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Dynamical Analysis of Radiation and Heat Transfer on MHD Second Grade Fluid

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ABSTRACT

Convective flow is a self-sustained flow with the effect of the temperature gradient. The density is non-uniform due to the variation of temperature. The effect of the magnetic flux plays a major role in convective flow. The process of heat transfer is accompanied by a mass transfer process; for instance, condensation, evaporation, and chemical process. Due to the applications of the heat and mass transfer combined effects in a different field, the main aim of this paper is to do a comprehensive analysis of heat and mass transfer of MHD unsteady second-grade fluid in the presence of ramped boundary conditions near a porous surface. The dynamical analysis of heat transfer is based on classical differentiation with no memory effects. The non-dimensional form of the governing equations of the model is developed. These are solved by the classical integral (Laplace) transform technique/method with the convolution theorem and closed-form solutions are attained for temperature, concentration, and velocity. The physical aspects of distinct parameters are discussed via graph to see the influence on the fluid concentration, velocity, and temperature. Our results suggest that the velocity profile decrease by increasing the Prandtl number. The existence of a Prandtl number may reflect the control of the thickness of momentum and enlargement of thermal conductivity. Furthermore, to validate our results, some results are recovered from the literature.

KEYWORDS

Heat transfer; magnetic effect; ramped conditions; porous medium; Laplace transform; thermal radiation

1 Introduction

In the literature, different theories are made to see the phenomenon of heat transfer analysis. Radiation, convection, and conduction are three modes of heat transfer. Convection can be defined as heat transfer by the substance motion which may be air or water. It plays a central role in creating the weather clause on the plant. Convection consists of forced and natural convection. It happens when the medium divides the heat energy and is on its move. Heat is disbursed when a fan pushes air, and sometimes heat advection is referred to the forced convection. Heat is animated

to distress by means of itself hotness becomes the cause of natural convection by means of shifting source heat. Heat and mass are related to each other as heat transfer rate depends on mass transfer and mass transfer further depends upon concentration difference. Free convective flow is a self-reliant flow with the effect of heat transfer. In different industrial problems, the fluid flow phenomenon of free convection has achieved much consideration in recent years. The atmospheric, oceanic circulation, and emergency cooling system of advanced nuclear reactors is the most important application of free convection. The effect of heat transfer in non-Newtonian fluids plays a major role applying engineering such as drag reduction, the emergency cooling system of nuclear reactors, and thermal welding. In literature, different theories are made to see the phenomenon of heat transfer analysis. The convection heat transform between two heated cubes was discussed by Mousazadeh et al. [1]. Sajad et al. [2] investigated the heat transfer and magnetic effects on hybrid nanofluid. Iftikhar et al. [3] analyzed the influence of heat and mass transform with the existence of a magnetic field in the rate type fluid model. Convective flow is a self-reliant flow with the effect of the heat transfer. The influence of magnetic flux plays a significant role in convective flow. In the literature, different theories are made to see the phenomenon of heat transfer analysis. Radiation, convection and conduction are three modes of heat transfer. Convection can be defined as heat transfer by the substance motion which may be air or water. It plays a central role in creating the weather clause on the plant.

Qi et al. [4] investigated the mathematical analysis of the heat transfer model subject to the stretching plate with the velocity slip boundary for the Maxwell fluid model. They explored the exact solutions using the effect of the heat flux relaxation time, slip coefficient, and the Prandtl number. Literature shows more interest in developed identical studies in [5–8]. Ramped heating plays a good role in real-life problems such as prognosis diagnoses, heart function analysis, and blood vessel system [9–11]. Moreover, Kundu [12] investigated the thermal therapy based on ramped heating to destroy the cancer cells on the human structure. Researchers in [13–17] used the ramped heating to investigate the flows of Newtonian and non-Newtonian fluids. Initially, convective viscous fluid with ramped heating over vertical wall was analyzed by Schertz [18] and Hayday [19]. The flow of a viscous fluid with the impact of ramped temperature over a moving surface was analyzed by Das et al. [20]. The heat absorption ramped heating and thermal effect near a moving wall was discussed by Seth et al. [21]. The time-dependent motion with a slip effect on ramped surface was investigated by Maiti et al. [22]. Seth et al. [23,24] investigated the dynamical aspect of mass and heat transformation with Darcy's law, chemical reaction, and thermal conditions. Previously, there was less study which deals the simultaneous use of ramped heating with ramped velocity. It is complicated to apply these conditions simultaneously, but they have broad significance as a physical aspect. Recently, Tiwana et al. [25] and Anwar et al. [26] analyzed the effect of ramped temperature and ramped velocity in the existence of thermal radiation. Anwar et al. [27] analyzed the impact of ramped wall temperature and ramped velocity on Maxwell fluid. Anwar et al. [28] investigated the exact solution of second-grade fluid under the impact of ramped conditions. The different technical problems, the flow of fluid in the existence of heat radiation has been achieved consideration in recent years. The power plant for rocket ignition, missile and emergency gas-cooling system are the most important application of heat radiation. The influence of heat radiation on micro-polar nanofluid with magnetic effect was discussed by Izadi et al. [29]. The exact solution of magneto nanofluid with free convection and heat radiation effect was investigated by Das et al. [30]. Two phase model for nanofluid was investigated by Sheikholeslami et al. [31] using impact of heat radiation. Gangadhar et al. [32] investigated the role of heat radiation and Newtonian heating on ferrofluid. The Numerical results for Casson

fluid under the effect of heat radiation, and Newtonian & Joule heating was investigated by Seth et al. [33].

