliments are given below in Eqs. (9) and (10), respectively.

$$\left(\left(\mho_A^{(3,13)wc}\right)^{wc}\right) = \begin{pmatrix} \langle 0,(0,0,0;1,1,1;1,1,1)\rangle & \langle 0,(0,0,0;1,1,1;1,1,1)\rangle & \langle 0,(0,0,0;1,1,1;1,1,1)\rangle \\ \langle 6,(.1,.9,.7;.5,.3,0.2;.4,.6,.8)\rangle & \langle 0,(0,0,0;1,1,1;1,1,1)\rangle & \langle 0,(0,0,0;1,1,1;1,1,1)\rangle \\ \langle 0,(0,0,0;1,1,1;1,1,1)\rangle & \langle 0,(0,0,0;1,1,1;1,1,1)\rangle & \langle 4,(.4,.3,.4;.5,.6,.8;.7,.6,.5)\rangle \\ \langle 0,(0,0,0;1,1,1;1,1,1)\rangle & \langle 0,(0,0,0;1,1,1;1,1,1)\rangle & \langle 5,(.5,.4,.5;.6,.7,.7;.6,.4,.2)\rangle \\ \langle 7,(.2,.1,.2;.3,.4.5;.6,.7,.8)\rangle & \langle 0,(0,0,0;1,1,1;1,1,1)\rangle & \langle 0,(0,0,0;1,1,1;1,1,1)\rangle \\ \langle 0,(0,0,0;1,1,1;1,1,1)\rangle & \langle 0,(0,0,0;1,1,1;1,1,1)\rangle & \langle 7,(.6,.5,.6;.7,.8.9;.8,.7,.6)\rangle \\ \langle 7,(.3,.4,.5;.6,.7,.6;.5,.4,.3)\rangle & \langle 0,(0,0,0;1,1,1;1,1,1)\rangle & \langle 0,(0,0,0;1,1,1;1,1,1)\rangle \\ \langle 0,(0,0,0;1,1,1;1,1,1)\rangle & \langle 0,(0,0,0;1,1,1;1,1,1)\rangle & \langle 0,(0,0,0;1,1,1;1,1,1)\rangle \end{pmatrix} \tag{9}$$

$$\left(\left(\mho_A^{(3,10)wc}\right)^{wc}\right) = \begin{pmatrix} \langle 0,(0,0,0;1,1,1;1,1,1)\rangle & \langle 4,(.7,.6,.4,:.2,.1,.4;.3,.5,.7)\rangle & \langle 0,(0,0,0;1,1,1;1,1,1)\rangle \\ \langle 0,(0,0,0;1,1,1;1,1,1)\rangle & \langle 0,(0,0,0;1,1,1;1,1,1)\rangle & \langle 3,(.2,.1,.3;.6,.7,.9;.7,.3,.6)\rangle \\ \langle 0,(0,0,0;1,1,1;1,1,1)\rangle & \langle 0,(0,0,0;1,1,1;1,1,1)\rangle & \langle 0,(0,0,0;1,1,1;1,1,1)\rangle \\ \langle 0,(0,0,0;1,1,1;1,1,1)\rangle & \langle 7,(.8,.6,.5;.6,.4,.7;.5,.8,.9)\rangle & \langle 0,(0,0,0;1,1,1;1,1,1)\rangle \\ \langle 0,(0,0,0;1,1,1;1,1,1)\rangle & \langle 0,(0,0,0;1,1,1;1,1,1)\rangle & \langle 6,(.1,.2,.4;.8,.7,.1;.8,.3,.7)\rangle \\ \langle 0,(0,0,0;1,1,1;1,1,1)\rangle & \langle 0,(0,0,0;1,1,1;1,1,1)\rangle & \langle 0,(0,0,0;1,1,1;1,1,1)\rangle \\ \langle 0,(0,0,0;1,1,1;1,1,1)\rangle & \langle 0,(0,0,0;1,1,1;1,1,1)\rangle & \langle 5,(.2,.3,.6;.9,.8,.7;.8,.7,.5)\rangle \\ \langle 0,(0,0,0;1,1,1;1,1,1)\rangle & \langle 9,(.1,.3,.4;.8,.7,.4;.7,.3,.1)\rangle & \langle 0,(0,0,0;1,1,1;1,1,1)\rangle \end{pmatrix} \tag{10}$$

Hence from Eqs. (1), (2), (9) and (10). We conclude that $((\mho_A^{(3,13)})^{wc})^{wc} \neq \mho_A^{(3,13)}$ and $((\mho_B^{(3,10)})^{wc})^{wc} \neq \mho_B^{(3,10)}$.

Since null 3PDN10S set is

$$\mho_{\varnothing}^{(3,0)} = \begin{pmatrix} \langle 0,(0,0,0;1,1,1;1,1,1)\rangle & \langle 0,(0,0,0;1,1,1;1,1,1)\rangle & \langle 0,(0,0,0;1,1,1;1,1,1)\rangle \\ \langle 0,(0,0,0;1,1,1;1,1,1)\rangle & \langle 0,(0,0,0;1,1,1;1,1,1)\rangle & \langle 0,(0,0,0;1,1,1;1,1,1)\rangle \\ \langle 0,(0,0,0;1,1,1;1,1,1)\rangle & \langle 0,(0,0,0;1,1,1;1,1,1)\rangle & \langle 0,(0,0,0;1,1,1;1,1,1)\rangle \\ \langle 0,(0,0,0;1,1,1;1,1,1)\rangle & \langle 0,(0,0,0;1,1,1;1,1,1)\rangle & \langle 0,(0,0,0;1,1,1;1,1,1)\rangle \\ \langle 0,(0,0,0;1,1,1;1,1,1)\rangle & \langle 0,(0,0,0;1,1,1;1,1,1)\rangle & \langle 0,(0,0,0;1,1,1;1,1,1)\rangle \\ \langle 0,(0,0,0;1,1,1;1,1,1)\rangle & \langle 0,(0,0,0;1,1,1;1,1,1)\rangle & \langle 0,(0,0,0;1,1,1;1,1,1)\rangle \\ \langle 0,(0,0,0;1,1,1;1,1,1)\rangle & \langle 0,(0,0,0;1,1,1;1,1,1)\rangle & \langle 0,(0,0,0;1,1,1;1,1,1)\rangle \end{pmatrix} \tag{11}$$

