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ARTICLE

A New Attempt to Neutrosophic Soft Bi-Topological Spaces

Arif Mehmood[1], **Muhammad Aslam**[2], **Muhammad Imran Khan**[3], **Humera Qureshi**[3], **Choonkil Park**[4,*] **and Jung Rye Lee**[5]

[1]Department of Mathematics & Statistics, Riphah International University, Sector I-14, Islamabad, 44000, Pakistan

[2]Department of Mathematics, College of Sciences, King Khalid University, Abha, 61413, Saudi Arabia

[3]Department of Pure and Applied Mathematics (Statistics), University of Haripur, Haripur, 22630, Pakistan

[4]Research Institute for Natural Sciences, Hanyang University, Seoul, 04763, Korea

[5]Department of Data Science, Daejin University, Kyunggi, 11159, Korea

[*]Corresponding Author: Choonkil Park. Email: baak@hanyang.ac.kr

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ABSTRACT

In this article, new generalized neutrosophic soft $*_b$ open set is introduced in neutrosophic soft bi-topological structurers (NSBTS) concerning soft points of the space. This new set is produced by making the marriage of soft semi-open set with soft pre-open set in neutrosophic soft topological structure. An ample of results are investigated in NSBTS on the basis of this new neutrosophic soft $*_b$ open set. Proper examples are settled for justification of these results. The non-validity of some results is vindicated with examples.

KEYWORDS

Neutrosophic soft set (NSS); neutrosophic soft points (NSP); neutrosophic soft bi-topological structurers (NSBTS); neutrosophic soft $*_b$-open set and neutrosophic soft $*_b$-separation axioms

1 Introduction

Fuzzy set theory [1] is the most importantly effective way to deal with vagueness and incomplete data and it is being developed and used in various fields of science. Fuzzy set (FS), which is directly an extension of the crisp sets. However, it has a shortcoming, i.e., it only addresses membership value and is unable to address the non-membership value. Since fuzzy set theory (FST) was too young at that time and researchers were working actively. Finally, Atanassov [2] addressed the deficiency in fuzzy set theory in sophisticated way and resulted in bouncing up a new idea with new wording "intusionistic fuzzy set theory (IFST)". This approach supposes membership value and non-membership value.

Molodtsov [3] opened a new window of research and inaugurated the concept of soft set theory (SST) to address the uncertainty, which links a crisp set with another set of parameters. Soft set theory has a big hand as an application in many fields like function smoothness, Riemann integration, measurement theory, game theory, etc. [4]. The second leading cause of cancer death

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among men in most industrialized countries is prostate cancer which depends upon various factors, such as family history, age, ethnicity, and prostate-specific blood level (PSA).

PSA levels in the blood are an important way of diagnosing patients initially [5–7]. Yuksel et al. [8] discussed prostate cancer (PSA), prostrate volume (PV) and age factors of patients on the basis of fuzzy set and soft sets.

Wei et al. [9] leaked out the concept of VSS which is an extension to Huang et al. [10] deeply studied [9] and pointed out some incorrect results. They verified the incorrect result with examples and gave some more new definitions. Wang et al. [11] initiated the concept of vague soft topological structures with title topological structure of vague soft sets. They authors discussed the basic concepts related to vague soft topological and studied the results in vague soft topology. The soft set to the hyper soft set was generalized by Smarandache [12]. In addition to this, the author introduced the hybrids of crisp, fuzzy, intuitionistic fuzzy, neutrosophic, and plithogenic hyper-soft set. Bera et al. [13] opened the door to a new world of mathematics and inaugurated the conception of new structure (neutrosophic soft topology) on the basis of neutrosophic soft set (NSS). He discussed all the fundamentals in polite way and on the basis of these fundamentals he moved to the fundamentals results and for better understanding suitable examples were put forwarded. Ozturk et al. [14] are pioneered in new operations on neutrosophic soft sets and then new approach to neutrosophic soft topology on the basis of these new operations. Ozturk et al. [15] leaked out concept of neutrosophic soft mapping, neutrosophic soft open mapping and neutrosophic soft homeomorphism on the basis of operation defined in [14]. Mehmood et al. [16] generated new open set in NSTS, known as left b star open sets. Neutrosophic soft separation axioms in NSTS are switched on to different results which are countability theorems, linking of these theorems with Hausdorff spaces, convergency of sequences, continuity, product of spaces, Bolzano Weirstrass property, compactness and sequentially compactness, etc. AL-Nafee [17] introduced new family of NSS. The author defined new operation on neutrosophic set. These operations are union, intersection etc. On the basis of these new operations the author defined NSTS. AL-Nafee et al. [18] continued his work and extended the NSTS to NSBTS on the basis of operations defined in [18]. The authors regenerated all the fundamental results of NSBTS on these basic operations. Hasan et al. [19] inaugurated NSBTS on the basis of the operations defined in [13]. The authors introduced pair-wise neutrosophic soft (closed) sets in NSBTS. These references [13–19] became source of motivation for my new research.

In our study, we worked with the operations given in references [14–16] which are entirely different from references [13,17]. Then unlike [18,19], neutrosophic soft bi-topological structure is reconstructed. In Section 2, some basic recipes are inaugurated. In Section 3, Neutrosophic soft bi-topology (NSBT) is addressed with examples. Some results, union and intersection are also studied in (NSBT). In Section 4, some main results are addressed. Section 5, more main results are addressed. In Section 6, some concluding remarks and future work are announced.

2 Related Work

In this section, fundamental concepts are addressed.

Definition 2.1. [20] A neutrosophic set (NS) symbolized by A on the key set π is defined as:
$$A = \{\langle \kappa, \urcorner \langle\kappa\rangle, \mathrm{i}_A\langle\kappa\rangle, \mathrm{F}_A\langle\kappa\rangle : \kappa \in \pi\rangle\}\, where, \begin{bmatrix} \urcorner : \pi \longrightarrow \left(0^-, 1^+\right) \\ \mathrm{i} : \pi \longrightarrow \left(0^-, 1^+\right), \\ \mathrm{F} : \pi \longrightarrow \left(0^-, 1^+\right) \\ 0^- \preccurlyeq \urcorner_A\langle\kappa\rangle + \mathrm{i}_A\langle\kappa\rangle \\ +\mathrm{F}_A\langle\kappa\rangle \preccurlyeq 3^+. \\ , \end{bmatrix}.$$

Definition 2.2. [3] π be key set, θ be a set of all parameters, and $\mathcal{L}(\pi)$ symbolizes the key set of π. A pair $\langle \mathfrak{f}, \theta\rangle$ is referred to as a soft set (SS) over π, where $\mathfrak{f}$ is a mapping given by $\mathfrak{f} : \theta \to \mathcal{L}(\pi)$.

Definition 2.3. [21,22] Let π be KS and θ set of parameters. Let $\mathcal{L}(\pi)$ signifies power set of all neutrosophic sets on π. Then a neutrosophic soft set $\left(\tilde{\mathfrak{f}}, \theta\right)$ over π is a set defined by a set valued function $\tilde{\mathfrak{f}}$ representing a mapping $\tilde{\mathfrak{f}} : \theta \to \mathcal{L}(\pi)$ where $\tilde{\mathfrak{f}}$ is called approximate function of the neutrosophic soft set $\left(\tilde{\mathfrak{f}}, \theta\right)$. It can be written as a set of ordered pairs: $(\tilde{\mathfrak{f}}, \theta) = \{((n, [\![\kappa, \mathbb{T}_{\tilde{\mathfrak{f}}(n)^{(\kappa)}}, \mathbb{I}_{\tilde{\mathfrak{f}}(n)^{(\kappa)}}, \mathbb{F}_{\tilde{\mathfrak{f}}(n)^{(\kappa)}} : \kappa \in \pi]\!]) : n \in \theta\}$.

Definition 2.4. [13] Let $\left(\tilde{\mathfrak{f}}, \theta\right)$ be a neutrosophic soft over the key set π then the complement of $\left(\tilde{\mathfrak{f}}, \theta\right)$ is signified $\left(\tilde{\mathfrak{f}}, \theta\right)^c$ and is defined as follows:

$\left(\tilde{\mathfrak{f}}, \theta\right)^c = \{((n, [\![\kappa, \urcorner_{\tilde{\mathfrak{f}}(n)^{(v)}}, 1 - \mathrm{i}_{\tilde{\mathfrak{f}}(n)^{(\kappa)}}, \mathbb{F}_{\tilde{\mathfrak{f}}(n)^{(\kappa)}} : \kappa \in \pi]\!]) : n \in \theta\}$. It's clear that

$\left(\left(\tilde{\mathfrak{f}}, \theta\right)^c\right)^c = \left(\tilde{\mathfrak{f}}, \theta\right)$.

Definition 2.5. [22] Let $(\tilde{\mathfrak{f}}, \theta)$, $(\tilde{\rho}, \theta)$ two neutrosophic soft over the key set π. $\left(\tilde{\mathfrak{f}}, \theta\right)$ is supposed to be neutrosophic soft sub-set of $(\tilde{\rho}, \theta)$ if $\urcorner_{\tilde{\mathfrak{f}}(n)^{(\kappa)}} \preccurlyeq \urcorner_{\tilde{\rho}(n)^{(\kappa)}}$, $\mathrm{i}_{\tilde{\mathfrak{f}}(n)^{(\kappa)}} \preccurlyeq \mathrm{i}_{\tilde{\rho}(n)^{(\kappa)}}$, $\mathrm{F}_{\tilde{\mathfrak{f}}(n)^{(\kappa)}} \succcurlyeq \mathrm{F}_{\tilde{\rho}(n)^{(\kappa)}}$, $\forall n \in \theta \& \forall\ \kappa \in \pi$. It is denoted by $(\tilde{\mathfrak{f}}, \theta) \sqsubseteq (\tilde{\rho}, \theta)$. $(\tilde{\mathfrak{f}}, \theta)$ is said to be neutrosophic soft equal to $(\tilde{\rho}, \theta)$ if $(\tilde{\mathfrak{f}}, \theta)$ is neutrosophic soft sub-set of $(\tilde{\rho}, \theta)$ and $(\tilde{\rho}, \theta)$ is neutrosophic soft sub-set of $\left(\tilde{\mathfrak{f}}, \theta\right)$. It is denoted by $(\tilde{\mathfrak{f}}, \theta) = (\tilde{\rho}, \theta)$.

Definition 2.6. [16] Let $\left(\tilde{\mathfrak{f}}_1, \theta\right), \left(\tilde{\mathfrak{f}}_2, \theta\right)$ be two neutrosophic soft sub-sets over the key set π so that $(\tilde{\mathfrak{f}}_1, \theta) \neq (\tilde{\mathfrak{f}}_2, \theta)$. Then their union is denoted by $(\tilde{\mathfrak{f}}_1, \theta) \tilde{\bigsqcup} (\tilde{\mathfrak{f}}_2, \theta) = (\tilde{\mathfrak{f}}_3, \theta)$ and is defined as $(\tilde{\mathfrak{f}}_3, \theta) = \{((n, [\![\kappa, \urcorner_{\tilde{\mathfrak{f}}_3(n)^{(\kappa)}}, \mathrm{i}_{\tilde{\mathfrak{f}}_3(n)^{(\kappa)}}, \mathrm{F}_{\tilde{\mathfrak{f}}_3(n)^{(\kappa)}} : \kappa \in \pi]\!]) : n \in \theta\}$. where

$$\begin{bmatrix} \urcorner_{\tilde{\mathfrak{f}}_3(n)^{\langle\kappa\rangle}} = max\left[\urcorner_{\tilde{\mathfrak{f}}_1(n)^{\langle\kappa\rangle}}, \urcorner_{\tilde{\mathfrak{f}}_2(n)^{\langle\kappa\rangle}}\right., \\ \mathrm{i}_{\tilde{\mathfrak{f}}_3(n)^{\langle\kappa\rangle}} = max\left[\mathrm{i}_{\tilde{\mathfrak{f}}_1(n)^{\langle\kappa\rangle}}, \mathrm{i}_{\tilde{\mathfrak{f}}_2(n)^{\langle\kappa\rangle}}\right., \\ \mathrm{F}_{\tilde{\mathfrak{f}}_3(n)^{\langle\kappa\rangle}} = min\left[\mathrm{F}_{\tilde{\mathfrak{f}}_1(n)^{\langle\kappa\rangle}}, \mathrm{F}_{\tilde{\mathfrak{f}}_2(n)^{\langle\kappa\rangle}}\right.. \end{bmatrix}$$

Definition 2.7. [16] Let $\left(\tilde{\mathfrak{f}}_{\mathbf{1}},\theta\right)$, $\left(\tilde{\mathfrak{f}}_{\mathbf{2}},\theta\right)$ be two neutrosophic soft sub-sets over key set π such that $\left(\tilde{\mathfrak{f}}_{\mathbf{1}},\theta\right)\neq\left(\tilde{\mathfrak{f}}_{\mathbf{2}},\theta\right)$. Then their intersection is denoted by $\left(\tilde{\mathfrak{f}}_{\mathbf{1}},\theta\right)\tilde{\sqcap}\left(\tilde{\mathfrak{f}}_{\mathbf{2}},\theta\right)=\left(\tilde{\mathfrak{f}}_{\mathbf{3}},\theta\right)$, is defined as $[(\tilde{\mathfrak{f}}_{\mathbf{3}},n)=\{((n,[\![\kappa,\urcorner_{\tilde{\mathfrak{f}}_{\mathbf{3}}(n)^{\langle\kappa\rangle}},\mathrm{i}_{\tilde{\mathfrak{f}}_{\mathbf{3}}(n)^{\langle\kappa\rangle}},\mathrm{F}_{\tilde{\mathfrak{f}}_{\mathbf{3}}(n)^{\langle\kappa\rangle}}:\kappa\in\pi]\!]):n\in\theta\}.]$ where,

$$where\begin{bmatrix}\urcorner_{\tilde{\mathfrak{f}}_{\mathbf{3}}(n)^{\langle\kappa\rangle}}=min\left[\urcorner_{\tilde{\mathfrak{f}}_{\mathbf{1}}(n)^{\langle\kappa\rangle}},\urcorner_{\tilde{\mathfrak{f}}_{\mathbf{2}}(n)^{\langle\kappa\rangle}}\right.,\\ \mathrm{i}_{\tilde{\mathfrak{f}}_{\mathbf{3}}(n)^{\langle\kappa\rangle}}=min\left[\mathrm{i}_{\tilde{\mathfrak{f}}_{\mathbf{1}}(n)^{\langle\kappa\rangle}},\mathrm{i}_{\tilde{\mathfrak{f}}_{\mathbf{2}}(n)^{\langle\kappa\rangle}}\right.,\\ \mathrm{F}_{\tilde{\mathfrak{f}}_{\mathbf{3}}(n)^{\langle\kappa\rangle}}=max\left[\mathrm{F}_{\tilde{\mathfrak{f}}_{\mathbf{1}}(n)^{\langle\kappa\rangle}},\mathrm{F}_{\tilde{\mathfrak{f}}_{\mathbf{2}}(n)^{\langle\kappa\rangle}}.\right.\end{bmatrix}$$

Definition 2.8. [14] Neutrosophic soft set $\left(\tilde{\mathfrak{f}},\theta\right)$ overkey set π is said to be a neutrosophic soft null set If $\urcorner_{\mathfrak{f}(n)^{\langle\kappa\rangle}}=0,\mathrm{i}_{\mathfrak{f}(n)^{\langle\kappa\rangle}}=0,\mathrm{F}_{\mathfrak{f}(n)^{\langle\kappa\rangle}}=1;,\forall n\in\theta,\forall\kappa\in\pi$.

It is signified as $0_{(\pi,\theta)}$.