Although the research on the Second grade model can be continued, we include here some related studies based on heat transfer, fractional models of fluids, magnetized fluids, and few others therein [34–40]. Motivating by above discussion, this manuscript aims to explore the unique and exact solution of MHD Second grade model with simultaneous use of ramped heating with ramped velocity near a porous surface. Laplace integral transformation is used to obtain the unique solution of velocity, temperature, and concentration under impact of ramped heating. In Section 2, the dimensionless governing equations are developed. In Section 3, Laplace integral transform is used to find the exact solution of the temperature, velocity, and concentration field. In Section 4, some existing results are recovered as the limiting cases to justify our results. In Section 5, the effect of physical parameters is analyzed graphically. The concluding observation is listed at the end.

2 Development of Governing Equations Based on Problem Statement

Consider the MHD Second grade fluid flow in a porous medium over an limitless vertical plate. Initially, the fluid is at rest at $t = 0$. After $t < t_0$, the $T_\infty - \left(T_\infty - T_w \frac{t}{t_0} \right)$ is the plate temperature with velocity $w_0 \frac{t}{t_0}$. At $t > t_0$, it give constant velocity w_0 with temperature T_∞. The fluid flow is one dimensional. The flow direction of radiative flux is negligible. Finally in energy equation, the dispersion term is neglected. The geometrical presentation of considered model is provided in Fig. 1. Under some these presumptions, the governing equation for MHD Second grade fluid with appropriate conditions are given below [28]:

$$\frac{\partial w(\zeta,t)}{\partial t} = \upsilon \left(1 + \frac{\alpha_1}{\mu} \frac{\partial}{\partial t} \right) \frac{\partial^2 w(\zeta,t)}{\partial \zeta^2} + g\beta_T (T - T_\infty) + g\beta_C (C - C_\infty) - \frac{\sigma_0 M_0^2}{\rho} w(\zeta,t)$$

$$- \frac{\upsilon\phi}{k_0} \left(1 + \frac{\alpha_1}{\mu} \frac{\partial}{\partial t} \right) w(\zeta,t), \tag{1}$$

$$\frac{\partial T}{\partial t} = \frac{k}{\rho C_p} \left(1 + \frac{16\sigma_1 T_\infty^3}{3kK_1} \right) \frac{\partial^2 T}{\partial \zeta^2}, \tag{2}$$

$$\frac{\partial C}{\partial t} = D_m \frac{\partial^2 C}{\partial \zeta^2} - D_m (C - C_\infty), \tag{3}$$

with appropriate conditions

$$w(\zeta,0) = 0, \quad T(\zeta,0) = T_\infty, \quad C(\zeta,0) = C_\infty, \quad \frac{\partial w(\zeta,0)}{\partial t} = 0, \quad \zeta \ge 0, \tag{4}$$

$$C(0,t) = C_w, \quad w(0,t) = f_1(t), \quad T(0,t) = f_2(t), \tag{5}$$

where

$$f_1(t) = \begin{Bmatrix} w_0 \frac{t}{t_0}, & 0 < t \le t_0; \\ w_0, & t > t_0 \end{Bmatrix} \text{ and } f_2(t) = \begin{Bmatrix} T_\infty + (T_w - T_\infty) \frac{t}{t_0}, & 0 < t \le t_0; \\ T(0,t) = T_w, & t > t_0 \end{Bmatrix},$$

$$w(\zeta,t) \to 0, T(\zeta,t) \to \infty, C(\zeta,t) \to \infty \text{ as } \zeta \to \infty. \tag{6}$$

Consider the different dimensionless quantities

$$\psi = \frac{w_0^3}{\upsilon}\zeta, \quad \tau = \frac{w_0^2}{\upsilon}t, \quad V = \frac{w}{w_0^3}, \quad \theta = \frac{T - T_\infty}{T_w - T_\infty}, \quad \phi = \frac{C - C_\infty}{C_w - C_\infty}, \quad G_r = \frac{g\beta_T t_0 \left(T - T_\infty\right)}{w_0^3},$$

$$G_m = \frac{g\beta_C t_0 \left(C - C_\infty\right)}{w_0^3}, \quad M = \frac{\sigma_0 M_0^2 \upsilon}{\rho w_0^2}, \quad N_r = \frac{16\sigma_1 T_\infty^3}{3k K_1}, \quad P_r = \frac{\upsilon C_p}{k}, \quad P_{reff} = \frac{P_r}{1 + N_r}, \tag{7}$$

$$\frac{1}{K} = \frac{\upsilon^2 \phi}{k_0 w_0^2}, \quad \alpha = \frac{\alpha_1 \rho w_0^2}{\mu^2}, \quad Sc = \frac{\upsilon}{D_m}, \quad r = M + \frac{1}{K}, \quad s = \frac{\alpha}{K}, \quad \lambda = \frac{\upsilon^2}{w_0^2}.$$