Since absolute 3PDN10S set is

$$\mho_{E}^{(3,10)} = \begin{pmatrix} \langle 9,(1,1,1;0,0,0;0,0,0)\rangle & \langle 9,(1,1,1;0,0,0;0,0,0)\rangle & \langle 9,(1,1,1;0,0,0;0,0,0)\rangle \\ \langle 9,(1,1,1;0,0,0;0,0,0)\rangle & \langle 9,(1,1,1;0,0,0;0,0,0)\rangle & \langle 9,(1,1,1;0,0,0;0,0,0)\rangle \\ \langle 9,(1,1,1;0,0,0;0,0,0)\rangle & \langle 9,(1,1,1;0,0,0;0,0,0)\rangle & \langle 9,(1,1,1;0,0,0;0,0,0)\rangle \\ \langle 9,(1,1,1;0,0,0;0,0,0)\rangle & \langle 9,(1,1,1;0,0,0;0,0,0)\rangle & \langle 9,(1,1,1;0,0,0;0,0,0)\rangle \\ \langle 9,(1,1,1;0,0,0;0,0,0)\rangle & \langle 9,(1,1,1;0,0,0;0,0,0)\rangle & \langle 9,(1,1,1;0,0,0;0,0,0)\rangle \\ \langle 9,(1,1,1;0,0,0;0,0,0)\rangle & \langle 9,(1,1,1;0,0,0;0,0,0)\rangle & \langle 9,(1,1,1;0,0,0;0,0,0)\rangle \\ \langle 9,(1,1,1;0,0,0;0,0,0)\rangle & \langle 9,(1,1,1;0,0,0;0,0,0)\rangle & \langle 9,(1,1,1;0,0,0;0,0,0)\rangle \end{pmatrix} \tag{12}$$

Weak complement of $\mho_{\varnothing}^{(3,0)}$ and $\mho_{E}^{(3,10)}$ are given below in Eqs. (13) and (14), respectively.

$$(\mho_{\Phi}^{(3,0)})^{wc} = \begin{pmatrix} \langle 7,(1,1,1;0,0,0;0,0,0)\rangle & \langle 6,(1,1,1;0,0,0;0,0,0)\rangle & \langle 4,(1,1,1;0,0,0;0,0,0)\rangle \\ \langle 1,(1,1,1;0,0,0;0,0,0)\rangle & \langle 8,(1,1,1;0,0,0;0,0,0)\rangle & \langle 7,(1,1,1;0,0,0;0,0,0)\rangle \\ \langle 6,(1,1,1;0,0,0;0,0,0)\rangle & \langle 7,(1,1,1;0,0,0;0,0,0)\rangle & \langle 3,(1,1,1;0,0,0;0,0,0)\rangle \\ \langle 5,(1,1,1;0,0,0;0,0,0)\rangle & \langle 2,(1,1,1;0,0,0;0,0,0)\rangle & \langle 4,(1,1,1;0,0,0;0,0,0)\rangle \\ \langle 7,(1,1,1;0,0,0;0,0,0)\rangle & \langle 8,(1,1,1;0,0,0;0,0,0)\rangle & \langle 6,(1,1,1;0,0,0;0,0,0)\rangle \\ \langle 8,(1,1,1;0,0,0;0,0,0)\rangle & \langle 7,(1,1,1;0,0,0;0,0,0)\rangle & \langle 2,(1,1,1;0,0,0;0,0,0)\rangle \\ \langle 1,(1,1,1;0,0,0;0,0,0)\rangle & \langle 6,(1,1,1;0,0,0;0,0,0)\rangle & \langle 5,(1,1,1;0,0,0;0,0,0)\rangle \end{pmatrix} \tag{13}$$

$$(\mho_E^{(3,10)})^{wc} = \begin{pmatrix} \langle 9,(0,0,0;1,1,1;1,1,1)\rangle & \langle 5,(0,0,0;1,1,1;1,1,1)\rangle & \langle 5,(0,0,0;1,1,1;1,1,1)\rangle \\ \langle 1,(0,0,0;1,1,1;1,1,1)\rangle & \langle 3,(0,0,0;1,1,1;1,1,1)\rangle & \langle 9,(0,0,0;1,1,1;1,1,1)\rangle \\ \langle 8,(0,0,0;1,1,1;1,1,1)\rangle & \langle 9,(0,0,0;1,1,1;1,1,1)\rangle & \langle 6,(0,0,0;1,1,1;1,1,1)\rangle \\ \langle 2,(0,0,0;1,1,1;1,1,1)\rangle & \langle 5,(0,0,0;1,1,1;1,1,1)\rangle & \langle 5,(0,0,0;1,1,1;1,1,1)\rangle \\ \langle 7,(0,0,0;1,1,1;1,1,1)\rangle & \langle 4,(0,0,0;1,1,1;1,1,1)\rangle & \langle 7,(0,0,0;1,1,1;1,1,1)\rangle \\ \langle 4,(0,0,0;1,1,1;1,1,1)\rangle & \langle 6,(0,0,0;1,1,1;1,1,1)\rangle & \langle 5,(0,0,0;1,1,1;1,1,1)\rangle \\ \langle 1,(0,0,0;1,1,1;1,1,1)\rangle & \langle 8,(0,0,0;1,1,1;1,1,1)\rangle & \langle 9,(0,0,0;1,1,1;1,1,1)\rangle \end{pmatrix} \tag{14}$$

From Eqs. (11)–(14), we have $\left(\mho_{\varnothing}^{(3,0)}\right)^{wc} \neq \mho_E^{(3,10)}$ and $\left(\mho_E^{(3,10)}\right)^{wc} \neq \mho_{\varnothing}^{(3,0)}$. Hence, we have following result.

Remark 4.2 Let $\mho_A^{(m,N)}$ be MPDNNS sets, $\mho_{\varnothing}^{(m,0)}$ be null MPDNNS and $\mho_E^{(m,N-1)}$ absolute MPDNNS over X, then following results that hold in crisp set theory but not hold in MPDNNS set theory

(*i*) $(\mho_{\varnothing}^{(m,0)})^{wc} \neq \mho_E^{(m,N-1)}$.

(*ii*) $(\mho_E^{(m,N-1)})^{wc} \neq \mho_{\varnothing}^{(m,0)}$.

(*iii*) $((\mho_A^{(m,N)})^{wc})^{wc} \neq \mho_A^{(m,N)}$.

Definition 4.2. Let $\mho_A^{(m,N)}$ be MPDNNS set over X. Then their *top weak complement* is represented as $(\mho_A^{(m,N)})^{twc}$ and defined by

$$(\mho_A^{(m,N)})^{twc} = \left\{\left(e, \left\{\frac{\langle \rho, \mathfrak{l}_A^{twc}(\rho)\rangle}{(\mu_{1,A}^{twc}(\rho), \mu_{2,A}^{twc}(\rho), \cdots, \mu_{m,A}^{twc}(\rho); \lambda_{1,A}^{twc}(\rho), \lambda_{2,A}^{twc}(\rho), \cdots, \lambda_{m,A}^{twc}(\rho); \nu_{1,A}^{twc}(\rho), \nu_{2,A}^{twc}(\rho) \cdots, \nu_{m,A}^{twc}(\rho)}\right\}\right) \middle| e \in A, \rho \in X, \mathfrak{l}_A^{twc}(\rho) \in \mathfrak{L}\right\}$$

where,

- $\mu_{i,A}^{twc}(\rho) = \nu_{i,A}(\rho),\ 1 \leq i \leq m,$
- $\lambda_{i,A}^{twc}(\rho) = 1 - \lambda_{i,A}(\rho),\ 1 \leq i \leq m,$
- $\nu_{i,A}^{twc}(\rho) = \mu_{i,A}(\rho),\ 1 \leq i \leq m,$ and
- $\mathfrak{l}_A^{twc}(\rho) = \begin{cases} N-1, & \text{if } \mathfrak{l}_A(\rho) < N-1, \\ 0, & \text{if } \mathfrak{l}_A(\rho) = N-1, \forall e \in A. \end{cases}$