Definition 2.9. [14] Neutrosophic soft set $\left(\tilde{\mathfrak{f}},\theta\right)$ over key set π is an absolute neutrosophic soft set if $\mathbb{T}_{\mathfrak{f}(n)^{\langle\kappa\rangle}}=1,\mathrm{i}_{\mathfrak{f}(n)^{\langle\kappa\rangle}}=1,\mathbb{F}_{\mathfrak{f}(n)^{\langle\kappa\rangle}}=0;,\forall n\in\theta\&\forall\kappa\in\pi$.

It is signified as $1_{(\pi,n)}$. Clearly, ${0_{(\pi,n)}}^{c}=1_{(\pi,n)}$, ${1_{(\pi,n)}}^{c}=0_{(\pi,n)}$.

Definition 2.10. [14] Let neutrosophic soft set $(\tilde{\pi},\theta)$ be the family of all NS soft sets and $\tau\subset NSS(\tilde{\pi},\theta)$. Then τ is said to be a neutrosophic soft topology on $\tilde{\pi}$ if (1). $0_{(\langle\pi\rangle,n)},1_{(\langle\pi\rangle,n)}\in\tau$, (2). The union of any number of neutrosophic set soft sets in $\tau\in\tau$, (3). The intersection of a finite number of neutrosophic soft sets in $\tau\in\tau$, then $(\tilde{\pi},\tau,\theta)$ is said to be neutrosophic soft topological space over $\tilde{\pi}$.

Definition 2.11. [14] Let neutrosophic soft set $\widetilde{(\pi,\theta)}$ be the family of all neutrosophic set over $\tilde{\pi}$, $\kappa\in\tilde{\pi}$, then $NS\kappa_{\langle\mathfrak{y}_1,\mathfrak{y}_2,\mathfrak{y}_3\rangle}$ is NS point, for $0<\mathfrak{y}_1,\mathfrak{y}_2,\mathfrak{y}_3\leq 1$ and is defined as follows:

$$\kappa_{(\langle\mathfrak{y}_1,\mathfrak{y}_2,\mathfrak{y}_3\rangle(y)}=\left\lfloor\begin{cases}\langle\mathfrak{y}_1,\mathfrak{y}_2,\mathfrak{y}_3\rangle & if\ y=\kappa\}\\ (0,0,1) & if\ y\neq\kappa\}\end{cases}\right\rfloor$$

Definition 2.12. [14] Let neutrosophic soft set $\widetilde{(\pi,\theta)}$ be the family of all neutrosophic soft sets over key set π Then $NSS(\kappa_{\langle\mathfrak{y}_1,\mathfrak{y}_2,\mathfrak{y}_3\rangle})^{e}$ is called a neutrosophic soft point, for every $\kappa\in\widetilde{\pi},0\prec\{\mathfrak{y}_1,\mathfrak{y}_2,\mathfrak{y}_3\}\preccurlyeq 1,\mathrm{e}\in\theta$, and is defined as follows:

$$\kappa^{e}{}_{\langle\mathfrak{y}_1,\mathfrak{y}_2,\mathfrak{y}_3\rangle^{e^{`}(y)}}=\begin{bmatrix}\langle\mathfrak{y}_1,\mathfrak{y}_2,\mathfrak{y}_3\rangle & if\ e^{`}=e\wedge y=\kappa\\ (0,0,1) & if\ e^{`}\neq e\wedge y\neq\kappa\end{bmatrix}$$

Definition 2.13. [14] Let $\left(\tilde{\mathfrak{f}},\theta\right)$ be a NSS over key set π. $\kappa^{e}{}_{\langle\mathfrak{y}_1,\mathfrak{y}_2,\mathfrak{y}_3\rangle}\in NSS\left(\tilde{\mathfrak{f}},\theta\right)$ if $\mathfrak{y}_1\preccurlyeq\urcorner_{\tilde{\mathfrak{f}}(n)^{\langle\kappa\rangle}}$, $\mathfrak{y}_2\preccurlyeq\mathrm{i}_{\tilde{\mathfrak{f}}(n)^{\langle\kappa\rangle}}$, $\mathfrak{y}_3\succcurlyeq\mathrm{F}_{\tilde{\mathfrak{f}}(n)^{\langle\kappa\rangle}}$.

Definition 2.14. [14] Let $(\tilde{\pi},\tau,\theta)$ be a $NSTS$ over π. Neutrosophic soft set $\left(\tilde{\mathfrak{f}},\theta\right)$ in (π,τ,θ) is called a neutrosophic soft neighbourhood of the neutrosophic soft point $\kappa^{\curlywedge}{}_{\langle\mathfrak{y}_1,\mathfrak{y}_2,\mathfrak{y}_3\rangle}\in\left(\tilde{\mathfrak{f}},\theta\right)$, if there exists a neutrosophic soft open set $\left(\tilde{g},\theta\right)$ such that $\kappa^{\curlywedge}{}_{\langle\mathfrak{y}_1,\mathfrak{y}_2,\mathfrak{y}_3\rangle}\in\left(\tilde{g},\theta\right)$.

3 Neutrosophic Soft Bi-Topology

In this part, the concept of *NSBTS* is defined. Furthermore, new types of open and closed sets have been introduced in neutrosophic soft bitopological spaces.

Definition 3.1. *If* $\langle\pi,\tau_1,\theta\rangle$, $\langle\pi,\tau_2,\theta\rangle$ are two NSTS, then $\langle\pi,\tau_1,\tau_2,\theta\rangle$ is called NSBTS. *If* $\langle\pi,\tau_1,\tau_2,\theta\rangle$ be *NSBTS*. Neutrosophic soft sub-set (f,θ) is said to be open in $\langle\pi,\tau_1,\tau_2,\theta\rangle$ if there exists a neutrosophic soft open set (f_1,θ) in τ_1 and neutrosophic soft open set (f_2,θ) in τ_2 such that $(f,\theta)=(f_1,\theta)\cup(f_2,\theta)$.

Example 3.2. Let $\pi=\{\kappa_1,\kappa_2,\kappa_3\}$, $\theta=\{e_1,e_2\}$ and $\tau_1=\{0_{(\pi,\theta)},1_{(\pi,\theta)},(\omega_1,\theta),(\omega_2,\theta)\}$,

$\tau_1=\{0_{(\pi,\theta)},1_{(\pi,\theta)},(\measuredangle_1,\theta),(\measuredangle_2,\theta)\}$, where $(\omega_1,\theta),(\omega_2,\theta),(\measuredangle_1,\theta)$ ve $(\measuredangle_2,\theta)$ being neutrosophic soft sub-sets as following:

$$f_{(\omega_1,\theta)}(e_1)=\left[\left\langle\kappa_1,\frac{02}{10},\frac{03}{10},\frac{08}{10}\right\rangle,\left\langle\kappa_2,\frac{04}{10},\frac{04}{10},\frac{04}{10}\right\rangle,\left\langle\kappa_3,\frac{02}{10},\frac{04}{10},\frac{03}{10}\right\rangle\right]$$

$$f_{(\omega_1,\theta)}(e_2)=\left[\left\langle\kappa_1,\frac{03}{10},\frac{02}{10},\frac{06}{10}\right\rangle,\left\langle\kappa_2,\frac{01}{10},\frac{05}{10},\frac{05}{10}\right\rangle,\left\langle\kappa_3,\frac{04}{10},\frac{03}{10},\frac{04}{10}\right\rangle\right]$$

$$f_{(\omega_2,\theta)}(e_1)=\left[\left\langle\kappa_1,\frac{04}{10},\frac{03}{10},\frac{06}{10}\right\rangle,\left\langle\kappa_2,\frac{04}{10},\frac{05}{10},\frac{03}{10}\right\rangle,\left\langle\kappa_3,\frac{03}{10},\frac{05}{10},\frac{02}{10}\right\rangle\right]$$

$$f_{(\omega_2,\theta)}(e_2)=\left[\left\langle\kappa_1,\frac{03}{10},\frac{04}{10},\frac{05}{10}\right\rangle,\left\langle\kappa_2,\frac{02}{10},\frac{06}{10},\frac{04}{10}\right\rangle,\left\langle\kappa_3,\frac{04}{10},\frac{06}{10},\frac{03}{10}\right\rangle\right]$$

$$f_{(\measuredangle_1,\theta)}(e_1)=\left[\left\langle\kappa_1,\frac{05}{10},\frac{04}{10},\frac{04}{10}\right\rangle,\left\langle\kappa_2,\frac{06}{10},\frac{06}{10},\frac{02}{10}\right\rangle,\left\langle\kappa_3,\frac{04}{10},\frac{06}{10},\frac{01}{10}\right\rangle\right]$$

$$f_{(\measuredangle_1,\theta)}(e_2)=\left[\left\langle\kappa_1,\frac{04}{10},\frac{06}{10},\frac{03}{10}\right\rangle,\left\langle\kappa_2,\frac{03}{10},\frac{07}{10},\frac{03}{10}\right\rangle,\left\langle\kappa_3,\frac{05}{10},\frac{07}{10},\frac{01}{10}\right\rangle\right]$$

$$f_{(\measuredangle_2,\theta)}(e_1)=\left[\left\langle\kappa_1,\frac{01}{10},\frac{02}{10},\frac{07}{10}\right\rangle,\left\langle\kappa_2,\frac{03}{10},\frac{03}{10},\frac{03}{10}\right\rangle,\left\langle\kappa_3,\frac{01}{10},\frac{02}{10},\frac{02}{10}\right\rangle\right]$$

$$f_{(\measuredangle_2,\theta)}(e_2)=\left[\left\{\left\langle\kappa_1,\frac{01}{10},\frac{02}{10},\frac{07}{10}\right\rangle,\left\langle\kappa_2,\frac{03}{10},\frac{03}{10},\frac{03}{10}\right\rangle,\left\langle\kappa_3,\frac{01}{10},\frac{02}{10},\frac{02}{10}\right\rangle\right\}\right].$$

Then $(\omega_1,\theta)\cup(\omega_2,\theta)=(\omega_2,\theta)$, $(\omega_1,\theta)\cup(\measuredangle_1,\theta)=(\measuredangle_1,\theta)$, $(\omega_1,\theta)\cup(\measuredangle_2,\theta)=(\omega_2,\theta)$, $(\measuredangle_1,\theta)\cup(\measuredangle_2,\theta)=(\measuredangle_1,\theta)$, $(\omega_2,\theta)\cup(\measuredangle_2,\theta)=(\omega_2,\theta)$ and $(\omega_1,\theta)\cap(\omega_2,\theta)=(\omega_2,\theta)$, $(\omega_1,\theta)\cap\measuredangle=(\measuredangle_1,\theta)$, $(\omega_1,\theta)\cap(\measuredangle_2,\theta)=(\omega_2,\theta)$, $(\measuredangle_1,\theta)\cap(\measuredangle_2,\theta)=(\measuredangle_1,\theta)$, $(\omega_2,\theta)\cap(\measuredangle_2,\theta)=(\measuredangle_2,\theta)$.

Therefore, τ_1, τ_2 are *NSBTS* on π so $(\pi,\tau_1,\tau_2,\theta)$ is a NSBTS.

Theorem 3.3. Let $(\pi,\tau_1,\tau_2,\theta)$ be a NSBTS. Then $\tau_1\cap\tau_2$ is a *NSBTS* on π.

Proof: $T1$ and $NST3$ are clear. *For* $NST2$, let $\{(\omega_i,\theta);i\in I\}\in\tau_1\cap\tau_2$. Then $(\omega_i,\theta)\in\tau_1$, $(\omega_i,\theta)\in\tau_2$. As τ_2, τ_2 are Neutrosophic soft topologies on π, then $\cup_i(\omega_i,\theta)\in\tau_1$, $\cup_i(\omega_i,\theta)\in\tau_2$. Therefore $\cup_i(\omega_i,\theta)\in\tau_1\cap\tau_2$.

Remark 3.4. Let $(\pi,\tau_1,\tau_2,\theta)$ be a NSBTS, then $\tau_1\cup\tau_2$ need not be a NSBTS on π.

Example 3.5. Let $\pi = \{\kappa_1, \kappa_2, \kappa_3\}$, $\theta = \{e_1, e_2\}$ and $\tau_1 = \{0_{(\pi,\theta)}, 1_{(\pi,\theta)}, (\omega_1, \theta), (\omega_2, \theta), (\omega_3, \theta)\}$, $\tau_2 = \{0_{(\pi,\theta)}, 1_{(\pi,\theta)}, (\angle_1, \theta), (\angle_2, \theta)$ where $(\omega_1, \theta), (\omega_2, \theta), (\angle_1, \theta)$ ve $(\angle_2, \theta)$ being *NSSs* are as following:

$$f_{(\omega_1,\theta)}(e_1) = \left[\left\langle \kappa_1, \frac{02}{10}, \frac{03}{10}, \frac{08}{10} \right\rangle, \left\langle \kappa_2, \frac{04}{10}, \frac{04}{10}, \frac{04}{10} \right\rangle, \left\langle \kappa_3, \frac{02}{10}, \frac{04}{10}, \frac{03}{10} \right\rangle\right]$$

$$f_{(\omega_1,\theta)}(e_2) = \left[\left\langle \kappa_1, \frac{03}{10}, \frac{02}{10}, \frac{06}{10} \right\rangle, \left\langle \kappa_2, \frac{01}{10}, \frac{05}{10}, \frac{05}{10} \right\rangle, \left\langle \kappa_3, \frac{04}{10}, \frac{03}{10}, \frac{05}{10} \right\rangle\right]$$

$$f_{(\omega_2,\theta)}(e_1) = \left[\left\langle \kappa_1, \frac{04}{10}, \frac{03}{10}, \frac{06}{10} \right\rangle, \left\langle \kappa_2, \frac{04}{10}, \frac{05}{10}, \frac{03}{10} \right\rangle, \left\langle \kappa_3, \frac{03}{10}, \frac{05}{10}, \frac{02}{10} \right\rangle\right]$$

$$f_{(\omega_2,\theta)}(e_2) = \left[\left\langle \kappa_1, \frac{03}{10}, \frac{04}{10}, \frac{05}{10} \right\rangle, \left\langle \kappa_2, \frac{02}{10}, \frac{06}{10}, \frac{04}{10} \right\rangle, \left\langle \kappa_3, \frac{04}{10}, \frac{06}{10}, \frac{03}{10} \right\rangle\right]$$

$$f_{(\omega_3,\theta)}(e_1) = \left[\left\langle \kappa_1, \frac{05}{10}, \frac{04}{10}, \frac{04}{10} \right\rangle, \left\langle \kappa_2, \frac{06}{10}, \frac{06}{10}, \frac{02}{10} \right\rangle, \left\langle \kappa_3, \frac{04}{10}, \frac{06}{10}, \frac{0}{10} \right\rangle\right]$$

$$f_{(\omega_3,\theta)}(e_2) = \left[\left\langle \kappa_1, \frac{04}{10}, \frac{06}{10}, \frac{03}{10} \right\rangle, \left\langle \kappa_2, \frac{03}{10}, \frac{07}{10}, \frac{03}{10} \right\rangle, \left\langle \kappa_3, \frac{05}{10}, \frac{07}{10}, \frac{01}{10} \right\rangle\right]$$

$$f_{(\angle_1,\theta)}(e_1) = \left[\left\langle \kappa_1, \frac{05}{10}, \frac{04}{10}, \frac{04}{10} \right\rangle, \left\langle \kappa_2, \frac{06}{10}, \frac{06}{10}, \frac{02}{10} \right\rangle, \left\langle \kappa_3, \frac{04}{10}, \frac{06}{10}, \frac{01}{10} \right\rangle\right]$$

$$f_{(\angle_1,\theta)}(e_2) = \left[\left\langle \kappa_1, \frac{04}{10}, \frac{06}{10}, \frac{03}{10} \right\rangle, \left\langle \kappa_2, \frac{03}{10}, \frac{07}{10}, \frac{03}{10} \right\rangle, \left\langle \kappa_3, \frac{05}{10}, \frac{07}{10}, \frac{01}{10} \right\rangle\right]$$

$$f_{(\angle_2,\theta)}(e_1) = \left[\left\langle \kappa_1, \frac{01}{10}, \frac{02}{10}, \frac{07}{10} \right\rangle, \left\langle \kappa_2, \frac{03}{10}, \frac{03}{10}, \frac{03}{10} \right\rangle, \left\langle \kappa_3, \frac{01}{10}, \frac{02}{10}, \frac{02}{10} \right\rangle\right]$$

$$f_{(\angle_2,\theta)}(e_2) = \left[\left\langle \kappa_1, \frac{01}{10}, \frac{02}{10}, \frac{07}{10} \right\rangle, \left\langle \kappa_2, \frac{03}{10}, \frac{03}{10}, \frac{03}{10} \right\rangle, \left\langle \kappa_3, \frac{01}{10}, \frac{02}{10}, \frac{02}{10} \right\rangle\right].$$

Here $\tau_1 \cup \tau_2 = \{0_{(Y,\theta)}, 1_{(\pi,\theta)}, (\omega_1, \theta), (\omega_2, \theta), (\omega_3, \theta), (\angle_1, \theta), (\angle_2, \theta)\}$ is not aneutrosophic soft topology on π because $(\omega_3, \theta) \cup (\angle_2, \theta) \notin \tau_1 \cup \tau_2$.