Figure 1: Geometrical presentation for second grade model

After simplification, we have the set of dimensionless governing equations and their corresponding conditions are:

$$\frac{\partial V\left(\psi, \tau\right)}{\partial \tau} = \frac{\partial^2 V\left(\psi, \tau\right)}{\partial \psi^2} + G_r\theta\left(\psi, \tau\right) + G_m\phi\left(\psi, \tau\right) - rV\left(\psi, \tau\right) - s\frac{\partial V\left(\psi, \tau\right)}{\partial \tau} + \alpha\frac{\partial^3 V\left(\psi, \tau\right)}{\partial \tau \partial \psi^2}, \tag{8}$$

$$\frac{\partial \theta (\psi, \tau)}{\partial \tau} = \frac{1}{P_{reff}} \frac{\partial^2 \theta (\psi, \tau)}{\partial \psi^2}, \tag{9}$$

$$\frac{\partial \phi (\psi, \tau)}{\partial \tau} = \frac{1}{Sc} \frac{\partial^2 \phi (\psi, \tau)}{\partial \psi^2} - \frac{\lambda}{Sc} \phi (\psi, \tau), \tag{10}$$

$$V (\psi, 0) = \theta (\psi, 0) = \phi (\psi, 0) = 0, \tag{11}$$

$$V (0, \tau) = \theta (0, \tau) = f (\tau) \ where \ f (\tau) = \begin{cases} \tau, & 0 < \tau \leq 1 \\ 1, & \tau > 1 \end{cases}, \quad \phi (0, \tau) = 1, \tag{12}$$

$$V (\infty, \tau) \to 0, \quad \theta (\infty, \tau) \to 0, \quad \phi (\infty, \tau) \to 0, \quad \psi \to \infty. \tag{13}$$

3 Exact Solution of the Problem

3.1 Solution of Heat Profile

Applying Laplace transformation to get the solution of Eq. (9) using Eqs. (11)–(13), we have

$$\frac{\partial^2 \theta^c (\psi, m)}{\partial \psi^2} - m P_{reff} \theta^c (\psi, m) = 0. \tag{14}$$

The solution of above Eq. (14) is

$$\theta^c (\psi, m) = c_1 e^{\psi \sqrt{m P_{reff}}} + c_2 e^{-\psi \sqrt{m P_{reff}}}, \tag{15}$$

using appropriate conditions are used to find the value of constant,

$$\theta^c (\psi, m) = \left(\frac{1 - e^{-m}}{m^2} \right) e^{-\psi \sqrt{m P_{reff}}}, \tag{16}$$

$$\theta^c (\psi, m) = \left(\frac{e^{-\psi \sqrt{m P_{reff}}}}{m^2} \right) - e^{-m} \left(\frac{e^{-\psi \sqrt{m P_{reff}}}}{m^2} \right) = \theta_r^c (\psi, m) - e^{-m} \theta_r^c (\psi, m). \tag{17}$$

The inverse Laplace transformation of Eq. (17) is

$$\theta (\psi, \tau) = \theta_r (\psi, \tau) - \theta_r (\psi, \tau_0) H (\tau_0), \tag{18}$$

$$\theta_r (\psi, \tau) = \left(\frac{P_{reff}}{2} \psi^2 + \tau \right) erfc \left(\sqrt{\frac{P_{reff}}{4\tau}} \psi \right) - \left(\sqrt{\frac{P_{reff} \tau}{\pi}} \psi \right) e^{\frac{-P_{reff} \psi^2}{4\tau}}, \tag{19}$$

where $H(\tau_0)$ represent a standard Heaviside function with $\tau_0 = \tau - 1$.

3.1.1 Nusselt Number

To estimate the rate of heat transfer can be obtained as

$$N_u = -\frac{\partial \theta (\psi, \tau)}{\partial \psi} |_{\psi=0}. \tag{20}$$

3.2 Solution of Mass Profile

Applying the Laplace transformation to Eq. (10) with suitable initial condition on concentration

$$\frac{\partial^2 \phi^c (\psi, m)}{\partial \psi^2} - (m S_c + \lambda) \phi^c (\psi, m) = 0. \tag{21}$$

The required solution of second order differential Eq. (21) with help of (11)–(13) on concentration is given by

$$\phi^c (\psi, p) = \frac{1}{m} e^{-\psi \sqrt{m S_c + \lambda}}. \tag{22}$$

To obtain the exact solution, we take the inverse Laplace transformation, we have

$$\phi (\psi, \tau) = \frac{1}{2} \left(e^{-\psi \sqrt{S_c} \sqrt{\frac{\lambda}{S_c}}} erfc \left(\frac{\psi \sqrt{S_c}}{2\sqrt{\tau}} - \sqrt{\frac{\lambda}{S_c} \tau} \right) \right) + \frac{1}{2} \left(e^{\psi \sqrt{S_c} \sqrt{\frac{\lambda}{S_c}}} erfc \left(\frac{\psi \sqrt{S_c}}{2\sqrt{\tau}} + \sqrt{\frac{\lambda}{S_c} \tau} \right) \right), \tag{23}$$

using from literature

$$L^{-1} \left\{ \frac{e^{-\psi \sqrt{c} \sqrt{p+b}}}{p - a} \right\} = \frac{e^{a\tau}}{2} \left(e^{-\psi \sqrt{c} \sqrt{a+b}} erfc \left(\frac{\psi \sqrt{c}}{2\sqrt{\tau}} - \sqrt{(a+b)\tau} \right) \right)$$

$$+ \frac{e^{a\tau}}{2} \left(e^{\psi \sqrt{c} \sqrt{a+b}} erfc \left(\frac{\psi \sqrt{c}}{2\sqrt{\tau}} + \sqrt{(a+b)\tau} \right) \right). \tag{24}$$