The top weak complement of $\mho_A^{(3,13)}$ and $\mho_B^{(3,10)}$ as given in Example 3.2 by Eqs. (1) and (2) are given below in Eqs. (15) and (16), respectively.

$$(\mho_A^{(3,13)})^{twc} = \begin{pmatrix} \langle 0,(0,0,0;1,1,1;1,1,1)\rangle & \langle 0,(0,0,0;1,1,1;1,1,1)\rangle & \langle 0,(0,0,0;1,1,1;1,1,1)\rangle \\ \langle 12,(.4,.6,.8;.5,.7,0.8;.1,.9,.7)\rangle & \langle 0,(0,0,0;1,1,1;1,1,1)\rangle & \langle 0,(0,0,0;1,1,1;1,1,1)\rangle \\ \langle 0,(0,0,0;1,1,1;1,1,1)\rangle & \langle 0,(0,0,0;1,1,1;1,1,1)\rangle & \langle 12,(.7,.6,.5;.5,.4,.2;.4,.3,.4)\rangle \\ \langle 0,(0,0,0;1,1,1;1,1,1)\rangle & \langle 0,(0,0,0;1,1,1;1,1,1)\rangle & \langle 12,(.6,.4,.2;.4,.3,.3;.5,.4,.5)\rangle \\ \langle 12,(.6,.7,.8;.7,.6.5;.2,.1,.2)\rangle & \langle 0,(0,0,0;1,1,1;1,1,1)\rangle & \langle 0,(0,0,0;1,1,1;1,1,1)\rangle \\ \langle 0,(0,0,0;1,1,1;1,1,1)\rangle & \langle 0,(0,0,0;1,1,1;1,1,1)\rangle & \langle 12,(.8,.7,.6;.3,.2.1;.6,.5,.6)\rangle \\ \langle 12,(.5,.4,.3;.4,.3,.4;.3,.4,.5)\rangle & \langle 0,(0,0,0;1,1,1;1,1,1)\rangle & \langle 0,(0,0,0;1,1,1;1,1,1)\rangle \\ \langle 0,(0,0,0;1,1,1;1,1,1)\rangle & \langle 0,(0,0,0;1,1,1;1,1,1)\rangle & \langle 0,(0,0,0;1,1,1;1,1,1)\rangle \end{pmatrix} \tag{15}$$

$$(\mho_B^{(3,10)})^{twc} = \begin{pmatrix} \langle 0,(0,0,0;1,1,1;1,1,1)\rangle & \langle 12,(.3,.5,.7;.8,.9,.6;.7,.6,.4)\rangle & \langle 0,(0,0,0;1,1,1;1,1,1)\rangle \\ \langle 0,(0,0,0;1,1,1;1,1,1)\rangle & \langle 0,(0,0,0;1,1,1;1,1,1)\rangle & \langle 12,(.7,.3,.6;.4,.3,.1;.2,.1,.3)\rangle \\ \langle 0,(0,0,0;1,1,1;1,1,1)\rangle & \langle 0,(0,0,0;1,1,1;1,1,1)\rangle & \langle 0,(0,0,0;1,1,1;1,1,1)\rangle \\ \langle 0,(0,0,0;1,1,1;1,1,1)\rangle & \langle 12,(.5,.8,.9;.4,.6,.3;.8,.6,.5)\rangle & \langle 0,(0,0,0;1,1,1;1,1,1)\rangle \\ \langle 0,(0,0,0;1,1,1;1,1,1)\rangle & \langle 0,(0,0,0;1,1,1;1,1,1)\rangle & \langle 12,(.8,.3,.7;.2,.3,.9;.1,.2,.4)\rangle \\ \langle 0,(0,0,0;1,1,1;1,1,1)\rangle & \langle 0,(0,0,0;1,1,1;1,1,1)\rangle & \langle 0,(0,0,0;1,1,1;1,1,1)\rangle \\ \langle 0,(0,0,0;1,1,1;1,1,1)\rangle & \langle 0,(0,0,0;1,1,1;1,1,1)\rangle & \langle 12,(.8,.7,.5;.1,.2,.3;.2,.3,.6)\rangle \\ \langle 0,(0,0,0;1,1,1;1,1,1)\rangle & \langle 12,(.7,.3,.1;.8,.7,.4;.1,.3,.4)\rangle & \langle 0,(0,0,0;1,1,1;1,1,1)\rangle \end{pmatrix} \tag{16}$$

Proposition 4.1 Let $\mho_A^{(m,N)}$ be MPDNNS set, $\mho_\varnothing^{(m,0)}$ be null MPDNNS and $\mho_E^{(m,N-1)}$ absolute MPDNNS over X, then

(*i*) $(\mho_\varnothing^{(m,0)})^{twc} = \mho_E^{(m,N-1)}$.

(*ii*) $(\mho_E^{(m,N-1)})^{twc} = \mho_\varnothing^{(m,0)}$.

Remark 4.3. Again taking top weak complement of $\left(\mho_A^{(3,13)}\right)^{twc}$ given in Eq. (15) we have Eq. (17)

$$((\mho_A^{(3,13)})^{twc})^{twc} = \begin{pmatrix} \langle 0,(0,0,0;1,1,1;1,1,1)\rangle & \langle 0,(0,0,0;1,1,1;1,1,1)\rangle & \langle 0,(0,0,0;1,1,1;1,1,1)\rangle \\ \langle 0,(.1,.9,.7;.5,.3,0.2;.4,.6,.8)\rangle & \langle 0,(0,0,0;1,1,1;1,1,1)\rangle & \langle 0,(0,0,0;1,1,1;1,1,1)\rangle \\ \langle 0,(0,0,0;1,1,1;1,1,1)\rangle & \langle 0,(0,0,0;1,1,1;1,1,1)\rangle & \langle 0,(.4,.3,.4;.5,.6,.8;.7,.6,.5)\rangle \\ \langle 0,(0,0,0;1,1,1;1,1,1)\rangle & \langle 0,(0,0,0;1,1,1;1,1,1)\rangle & \langle 0,(.5,.4,.5;.6,.7,.7;.6,.4,.2)\rangle \\ \langle 0,(.2,.1,.2;.3,.4.5;.6,.7,.8)\rangle & \langle 0,(0,0,0;1,1,1;1,1,1)\rangle & \langle 0,(0,0,0;1,1,1;1,1,1)\rangle \\ \langle 0,(0,0,0;1,1,1;1,1,1)\rangle & \langle 0,(0,0,0;1,1,1;1,1,1)\rangle & \langle 0,(.6,.5,.6;.7,.8.9;.8,.7,.6)\rangle \\ \langle 0,(.3,.4,.5;.6,.7,.6;.5,.4,.3)\rangle & \langle 0,(0,0,0;1,1,1;1,1,1)\rangle & \langle 0,(0,0,0;1,1,1;1,1,1)\rangle \\ \langle 0,(0,0,0;1,1,1;1,1,1)\rangle & \langle 0,(0,0,0;1,1,1;1,1,1)\rangle & \langle 0,(0,0,0;1,1,1;1,1,1)\rangle \end{pmatrix} \tag{17}$$