Definition 3.6. Let $(\pi, \tau_1, \tau_2, \theta)$ be a NSBTS. Then aneutrosophic soft set

$$(\ltimes, \theta) = \{(e, \{\langle \kappa, T_{\ltimes(e)}(\kappa), I_{\ltimes(e)}(\kappa), F_{\ltimes(e)}(\kappa)\rangle\} : \kappa \in \pi, e \in \theta\}$$

is called as a pairwise neutrosophic soft open set (*PNSOS*) if there exist a NSOS $(\ltimes_1, \theta)$ in τ_1 and a *NSOS* $(\ltimes_2, \theta)$ in τ_2 such that for all $x \in U$

$$(\ltimes, \theta) = (\ltimes_1, \theta) \cup (\ltimes_2, \theta) = \left\{\left(e, \left\{\begin{array}{c} \langle x, \max\left\{T_{\ltimes(e)}(\kappa), T_{G(e)}(\kappa)\right\}, \\ \max\left\{I_{\ltimes(e)}(x), I_{G(e)}(\kappa)\right\}, \min\left\{F_{\ltimes(e)}(\kappa), F_{G(e)}(\kappa)\right\}\rangle \end{array}\right\}\right) : e \in \theta\right\}.$$

Definition 3.7. Let $(\pi, \tau_1, \tau_2, \theta)$ be a NSBTS. Then a neutrosophic soft set

$$(\ltimes, \theta) = \left\{\left(e, \left\{\langle \kappa, T_{\ltimes(e)}(\kappa), I_{\ltimes(e)}(\kappa), F_{\ltimes(e)}(\kappa)\rangle\right\}\right) : \kappa \in \pi, e \in \theta\right\}$$

is called as a PNSOS if there exist a NSOS $(\ltimes_1,\theta)$ in τ_1 and a $NSOS(\ltimes_2,\theta)$ in τ_2 such that for all $\kappa\in\pi$

$$(\ltimes,\theta)=(\ltimes_1,\theta)\cup(\ltimes_2,\theta)=\left\{\left(\mathrm{e},\left\{\begin{matrix}\langle \mathrm{x},\max\left\{\mathrm{T}_{\mathrm{H(e)}}(\kappa),\mathrm{T}_{\mathrm{G(e)}}(\kappa)\right\},\max\left\{\mathrm{I}_{\ltimes(\mathrm{e})}(\kappa),\mathrm{I}_{\mathrm{G(e)}}(\kappa)\right\},\\ \min\left\{\mathrm{F}_{\ltimes(\mathrm{e})}(\kappa),\mathrm{F}_{\mathrm{G(e)}}(\kappa)\right\}\rangle\end{matrix}\right\}\right):\mathrm{e}\in\theta\right\}.$$

The set of all pairwise neutrosophic open sets in $(\pi,\tau_1,\tau_2,\theta)$ is denoted by PNSO $(\pi,\tau_1,\tau_2,\theta)$.

Definition 3.8. Let $(\pi,\tau_1,\tau_2,\theta)$ be a NSBTS. Then a NSS

$$(\ltimes,\theta)=\left\{\left(\mathrm{e},\left\{\langle\kappa,\mathrm{T}_{\ltimes(\mathrm{e})}(\kappa),\mathrm{I}_{\ltimes(\mathrm{e})}(\kappa),\mathrm{F}_{\ltimes(\mathrm{e})}(\mathrm{x})\rangle\right\}\right):\kappa\in\pi,\mathrm{e}\in\theta\right\}$$

is called as a pairwise neutrosophic soft closed set (PNSC) if $(\ltimes,\theta)^{c}$ is a *PNSO*. It is clear that $(\ltimes,\theta)$ is a PNSC set if there exist a *NSOC* $(\ltimes_1,\theta)$ in τ_1 and a NSOC $(\ltimes_2,\theta)$ in τ_2 such that for all $\kappa\in\pi$

$$(\ltimes,\theta)=(\ltimes_1,\theta)\cap(\ltimes_2,\theta)=\left\{\left(\mathrm{e},\left\{\begin{matrix}\langle\kappa,\min\left\{\ltimes(\kappa),\mathrm{T}_{\mathrm{G(e)}}(\kappa)\right\},\min\left\{\mathrm{I}_{\ltimes(\mathrm{e})}(\kappa),\mathrm{I}_{\mathrm{G(e)}}(\kappa)\right\},\\ \max\left\{\mathrm{F}_{\ltimes(\mathrm{e})}(\kappa),\mathrm{F}_{\mathrm{G(e)}}(\kappa)\right\}\rangle\end{matrix}\right\}\right):\mathrm{e}\in\theta\right\}$$

The set of all *PNSC* in $(\pi,\tau_1,\tau_2,\theta)$ is denoted by *PNSC* $(\pi,\tau_1,\tau_2,\theta)$.

Example 3.9. Let $\pi=\{\kappa_1,\kappa_2,\kappa_3\}$, $\theta=\{\mathrm{e}_1,\mathrm{e}_2\}$, $\tau_1=\{0_{(\pi,\theta)},1_{(\pi,\theta)},(\omega_1,\theta)\}$, $\tau_2=\{0_{(\pi,\theta)},1_{(\pi,\theta)},(\omega_2,\theta)\}$ where (ω_1,θ), (ω_2,θ) are defined as

$$\mathrm{f}_{(\omega_1,\theta)}(\mathrm{e}_1)=\left[\left\langle\kappa_1,\frac{02}{10},\frac{03}{10},\frac{08}{10}\right\rangle,\left\langle\kappa_2,\frac{04}{10},\frac{04}{10},\frac{04}{10}\right\rangle,\left\langle\kappa_3,\frac{02}{10},\frac{04}{10},\frac{03}{10}\right\rangle\right]$$

$$\mathrm{f}_{(\omega_1,\theta)}(\mathrm{e}_2)=\left[\left\langle\kappa_1,\frac{03}{10},\frac{02}{10},\frac{06}{10}\right\rangle,\left\langle\kappa_2,\frac{01}{10},\frac{05}{10},\frac{05}{10}\right\rangle,\left\langle\kappa_3,\frac{04}{10},\frac{03}{10},\frac{05}{10}\right\rangle\right]$$

$$\mathrm{f}_{(\omega_2,\theta)}(\mathrm{e}_1)=\left[\left\langle\kappa_1,\frac{04}{10},\frac{02}{10},\frac{06}{10}\right\rangle,\left\langle\kappa_2,\frac{01}{10},\frac{05}{10},\frac{05}{10}\right\rangle,\left\langle\kappa_3,\frac{04}{10},\frac{03}{10},\frac{05}{10}\right\rangle\right]$$

$$\mathrm{f}_{(\omega_2,\theta)}(\mathrm{e}_2)=\left[\left\langle\kappa_1,\frac{04}{10},\frac{03}{10},\frac{06}{10}\right\rangle,\left\langle\kappa_2,\frac{04}{10},\frac{05}{10},\frac{03}{10}\right\rangle,\left\langle\kappa_3,\frac{03}{10},\frac{05}{10},\frac{02}{10}\right\rangle\right]$$

Then $(\omega_1,\theta)\cup(\omega_2,\theta)=\{(\mathrm{e}_1,\{\langle\kappa_1,\frac{02}{10},\frac{02}{10},\frac{08}{10}\rangle,\langle\kappa_2,\frac{01}{10},\frac{04}{10},\frac{05}{10}\rangle,\langle\kappa_3,\frac{02}{10},\frac{03}{10},\frac{05}{10}\rangle\}),\ (\mathrm{e}_2,\{\langle\kappa_1,\frac{03}{10},\frac{02}{10},\frac{06}{10}\rangle,\langle\kappa_2,\frac{01}{10},\frac{05}{10},\frac{05}{10}\rangle,\langle\kappa_3,\frac{03}{10},\frac{03}{10},\frac{05}{10}\rangle\})\}$ is a PNSOS. Also

$$\mathrm{f}_{(\omega_1,\theta)^c}(\mathrm{e}_1)=\left\{\left[\left\langle\kappa_1,\frac{08}{10},\frac{07}{10},\frac{02}{10}\right\rangle,\left\langle\kappa_2,\frac{06}{10},\frac{06}{10},\frac{06}{10}\right\rangle,\left\langle\kappa_3,\frac{03}{10},\frac{06}{10},\frac{02}{10}\right\rangle\right]\right\}$$

$$\mathrm{f}_{(\omega_1,\theta)^c}(\mathrm{e}_2)=\left\{\left[\left\langle\kappa_1,\frac{06}{10},\frac{08}{10},\frac{03}{10}\right\rangle,\left\langle\kappa_2,\frac{05}{10},\frac{05}{10},\frac{01}{10}\right\rangle,\left\langle\kappa_3,\frac{05}{10},\frac{07}{10},\frac{04}{10}\right\rangle\right]\right\}$$

$$\mathrm{f}_{(\omega_2,\theta)^c}(\mathrm{e}_2)=\left\{\left[\left\langle\kappa_1,\frac{06}{10},\frac{08}{10},\frac{03}{10}\right\rangle,\left\langle\kappa_2,\frac{05}{10},\frac{05}{10},\frac{01}{10}\right\rangle,\left\langle\kappa_3,\frac{03}{10},\frac{07}{10},\frac{04}{10}\right\rangle\right]\right\}$$

$$\mathrm{f}_{(\omega_2,\theta)^c}(\mathrm{e}_1)=\left\{\left[\left\langle\kappa_1,\frac{06}{10},\frac{07}{10},\frac{04}{10}\right\rangle,\left\langle\kappa_2,\frac{03}{10},\frac{05}{10},\frac{04}{10}\right\rangle,\left\langle\kappa_3,\frac{02}{10},\frac{05}{10},\frac{03}{10}\right\rangle\right]\right\}.$$

Therefore,

$$(\omega_1,\theta)^c \cap (\omega_2,\theta)^c = \left\{ \left(e_1, \left\{ \left\langle \kappa_1, \frac{08}{10}, \frac{07}{10}, \frac{02}{10} \right\rangle, \left\langle \kappa_2, \frac{06}{10}, \frac{06}{10}, \frac{06}{10} \right\rangle, \left\langle \kappa_3, \frac{08}{10}, \frac{06}{10}, \frac{07}{10} \right\rangle \right\} \right) \right.$$
$$\left. \left(e_2, \left\{ \left\langle \kappa_1, \frac{07}{10}, \frac{08}{10}, \frac{04}{10} \right\rangle, \left\langle \kappa_2, \frac{09}{10}, \frac{05}{10}, \frac{05}{10} \right\rangle, \left\langle \kappa_3, \frac{06}{10}, \frac{07}{10}, \frac{01}{10} \right\rangle \right\} \right) \right\}$$

is a PNSC.

Theorem 3.10. Let $(\pi, \tau_1, \tau_2, \theta)$ be a $NSBTS$. In this case

1. $0_{(\pi,\theta)}, 1_{(\pi,\theta)} \in \text{PNSO}\ (\pi, \tau_1, \tau_2, \theta)$.
2. If $\{(\ltimes_i, \theta) | i \in I\} \subseteq \text{PNSO}\ (\pi, \tau_1, \tau_2, \theta)$ then $\bigcup_{i \in I} (\ltimes_i, \theta) \in \text{PNSO}\ (\pi, \tau_1, \tau_2, \theta)$.
3. If $\{(G_i, \theta) | i \in I\} \subseteq \text{PNSC}\ (\pi, \tau_1, \tau_2, \theta)$ then $\bigcap_{i \in I} (G_i, \theta) \in \text{PNSC}\ (\pi, \tau_1, \tau_2, \theta)$.

Proof.

1. Since $0_{(\pi,\theta)} \cup 0_{(\pi,\theta)} = 0_{(\pi,\theta)}$, $1_{(\pi,\theta)} \cup 1_{(\pi,\theta)} = 1_{(\pi,\theta)}$ then $0_{(\pi,\theta)}$, $1_{(\pi,\theta)}$ are $PNSC$.
2. Since $(\ltimes_i, \theta) \in \text{PNSO}\ (\pi, \tau_1, \tau_2, \theta)$, there exist $(\ltimes_i^1, \theta) \in \tau_1$, $(\ltimes_i^1, \theta) \in \tau_2$ such that $(\ltimes_i, \theta) = (\ltimes_i^1, \theta) \cup (\ltimes_i^1, \theta)$ for all $i \in I$. Then

$\bigcup_{i \in I} (\ltimes_i, \theta) = \bigcup_{i \in I} \left((\ltimes_i^1, \theta) \cup (\ltimes_i^1, \theta) \right) = \left(\bigcup_{i \in I} (\ltimes_i^1, \theta) \right) \cup \left(\bigcup_{i \in I} (\ltimes_i^2, \theta) \right)$.

As τ_1, τ_2 are NST on π, $\bigcup_{i \in I} (\ltimes_i^1, \theta) \in \tau_1$, $\bigcup_{i \in I} (\ltimes_i^2, \theta) \in \tau_1$.

Therefore, $\bigcup_{i \in I} (\ltimes_i, \theta) \in \text{PNSO}\ (\pi, \tau_1, \tau_2, \theta)$.

3. Since $(G_i, \theta) \in \text{PNSC}\ (\pi, \tau_1, \tau_2, \theta)$, there exist $(G_i^1, \theta)^c \in \tau_1$ and $(G_i^2, \theta)^c \in \tau_2$ such that $(G_i, \theta) = (G_i^1, \theta) \cap (G_i^2, \theta)$ for all $i \in I$. Then

$\bigcap_{i \in I} (G_i, \theta) = \bigcap_{i \in I} \left((G_i^1, \theta) \cap (G_i^2, \theta) \right) = (\bigcap_{i \in I} (G_i^1, \theta)) \cap (\bigcap_{i \in I} (G_i^2, \theta))$.

Then $\bigcap_{i \in I} (G_i, \theta) \in \text{PNSC}\ (\pi, \tau_1, \tau_2, \theta)$ as $\left(\bigcap_{i \in I} (G_i^1, \theta) \right)^c \in \tau_1$, $\left(\bigcap_{i \in I} (G_i^2, \theta) \right)^c \in \tau_1$.

Theorem 3.11. Let $(\pi, \tau_1, \tau_2, \theta)$ be a NSBTS. Then every neutrosophic soft i-open set is a $PNSOS$.

Proof. Let $(\ltimes, \theta) \in \tau_I$ or $(\ltimes, \theta) \in \tau_2$. Since $(\ltimes, \theta) = (\ltimes, \theta) \cup 0_{(X,\theta)}$, then $(\ltimes, \theta) \in \text{PNSO}\ (\pi)$.