3.3 Solution of Velocity Profile

The solution of Eq. (8) by using Laplace transform, we have

$$(r + (1+s) m) V^c (\psi, m) = (1 + \alpha m) \frac{\partial^2 V^c (\psi, m)}{\partial \psi^2} + G_r \theta^c (\psi, m) + G_m \phi^c (\psi, m), \tag{25}$$

by using the Eqs. (16) and (22) for the values $\theta^c (\psi, m)$ and $\phi^c (\psi, m)$, the solution of Eq. (25) can be written as

$$V^c (\psi, m) = c_1 e^{\psi \sqrt{\frac{s+(1+s)m}{1+\alpha m}}} + c_2 e^{-\psi \sqrt{\frac{s+(1+s)m}{1+\alpha m}}} - \left(\frac{G_s (1 - e^{-m})}{P_{reff} m - \left(\frac{s+(1+s)m}{1+\alpha m} \right)} \right) \frac{e^{-\psi \sqrt{P_{reff} m}}}{(1 + \alpha m) m^2}$$

$$- \left(\frac{G_m}{S_c m + \lambda - \left(\frac{s+(1+s)m}{1+\alpha m} \right)} \right) \frac{e^{-\psi \sqrt{S_c m + \lambda}}}{(1 + \alpha m) m}, \tag{26}$$

$$V^c (\psi, m) = \left(\frac{1 - e^{-m}}{m^2} \right) e^{-\psi \sqrt{\frac{r+(1+s)m}{1+\alpha m}}} + \left(\frac{1 - e^{-m}}{m^2} \right) \left(\frac{G_r}{\alpha P_{reff}} \right) \times \left(\frac{e^{-\psi \sqrt{\frac{r+(1+s)m}{1+\alpha m}}} - e^{-\psi \sqrt{P_{reff} m}}}{(m + n_1)^2 - (n_2)^2} \right)$$

$$+ \left(\frac{G_m}{m \alpha S_c} \right) \times \left(\frac{e^{-\psi \sqrt{\frac{r+(1+s)m}{1+\alpha m}}} - e^{-\psi \sqrt{S_c m + \lambda}}}{(m + n_3)^2 - (n_4)^2} \right), \tag{27}$$

where

$$n_1 = \frac{P_{reff} - (1+s)}{2\alpha P_{reff}}, \quad n_2 = \sqrt{\left(\frac{P_{reff} - (1+s)}{2\alpha P_{reff}}\right)^2 + \frac{r}{\alpha P_{reff}}},$$

$$n_3 = \frac{S_c + \alpha\lambda - (1+s)}{2\alpha S_c}, \quad n_4 = \sqrt{\left(\frac{S_c + \alpha\lambda - (1+s)}{2\alpha S_c}\right)^2 - \frac{\lambda r}{\alpha S_c}}. \tag{28}$$

Eq. (27) can be written as in the following form:

$$V^c(\psi, m) = S^c(\psi, m) - e^{-m} S^c(\psi, m) + R^c(\psi, m), \tag{29}$$

where

$$S^c(\psi, m) = \left(\frac{e^{-\psi\sqrt{\frac{r+(1+s)m}{1+\alpha m}}}}{m^2}\right) + \left(\frac{G_r}{\alpha P_{reff} m^2}\right)\left(\frac{e^{-\psi\sqrt{\frac{r+(1+s)m}{1+\alpha m}}} - e^{-\psi\sqrt{P_{reff}m}}}{(m+n_1)^2 - (n_2)^2}\right), \tag{30}$$

$$R^c(\psi, m) = \left(\frac{G_m}{m\alpha S_c}\right)\left(\frac{e^{-\psi\sqrt{\frac{r+(1+s)m}{1+\alpha m}}} - e^{-\psi\sqrt{S_c m+\lambda}}}{(m+n_3)^2 - (n_4)^2}\right), \tag{31}$$

Further, $S^c(\psi, m)$ can be written as

$$S^c(\psi, m) = S_1^c(\psi, m) + [S_1^c(\psi, m) - \theta_r^c(\psi, m)] S_2^c(m), R^c(\psi, m) = [R_1^c(\psi, p) - \phi^c(\psi, m)] R_2^c(m), \tag{32}$$

where

$$S_1^c(\psi, m) = \left(\frac{e^{-\psi\sqrt{\frac{r+(1+s)m}{1+\alpha m}}}}{m^2}\right), \quad S_2^c(m) = \left(\frac{G_r}{\alpha P_{reff}}\right)\left(\frac{1}{(m+n_1)^2 - (n_2)^2}\right),$$

$$R_1^c(\psi, m) = \left(\frac{e^{-\psi\sqrt{\frac{r+(1+s)m}{1+\alpha m}}}}{m}\right) \text{ and } R_2^c(m) = \left(\frac{G_m}{\alpha S_c}\right)\left(\frac{1}{(m+n_3)^2 - (n_4)^2}\right). \tag{33}$$

Applying inverse Laplace transformation with convolution product on Eq. (32), we get the required solution

$$S_1(\psi, \tau) = \frac{1+s}{\alpha}\int_0^\infty \int_0^\tau e^{-\frac{(w(1+s)+z)}{\alpha}} erfc\left(\frac{\psi}{2\sqrt{z}}\right) I_0\left(\frac{2}{\alpha}\sqrt{(1+s-r\alpha)wz}\right) dz dw$$

$$+ \frac{r}{\alpha}\int_0^\infty \int_0^v \int_0^\tau e^{-\frac{(w(1+r)+m)}{\alpha}} erfc\left(\frac{\psi}{2\sqrt{z}}\right) I_0\left(\frac{2}{\alpha}\sqrt{(1+s-r\alpha)qw}\right) dz dq dw, \tag{34}$$

$$S_2(\tau) = \frac{G_r e^{-n_1\tau}}{P_{reff}}\left[\frac{sinh(n_2\tau)}{\alpha n_2}\right], \tag{35}$$

$$R_1(\psi, \tau) = e^{-\psi\sqrt{e_1}} - \frac{\psi\sqrt{e_2}}{2\pi}\int_0^\infty \int_0^\tau \frac{1}{\sqrt{\tau}}e^{-\left(e_3\tau + \frac{\psi^2}{4v} + e_1 v\right)} I_1\left(2\sqrt{e_2 v\tau}\right) d\tau dv, \tag{36}$$

$$R_2(\tau) = \frac{Gme^{-n_3\tau}}{Sc}\left[\frac{sinh(n_4\tau)}{\alpha n_4}\right]. \tag{37}$$