Hence from Eqs. (1) and (17) we conclude that $\left(\left(\mho_A^{(3,13)}\right)^{twc}\right)^{twc} \neq \mho_A^{(3,13)}$. Hence we have the following result:

Remark 4.4. Let $\mho_A^{(m,N)}$ be MPDNNS set, over X, then $((\mho_A^{(m,N)})^{twc})^{twc} \neq \mho_A^{(m,N)}$.

Definition 4.3. Let $\mho_A^{(m,N)}$ be MPDNNS set over X. Then their *bottom weak complement* is represented as $\left(\mho_A^{(m,N)}\right)^{bwc}$ and defined by

$$(\mho_A^{(m,N)})^{bwc} = \left\{\left(e,\left\{\frac{\langle \rho, \mathfrak{l}_A^{bwc}(\rho)\rangle}{(\mu_{1,A}^{bwc}(\rho),\mu_{2,A}^{bwc}(\rho),\cdots,\mu_{m,A}^{bwc}(\rho);\lambda_{1,A}^{bwc}(\rho),\lambda_{2,A}^{bwc}(\rho),\cdots,\lambda_{m,A}^{bwc}(\rho);\nu_{1,A}^{bwc}(\rho),\nu_{2,A}^{bwc}(\rho)\cdots,\nu_{m,A}^{bwc}(\rho))}\right\}\right)\right.$$
$$\left. | e \in A, \rho \in X, \mathfrak{l}_A^{bwc}(\rho) \in \mathfrak{L}\right\}$$

where,

- $\mu_{i,A}^{bwc}(\rho) = \nu_{i,A}(\rho),\ 1 \le i \le m,$
- $\lambda_{i,A}^{bwc}(\rho) = 1 - \lambda_{i,A}(\rho),\ 1 \le i \le m,$
- $\nu_{i,A}^{bwc}(\rho) = \mu_{i,A}(\rho),\ 1 \le i \le m$, and
- $\mathfrak{l}_A^{bwc}(\rho) = \begin{cases} N-1, & \text{if } \mathfrak{l}_A(\rho) = 0, \\ 0, & \text{if} \mathfrak{l}_A(\rho) > 0, \forall e \in A. \end{cases}$

Example 4.2. Consider $\mho_A^{(3,13)}$ and $\mho_B^{(3,10)}$ as given in Example 3.2 by Eqs. (1) and (2), respectively. Bottom weak complement of $\mho_A^{(3,13)}$ and $\mho_B^{(3,10)}$ are given below in Eqs. (18) and (19):

$$(\mho_A^{(3,13)})^{bwc} = \begin{pmatrix} \langle 0,(0,0,0;1,1,1;1,1,1)\rangle & \langle 0,(0,0,0;1,1,1;1,1,1)\rangle & \langle 0,(0,0,0;1,1,1;1,1,1)\rangle \\ \langle 0,(.4,.6,.8;.5,.7,0.8;.1,.9,.7)\rangle & \langle 0,(0,0,0;1,1,1;1,1,1)\rangle & \langle 0,(0,0,0;1,1,1;1,1,1)\rangle \\ \langle 0,(0,0,0;1,1,1;1,1,1)\rangle & \langle 0,(0,0,0;1,1,1;1,1,1)\rangle & \langle 0,(.7,.6,.5;.5,.4,.2;.4,.3,.4)\rangle \\ \langle 0,(0,0,0;1,1,1;1,1,1)\rangle & \langle 0,(0,0,0;1,1,1;1,1,1)\rangle & \langle 0,(.6,.4,.2;.4,.3,.3;.5,.4,.5)\rangle \\ \langle 0,(.6,.7,.8;.7,.6.5;.2,.1,.2)\rangle & \langle 0,(0,0,0;1,1,1;1,1,1)\rangle & \langle 0,(0,0,0;1,1,1;1,1,1)\rangle \\ \langle 0,(0,0,0;1,1,1;1,1,1)\rangle & \langle 0,(0,0,0;1,1,1;1,1,1)\rangle & \langle 0,(.8,.7,.6;.3,.2.1;.6,.5,.6)\rangle \\ \langle 0,(.5,.4,.3;.4,.3,.4;.3,.4,.5)\rangle & \langle 0,(0,0,0;1,1,1;1,1,1)\rangle & \langle 0,(0,0,0;1,1,1;1,1,1)\rangle \\ \langle 0,(0,0,0;1,1,1;1,1,1)\rangle & \langle 0,(0,0,0;1,1,1;1,1,1)\rangle & \langle 0,(0,0,0;1,1,1;1,1,1)\rangle \end{pmatrix} \tag{18}$$

$$(\mho_B^{(3,13)})^{bwc} = \begin{pmatrix} \langle 0,(0,0,0;1,1,1;1,1,1)\rangle & \langle 0,(.3,.5,.7;.8,.9,.6;.7,.6,.4)\rangle & \langle 0,(0,0,0;1,1,1;1,1,1)\rangle \\ \langle 0,(0,0,0;1,1,1;1,1,1)\rangle & \langle 0,(0,0,0;1,1,1;1,1,1)\rangle & \langle 0,(.7,.3,.6;.4,.3,.1;.2,.1,.3)\rangle \\ \langle 0,(0,0,0;1,1,1;1,1,1)\rangle & \langle 0,(0,0,0;1,1,1;1,1,1)\rangle & \langle 0,(0,0,0;1,1,1;1,1,1)\rangle \\ \langle 0,(0,0,0;1,1,1;1,1,1)\rangle & \langle 0,(.5,.8,.9;.4,.6,.3;.8,.6,.5)\rangle & \langle 0,(0,0,0;1,1,1;1,1,1)\rangle \\ \langle 0,(0,0,0;1,1,1;1,1,1)\rangle & \langle 0,(0,0,0;1,1,1;1,1,1)\rangle & \langle 0,(.8,.3,.7;.2,.3,.9;.1,.2,.4)\rangle \\ \langle 0,(0,0,0;1,1,1;1,1,1)\rangle & \langle 0,(0,0,0;1,1,1;1,1,1)\rangle & \langle 0,(0,0,0;1,1,1;1,1,1)\rangle \\ \langle 0,(0,0,0;1,1,1;1,1,1)\rangle & \langle 0,(0,0,0;1,1,1;1,1,1)\rangle & \langle 0,(.8,.7,.5;.1,.2,.3;.2,.3,.6)\rangle \\ \langle 0,(0,0,0;1,1,1;1,1,1)\rangle & \langle 0,(.7,.3,.1;.8,.7,.4;.1,.3,.4)\rangle & \langle 0,(0,0,0;1,1,1;1,1,1)\rangle \end{pmatrix} \tag{19}$$

Proposition 4.2. Let $\mho_A^{(m,N)}$ be MPDNNS set, $\mho_\varnothing^{(m,0)}$ be null MPDNNS and $\mho_E^{(m,N-1)}$ absolute MPDNNS over X, then

(*i*) $(\mho_\varnothing^{(m,0)})^{bwc} = \mho_E^{(m,N:\,amp:\,minus;1)}$.