Theorem 3.12. Let $(\pi, \tau_1, \tau_2, \theta)$ be a $NSBTS$ and $(\measuredangle, \theta), (\omega, \theta) \in \text{NSS}(\pi)$. Then,

1. $cl_P^{NSS}\left(0_{(\pi,\theta)}\right) = 0_{(\pi,\theta)}$ and $cl_P^{NSS}\left(1_{(\pi,\theta)}\right) = 1_{(\pi,\theta)}$
2. $(\measuredangle, \theta) \subseteq cl_P^{NSS}(\measuredangle, \theta)$
3. $(\measuredangle, \theta)$ is a PNSCS if $cl_P^{NSS}(\measuredangle, \theta) = (\measuredangle, \theta)$
4. $cl_P^{NSS}(\measuredangle, \theta) \subseteq cl_P^{NSS}(\omega, \theta)$ if $(\measuredangle, \theta) \subseteq (\omega, \theta)$
5. $cl_P^{NSS}(\measuredangle, \theta) \cup cl_P^{NSS}(\omega, \theta) \subset cl_P^{NSS}((\measuredangle, \theta) \cup (\omega, \theta))$
6. $cl_P^{NSS}\left(cl_P^{NSS}(\measuredangle, \theta)\right) = cl_P^{NSS}(\measuredangle, \theta)$, i.e., $cl_P^{NSS}(\measuredangle, \theta)$ is a PNSCS.

Proof. It is obvious.

Theorem 3.13. Let $(\pi, \tau_1, \tau_2, \theta)$ be a *NSBTS* over π and $(\omega, \theta) \in \mathrm{NSS}(\pi)$. Then, $\kappa^{e}{}_{(\eta_1,\eta_2,\eta_3)} \in cl_P^{NSS}(\omega,\theta)$ if and only if for all $U_{x^{e}{}_{(\eta_1,\eta_2,\eta_3)}} \in \tau_{12}\left(\kappa^{e}{}_{(\eta_1,\eta_2,\eta_3)}\right)$ where $U_{\kappa^{e}{}_{(\eta_1,\eta_2,\eta_3)}}$ is any *PNSOS* contains $\kappa^{e}{}_{(\eta_1,\eta_2,\eta_3)}$ and $\tau_{12}\left(\kappa^{e}{}_{(\eta_1,\eta_2,\eta_3)}\right)$ is the family of all *PNSOS* contains $\kappa^{e}{}_{(\eta_1,\eta_2,\eta_3)}$, $U_{x^{e}{}_{(\eta_1,\eta_2,\eta_3)}} \cap (\curlywedge, \theta) \neq 0_{(\pi,\theta)}$.

Proof. Let $\kappa^{e}{}_{(\eta_1,\eta_2,\eta_3)} \in \mathrm{cl}_P^{NSS}(\omega,\theta)$ and suppose that there exists $U_{\kappa^{e}{}_{(\eta_1,\eta_2,\eta_3)}} \in \tau_{12}\left(x^{e}{}_{(\eta_1,\eta_2,\eta_3)}\right)$ such that $U_{\kappa^{e}{}_{(\eta_1,\eta_2,\eta_3)}} \cap (\curlywedge,\theta) = 0_{(\pi,\theta)}$. Then $(\curlywedge,\theta) \subset \left(U_{\kappa^{e}{}_{(\eta_1,\eta_2,\eta_3)}}\right)^c$. Thus $\mathrm{cl}_P^{NSS}(\curlywedge,\theta) \subset \mathrm{cl}_P^{NSS}\left(U_{\kappa^{e}{}_{(\eta_1,\eta_2,\eta_3)}}\right)^c = \left(U_{\kappa^{e}{}_{(\eta_1,\eta_2,\eta_3)}}\right)^c$ which implies $\mathrm{cl}_P^{NSS}(\omega,\theta) \cap U^c_{\kappa^{e}{}_{(\eta_1,\eta_2,\eta_3)}} = 0_{(\pi,\theta)}$, a contradiction.

Conversely, that $\kappa^{e}{}_{(\eta_1,\eta_2,\eta_3)} \notin \mathrm{cl}_P^{NSS}(\curlywedge,\theta)$, then $\kappa^{e}{}_{(\eta_1,\eta_2,\eta_3)} \in \left(\mathrm{cl}_P^{NSS}(\curlywedge,\theta)\right)^c \in \tau_{12}\left(x^{e}{}_{(\eta_1,\eta_2,\eta_3)}\right)$. Therefore, by hypothesis, $\left(\mathrm{cl}_P^{NSS}(\curlywedge,\theta)\right) \cap (\curlywedge,\theta) \neq 0_{(\pi,\theta)}$, a contradiction.

4 Main Results

In this section, some new definitions are introduced which are necessary for the up-coming sections.

Definition 4.1. Let $\langle \pi, \tau_1, \tau_2, \theta \rangle$ be a *NSBTS* over π, $\left(\tilde{\mathfrak{f}}, \theta\right)$ be a neutrosophic soft set over π. Then $\left(\tilde{\mathfrak{f}}, \theta\right)$ is

(1) Neutrosophic soft semi-open if $\left(\tilde{\mathfrak{f}}, \theta\right) \subseteq VScl\left(VSint\left(\tilde{\mathfrak{f}}, \theta\right)\right)$.

(2) Neutrosophic soft pre-open if $\left(\tilde{\mathfrak{f}}, \theta\right) \subseteq VSint\left(VScl\left(\tilde{\mathfrak{f}}, \theta\right)\right)$.

(3) Neutrosophic soft $*_b$ open if $\left(\tilde{\mathfrak{f}}, \theta\right) \subseteq VScl\left(VSint\left(\tilde{\mathfrak{f}}, \theta\right)\right) \sqcup VSint\left(VScl\left(\tilde{\mathfrak{f}}, \theta\right)\right)$ and neutrosophic soft $*_b$ close if $\left(\tilde{\mathfrak{f}}, \theta\right) \supseteq VScl\left(VSint\left(\tilde{\mathfrak{f}}, \theta\right)\right) \sqcap VSint\left(VScl\left(\tilde{\mathfrak{f}}, \theta\right)\right)$.

Definition 4.2. Let $\langle \pi, \tau_1, \tau_2, \theta \rangle$ be a *NSBTS* over π, $\kappa_1{}^{\curlywedge}{}_{\langle \eta_1,\eta_2,\eta_3 \rangle} \neq \kappa_2{}^{\curlywedge/}{}_{\langle \eta_1{}',\eta_2{}',\eta_3{}' \rangle}$ are neutrosophic soft points. If there exist $NS*_b$ open sets $\left(\tilde{\mathfrak{f}}, \theta\right) \& \left(\tilde{\mathscr{g}}, \theta\right)$ such that $\kappa_1{}^{\curlywedge}{}_{\langle \eta_1,\eta_2,\eta_3 \rangle} \in \left(\tilde{\mathfrak{f}}, \theta\right), \kappa_1{}^{\curlywedge}{}_{\langle \eta_1,\eta_2,\eta_3 \rangle} \sqcap \left(\tilde{\mathscr{g}}, \theta\right) = 0_{\left(\langle \tilde{\pi} \rangle, \theta\right)}$ or $\kappa_2{}^{\curlywedge/}{}_{\langle \eta_1{}',\eta_2{}',\eta_3{}' \rangle} \in \left(\tilde{\mathscr{g}}, \theta\right), \kappa_2{}^{\curlywedge/}{}_{\langle \eta_1{}',\eta_2{}',\eta_3{}' \rangle} \sqcap \left(\tilde{\mathfrak{f}}, \theta\right) = 0_{\left(\langle \tilde{\pi} \rangle, \theta\right)}$, Then $\langle \pi, \tau_1, \tau_2, \theta \rangle$ is called a $NSB*_{b0}$ space.

Definition 4.3. Let $\langle \pi, \tau_1, \tau_2, \theta \rangle$ be a *NSBTS* over π, $\kappa_1{}^{\curlywedge}{}_{\langle \eta_1,\eta_2,\eta_3 \rangle} \neq \kappa_2{}^{\curlywedge/}{}_{\langle \eta_1{}',\eta_2{}',\eta_3{}' \rangle}$ are neutrosophic soft points. If there exist $NS*_b$-open sets $\left(\tilde{\mathfrak{f}}, \theta\right), \left(\tilde{\mathscr{g}}, \theta\right)$ such that $\kappa_1{}^{\curlywedge}{}_{\langle \eta_1,\eta_2,\eta_3 \rangle} \in \left(\tilde{\mathfrak{f}}, \theta\right), \kappa_1{}^{\curlywedge}{}_{\langle \eta_1,\eta_2,\eta_3 \rangle} \sqcap \left(\tilde{\mathscr{g}}, \theta\right) = 0_{\left(\langle \tilde{\pi} \rangle, \theta\right)}$ and $\kappa_2{}^{\curlywedge/}{}_{\langle \Delta_1{}',\Delta_2{}',\Delta_3{}' \rangle} \in \left(\tilde{\mathscr{g}}, \theta\right), \kappa_2{}^{\curlywedge/}{}_{\langle \eta_1{}',\eta_2{}',\eta_3{}' \rangle} \sqcap \left(\tilde{\mathfrak{f}}, \theta\right) = 0_{\left(\langle \tilde{\pi} \rangle, \theta\right)}$, Then $\langle \pi, \tau_1, \tau_2, \theta \rangle$ is called a $NSB*_{b1}$ space.

Definition 4.4. Let $\langle \pi, \tau_1, \tau_2, \theta \rangle$ be a *NSBTS* over $\pi, \kappa_1{}^{\curlywedge}{}_{\langle \eta_1,\eta_2,\eta_3 \rangle} \neq \kappa_2{}^{\curlywedge'}{}_{\langle \eta_1',\eta_2',\eta_3' \rangle}$ are neutrosophic soft points. If there exists $NS*_b$ open set $\left(\tilde{\mathfrak{f}}, \theta\right)$, such that $\kappa_1{}^{\curlywedge}{}_{\langle \eta_1,\eta_2,\eta_3 \rangle} \in \left(\tilde{\mathfrak{f}}, \theta\right)$, $\kappa_2{}^{\curlywedge'}{}_{\langle \eta_1',\eta_2',\eta_3' \rangle} \in \left(\tilde{\mathfrak{g}}, \theta\right)$ & $\left(\tilde{\mathfrak{f}}, \theta\right) \sqcap \left(\tilde{\mathfrak{g}}, \theta\right) = 0_{\left(\langle \tilde{\pi} \rangle, \theta\right)}$. Then $\langle \pi, \tau_1, \tau_2, \theta \rangle$ is called a $NSB*_{b2}$ space.

Example 4.5. let $\tilde{\pi} = \{x_1, x_2\}$, the set of conditions by $\theta = \{e_1, e_2\}$. Let us consider neutrosophic set and $x^{e_1}{}_{1(0.1,0.4,0.7)}$, $x^{e_2}{}_{1(0.2,0.5,0.6)}$, $x^{e_1}{}_{2(0.3,0.3,0.5)}$ and $x^{e_1}{}_{2(0.4,0.4,0.4)}$ be neutrosophic soft points. Then the family $\tau_1 = \{0_{\left(\langle \tilde{\pi} \rangle, \theta\right)}, 1_{\left(\langle \tilde{\pi} \rangle, \theta\right)}, \left(\tilde{\mathfrak{f}_1}, \theta\right), \left(\tilde{\mathfrak{f}_2}, \theta\right), \left(\tilde{\mathfrak{f}_3}, \theta\right), \left(\tilde{\mathfrak{f}_4}, \theta\right), \left(\tilde{\mathfrak{f}_5}, \theta\right), \left(\tilde{\mathfrak{f}_6}, \theta\right),$ $\left(\tilde{\mathfrak{f}_7}, \theta\right), \left(\tilde{\mathfrak{f}_8}, \theta\right), \ldots \left(\widetilde{\mathfrak{f}_{15}}, \theta\right)\}$, where $\left(\tilde{\mathfrak{f}_1}, \theta\right) = x^{e_1}{}_{1(0.1,0.4,0.7)}$, $\left(\tilde{\mathfrak{f}_2}, \theta\right) = x^{e_2}{}_{1(0.2,0.5,0.6)}$, $\left(\tilde{\mathfrak{f}_3}, \theta\right) = x^{e_1}{}_{2(0.3,0.3,0.5)}$, $\left(\tilde{\mathfrak{f}_4}, \theta\right) = x^{e_2}{}_{2(0.4,0.4,0.4)}$ $\left(\tilde{\mathfrak{f}_5}, \theta\right) = \left(\tilde{\mathfrak{f}_1}, \theta\right) \cup \left(\tilde{\mathfrak{f}_2}, \theta\right), \left(\tilde{\mathfrak{f}_6}, \theta\right) = \left(\tilde{\mathfrak{f}_1}, \theta\right) \cup \left(\tilde{\mathfrak{f}_3}, \theta\right), \left(\tilde{\mathfrak{f}_7}, \theta\right) = \left(\tilde{\mathfrak{f}_2}, \theta\right) \cup \left(\tilde{\mathfrak{f}_4}, \theta\right), \left(\tilde{\mathfrak{f}_8}, \theta\right) = \left(\tilde{\mathfrak{f}_2}, \theta\right) \cup \left(\tilde{\mathfrak{f}_3}, \theta\right), \left(\tilde{\mathfrak{f}_9}, \theta\right) = \left(\tilde{\mathfrak{f}_2}, \theta\right) \cup \left(\tilde{\mathfrak{f}_4}, \theta\right), \left(\widetilde{\mathfrak{f}_{10}}, \theta\right) = \left(\tilde{\mathfrak{f}_3}, \theta\right) \cup \left(\tilde{\mathfrak{f}_4}, \theta\right), \left(\widetilde{\mathfrak{f}_{11}}, \theta\right) = \left(\tilde{\mathfrak{f}_1}, \theta\right) \cup \left(\tilde{\mathfrak{f}_2}, \theta\right) \cup \left(\tilde{\mathfrak{f}_3}, \theta\right), \left(\widetilde{\mathfrak{f}_{12}}, \theta\right) = \left(\tilde{\mathfrak{f}_1}, \theta\right) \cup \left(\tilde{\mathfrak{f}_2}, \theta\right) \cup \left(\tilde{\mathfrak{f}_4}, \theta\right), \left(\widetilde{\mathfrak{f}_{13}}, \theta\right) = \left(\tilde{\mathfrak{f}_2}, \theta\right) \cup \left(\tilde{\mathfrak{f}_3}, \theta\right) \cup \left(\tilde{\mathfrak{f}_4}, \theta\right), \left(\widetilde{\mathfrak{f}_{14}}, \theta\right) = \left(\tilde{\mathfrak{f}_1}, \theta\right) \cup \left(\tilde{\mathfrak{f}_3}, \theta\right) \cup \left(\tilde{\mathfrak{f}_4}, \theta\right)$, $\left(\widetilde{\mathfrak{f}_{15}}, \theta\right) = \{x^{e_1}{}_{1(0.1,0.4,0.7)}, x^{e_2}{}_{1(0.2,0.5,0.6)}, x^{e_1}{}_{2(0.3,0.3,0.5)}, x^{e_2}{}_{2(0.4,0.4,0.4)}\}$ is a *NSTS*. Also $\tau_2 = \left\{0_{\left(\langle \tilde{\pi} \rangle, \theta\right)}, 1_{\left(\langle \tilde{\pi} \rangle, \theta\right)}, \left(\tilde{\mathfrak{f}_1}, \theta\right)\right\}$ *NSTS*. Thus $(\pi, \tau_1, \tau_2, \theta)$ be a *NSBTS*. Also $(\pi, \tau_1, \tau_2, \theta)$ is $VSB*_{b2}$ structure.