Substituting all the above functions, the equation of velocity field becomes in the form

$$V(\psi,\tau) = S(\psi,\tau) - S(\psi,\tau_0) * H(\tau_0) + R(\psi,\tau). \tag{38}$$

3.3.1 Skin Friction

The following relation used to calculate the skin friction as

$$C_f = -\frac{\partial V(\psi,\tau)}{\partial \psi}\big|_{\psi=0}. \tag{39}$$

4 Limiting Cases

We recover the same result for velocity field in the absence of mass Grashof number $G_m = 0$ obtained by Anwar et al. [28]. Also when $N_r = 0$ and $G_m = 0$, we get the same expression for velocity as discussed by Samiulhaq et al. [34].

5 Graphical Discussion

This part is devoted for physical interpretation of heat transfer is executed on the motion of Second-grade fluid near a porous surface. The impact of thermal radiation, magnetic field, and ramped temperature conditions are also analyzed via Laplace integral transform to obtain a unique and exact solution. The graphical representations are depicted for showing the impact of physical parameters such as P_r, N_r, S_c, G_m, G_r, K and M on energy, concentration and velocity profile. Fig. 2 is plotted for the impact of P_r on energy profile. It is seen that thermal layer and temperature decreases by large value of P_r. As P_r increase, the temperature profile reduce more rapidly. Physically, for less value of P_r heat conductivity enhances. Fig. 3 shows the impact of heat radiation N_r. By enhancing the value of N_r, the required energy enhance. The domination of N_r, enhanced due to transportation of energy.
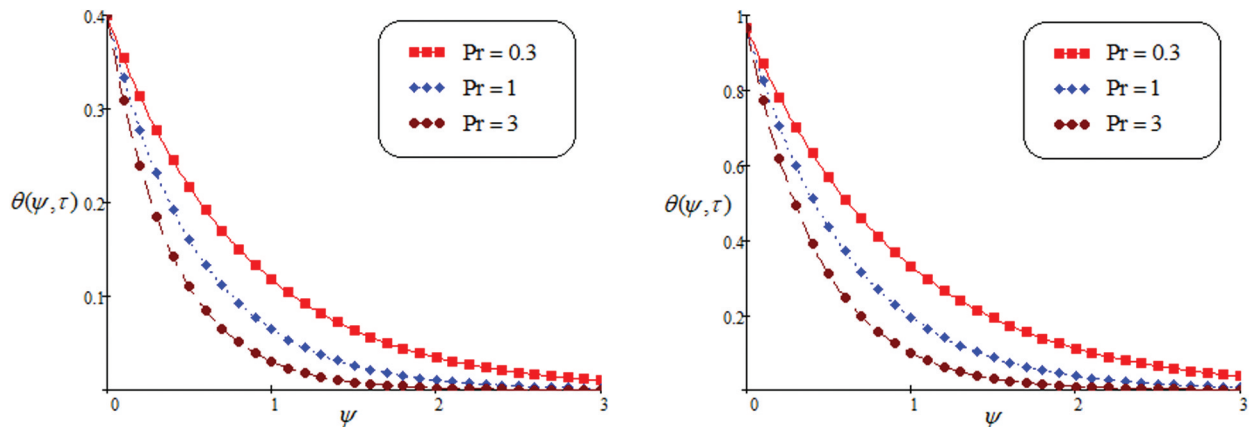

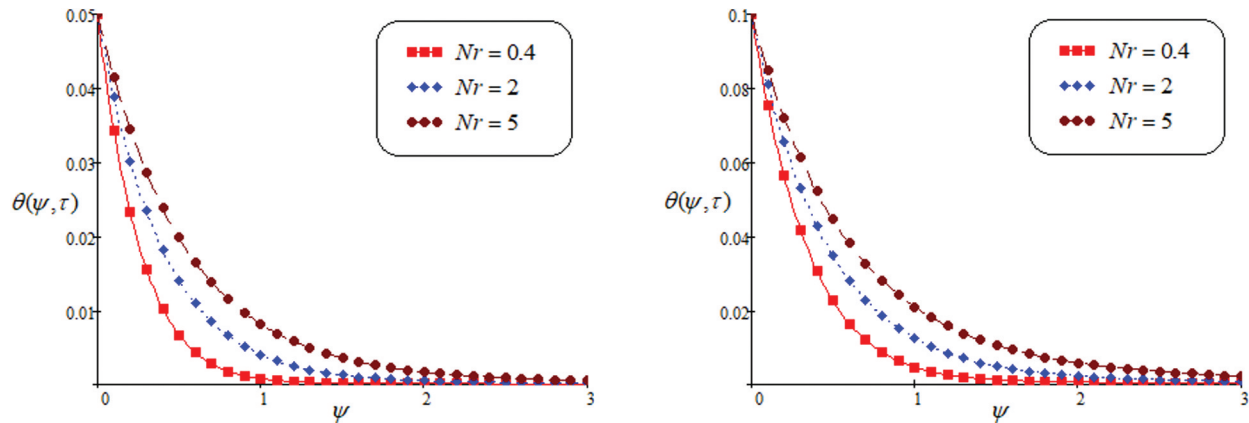**Figure 2:** Temprature for altered values of P_r