(*ii*) $(\mho_E^{(m,N-1)})^{bwc} = \mho_\varnothing^{(m,0)}$.

Remark 4.5. Again taking bottom weak complement of $(\mho_A^{(3,13)})^{bwc}$ as given above in Eq. (18) we have Eq. (20) given below:

$$((\mho_A^{(3,13)})^{bwc})^{bwc} = \begin{pmatrix} \langle 0,(0,0,0;1,1,1;1,1,1)\rangle & \langle 0,(0,0,0;1,1,1;1,1,1)\rangle & \langle 0,(0,0,0;1,1,1;1,1,1)\rangle \\ \langle 12,(.1,.9,.7;.5,.3,0.2;.4,.6,.8)\rangle & \langle 0,(0,0,0;1,1,1;1,1,1)\rangle & \langle 0,(0,0,0;1,1,1;1,1,1)\rangle \\ \langle 0,(0,0,0;1,1,1;1,1,1)\rangle & \langle 0,(0,0,0;1,1,1;1,1,1)\rangle & \langle 12,(.4,.3,.4;.5,.6,.8;.7,.6,.5)\rangle \\ \langle 0,(0,0,0;1,1,1;1,1,1)\rangle & \langle 0,(0,0,0;1,1,1;1,1,1)\rangle & \langle 12,(.5,.4,.5;.6,.7,.7;.6,.4,.2)\rangle \\ \langle 12,(.2,.1,.2;.3,.4.5;.6,.7,.8)\rangle & \langle 0,(0,0,0;1,1,1;1,1,1)\rangle & \langle 0,(0,0,0;1,1,1;1,1,1)\rangle \\ \langle 0,(0,0,0;1,1,1;1,1,1)\rangle & \langle 0,(0,0,0;1,1,1;1,1,1)\rangle & \langle 12,(.6,.5,.6;.7,.8.9;.8,.7,.6)\rangle \\ \langle 12,(.3,.4,.5;.6,.7,.6;.5,.4,.3)\rangle & \langle 0,(0,0,0;1,1,1;1,1,1)\rangle & \langle 0,(0,0,0;1,1,1;1,1,1)\rangle \\ \langle 0,(0,0,0;1,1,1;1,1,1)\rangle & \langle 0,(0,0,0;1,1,1;1,1,1)\rangle & \langle 0,(0,0,0;1,1,1;1,1,1)\rangle \end{pmatrix} \tag{20}$$

Hence from Eqs. (1) and (20) we conclude that $\left(\left(\mho_A^{(3,13)}\right)^{bwc}\right)^{bwc} \neq \mho_A^{(3,13)}$.

5 Relationships with Existing Models and Application in Multi-Attribute Group Decision-Making

The comparison with existing sets is shown in Table 1.

The following proposed algorithm for the choice values of MPDNNSs:

An Algorithm for the Choice Values of MPDNNSs

1: $S = \{\rho_1, \rho_2, \ldots, \rho_m\}$ is the universal set.

2: $T = \{e_1, e_2, \ldots, e_n\}$ set of attributes.

3: Input MPDFNSs $\psi_A^{(m,n)}$ with $N = \{0, 1, 2, \ldots, N - 1\}$.

4: Calculate $M_i^m = \left(\sum_{j=1}^m d_{\alpha_{ij}}, \sum_{j=1}^m R_{a_{ij}}\right)$. Here,

$$d_{a_{ij}} = 1 - \frac{2\theta_a}{\pi}, \quad R_{a_{ij}} = \frac{1}{2} + r_a\left(\frac{1}{2} - \frac{2\theta_a}{\pi}\right),$$

$$r_a = \left(\mu_1^2 + \mu_2^2 + \cdots + \mu_m^2 + \lambda_1^2 + \lambda_2^2 + \cdots + \lambda_m^2 + \nu_1^2 + \nu_2^2 + \cdots + \nu_m^2\right)^{1/2},$$

$$\theta_a = Tan^{-1}\left(\frac{< (\lambda_1, \lambda_2, \cdots, \lambda_m), (\nu_1, \nu_2, \cdots, \nu_m) >}{\|(\mu_1, \mu_2, \cdots, \mu_m)\|}\right),$$

where, $<(\lambda_1, \lambda_2, \ldots, \lambda_m), (\nu_1, \nu_2, \ldots, \nu_m)> = \lambda_1\nu_1 + \lambda_2\nu_2 + \ldots, \lambda_m\nu_m$.

5: Calculate $M_t^m = max\left\{M_i^m\right\}$ with $i = 1, 2, \ldots, n$.

6: $M_t^m = max\left\{M_i^m\right\}$ can be chosen for any alternative.

Table 1: Comparison of proposed model with existing models

Sets	Truth membership	Falsity membership	Indeterminacy	Parametrization	Non-binary evaluation	Multi-polarity
Fuzzy set	✓	×	×	×	×	×
Intutionistic fuzzy set	✓	✓	×	×	×	×
Neutrosopic set	✓	✓	✓	×	×	×
Soft set	×	×	×	✓	×	×
N-soft set	×	×	×	✓	✓	×
Fuzzy N-soft set	✓	×	×	✓	✓	×
Intutionistic N-soft set	✓	✓	×	✓	✓	×
Neutrosopic N-soft set	✓	✓	✓	✓	✓	×
m-polar fuzzy set	✓	×	×	×	×	✓
m-polar diophantine neutrosopic N-soft set	✓	✓	✓	✓	✓	✓

Example 5.1. We assume that there are five candidates which appear in the interview and their interviews will be completed in three days. There are three judges which are observed to each candidate separately. The following data are elaborated in Table 2, of each candidate c_i, $i = 1, 2, 3, 4, 5$ which are observed by from judges j_i, $i = 1, 2, 3$.