Theorem 4.6. Let $\langle \pi, \tau_1, \tau_2, \theta \rangle$ be a *NSBTS*. Then $\langle \pi, \tau_1, \tau_2, \theta \rangle$ be a $NSB*_{b1}$ structure if and only if each neutrosophic soft point is a $NS*_b$-close.

Proof. Let $\langle \pi, \tau_1, \tau_2, \theta \rangle$ be a *NSBTS* over $\pi . \kappa_1{}^{\curlywedge}{}_{\langle \eta_1,\eta_2,\eta_3 \rangle}$ be an arbitrary neutrosophic soft point. We establish $\kappa_1{}^{\curlywedge}{}_{\langle \eta_1,\eta_2,\eta_3 \rangle}$ is a neutrosophic soft $*_b$-open set. Let $\kappa_2{}^{\curlywedge'}{}_{\langle \eta_1',\eta_2',\eta_3' \rangle} \in \kappa_1{}^{\curlywedge}{}_{\langle \eta_1,\eta_2,\eta_3 \rangle}$. Then either $\kappa_2{}^{\curlywedge'}{}_{\langle \eta_1',\eta_2',\eta_3' \rangle} \succ \kappa_1{}^{\curlywedge}{}_{\langle \eta_1,\eta_2,\eta_3 \rangle}$ or $\kappa_2{}^{\curlywedge'}{}_{\langle \eta_1',\eta_2',\eta_3' \rangle} \prec \kappa_2{}^{\curlywedge'}{}_{\langle \eta_1',\eta_2',\eta_3' \rangle}$ or $\kappa_2{}^{\curlywedge'}{}_{\langle \eta_1',\eta_2',\eta_3' \rangle} \succ\succ \kappa_1{}^{\curlywedge}{}_{\langle \eta_1,\eta_2,\eta_3 \rangle}$ or $\kappa_2{}^{\curlywedge'}{}_{\langle \eta_1',\eta_2',\eta_3' \rangle} \prec\prec \kappa_1{}^{\curlywedge}{}_{\langle \eta_1,\eta_2,\eta_3 \rangle}$. This means that $\kappa_2{}^{\curlywedge'}{}_{\langle \eta_1',\eta_2',\eta_3' \rangle}$ and $\kappa_1{}^{\curlywedge}{}_{\langle \eta_1,\eta_2,\eta_3 \rangle}$ are two are distinct *NS* points. Thus $\kappa_1 \succ \kappa_2$ or $\kappa_1 \prec \kappa_2$ or $\curlywedge' \succ \curlywedge$ or $\curlywedge' \prec \curlywedge$ or $\kappa_1 \succ\succ \kappa_2$ or $\kappa_1 \prec\prec \kappa_2$ or $\curlywedge' \succ\succ \curlywedge$ or $\curlywedge' \prec\prec \curlywedge$. Since $\langle \pi, \tau_1, \tau_2, \theta \rangle$ be a $NS*_{b1}$ structure, $\exists$ a $NS*_b$-open set $\left(\tilde{\mathfrak{g}}, \theta\right)$ so that $\kappa_2{}^{\curlywedge'}{}_{\langle \eta_1',\eta_2',\eta_3' \rangle} \in \left(\tilde{\mathfrak{g}}, \theta\right)$ and $\kappa_1{}^{\curlywedge}{}_{\langle \eta_1,\eta_2,\eta_3 \rangle} \sqcap \left(\tilde{\mathfrak{g}}, \theta\right) = 0_{\left(\langle \tilde{\pi} \rangle, \theta\right)}$. Since, $\kappa_1{}^{\curlywedge}{}_{\langle \eta_1,\eta_2,\eta_3 \rangle} \sqcap \left(\tilde{\mathfrak{g}}, \theta\right) = 0_{\left(\langle \tilde{\pi} \rangle, \theta\right)}$. So $\kappa_2{}^{\curlywedge'}{}_{\langle \eta_1',\eta_2',\eta_3' \rangle} \in \left(\tilde{\mathfrak{g}}, \theta\right) \subset \kappa_1{}^{\curlywedge}{}_{\langle \eta_1,\eta_2,\eta_3 \rangle}$. Thus $\kappa_1{}^{\curlywedge}{}_{\langle \eta_1,\eta_2,\eta_3 \rangle}$ is a $NS*_b$-open set, i.e., $\kappa_1{}^{\curlywedge}{}_{\langle \eta_1,\eta_2,\eta_3 \rangle}$ is a $NS*_b$-closed set. Suppose that each neutrosophic soft point $\kappa_1{}^{\curlywedge}{}_{\langle \eta_1,\eta_2,\eta_3 \rangle}$ is a $NS*_b$-closed. Then $\left(\kappa_1{}^{\curlywedge}{}_{\langle \eta_1,\eta_2,\eta_3 \rangle}\right)^c$ is a $NS*_b$-open set. Let $\kappa_1{}^{\curlywedge}{}_{\langle \eta_1,\eta_2,\eta_3 \rangle} \sqcap \kappa_2{}^{\curlywedge'}{}_{\langle \eta_1',\eta_2',\eta_3' \rangle} = 0_{\left(\langle \tilde{\pi} \rangle, \theta\right)}$. Thus $\left(\kappa_2{}^{\curlywedge'}{}_{\langle \eta_1',\eta_2',\eta_3' \rangle}, \theta\right) \in \left(\kappa_1{}^{\curlywedge}{}_{\langle \eta_1,\eta_2,\eta_3 \rangle}\right)^c$, $\kappa_1{}^{\curlywedge}{}_{\langle \eta_1,\eta_2,\eta_3 \rangle} \sqcap \left(\kappa_1{}^{\curlywedge}{}_{\langle \eta_1,\eta_2,\eta_3 \rangle}\right)^c = 0_{\left(\langle \tilde{\pi} \rangle, \theta\right)}$. So $\langle \pi, \tau_1, \tau_2, \theta \rangle$ be a $NSB\text{-}*_{b1}$ space.

Theorem 4.7. Let $\langle \pi, \tau_1, \tau_2, \theta \rangle$ be a *NSTBS* over the father set π Then (π, τ, θ) is $NS\text{-}*_{b2}$ space if and only if for distinct neutrosophic soft points $\kappa_1{}^{\curlywedge}{}_{\langle \eta_1,\eta_2,\eta_3 \rangle}$, $\kappa_2{}^{\curlywedge'}{}_{\langle \eta_1',\eta_2',\eta_3' \rangle}$, there exists a $NS*_b$-open set $\left(\tilde{\mathfrak{f}}, \theta\right)$ containing but not $\kappa_2{}^{\curlywedge'}{}_{\langle \eta_1',\eta_2',\eta_3' \rangle}$ such that $\kappa_2{}^{\curlywedge'}{}_{\langle \eta_1',\eta_2',\eta_3' \rangle} \notin \overline{\left(\tilde{\mathfrak{f}}, \theta\right)}$.

Proof. Let $\kappa_1{}^{\curlywedge}{}_{\langle \mathfrak{y}_1,\mathfrak{y}_2,\mathfrak{y}_3\rangle} \succ \kappa_2{}^{\curlywedge/}{}_{\langle \mathfrak{y}_1/,\mathfrak{y}_2/,\mathfrak{y}_3/\rangle}$ be two neutrosophic soft points in $NS*_{b2}$ space. Then there exists disjoint $NS*_b$ open sets $\left(\tilde{\mathfrak{f}},\theta\right),(\tilde{\mathscr{g}},\theta)$ such that $\kappa_1{}^{\curlywedge}{}_{\langle \mathfrak{y}_1,\mathfrak{y}_2,\mathfrak{y}_3\rangle} \in \left(\tilde{\mathfrak{f}},\theta\right) \& \kappa_2{}^{\curlywedge/}{}_{\langle \mathfrak{y}_1/,\mathfrak{y}_2/,\mathfrak{y}_3/\rangle} \in (\tilde{\mathscr{g}},\theta)$. Since $\left\langle \kappa_1{}^{\curlywedge}{}_{\langle \mathfrak{y}_1,\mathfrak{y}_2,\mathfrak{y}_3\rangle},\theta\right\rangle \sqcap \kappa_2{}^{\curlywedge/}{}_{\langle \mathfrak{y}_1/,\mathfrak{y}_2/,\mathfrak{y}_3/\rangle} = 0_{\left(\langle\tilde{\pi}\rangle,\theta\right)}$ and $\left(\tilde{\mathfrak{f}},\theta\right) \sqcap (\tilde{\mathscr{g}},\theta) = 0_{\left(\langle\tilde{\pi}\rangle,\theta\right)}.\kappa_2{}^{\curlywedge/}{}_{\langle \mathfrak{y}_1/,\mathfrak{y}_2/,\mathfrak{y}_3/\rangle} \notin \left(\tilde{\mathfrak{f}},\theta\right) \Longrightarrow \kappa_2{}^{\curlywedge/}{}_{\langle \mathfrak{y}_1/,\mathfrak{y}_2/,\mathfrak{y}_3/\rangle} \notin \overline{\left(\tilde{\mathfrak{f}},\theta\right)}$. Next suppose that, $\kappa_1{}^{\curlywedge}{}_{\langle \mathfrak{y}_1,\mathfrak{y}_2,\mathfrak{y}_3\rangle} \succ \kappa_2{}^{\curlywedge/}{}_{\langle \mathfrak{y}_1/,\mathfrak{y}_2/,\mathfrak{y}_3/\rangle}$, there exists a $NS*_b$ open set $\left(\tilde{\mathfrak{f}},\theta\right)$ containing $\kappa_1{}^{\curlywedge}{}_{\langle \mathfrak{y}_1,\mathfrak{y}_2,\mathfrak{y}_3\rangle}$ but not $\kappa_2{}^{\curlywedge/}{}_{\langle \mathfrak{y}_1/,\mathfrak{y}_2/,\mathfrak{y}_3/\rangle}$ such that $\kappa_2{}^{\curlywedge/}{}_{\langle \mathfrak{y}_1/,\mathfrak{y}_2/,\mathfrak{y}_3/\rangle} \notin \overline{\left(\tilde{\mathfrak{f}},\theta\right)}^c$ that is $\left(\tilde{\mathfrak{f}},\theta\right)$ and $\overline{\left(\tilde{\mathfrak{f}},\theta\right)}^c$ are mutually exclusive $NS*_b$ open sets supposing $\kappa_1{}^{\curlywedge}{}_{\langle \mathfrak{y}_1,\mathfrak{y}_2,\mathfrak{y}_3\rangle}$ and $\kappa_2{}^{\curlywedge/}{}_{\langle \mathfrak{y}_1/,\mathfrak{y}_2/,\mathfrak{y}_3/\rangle}$ in turn.

Theorem 4.8. Let $\langle \pi,\tau_1,\tau_2,\theta\rangle$ be a *NSBTS*. Then $\langle \pi,\tau_1,\tau_2,\theta\rangle$ is $NS*_{b1}$ space if every neutrosophic soft point $\kappa_1{}^{\curlywedge}{}_{\langle \mathfrak{y}_1,\mathfrak{y}_2,\mathfrak{y}_3\rangle} \in \left(\tilde{\mathfrak{f}},\theta\right) \in (\pi,\tau,\theta)$. If there exists $NS*_b$ open set $(\tilde{\mathscr{g}},\theta)$ such that $\kappa_1{}^{\curlywedge}{}_{\langle \mathfrak{y}_1,\mathfrak{y}_2,\mathfrak{y}_3\rangle} \in (\tilde{\mathscr{g}},\theta) \subset \overline{(\tilde{\mathscr{g}},\theta)} \subset \left(\tilde{\mathfrak{f}},\theta\right)$, Then $\langle \pi,\tau_1,\tau_2,\theta\rangle$ a $NS*_{b2}$ space.

Proof. Suppose $\kappa_1{}^{\curlywedge}{}_{\langle \mathfrak{y}_1,\mathfrak{y}_2,\mathfrak{y}_3\rangle} \sqcap \kappa_2{}^{\curlywedge/}{}_{\langle \mathfrak{y}_1/,\mathfrak{y}_2/,\mathfrak{y}_3/\rangle} = 0_{\left(\langle\tilde{\pi}\rangle,\theta\right)}$. Since $\langle \pi,\tau_1,\tau_2,\theta\rangle$ is $NS*_{b1}$ space. $\kappa_1{}^{\curlywedge}{}_{\langle \mathfrak{y}_1,\mathfrak{y}_2,\mathfrak{y}_3\rangle}$ and $\kappa_2{}^{\curlywedge/}{}_{\langle \mathfrak{y}_1/,\mathfrak{y}_2/,\mathfrak{y}_3/\rangle}$ are $NS*_b$ closed sets in $\langle \pi,\tau_1,\tau_2,\theta\rangle$. Then $\kappa_1{}^{\curlywedge}{}_{\langle \mathfrak{y}_1,\mathfrak{y}_2,\mathfrak{y}_3\rangle} \in \left(\kappa_2{}^{\curlywedge/}{}_{\langle \mathfrak{y}_1/,\mathfrak{y}_2/,\mathfrak{y}_3/\rangle}\right)^c \in \langle \pi,\tau_1,\tau_2,\theta\rangle$. Thus there exists a $NS*_b$ open set $(\tilde{\mathscr{g}},\theta) \in \langle \pi,\tau_1,\tau_2,\theta\rangle$ such that $\kappa_1{}^{\curlywedge}{}_{\langle \mathfrak{y}_1,\mathfrak{y}_2,\mathfrak{y}_3\rangle} \in (\tilde{\mathscr{g}},\theta) \subset \overline{(\tilde{\mathscr{g}},\theta)} \subset \left(\kappa_2{}^{\curlywedge/}{}_{\langle \mathfrak{y}_1/,\mathfrak{y}_2/,\mathfrak{y}_3/\rangle}\right)^c$. So we have $\kappa_2{}^{\curlywedge/}{}_{\langle \mathfrak{y}_1/,\mathfrak{y}_2/,\mathfrak{y}_3/\rangle} \in (\tilde{\mathscr{g}},\theta)$ and $(\tilde{\mathscr{g}},\theta) \sqcap \left((\tilde{\mathscr{g}},\theta)\right)^c = 0_{\left(\langle\tilde{\pi}\rangle,\theta\right)}$, that is $\langle \pi,\tau_1,\tau_2,\theta\rangle$ is a neutrosophic soft $*_{b2}$ space.

Definition 4.9. Let $\langle \pi,\tau_1,\tau_2,\theta\rangle$ be a *NSBTS*. $\left(\tilde{\mathfrak{f}},\theta\right)$ be a $NS*_b$ closed set and $\kappa_1{}^{\curlywedge}{}_{\langle \mathfrak{y}_1,\mathfrak{y}_2,\mathfrak{y}_3\rangle} \sqcap \left(\tilde{\mathfrak{f}},\theta\right) = 0_{\left(\langle\tilde{\mathcal{X}}\rangle,\theta\right)}$. If there exists $NS*_b$-open sets $(\tilde{\mathscr{g}}_1,\theta)$ and $(\tilde{\mathscr{g}}_2,\theta)$ such that $\kappa_1{}^{\curlywedge}{}_{\langle \mathfrak{y}_1,\mathfrak{y}_2,\mathfrak{y}_3\rangle} \in (\tilde{\mathscr{g}}_1,\theta), \left(\tilde{\mathfrak{f}},\theta\right) \subset (\tilde{\mathscr{g}}_2,\theta)$ & $\kappa_1{}^{\curlywedge}{}_{\langle \mathfrak{y}_1,\mathfrak{y}_2,\mathfrak{y}_3\rangle} \sqcap (\tilde{\mathscr{g}}_1,\theta) = 0_{\left(\langle\tilde{\pi}\rangle,\theta\right)}$, then $\langle \pi,\tau_1,\tau_2,\theta\rangle$ is called a $NBS*_b$-regular space. $\langle \pi,\tau_1,\tau_2,\theta\rangle$ is said to be $NS*_{b3}$ space, if it is both a *NS* regular and $NSB*_{b1}$ space.