Figure 3: Temprature for altered values of N_r

In Figs. 4 and 5, we discussed the behavior of λ and S_c on concentration profile. It is noted that the greater value of λ, the concentration is increasing while in case of S_c, the concentration reduce as enhance the value of S_c. Contribution of concentration component on the fluid motion is important and it cannot be ignored.
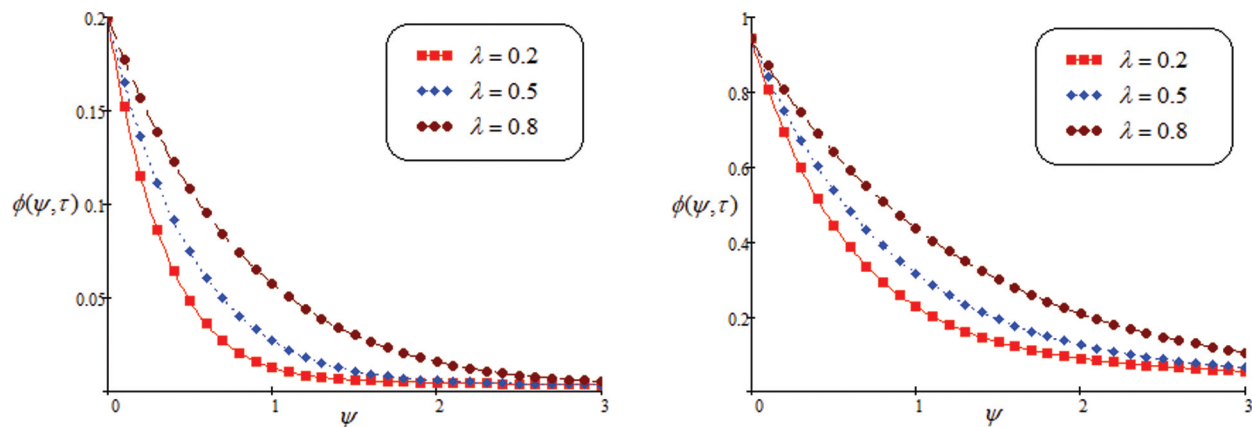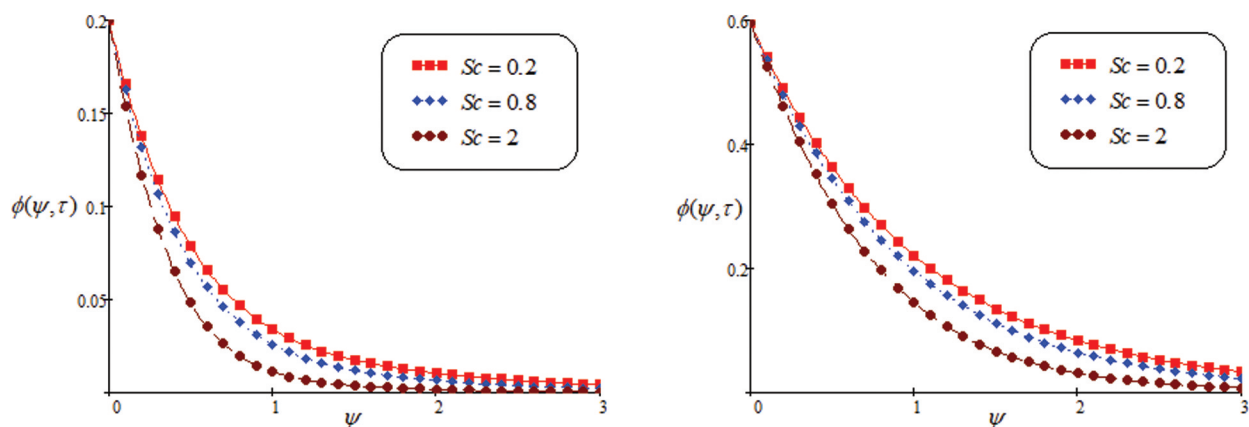

Figure 4: Concentration for altered values of λ

Figure 5: Concentration for altered values of S_c

Fig. 6 shows the influence of G_m. It is the ratio of mass force to viscous force, which causes free convection. It is noted that the fluid velocity is increasing, if we increase the value of G_m. Fig. 7 shows that the impact on G_r for velocity field. The curve behavior of G_r is same as G_m. It is noted that the velocity field enhance by increasing in G_r. Physically, when the G_r is increased, then fluid flow increase due to the thermal effects.
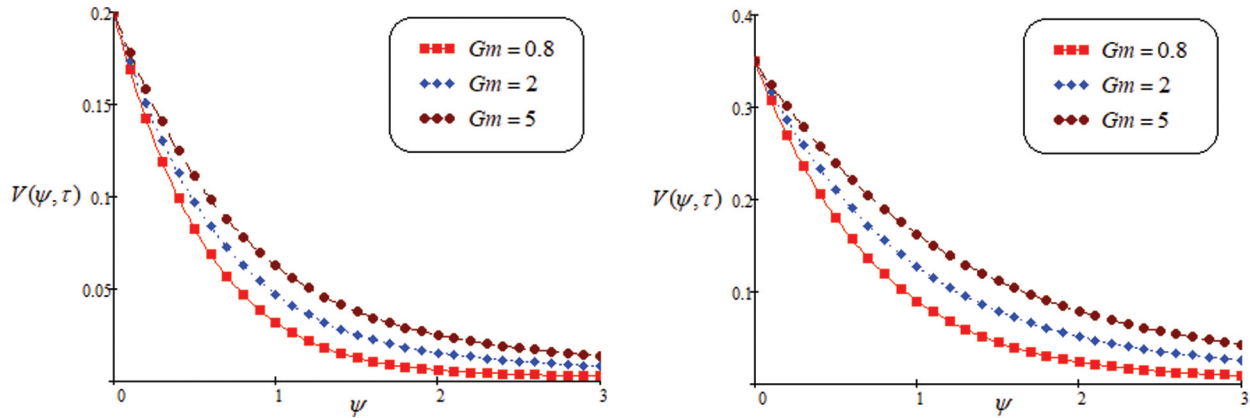