Table 2: Data of interview of each candidates observed by judges

$\mho_C^{(3,11)}$	j_1	j_2	j_3
c_1	$\langle 4, (.3, .2, .5; .2, .7, .8; .1, .4, .6)\rangle$	$\langle 6, (.5, .4, .7; .6, .5, .4; .3, .6, .5)\rangle$	$\langle 7, (.3, .5, .5; .3, .6, .7; .6, .4, .3)\rangle$
c_2	$\langle 6, (.6, .4, .6; .3, .5, .7; .5, .4, .3)\rangle$	$\langle 5, (.2, .5, .6; .4, .6, .9; .1, .2, .3)\rangle$	$\langle 8, (.2, .6, .5;.5, .8, .6; .3, .1, .4)\rangle$
c_3	$\langle 7, (.4, .6, .5; .3, .5, .7; .1, .4, .3)\rangle$	$\langle 2, (.4, .5, .3; .5, .6, .2; .5, .4, .5)\rangle$	$\langle 3, (.5, .3, .5; .4, .3, .7; .4, .3, .6)\rangle$
c_4	$\langle 3, (.5, .3, .4; .4, .5, .6; .3, .2, .5)\rangle$	$\langle 8, (.3, .6, .4; .4, .9, .6; .5, .4, .3)\rangle$	$\langle 9, (.5, .4, .5; .3, .7, .6; .4, .3, .2)\rangle$
c_5	$\langle 1, (.6, .5, .4; .3, .5, .7; .3, .4, .4)\rangle$	$\langle 4, (.6, .5, .4; .4, .7, .1; .3, .5, .5)\rangle$	$\langle 7, (.4, .3, .5; .2, .5, .7; .4, .6, .4)\rangle$

In Table 3, we find the choice values of all candidates by using proposed algorithm.

From Table 4, the candidates c_5, c_2, c_2 are leading with respect to days first, second, and third, respectively.

The working of proposed algorithm is shown in Fig. 1.

Table 3: Tabular representation of choice value of $\psi_C^{(3,11)}$

$\mho_C^{(3,11)}$	j_1	j_2	j_3	M_i^1	M_i^2	M_i^3
c_1	⟨4, (.3, .2, .5; .2, .7, .8; .1, .4, .6)⟩	⟨6, (.5, .4, .7; .6, .5, .4; .3, .6, .5)⟩	⟨7, (.3, .5, .5; .3, .6, .7; .6, .4, .3)⟩	(17, 2.59680)	(17, 1.78017)	(17, 1.64489)
c_2	⟨6, (.6, .4, .6; .3, .5, .7; .5, .4, .3)⟩	⟨5, (.2, .5, .6; .4, .6, .9; .1, .2, .3)⟩	⟨8, (.2, .6, .5;.5, .8, .6; .3, .1, .4)⟩	(19, 2.30871)	(19, 2.36016)	(19, 2.24883)
c_3	⟨7, (.4, .6, .5; .3, .5, .7; .1, .4, .3)⟩	⟨2, (.4, .5, .3; .5, .6, .2; .5, .4, .5)⟩	⟨3, (.5, .3, .5; .4, .3, .7; .4, .3, .6)⟩	(12, 2.39995)	(12, 2.13054)	(12, 1.91588)
c_4	⟨3, (.5, .3, .4; .4, .5, .6; .3, .2, .5)⟩	⟨8, (.3, .6, .4; .4, .9, .6; .5, .4, .3)⟩	⟨9, (.5, .4, .5; .3, .7, .6; .4, .3, .2)⟩	(20, 2.32576)	(20, 2.05034)	(20, 1.89522)
c_5	⟨1, (.6, .5, .4; .3, .5, .7; .3, .4, .4)⟩	⟨4, (.6, .5, .4; .4, .7, .1; .3, .5, .5)⟩	⟨7, (.4, .3, .5; .2, .5, .7; .4, .6, .4)⟩	(12, 2.65388)	(12, 2.08484)	(12, 1.94623)

Table 4: Comparison table of MPDFNSs with pervious models

$\mho_C^{(3,11)}$	$NSS(\sigma_i)$	$FNSS(Q_i)$	$IFNSS(S_i)$	$PFNSS(H_i)$	M_i^1	M_i^2	M_i^3
c_1	17	(17, 0.89)	(17, −0.03)	(17, 1.7462)	(17, 2.59680)	(17, 1.78017)	(17, 1.64489)
c_2	19	(19, 0.79)	(19, 0.09)	(19, 1.6368)	(19, 2.30871)	(19, 2.36016)	(19, 2.24883)
c_3	12	(12, 0.99)	(12, 0.15)	(12, 1.8940)	(12, 2.39995)	(12, 2.13054)	(12, 1.91588)
c_4	20	(20, 1.09)	(20, 0.09)	(20, 1.5705)	(20, 2.32576)	(20, 2.05034)	(20, 1.89522)
c_5	12	(12, 1.22)	(12, 0.54)	(12, 1.9097)	(12, 2.65388)	(12, 2.08484)	(12, 1.94623)

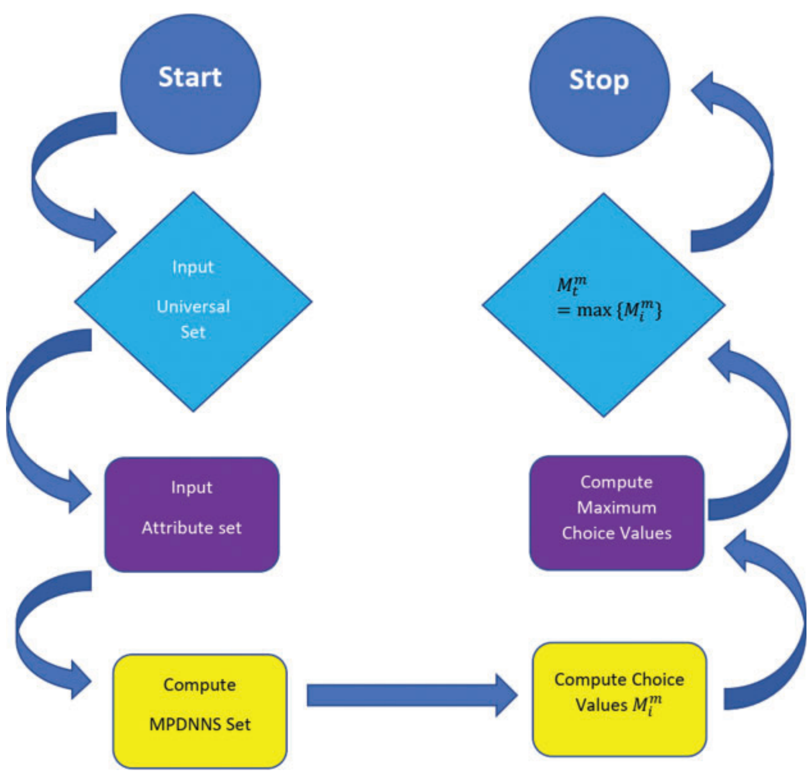

Figure 1: Flow chart of proposed algorithm

5.1 Relationships with Existing Sets

In this subsection, we establish the relationship with existing sets.