Theorem 4.10. Let $\langle \pi,\tau_1,\tau_2,\theta\rangle$ be a *NSBTS*. (π,τ,θ) is soft $*_{b3}$ space if and only if for every $\kappa_1{}^{\curlywedge}{}_{\langle \mathfrak{y}_1,\mathfrak{y}_2,\mathfrak{y}_3\rangle} \in \left(\tilde{\mathfrak{f}},\theta\right)$ that is $(\tilde{\mathscr{g}},\theta) \in (\langle \pi,\tau_1,\tau_2,\theta\rangle$ such that $\kappa_1{}^{\curlywedge}{}_{\langle \mathfrak{y}_1,\mathfrak{y}_2,\mathfrak{y}_3\rangle} \in (\tilde{\mathscr{g}},\theta) \subset \overline{(\tilde{\mathscr{g}},\theta)} \subset \left(\tilde{\mathfrak{f}},\theta\right)$.

Proof. Let $\langle \pi,\tau_1,\tau_2,\theta\rangle$ is $NS*_{b3}$ space and $\kappa_1{}^{\curlywedge}{}_{\langle \mathfrak{y}_1,\mathfrak{y}_2,\mathfrak{y}_3\rangle} \in \left(\tilde{\mathfrak{f}},\theta\right) \in \langle \pi,\tau_1,\tau_2,\theta\rangle$. Since $\langle \pi,\tau_1,\tau_2,\theta\rangle$ is $NS*_{b3}$ space for the neutrosophic soft point $\kappa_1{}^{\curlywedge}{}_{\langle \mathfrak{y}_1,\mathfrak{y}_2,\mathfrak{y}_3\rangle}$ and $NS*_b$ closed set $\left(\tilde{\mathfrak{f}},\theta\right)^c$, there exists $(\tilde{\mathscr{g}}_1,\theta)$ and $(\tilde{\mathscr{g}}_2,\theta)$ such that $\kappa_1{}^{\curlywedge}{}_{\langle \mathfrak{y}_1,\mathfrak{y}_2,\mathfrak{y}_3\rangle} \in (\tilde{\mathscr{g}}_1,\theta), \left(\tilde{\mathfrak{f}},\theta\right)^c \subset (\tilde{\mathscr{g}}_2,\theta)$ and $(\tilde{\mathscr{g}}_1,\theta) \cap (\tilde{\mathscr{g}}_2,\theta) = 0_{\left(\langle\tilde{\pi}\rangle,\theta\right)}$. Then we have $\left\langle \kappa_1{}^{\curlywedge}{}_{\langle \mathfrak{y}_1,\mathfrak{y}_2,\mathfrak{y}_3\rangle},\theta\right\rangle \in (\tilde{\mathscr{g}}_1,\theta) \subset (\tilde{\mathscr{g}}_2,\theta)^c \subset \left(\tilde{\mathfrak{f}},\theta\right)$. Since $(\tilde{\mathscr{g}}_2,\theta)^c\, NS*_b$ closed set $\overline{(\tilde{\mathscr{g}}_1,\theta)} \subset (\tilde{\mathscr{g}}_2,\theta)^c$. Conversely, let $\left\langle \kappa_1{}^{\curlywedge}{}_{\langle \mathfrak{y}_1,\mathfrak{y}_2,\mathfrak{y}_3\rangle},\theta\right\rangle \cap \left(\tilde{\mathcal{H}},\theta\right) = 0_{\left(\langle\tilde{\pi}\rangle,\theta\right)}$ and $\left(\tilde{\mathcal{H}},\theta\right)$ be a $NS*_b$ closed set. $\kappa_1{}^{\curlywedge}{}_{\langle\mathfrak{y}_1,\mathfrak{y}_2,\mathfrak{y}_3\rangle} \in \left(\tilde{\mathcal{H}},\theta\right)^c$ and from the condition of the theorem, we have $\kappa_1{}^{\curlywedge}{}_{\langle\mathfrak{y}_1,\mathfrak{y}_2,\mathfrak{y}_3\rangle} \in \left(\tilde{g},\theta\right) \subset \overline{\left(\tilde{g},\theta\right)} \subset \left(\tilde{\mathcal{H}},\theta\right)^c$. Thus $\kappa_1{}^{\curlywedge}{}_{\langle\mathfrak{y}_1,\mathfrak{y}_2,\mathfrak{y}_3\rangle} \in \left(\tilde{g},\theta\right), \left(\tilde{\mathcal{H}},\theta\right) \subset \overline{\left(\tilde{g},\theta\right)}^c$ and $\left(\tilde{g},\theta\right) \cap \overline{\left(\tilde{g},\theta\right)}^c = 0_{\left(\langle\tilde{\pi}\rangle,\theta\right)}$. So $(\langle\pi,\tau_1,\tau_2,\theta\rangle$ is $NSB*_{b3}$ space.

5 More Main Results

In this section, more main results are addressed. The structures in one space can be switched over to another space through soft functions satisfying some more conditions. These structures are separation axioms and other separation axioms, compactness and countably compactness.

Theorem 5.1. Let $\langle\pi,\tau_1,\tau_2,\theta\rangle$ be *NSBTS* such that it is $NSB*_b$ Hausdorff space and $\left(\langle\tilde{Y}\rangle,\mathfrak{F}_1,\mathfrak{F}_2,\theta\right)$ be an-other *NSBTS*. Let $\left\langle f,\theta\right\rangle$: $(\pi,\tau_1,\tau_2,\theta) \longrightarrow \left(\langle\tilde{Y}\rangle,\mathfrak{F}_1,\mathfrak{F}_2,\theta\right)$ be an-other *NSBTS* be a soft function such that it is soft monotone and continuous. Then $\left(\langle\tilde{Y}\rangle,\mathfrak{F}_1,\mathfrak{F}_2,\theta\right)$ is also of characteristic of $NSB*_b$ Hausdorffness.

Proof. Suppose $\kappa_1{}^{\curlywedge}{}_{\langle\mathfrak{y}_1,\mathfrak{y}_2,\mathfrak{y}_3\rangle_1}, \kappa_1{}^{\curlywedge}{}_{\langle\mathfrak{y}_1,\mathfrak{y}_2,\mathfrak{y}_3\rangle_2} \in \langle\tilde{\pi}\rangle$ such that either $\kappa_1{}^{\curlywedge}{}_{\langle\mathfrak{y}_1,\mathfrak{y}_2,\mathfrak{y}_3\rangle_1} \succ \kappa_1{}^{\curlywedge}{}_{\langle\mathfrak{y}_1,\mathfrak{y}_2,\mathfrak{y}_3\rangle_2}$ or $\kappa_1{}^{\curlywedge}{}_{\langle\mathfrak{y}_1,\mathfrak{y}_2,\mathfrak{y}_3\rangle_1} \prec \kappa_1{}^{\curlywedge}{}_{\langle\mathfrak{y}_1,\mathfrak{y}_2,\mathfrak{y}_3\rangle_2}$. Since $\left\langle f,\theta\right\rangle$ is soft monotone. Let us suppose the monotonically increasing case. So, $\kappa_1{}^{\curlywedge}{}_{\langle\mathfrak{y}_1,\mathfrak{y}_2,\mathfrak{y}_3\rangle_1} \succ \kappa_1{}^{\curlywedge}{}_{\langle\Delta_1,\Delta_2,\Delta_3\rangle_2}$ or $\kappa_1{}^{\curlywedge}{}_{\langle\mathfrak{y}_1,\mathfrak{y}_2,\mathfrak{y}_3\rangle_1} \prec \kappa_1{}^{\curlywedge}{}_{\langle\mathfrak{y}_1,\mathfrak{y}_2,\mathfrak{y}_3\rangle_2}$ implies that $f_{\left\langle\kappa_1{}^{\curlywedge}{}_{\langle\mathfrak{y}_1,\mathfrak{y}_2,\mathfrak{y}_3\rangle}\right\rangle_1} \succ f_{\left\langle\kappa_1{}^{\curlywedge}{}_{\langle\mathfrak{y}_1,\mathfrak{y}_2,\mathfrak{y}_3\rangle}\right\rangle_2}$ or $f_{\left\langle\kappa_1{}^{\curlywedge}{}_{\langle\mathfrak{y}_1,\mathfrak{y}_2,\mathfrak{y}_3\rangle}\right\rangle_1} \prec f_{\left\langle\kappa_1{}^{\curlywedge}{}_{\langle\mathfrak{y}_1,\mathfrak{y}_2,\mathfrak{y}_3\rangle}\right\rangle_2}$ respectively. Suppose $\kappa_2{}^{\curlywedge/}{}_{\langle\mathfrak{y}_1{}^/,\mathfrak{y}_2{}^/,\mathfrak{y}_3{}^/\rangle_1}, \kappa_2{}^{\curlywedge/}{}_{\langle\mathfrak{y}_1{}^/,\mathfrak{y}_2{}^/,\mathfrak{y}_3{}^/\rangle_2} \in \langle\tilde{Y}\rangle$ such that $\kappa_2{}^{\curlywedge/}{}_{\langle\mathfrak{y}_1{}^/,\mathfrak{y}_2{}^/,\mathfrak{y}_3{}^/\rangle_1} \succ \kappa_2{}^{\curlywedge/}{}_{\langle\mathfrak{y}_1{}^/,\mathfrak{y}_2{}^/,\mathfrak{y}_3{}^/\rangle_2}$ or $\kappa_2{}^{\curlywedge/}{}_{\langle\mathfrak{y}_1{}^/,\mathfrak{y}_2{}^/,\mathfrak{y}_3{}^/\rangle_1} \prec \kappa_2{}^{\curlywedge/}{}_{\langle\mathfrak{y}_1{}^/,\mathfrak{y}_2{}^/,\mathfrak{y}_3{}^/\rangle_2}$ so, $\kappa_2{}^{\curlywedge/}{}_{\langle\mathfrak{y}_1{}^/,\mathfrak{y}_2{}^/,\mathfrak{y}_3{}^/\rangle_1} \succ \kappa_2{}^{\curlywedge/}{}_{\langle\mathfrak{y}_1{}^/,\mathfrak{y}_2{}^/,\mathfrak{y}_3{}^/\rangle_2}$ or $\kappa_2{}^{\curlywedge/}{}_{\langle\mathfrak{y}_1{}^/,\mathfrak{y}_2{}^/,\mathfrak{y}_3{}^/\rangle_1} \prec \kappa_2{}^{\curlywedge/}{}_{\langle\mathfrak{y}_1{}^/,\mathfrak{y}_2{}^/,\mathfrak{y}_3{}^/\rangle_2}$ respectively such that $\kappa_2{}^{\curlywedge/}{}_{\langle\mathfrak{y}_1{}^/,\mathfrak{y}_2{}^/,\mathfrak{y}_3{}^/\rangle_1} = f_{\left\langle\kappa_1{}^{\curlywedge}{}_{\langle\mathfrak{y}_1,\mathfrak{y}_2,\mathfrak{y}_3\rangle}\right\rangle_1}, \kappa_2{}^{\curlywedge/}{}_{\langle\mathfrak{y}_1{}^/,\mathfrak{y}_2{}^/,\mathfrak{y}_3{}^/\rangle_2} = f_{\kappa_1{}^{\curlywedge}{}_{\langle\mathfrak{y}_1,\mathfrak{y}_2,\mathfrak{y}_3\rangle_2}}$, since, $(\pi,\tau_1,\tau_2,\theta)$ is $NSB*_b$ Hausdorff space so there exists mutually disjoint $NS*_b$ open sets $\langle k_1,\theta\rangle$ and $\langle k_2,\theta\rangle \in (\pi,\tau_1,\tau_2,\theta) \Longrightarrow f(\langle k_1,\theta\rangle)$ and $f(\langle k_2,\theta\rangle) \in \langle\tilde{Y}\rangle,\mathfrak{F},\theta)$. We claim that $f(\langle k_1,\theta\rangle)\tilde{\cap}f(\langle k_2,\theta\rangle) = \widetilde{0_{(\langle\tilde{\pi}\rangle,\theta}}$. Otherwise $f(\langle k_1,\theta\rangle)\tilde{\cap}f(\langle k_2,\theta\rangle) \neq \widetilde{0_{(\langle\tilde{\pi}\rangle,\theta)}}$. Suppose there exists $\kappa_3{}^{\curlywedge//}{}_{\langle\mathfrak{y}_1{}^{//},\mathfrak{y}_2{}^{//},\mathfrak{y}_3{}^{//}\rangle_1} \in f(\langle k_1,\theta\rangle)\tilde{\cap}f(\langle k_2,\theta\rangle) \Longrightarrow \kappa_3{}^{\curlywedge//}{}_{\langle\mathfrak{y}_1{}^{//},\mathfrak{y}_2{}^{//},\mathfrak{y}_3{}^{//}\rangle_1} \in f(\langle k_1,\theta\rangle)\kappa_3{}^{\curlywedge//}{}_{\langle\mathfrak{y}_1{}^{//},\mathfrak{y}_2{}^{//},\mathfrak{y}_3{}^{//}\rangle_1} \in f(\langle k_2,\theta\rangle), \kappa_3{}^{\curlywedge//}{}_{\langle\mathfrak{y}_1{}^{//},\mathfrak{y}_2{}^{//},\mathfrak{y}_3{}^{//}\rangle_1} \in f(\langle k_1,\theta\rangle), f$ is soft one-one and there exists $\kappa_3{}^{\curlywedge//}{}_{\langle\mathfrak{y}_1{}^{//},\mathfrak{y}_2{}^{//},\mathfrak{y}_3{}^{//}\rangle_2} \in \langle k_1,\theta\rangle$ such that $\kappa_3{}^{\curlywedge//}{}_{\langle\mathfrak{y}_1{}^{//},\mathfrak{y}_2{}^{//},\mathfrak{y}_3{}^{//}\rangle_1} = f(\kappa_3{}^{\curlywedge//}{}_{\langle\mathfrak{y}_1{}^{//},\mathfrak{y}_2{}^{//},\mathfrak{y}_3{}^{//}\rangle_2}), \kappa_3{}^{\curlywedge//}{}_{\langle\mathfrak{y}_1{}^{//},\mathfrak{y}_2{}^{//},\mathfrak{y}_3{}^{//}\rangle_1} \in f(\langle k_2,\theta\rangle) \Longrightarrow \exists\, \kappa_3{}^{\curlywedge//}{}_{\langle\mathfrak{y}_1{}^{//},\mathfrak{y}_2{}^{//},\mathfrak{y}_3{}^{//}\rangle_3} \in \langle k_2,\theta\rangle$ such that $f(\kappa_3{}^{\curlywedge//}{}_{\langle\mathfrak{y}_1{}^{//},\mathfrak{y}_2{}^{//},\mathfrak{y}_3{}^{//}\rangle_3}) \Longrightarrow f(\kappa_3{}^{\curlywedge//}{}_{\langle\mathfrak{y}_1{}^{//},\mathfrak{y}_2{}^{//},\mathfrak{y}_3{}^{//}\rangle_2}) = f(\kappa_3{}^{\curlywedge//}{}_{\langle\mathfrak{y}_1{}^{//},\mathfrak{y}_2{}^{//},\mathfrak{y}_3{}^{//}\rangle_3})$. Since, f is soft one-one $\Longrightarrow \kappa_3{}^{\curlywedge//}{}_{\langle\mathfrak{y}_1{}^{//},\mathfrak{y}_2{}^{//},\mathfrak{y}_3{}^{//}\rangle_2} = \kappa_3{}^{\curlywedge//}{}_{\langle\mathfrak{y}_1{}^{//},\mathfrak{y}_2{}^{//},\mathfrak{y}_3{}^{//}\rangle_3}$ implies that $\kappa_3{}^{\curlywedge//}{}_{\langle\mathfrak{y}_1{}^{//},\mathfrak{y}_2{}^{//},\mathfrak{y}_3{}^{//}\rangle_2} \in f(\langle k_1,\theta\rangle), \kappa_3{}^{\curlywedge//}{}_{\langle\mathfrak{y}_1{}^{//},\mathfrak{y}_2{}^{//},\mathfrak{y}_3{}^{//}\rangle_2} \in f(\langle k_2,\theta\rangle)$ implies that $\kappa_3{}^{\curlywedge//}{}_{\langle\mathfrak{y}_1{}^{//},\mathfrak{y}_2{}^{//},\mathfrak{y}_3{}^{//}\rangle_2} \in f(\langle k_1,\theta\rangle)\tilde{\cap}f(\langle k_2,\theta\rangle)$. This is contradiction because $\langle k_1,\theta\rangle\,\tilde{\cap}\,\langle k_2,\theta\rangle = 0_{(\langle\tilde{\pi}\rangle,\theta)}$. Therefore, $f(\langle k_1,\theta\rangle)\tilde{\cap}\, f(\langle k_2,\theta\rangle) = 0_{(\langle\tilde{\pi}\rangle,\theta)}$. Finally, $\kappa_1{}^{\curlywedge}{}_{\langle\mathfrak{y}_1,\mathfrak{y}_2,\mathfrak{y}_3\rangle_1} \succ \kappa_1{}^{\curlywedge}{}_{\langle\mathfrak{y}_1,\mathfrak{y}_2,\mathfrak{y}_3\rangle_2}$ or $\kappa_1{}^{\curlywedge}{}_{\langle\mathfrak{y}_1,\mathfrak{y}_2,\mathfrak{y}_3\rangle_1} \prec \kappa_1{}^{\curlywedge}{}_{\langle\mathfrak{y}_1,\mathfrak{y}_2,\mathfrak{y}_3\rangle_2} \Longrightarrow \kappa_1{}^{\curlywedge}{}_{\langle\mathfrak{y}_1,\mathfrak{y}_2,\mathfrak{y}_3\rangle_1} \neq \kappa_1{}^{\curlywedge}{}_{\langle\mathfrak{y}_1,\mathfrak{y}_2,\mathfrak{y}_3\rangle_2} \Longrightarrow f(\kappa_1{}^{\curlywedge}{}_{\langle\mathfrak{y}_1,\mathfrak{y}_2,\mathfrak{y}_3\rangle_1}) \succ f(\kappa_1{}^{\curlywedge}{}_{\langle\mathfrak{y}_1,\mathfrak{y}_2,\mathfrak{y}_3\rangle_2})$ or $f(\kappa_1{}^{\curlywedge}{}_{\langle\mathfrak{y}_1,\mathfrak{y}_2,\mathfrak{y}_3\rangle_1}) \prec f(\kappa_1{}^{\curlywedge}{}_{\langle\mathfrak{y}_1,\mathfrak{y}_2,\mathfrak{y}_3\rangle_2}) \Longrightarrow f(\kappa_1{}^{\curlywedge}{}_{\langle\mathfrak{y}_1,\mathfrak{y}_2,\mathfrak{y}_3\rangle_1}) \neq f(\kappa_1{}^{\curlywedge}{}_{\langle\mathfrak{y}_1,\mathfrak{y}_2,\mathfrak{y}_3\rangle_2})$. Given a pair of points $\kappa_2{}^{\curlywedge/}{}_{\langle\mathfrak{y}_1{}^/,\mathfrak{y}_2{}^/,\mathfrak{y}_3{}^/\rangle_1}, \kappa_2{}^{\curlywedge/}{}_{\langle\mathfrak{y}_1{}^/,\mathfrak{y}_2{}^/,\mathfrak{y}_3{}^/\rangle_2} \in \langle\tilde{Y}\rangle \ni \kappa_2{}^{\curlywedge/}{}_{\langle\mathfrak{y}_1{}^/,\mathfrak{y}_2{}^/,\mathfrak{y}_3{}^/\rangle_1} \neq \kappa_2{}^{\curlywedge/}{}_{\langle\mathfrak{y}_1{}^/,\mathfrak{y}_2{}^/,\mathfrak{y}_3{}^/\rangle_2}$ We are able to find out mutually exclusive $NS*_b$ open sets $\not{f}(\langle \not{k}_1,\theta\rangle), \not{f}(\langle \not{k}_2,\theta\rangle) \in (\langle \tilde{Y}\rangle, \mathfrak{F}_1, \mathfrak{F}_2, \theta)$ such that $\kappa_2^{\wedge'}{}_{\langle \mathfrak{y}_1', \mathfrak{y}_2', \mathfrak{y}_3'\rangle_1} \in \not{f}(\langle \not{k}_1,\theta\rangle), \kappa_2^{\wedge'}{}_{\langle \mathfrak{y}_1', \mathfrak{y}_2', \mathfrak{y}_3'\rangle_2} \in \not{f}(\langle \not{k}_2,\theta\rangle)$ this proves that $(\langle \tilde{Y}\rangle, \mathfrak{F}_1, \mathfrak{F}_2, \theta)$ is $NSB{*_b}$ Husdorff space.