Figure 6: Velocity for altered values of G_m
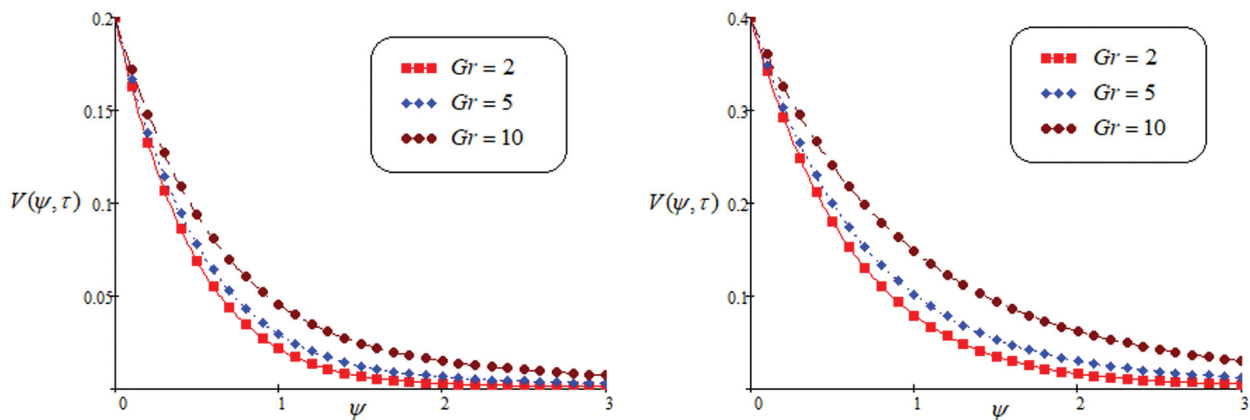
Figure 7: Velocity for altered values of G_r

Fig. 8 shows the effect of permeability on velocity curve. The fluid velocity enhance as escalate the value of K. Fig. 9 discusses the domination of M on the velocity curve. These graphical representation indicates that an increase in M, the velocity reduce due to Lorentz force. It behave as a drag force. By increasing the parameter of magnetic field, the Lorentz force also increase. Fluid flow on the boundary layer is slow down due to this force. Fig. 10 discusses the behavior of P_r. Specific heat and conductivity are depend on P_r. The thickness of momentum and boundary layer is controled by prandtl number. It seen from the graph, decreasing the velocity, observed by increase the value of P_r. The lower prandtl number enhance the thermal conductivity and increase the boundary layer. Fig. 11 shows the impact of S_c on fluid profile. It notices that the value of S_c enhance, the velocity of fluid flow decreased.
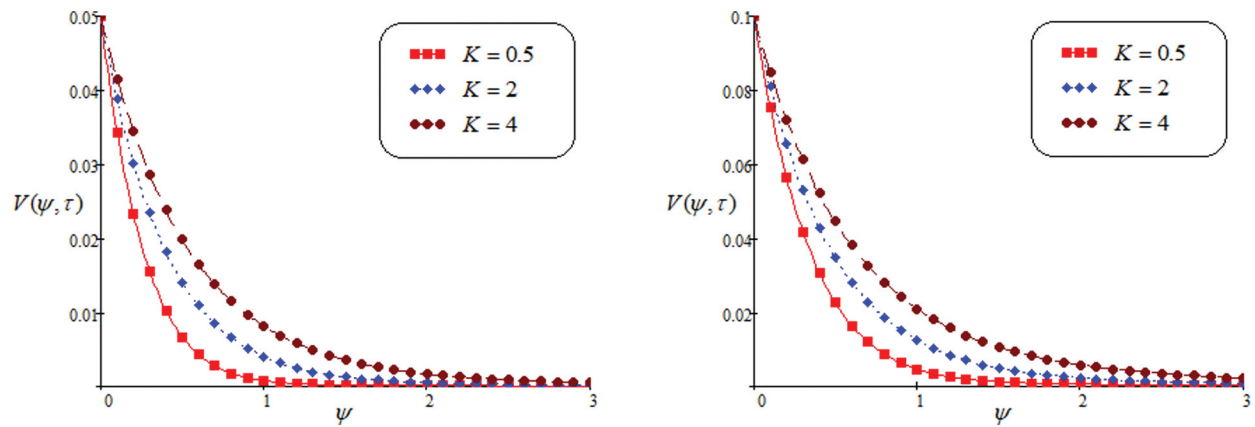
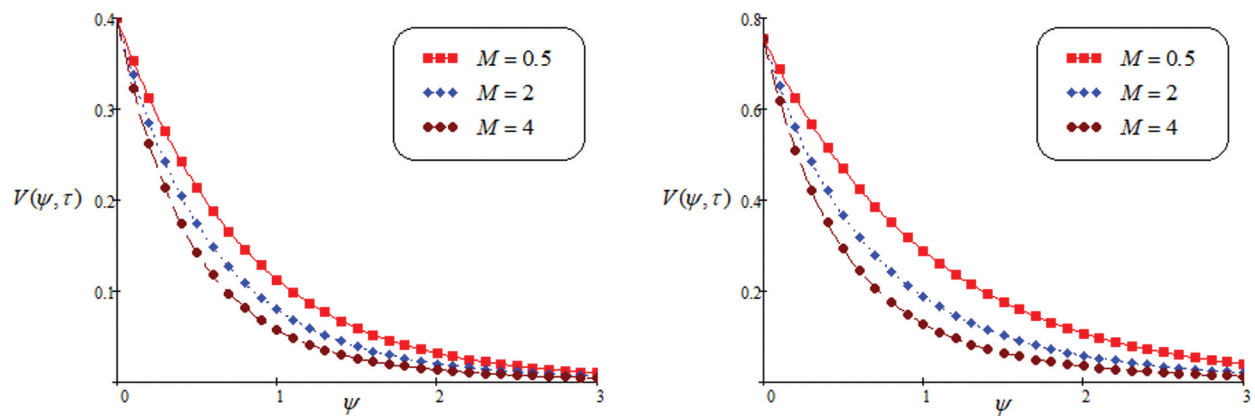**Figure 8:** Velocity for altered values of K