Definition 5.1. Let n be a threshold lies between 0 and N for the level, the PFNSS and PFSS over X associated with $\mho_A^{(m,N)}$ and n, symbolized by $\mho_A^{(n,m,N)}$, defined by, for all $a \in A$,

$$\mho_a^{(n,m,N)} = \begin{cases} \langle x, (\mu(a), \nu(a)) \rangle, & \text{if } m = 1, \\ (x, d_a)(\mu(a), \nu(a)), & \text{if } (x, d_a) \in F(a), and d_a \geq n, \\ (0, 0.5), & \text{if } d_a/N \geq 0.5, \\ (0, 1), & \text{if } d_a/N < 0.5. \end{cases}$$

Definition 5.2. Let k be a threshold with $k \in [-1, 1]$ for the score function, the N-soft over X associated with $\mho_A^{(m,N)}$ and k, symbolized by $\mho_A^{(k,m,N)}$, defined by, for all $a \in A$,

$$\mho_a^{(k,m,N)} = \begin{cases} (x, d_a), & \text{if } (x, d_a) \in F(a), \text{ and } S_a(x) \geq k, m = 1, \\ 1, & \text{if } S_a(x) > 0, m = 1, \\ 0, & \text{if } S_a(x) \leq 0, m = 1. \end{cases}$$

Definition 5.3. Let n be a threshold lies between 0 and N and threshold k with $k \in [-1, 1]$ for the score function, the soft over X associated with $\psi_A^{(m,N)}$ and (n, k), symbolized by $\mho_A^{((n,k),m,N)}$, defined by, for all $a \in A$,

$$\mho_A^{((n,k),1,N)} = \left\{ x \in X | S_a^{\mho^{(n,1,N)}}(x) > k, \right\}$$

For soft set associated with $\psi_A^{(3,6)}$ and threshold $(n, k) = (3, 0.3)$ is $\mho_A^{((3,0.3),1,6)} = \{\ \}$.

From Example 3.1, we have the following outcome which are elaborate in Table 5.

Table 5: Pythagorean fuzzy N-soft set associated with $\mho_A^{(3,6)}$ and $m = 1$

$\mho_A^{(1,7)}$	e_1	e_2	e_3
ρ_1	⟨4, (0.3, 0.1)⟩	⟨0, (0, 1)⟩	⟨0, (0, 1)⟩
ρ_2	⟨0, (0, 1)⟩	⟨0, (0, 1)⟩	⟨3, (0.2, 0.8)⟩
ρ_3	⟨0, (0, 1,)⟩	⟨0, (0, 1)⟩	⟨2, (0.5, 0.4)⟩
ρ_4	⟨5, (0.5, 0.3)⟩	⟨0, (0, 1)⟩	⟨0, (0, 1)⟩
ρ_5	⟨0, (0, 1)⟩	⟨0, (0, 1)⟩	⟨2, (0.4, 0.7)⟩
ρ_6	⟨0, (0, 1)⟩	⟨0, (0, 1)⟩	⟨0, (0, 1)⟩
ρ_7	⟨6, (0.6, 0.4)⟩	⟨0, (0, 1)⟩	⟨ 0, (0, 1)⟩

In Tables 6 and 7, we deduce Pythagorean fuzzy soft set and N-soft set from MPDFNSs.

Table 6: Pythagorean fuzzy soft set associated with $\mho_A^{(3,6)}$ and threshold $n = 3$

$\mho_A^{(3,1,6)}$	e_1	e_2	e_3
ρ_1	(0.3, 0.1)	(0, 1)	(0, 1)
ρ_2	(0, 1)	(0, 1)	(0.2, 0.8)
ρ_3	(0, 1,)	(0, 1)	(0, 1)
ρ_4	(0.5, 0.3)	(0, 1)	(0, 1)
ρ_5	(0, 1)	(0, 1)	(0, 1)
ρ_6	(0, 1)	(0, 1)	(0, 1)
ρ_7	(0.6, 0.4)	(0, 1)	(0, 1)

Table 7: N-soft set associated with $\mho_A^{(3,6)}$ and threshold $k = 0.3$

$\mho_A^{(0.3,1,6)}$	e_1	e_2	e_3
ρ_1	1	0	0
ρ_2	0	0	0
ρ_3	0	0	1
ρ_4	1	0	1
ρ_5	0	0	0
ρ_6	0	0	0
ρ_7	1	0	0

6 Conclusion

In this paper, we investigate a new set namely the m-polar Diophantine neutrosophic N-soft set which is based on neutrosophic set and soft set. We are discussed different types of compliments on the proposed set and elaborate these compliments with examples. The purposed set is a generalized form of fuzzy, soft, Pythagorean fuzzy, Pythagorean fuzzy soft, and Pythagorean fuzzy N-soft sets. Moreover, as an application, we proposed an algorithm for multi-attribute decision-making problems by defining the new score function. In future work, one can discuss algebraic structures and topological properties on m-polar Diophantine neutrosophic N-soft set. Moreover, ones can develop the concept of m-polar Diophantine neutrosophic N-soft graph and then discuss their properties.