Theorem 5.2. Let $(\tilde{\pi}, \tau_1, \tau_2, \theta)$ be $NSBTS$ and $\left(\langle \tilde{Y}\rangle, \mathfrak{F}_1, \mathfrak{F}_2, \theta\right)$ be an-other $NSBTS$ which satisfies one more condition of $NSB{*_b}$ Hausdorffness. Let $\langle \not{f}, \theta\rangle$: $(\tilde{\pi}, \tau_1, \tau_2, \theta) \longrightarrow \left(\langle \tilde{Y}\rangle, \mathfrak{F}_1, \mathfrak{F}_2, \theta\right)$ a soft function such that it is soft monotone and continuous. Then $(\tilde{\pi}, \tau_1, \tau_2, \theta)$ is also of characteristics of $NSB{*_b}$ Hausdorffness.

Proof. Suppose $\kappa_1^{\wedge}{}_{\langle \mathfrak{y}_1, \mathfrak{y}_2, \mathfrak{y}_3\rangle_1}, \kappa_1^{\wedge}{}_{\langle \mathfrak{y}_1, \mathfrak{y}_2, \mathfrak{y}_3\rangle_2} \in \tilde{\pi}$ such that either $\kappa_1^{\wedge}{}_{\langle \mathfrak{y}_1, \mathfrak{y}_2, \mathfrak{y}_3\rangle_1} \succ \kappa_1^{\wedge}{}_{\langle \mathfrak{y}_1, \mathfrak{y}_2, \mathfrak{y}_3\rangle_2}$ or $\kappa_1^{\wedge}{}_{\langle \mathfrak{y}_1, \mathfrak{y}_2, \mathfrak{y}_3\rangle_1} \prec \kappa_1^{\wedge}{}_{\langle \mathfrak{y}_1, \mathfrak{y}_2, \mathfrak{y}_3\rangle_2}$. Let $\kappa_1^{\wedge}{}_{\langle \mathfrak{y}_1, \mathfrak{y}_2, \mathfrak{y}_3\rangle_1} \succ \kappa_1^{\wedge}{}_{\langle \mathfrak{y}_1, \mathfrak{y}_2, \mathfrak{y}_3\rangle_2}$ or $\kappa_1^{\wedge}{}_{\langle \mathfrak{y}_1, \mathfrak{y}_2, \mathfrak{y}_3\rangle_1} \prec \kappa_1^{\wedge}{}_{\langle \mathfrak{y}_1, \mathfrak{y}_2, \mathfrak{y}_3\rangle_2}$ implies that $\not{f}(\kappa_1^{\wedge}{}_{\langle \mathfrak{y}_1, \mathfrak{y}_2, \mathfrak{y}_3\rangle_1}) \succ \not{f}(\kappa_1^{\wedge}{}_{\langle \mathfrak{y}_1, \mathfrak{y}_2, \mathfrak{y}_3\rangle_2})$ or $\not{f}(\kappa_1^{\wedge}{}_{\langle \mathfrak{y}_1, \mathfrak{y}_2, \mathfrak{y}_3\rangle_1}) \prec \not{f}(\kappa_1^{\wedge}{}_{\langle \mathfrak{y}_1, \mathfrak{y}_2, \mathfrak{y}_3\rangle_2})$ respectively. Suppose $\kappa_2^{\wedge'}{}_{\langle \mathfrak{y}_1', \mathfrak{y}_2', \mathfrak{y}_3'\rangle_1}, \kappa_2^{\wedge'}{}_{\langle \mathfrak{y}_1', \mathfrak{y}_2', \mathfrak{y}_3'\rangle_2} \in \langle \tilde{Y}\rangle$ such that $\kappa_2^{\wedge'}{}_{\langle \mathfrak{y}_1', \mathfrak{y}_2', \mathfrak{y}_3'\rangle_1} \succ \kappa_2^{\wedge'}{}_{\langle \mathfrak{y}_1', \mathfrak{y}_2', \mathfrak{y}_3'\rangle_2}$ or $\kappa_2^{\wedge'}{}_{\langle \mathfrak{y}_1', \mathfrak{y}_2', \mathfrak{y}_3'\rangle_1} \prec \kappa_2^{\wedge'}{}_{\langle \mathfrak{y}_1', \mathfrak{y}_2', \mathfrak{y}_3'\rangle_2}$. So, $\kappa_2^{\wedge'}{}_{\langle \mathfrak{y}_1', \mathfrak{y}_2', \mathfrak{y}_3'\rangle_1} \succ \kappa_2^{\wedge'}{}_{\langle \mathfrak{y}_1', \mathfrak{y}_2', \mathfrak{y}_3'\rangle_2}$ or $\kappa_2^{\wedge'}{}_{\langle \mathfrak{y}_1', \mathfrak{y}_2', \mathfrak{y}_3'\rangle_1} \prec \kappa_2^{\wedge'}{}_{\langle \mathfrak{y}_1', \mathfrak{y}_2', \mathfrak{y}_3'\rangle_2}$ respectively such that $\kappa_2^{\wedge'}{}_{\langle \mathfrak{y}_1', \mathfrak{y}_2', \mathfrak{y}_3'\rangle_2} = \not{f}((\kappa_1^{\wedge}{}_{\langle \mathfrak{y}_1, \mathfrak{y}_2, \mathfrak{y}_3\rangle})1), \kappa_2^{\wedge'}{}_{\langle \mathfrak{y}_1', \mathfrak{y}_2', \mathfrak{y}_3'\rangle_2} = \not{f}(\kappa_1^{\wedge}{}_{\langle \mathfrak{y}_1, \mathfrak{y}_2, \mathfrak{y}_3\rangle_2})$ such that $\kappa_1^{\wedge}{}_{\langle \mathfrak{y}_1, \mathfrak{y}_2, \mathfrak{y}_3\rangle_1} = \not{f}^{-1}(\kappa_2^{\wedge'}{}_{\langle \mathfrak{y}_1', \mathfrak{y}_2', \mathfrak{y}_3'\rangle_1})$ and $\kappa_1^{\wedge}{}_{\langle \mathfrak{y}_1, \mathfrak{y}_2, \mathfrak{y}_3\rangle_2} = \not{f}^{-1}(\kappa_2^{\wedge'}{}_{\langle \mathfrak{y}_1', \mathfrak{y}_2', \mathfrak{y}_3'\rangle_2})$. Since $\kappa_2^{\wedge'}{}_{\langle \mathfrak{y}_1', \mathfrak{y}_2', \mathfrak{y}_3'\rangle_1}, \kappa_2^{\wedge'}{}_{\langle \mathfrak{y}_1', \mathfrak{y}_2', \mathfrak{y}_3'\rangle_2} \in \langle \tilde{Y}\rangle$ but $(\langle \tilde{Y}\rangle, \mathfrak{F}_1, \mathfrak{F}_2, \theta)$ is $NSB{*_b}$ Hausdorff space. So according to definition $\kappa_2^{\wedge'}{}_{\langle \mathfrak{y}_1', \mathfrak{y}_2', \mathfrak{y}_3'\rangle_1} \succ \kappa_2^{\wedge'}{}_{\langle \mathfrak{y}_1', \mathfrak{y}_2', \mathfrak{y}_3'\rangle_2}$ or $\kappa_2^{\wedge'}{}_{\langle \mathfrak{y}_1', \mathfrak{y}_2', \mathfrak{y}_3'\rangle_1} \prec \kappa_2^{\wedge'}{}_{\langle \mathfrak{y}_1', \mathfrak{y}_2', \mathfrak{y}_3'\rangle_2}$. There definitely exists $NS{*_b}$ open sets $\langle \not{k}_1,\theta\rangle$ and $\langle \not{k}_2,\theta\rangle \in (\langle \tilde{Y}\rangle, \mathfrak{F}_1, \mathfrak{F}_2, \theta)$ such that $\kappa_2^{\wedge'}{}_{\langle \mathfrak{y}_1', \mathfrak{y}_2', \mathfrak{y}_3'\rangle_1} \in \langle \not{k}_1,\theta\rangle$ and $\kappa_2^{\wedge'}{}_{\langle \mathfrak{y}_1', \mathfrak{y}_2', \mathfrak{y}_3'\rangle_2} \in \langle \not{k}_2,\theta\rangle$ and these two $NS{*_b}$ open sets which are disjoint. Since $\not{f}^{-1}(\langle \not{k}_1,\theta\rangle)$ and $\not{f}^{-1}(\langle \not{k}_2,\theta\rangle)$ are $NS{*_b}$ open in $(\tilde{\pi}, \tau_1, \tau_2, \theta)$. Now, $\not{f}^{-1}(\langle \not{k}_1,\theta\rangle) \tilde{\cap} \not{f}^{-1}(\langle \not{k}_1,\theta\rangle) = \not{f}^{-1}(\langle \not{k}_1,\theta\rangle \tilde{\cap} \langle \not{k}_2,\theta\rangle) = \not{f}^{-1}(\tilde{\varnothing}) = \widetilde{0_{(\langle\widetilde{\pi}\rangle,\theta)}}$ and $(\kappa_2^{\wedge'}{}_{\langle \mathfrak{y}_1', \mathfrak{y}_2', \mathfrak{y}_3'\rangle})1 \in \langle \not{k}_1,\theta\rangle \Rightarrow \not{f}^{-1}((\kappa_2^{\wedge'}{}_{\langle \mathfrak{y}_1', \mathfrak{y}_2', \mathfrak{y}_3'\rangle})1) \in \not{f}^{-1}(\langle \not{k}_1,\theta\rangle) \Rightarrow \kappa_1^{\wedge}{}_{\langle \mathfrak{y}_1, \mathfrak{y}_2, \mathfrak{y}_3\rangle_1} \in (\langle \not{k}_1,\theta\rangle)$, $\kappa_2^{\wedge'}{}_{\langle \mathfrak{y}_1', \mathfrak{y}_2', \mathfrak{y}_3'\rangle_2} \in \langle \not{k}_2,\theta\rangle \Rightarrow \not{f}^{-1}(\kappa_2^{\wedge'}{}_{\langle \mathfrak{y}_1', \mathfrak{y}_2', \mathfrak{y}_3'\rangle_2}) \in \not{f}^{-1}(\langle \not{k}_2,\theta\rangle)$ implies that $\kappa_1^{\wedge}{}_{\langle \mathfrak{y}_1, \mathfrak{y}_2, \mathfrak{y}_3\rangle_2} \in (\langle \not{k}_2,\theta\rangle)$. $(\langle \not{k}_1,\theta\rangle)$, $\kappa_1^{\wedge}{}_{\langle \mathfrak{y}_1, \mathfrak{y}_2, \mathfrak{y}_3\rangle_2} \in \not{f}^{-1}(\langle \not{k}_2,\theta\rangle)$. Accordingly, $NSBTS$ is ${*_b}$ Hausdorff space.