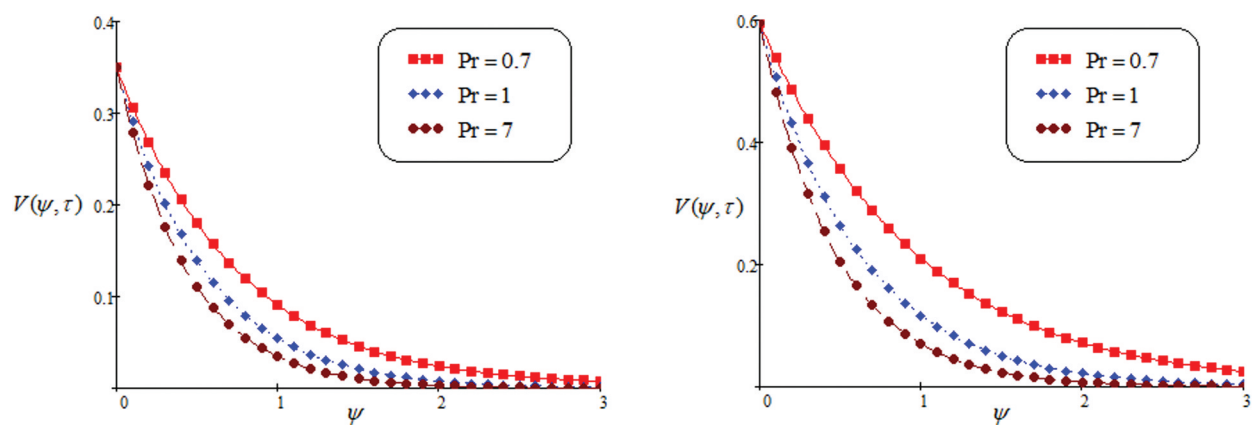**Figure 9:** Velocity for altered values of M

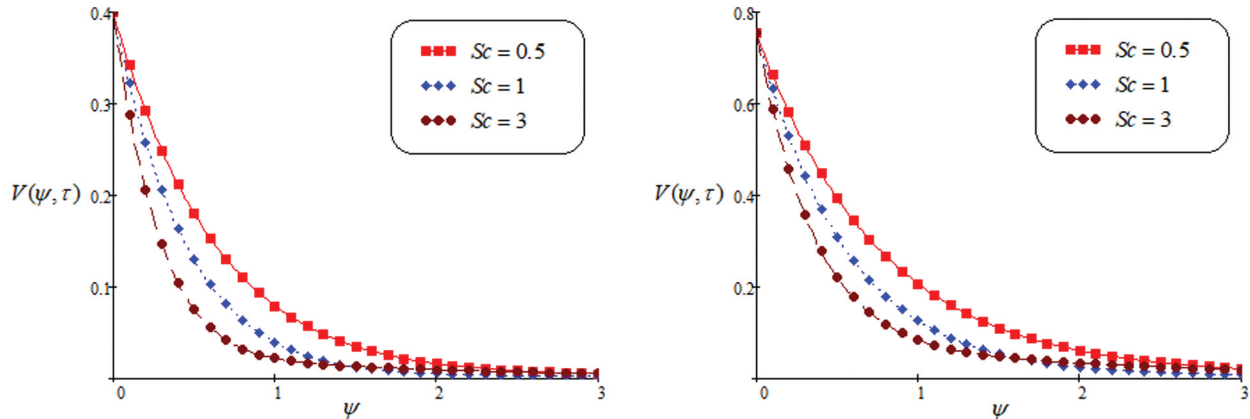**Figure 10:** Velocity for altered values of P_r

Figure 11: Velocity for altered values of S_c

6 Concluding Observation

This part is devoted for dynamical analysis of heat transfer on MHD Second-grade fluid near a porous surface. The impact of ramped temperature condition and magnetic effect are analyzed via Laplace integral transform to obtain a unique and exact solution of velocity profile. Some significant remarks and concluding results are given below:

- It is noted that the decay in temperature for increasing value of P_r while temperature is increasing as increase in N_r.
- The decay in concentration for growing the values of S_c.
- The fluid velocity decreases when the value of M increased.
- Enhance the values of G_r, G_m, the required fluid velocity is increasing.
- Enhance the values of K, the required fluid velocity is increasing.
- In case of large value of S_c and magnetic field M the fluid velocity is decelerated.
- Contribution of concentration component of fluid velocity on the fluid motion is important and it cannot be ignored.

Conflicts of Interest: The authors declare that they have no conflicts of interest to report regarding the present study.

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