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References

1. Zadeh, L. A. (1965). Information and control. *Fuzzy Sets, 8(3),* 338–353. DOI 10.1016/S0019-9958(65)90241-X.
2. Atanassov, K. T. (1986). Intuitionistic fuzzy set. *Fuzzy Sets and Systems, 20(1),* 87–96. DOI 10.1016/S0165-0114(86)80034-3.
3. Molodtsov, D. (1999). Soft set theory-first results. *Computers and Mathematics with Applications, 37(4–5),* 19–31. DOI 10.1016/S0898-1221(99)00056-5.
4. Ali, M. I., Feng, F., Liu, X., Min, W. K., Shabir, M. (2009). On some new operations in soft set theory. *Computers and Mathematics with Applications, 57(9),* 1547–1553. DOI 10.1016/j.camwa.2008.11.009.
5. Yager, R. R. (2013). Pythagorean fuzzy subsets. *Joint IFSA World Congress and NAFIPS Annual Meeting*, pp. 57–61. IEEE. DOI 10.1109/IFSA-NAFIPS.2013.6608375.
6. Yager, R. R., Abbasov, A. M. (2013). Pythagorean membership grades, complex numbers, and decision making. *International Journal of Intelligent Systems, 28(5),* 436–452. DOI 10.1002/int.21584.
7. Yager, R. R. (2013). Pythagorean membership grades in multicriteria decision making. *IEEE Transactions on Fuzzy Systems, 22(4),* 958–965. DOI 10.1109/TFUZZ.2013.2278989.
8. Peng, X., Yuan, H., Yang, Y. (2017). Pythagorean fuzzy information measures and their applications. *International Journal of Intelligent Systems, 32(10),* 991–1029. DOI 10.1002/int.21880.
9. Peng, X. D., Yang, Y., Song, J., Jiang, Y. (2015). Pythagorean fuzzy soft set and its application. *Computer Engineering, 41(7),* 224–229. DOI 10.3969/j.issn.1000-3428.2015.07.043.
10. Peng, X., Dai, J. (2018). Approaches to single-valued neutrosophic MADM based on MABAC, TOPSIS and new similarity measure with score function. *Neural Computing and Applications, 29(10),* 939–954. DOI 10.1007/s00521-016-2607-y.
11. Zeng, S., Shoaib, M., Ali, S., Smarandache, F., Rashmanlou, H. et al. (2021). Certain properties of single-valued neutrosophic graph with application in food and agriculture organization. *International Journal of Computational Intelligence Systems, 14(1),* 1516–1540. DOI 10.2991/ijcis.d.210413.001.
12. Gulfam, M., Mahmood, M. K., Smarandache, F., Ali, S. (2021). New Dombi aggregation operators on bipolar neutrosophic set with application in multi-attribute decision-making problems. *Journal of Intelligent & Fuzzy Systems,* 1–18 (Preprint).
13. Hameed, M. S., Mukhtar, S., Khan, H. N., Ali, S., Mateen, M. H. et al. (2021). Pythagorean fuzzy N-Soft groups. *International Journal of Electrical and Computer Engineering, 21(2),* 1030–1038.
14. Liu, J. B., Ali, S., Mahmood, M. K., Mateen, M. H. (2020). On m-polar Diophantine fuzzy N-soft set with applications. *Combinatorial Chemistry & High Throughput Screening, 23,* 1–18. DOI 10.2174/1386207323666201230092354.
15. Zeng, S., Shoaib, M., Ali, S., Abbas, Q., Nadeem, M. S. (2020). Complex vague graphs and their application in decision-making problems. *IEEE Access, 8,* 174094–174104. DOI 10.1109/ACCESS.2020.3025974.
16. Mahmood, M. K., Zeng, S., Gulfam, M., Ali, S., Jin, Y. (2020). Bipolar neutrosophic dombi aggregation operators with application in multi-attribute decision making problems. *IEEE Access, 8,* 156600–156614. DOI 10.1109/ACCESS.2020.3019485.
17. Akram, M., Amjad, U., Davvaz, B. (2021). Decision-making analysis based on bipolar fuzzy N-soft information. *Computational and Applied Mathematics, 40(6),* 1–39. DOI 10.1007/s40314-021-01570-y.
18. Akram, M., Luqman, A., Kahraman, C. (2021). Hesitant Pythagorean fuzzy ELECTRE-II method for multi-criteria decision-making problems. *Applied Soft Computing, 108(3),* 107479. DOI 10.1016/j.asoc.2021.107479.
19. Adeel, A., Akram, M., Cagman, N. (2021). Decision-making analysis based on hesitant fuzzy N-Soft ELECTRE-I approach. *Soft Computing* (Preprint).
20. Akram, M., Kahraman, C., Zahid, K. (2021). Extension of TOPSIS model to the decision-making under complex spherical fuzzy information. *Soft Computing, 25,* 1–25. DOI 10.1016/j.knosys.2021.106793.
21. Akram, M., Alsulami, S., Zahid, K. (2021). A hybrid method for complex pythagorean fuzzy decision making. *Mathematical Problems in Engineering, 2021(6),* 1–23. DOI 10.1155/2021/9915432.

22. Fatimah, F., Rosadi, D., Hakim, R. F., Alcantud, J. C. R. (2018). N-soft sets and their decision making algorithms. *Soft Computing, 22(12),* 3829–3842. DOI 10.1007/s00500-017-2838-6.
23. Alcantud, J. C. R., Laruelle, A. (2014). Dis-approval voting: A characterization. *Social Choice and Welfare, 43(1),* 1–10. DOI 10.1007/s00355-013-0766-7.
24. Ma, X., Liu, Q., Zhan, J. (2017). A survey of decision making methods based on certain hybrid soft set models. *Artificial Intelligence Review, 47(4),* 507–530. DOI 10.1007/s10462-016-9490-x.
25. Wang, H., Smarandache, F., Zhang, Y., Sunderraman, R. (2010). *Single valued neutrosophic sets*. Infinite Study.
26. Deli, I., Ali, M., Smarandache, F. (2015). Bipolar neutrosophic sets and their application based on multi-criteria decision making problems. *International Conference on Advanced Mechatronic Systems*, pp. 249–254. Beijing, China, IEEE. DOI 10.1109/ICAMechS.2015.7287068.
27. Zeng, S. Z., Hu, Y. J., Balezentis, T., Streimikiene, D. (2020). A multi-criteria sustainable supplier selection framework based on neutrosophic fuzzy data and entropy weighting. *Sustainable Development, 28(5),* 1431–1440. DOI 10.1002/sd.2096.
28. Zeng, S., Hu, Y., Xie, X. (2021). Q-rung orthopair fuzzy weighted induced logarithmic distance measures and their application in multiple attribute decision making. *Engineering Applications of Artificial Intelligence, 100(7),* 104167. DOI 10.1016/j.engappai.2021.104167.
29. Wang, J., Zeng, S., Zhang, C. (2020). Single-valued neutrosophic linguistic logarithmic weighted distance measures and their application to supplier selection of fresh aquatic products. *Mathematics, 8(3),* 439. DOI 10.3390/math8030439.
30. Zhang, C., Su, W., Zeng, S., Balezentis, T., Herrera-Viedma, E. (2021). A two-stage subgroup decision-making method for processing large-scale information. *Expert Systems with Applications, 171(3),* 114586. DOI 10.1016/j.eswa.2021.114586.
31. Zhang, H. D., Shu, L. (2016). Dual hesitant fuzzy soft set and its properties. *Fuzzy systems & operations research and management*, pp. 171–182. Cham: Springer.
32. Cagman, N., Enginoglu, S., Citak, F. (2011). Fuzzy soft set theory and its applications. *Iranian Journal of Fuzzy Systems, 8(3),* 137–147. DOI 10.22111/IJFS.2011.292.
33. Maji, P. K., Biswas, R. K., Roy, A. (2001). Fuzzy soft sets. *Journal of Fuzzy Mathematics, 9(3),* 589–602.
34. Maji, P. K., Biswas, R., Roy, A. R. (2001). Intuitionistic fuzzy soft sets. *Journal of Fuzzy Mathematics, 9(3),* 677–692.
35. Wei, G., Wei, Y. (2018). Similarity measures of Pythagorean fuzzy sets based on the cosine function and their applications. *International Journal of Intelligent Systems, 33(3),* 634–652. DOI 10.1002/int.21965.
36. Wan, S. P., Jin, Z., Dong, J. Y. (2018). Pythagorean fuzzy mathematical programming method for multi-attribute group decision making with Pythagorean fuzzy truth degrees. *Knowledge and Information Systems, 55(2),* 437–466. DOI 10.1007/s10115-017-1085-6.
37. Zhang, H., Jia-Hua, D., Yan, C. (2020). Multi-attribute group decision-making methods based on Pythagorean fuzzy N-soft sets. *IEEE Access, 8,* 62298–62309. DOI 10.1109/ACCESS.2020.2984583.
38. Smarandache, F. (1999). A unifying field in logics. *Neutrosophy: Neutrosophic probability, set and logic,* pp. 1–141. American Research Press.