Theorem 5.3. Let $(\tilde{\pi}, \tau_1, \tau_2, \theta)$ be $NSBTS$ and $\left(\langle \tilde{Y}\rangle, \mathfrak{F}_1, \mathfrak{F}_2, \theta\right)$ be an-other $NSBTS$. Let $\langle \not{f}, \theta\rangle$: $(\tilde{\pi}, \tau_1, \tau_2, \theta) \longrightarrow \left(\langle \tilde{Y}\rangle, \mathfrak{F}_1, \mathfrak{F}_2, \theta\right)$ be a soft mapping. Let $\left(\langle \tilde{Y}\rangle, \mathfrak{F}_1, \mathfrak{F}_2, \theta\right)$ is $NSB{*_b}$ Hausdorff space then it is guaranteed that $\left\{\left(\kappa_1^{\wedge}{}_{\langle \mathfrak{y}_1, \mathfrak{y}_2, \mathfrak{y}_3\rangle}, \kappa_2^{\wedge'}{}_{\langle \mathfrak{y}_1', \mathfrak{y}_2', \mathfrak{y}_3'\rangle}\right) : \not{f}\left(\kappa_1^{\wedge}{}_{\langle \mathfrak{y}_1, \mathfrak{y}_2, \mathfrak{y}_3\rangle}\right) = \not{f}\left(\kappa_2^{\wedge'}{}_{\langle \mathfrak{y}_1', \mathfrak{y}_2', \mathfrak{y}_3'\rangle}\right)\right\}$ is a $NS{*_b}$ closed sub-set of $(\tilde{\pi}, \tau_1, \tau_2, \theta) \times \left(\langle \tilde{Y}\rangle, \mathfrak{F}_1, \mathfrak{F}_2, \theta\right)$.

Proof. Given that $(\tilde{\pi}, \tau_1, \tau_2, \theta)$ be $NSBTS$ and $\left(\langle \tilde{Y}\rangle, \mathfrak{F}_1, \mathfrak{F}_2, \theta\right)$ be an-other $NSBTS$. Let $\langle \not{f}, \theta\rangle$: $(\tilde{\pi}, \tau_1, \tau_2, \theta) \longrightarrow \left(\langle \tilde{Y}\rangle, \mathfrak{F}_1, \mathfrak{F}_2, \theta\right)$ be a soft mapping such that it is continuous mapping $\left(\langle \tilde{Y}\rangle, \mathfrak{F}_1, \mathfrak{F}_2, \theta\right)$ is $NSB{*_b}$ Hausdorff space Then we will prove that $\left\{\left(\left(\left(\kappa_1^{\wedge}{}_{\langle \mathfrak{y}_1, \mathfrak{y}_2, \mathfrak{y}_3\rangle}\right)\right), \kappa_2^{\wedge'}{}_{\langle \mathfrak{y}_1', \mathfrak{y}_2', \mathfrak{y}_3'\rangle}\right)\right.$: $\left.\not{f}\left(\left(\kappa_1^{\wedge}{}_{\langle \mathfrak{y}_1, \mathfrak{y}_2, \mathfrak{y}_3\rangle}\right)\right) = \not{f}\left(\kappa_2^{\wedge'}{}_{\langle \mathfrak{y}_1', \mathfrak{y}_2', \mathfrak{y}_3'\rangle}\right)\right\}$ is a $NS{*_b}$ closed sub-set of $(\tilde{\pi}, \tau_1, \tau_2, \theta) \times \left(\langle \tilde{Y}\rangle, \mathfrak{F}_1, \mathfrak{F}_2, \theta\right)$.

Equavilintly, we will prove that $\left\{\left(\kappa_1{}^{\curlywedge}{}_{\langle\mathfrak{y}_1,\mathfrak{y}_2,\mathfrak{y}_3\rangle},\kappa_2{}^{\curlywedge/}{}_{\langle\mathfrak{y}_1{}',\mathfrak{y}_2{}',\mathfrak{y}_3{}'\rangle}\right):\not{f}\left(\left(\kappa_1{}^{\curlywedge}{}_{\langle\mathfrak{y}_1,\mathfrak{y}_2,\mathfrak{y}_3\rangle}\right)\right)=\left(\kappa_2{}^{\curlywedge/}{}_{\langle\mathfrak{y}_1{}',\mathfrak{y}_2{}',\mathfrak{y}_3{}'\rangle}\right)\right\}^c$ is $NS{*_b}$ open sub-set of $(\tilde{\pi},\tau_1,\tau_2,\theta)\longrightarrow\left(\langle\tilde{Y}\rangle,\mathfrak{F}_1,\mathfrak{F}_2,\theta\right)$. Let $\left(\left(\kappa_1{}^{\curlywedge}{}_{\langle\mathfrak{y}_1,\mathfrak{y}_2,\mathfrak{y}_3\rangle}\right),\kappa_2{}^{\curlywedge/}{}_{\langle\mathfrak{y}_1{}',\mathfrak{y}_2{}',\mathfrak{y}_3{}'\rangle}\right)\in\left\{\left(\kappa_1{}^{\curlywedge}{}_{\langle\mathfrak{y}_1,\mathfrak{y}_2,\mathfrak{y}_3\rangle},\kappa_2{}^{\curlywedge/}{}_{\langle\mathfrak{y}_1{}',\mathfrak{y}_2{}',\mathfrak{y}_3{}'\rangle}\right)\text{ with }\kappa_1{}^{\curlywedge}{}_{\langle\mathfrak{y}_1,\mathfrak{y}_2,\mathfrak{y}_3\rangle}\succ\kappa_2{}^{\curlywedge/}{}_{\langle\mathfrak{y}_1{}',\mathfrak{y}_2{}',\mathfrak{y}_3{}'\rangle}:\not{f}\left(\kappa_1{}^{\curlywedge}{}_{\langle\mathfrak{y}_1,\mathfrak{y}_2,\mathfrak{y}_3\rangle}\right)\succ\not{f}\left(\kappa_2{}^{\curlywedge/}{}_{\langle\mathfrak{y}_1{}',\mathfrak{y}_2{}',\mathfrak{y}_3{}'\rangle}\right)\right\}^c$ or $\left(\kappa_1{}^{\curlywedge}{}_{\langle\mathfrak{y}_1,\mathfrak{y}_2,\mathfrak{y}_3\rangle},\kappa_2{}^{\curlywedge/}{}_{\langle\mathfrak{y}_1{}',\mathfrak{y}_2{}',\mathfrak{y}_3{}'\rangle}\right)\in\left\{\left(\kappa_1{}^{\curlywedge}{}_{\langle\mathfrak{y}_1,\mathfrak{y}_2,\mathfrak{y}_3\rangle},\kappa_2{}^{\curlywedge/}{}_{\langle\mathfrak{y}_1{}',\mathfrak{y}_2{}',\mathfrak{y}_3{}'\rangle}\right)\text{ with }\kappa_1{}^{\curlywedge}{}_{\langle\mathfrak{y}_1,\mathfrak{y}_2,\mathfrak{y}_3\rangle}\prec\kappa_2{}^{\curlywedge/}{}_{\langle\mathfrak{y}_1{}',\mathfrak{y}_2{}',\mathfrak{y}_3{}'\rangle}:\not{f}\left(\kappa_1{}^{\curlywedge}{}_{\langle\mathfrak{y}_1,\mathfrak{y}_2,\mathfrak{y}_3\rangle}\right)\prec\not{f}\left(\kappa_2{}^{\curlywedge/}{}_{\langle\mathfrak{y}_1{}',\mathfrak{y}_2{}',\mathfrak{y}_3{}'\rangle}\right)\right\}^c$. Then, $\not{f}\left(\kappa_1{}^{\curlywedge}{}_{\langle\mathfrak{y}_1,\mathfrak{y}_2,\mathfrak{y}_3\rangle}\right)\succ\not{f}\left(\kappa_2{}^{\curlywedge/}{}_{\langle\mathfrak{y}_1{}',\mathfrak{y}_2{}',\mathfrak{y}_3{}'\rangle}\right)$ or $\prec\not{f}\left(\kappa_1{}^{\curlywedge}{}_{\langle\mathfrak{y}_1,\mathfrak{y}_2,\mathfrak{y}_3\rangle}\right)\prec\not{f}\left(\kappa_2{}^{\curlywedge/}{}_{\langle\mathfrak{y}_1{}',\mathfrak{y}_2{}',\mathfrak{y}_3{}'\rangle}\right)$ accordingly. Since, $\left(\langle\tilde{Y}\rangle,\mathfrak{F}_1,\mathfrak{F}_2,\theta\right)$ is ${*_b}$ Hausdorff space. Certainly, $\not{f}\left(\kappa_1{}^{\curlywedge}{}_{\langle\mathfrak{y}_1,\mathfrak{y}_2,\mathfrak{y}_3\rangle}\right),\not{f}\left(\kappa_2{}^{\curlywedge/}{}_{\langle\mathfrak{y}_1{}',\mathfrak{y}_2{}',\mathfrak{y}_3{}'\rangle}\right)$ are points of $\left(\langle\tilde{Y}\rangle,\mathfrak{F}_1,\mathfrak{F}_2,\theta\right)$, there exists $NS{*_b}$ open sets $\langle\mathcal{G},\theta\rangle,\langle\not{k},\theta\rangle\in\left(\langle\tilde{Y}\rangle,\mathfrak{F}_1,\mathfrak{F}_2,\theta\right)$ such that $\not{f}\left(\left(\kappa_1{}^{\curlywedge}{}_{\langle\mathfrak{y}_1,\mathfrak{y}_2,\mathfrak{y}_3\rangle}\right)\right)\in\langle\mathcal{G},\theta\rangle$, $\not{f}\left(\kappa_1{}^{\curlywedge}{}_{\langle\mathfrak{y}_1,\mathfrak{y}_2,\mathfrak{y}_3\rangle}\right)\in\langle\not{k},\theta\rangle$ provided $\langle\mathcal{G},\theta\rangle\tilde{\cap}\langle\not{k},\theta\rangle=\widetilde{0_{\left(\langle\widetilde{\pi}\rangle,\theta\right)_Y}}$. Since, $\langle\not{f},\theta\rangle$ is soft continuous, $\not{f}^{-1}(\langle\mathcal{G},\theta\rangle$ & $\not{f}^{-1}(\langle\not{k},\theta\rangle$ are $NS{*_b}$ open sets in $\langle\tilde{\pi},\tau_1,\tau_2,\theta\rangle$ supposing $\kappa_1{}^{\curlywedge}{}_{\langle\mathfrak{y}_1,\mathfrak{y}_2,\mathfrak{y}_3\rangle}$ and $\kappa_2{}^{\curlywedge/}{}_{\langle\mathfrak{y}_1{}',\mathfrak{y}_2{}',\mathfrak{y}_3{}'\rangle}$ respectively and so $\not{f}^{-1}(\langle\mathcal{G},\theta\rangle\times\not{f}^{-1}(\langle\mathcal{H},\theta\rangle$ is basic $NS{*_b}$ open set in $(\tilde{\pi},\tau_1,\tau_2,\theta)\longrightarrow\left(\langle\tilde{Y}\rangle,\mathfrak{F}_1,\mathfrak{F}_2,\theta\right)$ containing $\left(\left(\kappa_1{}^{\curlywedge}{}_{\langle\mathfrak{y}_1,\mathfrak{y}_2,\mathfrak{y}_3\rangle}\right),\kappa_2{}^{\curlywedge/}{}_{\langle\mathfrak{y}_1{}',\mathfrak{y}_2{}',\mathfrak{y}_3{}'\rangle}\right)$. Since $\langle\mathcal{G},\theta\rangle\tilde{\cap}\langle\not{k},\theta\rangle=\widetilde{0_{\theta\,Y}}$, it is clear by the definition of $\left\{\left(\left(\kappa_1{}^{\curlywedge}{}_{\langle\mathfrak{y}_1,\mathfrak{y}_2,\mathfrak{y}_3\rangle}\right),\kappa_2{}^{\curlywedge/}{}_{\langle\mathfrak{y}_1{}',\mathfrak{y}_2{}',\mathfrak{y}_3{}'\rangle}\right):\not{f}\left(\kappa_1{}^{\curlywedge}{}_{\langle\mathfrak{y}_1,\mathfrak{y}_2,\mathfrak{y}_3\rangle}\right)=\not{f}\left(\kappa_2{}^{\curlywedge/}{}_{\langle\mathfrak{y}_1{}',\mathfrak{y}_2{}',\mathfrak{y}_3{}'\rangle}\right)\right\}$ that is $\left\{\not{f}^{-1}(\langle\mathcal{G},\theta\rangle\,\&\,\not{f}^{-1}(\langle\not{k},\theta\rangle\right\}\tilde{\cap}\left\{\left(\left(\kappa_1{}^{\curlywedge}{}_{\langle\mathfrak{y}_1,\mathfrak{y}_2,\mathfrak{y}_3\rangle}\right),\kappa_2{}^{\curlywedge/}{}_{\langle\mathfrak{y}_1{}',\mathfrak{y}_2{}',\mathfrak{y}_3{}'\rangle}\right):\not{f}(x)=\not{f}\left(\kappa_2{}^{\curlywedge/}{}_{\langle\mathfrak{y}_1{}',\mathfrak{y}_2{}',\mathfrak{y}_3{}'\rangle}\right)\right\}=0_{(\pi,\theta)}$, that is $\not{f}^{-1}(\langle\mathcal{G},\theta\rangle\times\not{f}^{-1}(\langle\not{k},\theta\rangle\Subset\left\{\left(\kappa_1{}^{\curlywedge}{}_{\langle\mathfrak{y}_1,\mathfrak{y}_2,\mathfrak{y}_3\rangle},\kappa_2{}^{\curlywedge/}{}_{\langle\mathfrak{y}_1{}',\mathfrak{y}_2{}',\mathfrak{y}_3{}'\rangle}\right):\not{f}\left(\kappa_1{}^{\curlywedge}{}_{\langle\mathfrak{y}_1,\mathfrak{y}_2,\mathfrak{y}_3\rangle}\right)=\not{f}\left(\kappa_2{}^{\curlywedge/}{}_{\langle\mathfrak{y}_1{}',\mathfrak{y}_2{}',\mathfrak{y}_3{}'\rangle}\right)\right\}^c$. Hence, $\left\{\left(\kappa_1{}^{\curlywedge}{}_{\langle\mathfrak{y}_1,\mathfrak{y}_2,\mathfrak{y}_3\rangle},\kappa_2{}^{\curlywedge/}{}_{\langle\mathfrak{y}_1{}',\mathfrak{y}_2{}',\mathfrak{y}_3{}'\rangle}\right):\not{f}\left(\kappa_1{}^{\curlywedge}{}_{\langle\mathfrak{y}_1,\mathfrak{y}_2,\mathfrak{y}_3\rangle}\right)=\not{f}\left(\kappa_2{}^{\curlywedge/}{}_{\langle\mathfrak{y}_1{}',\mathfrak{y}_2{}',\mathfrak{y}_3{}'\rangle}\right)\right\}^c$ implies that $\left\{\left(\kappa_1{}^{\curlywedge}{}_{\langle\mathfrak{y}_1,\mathfrak{y}_2,\mathfrak{y}_3\rangle},\kappa_2{}^{\curlywedge/}{}_{\langle\mathfrak{y}_1{}',\mathfrak{y}_2{}',\mathfrak{y}_3{}'\rangle}\right):\not{f}\left(\kappa_1{}^{\curlywedge}{}_{\langle\mathfrak{y}_1,\mathfrak{y}_2,\mathfrak{y}_3\rangle}\right)=\not{f}\left(\kappa_2{}^{\curlywedge/}{}_{\langle\mathfrak{y}_1{}',\mathfrak{y}_2{}',\mathfrak{y}_3{}'\rangle}\right)\right\}$ is $NS{*_b}$ closed.

Theorem 5.4. Let $(\tilde{\pi},\tau_1,\tau_2,\theta)$ be $NSBTS$ and $\left(\langle\tilde{Y}\rangle,\mathfrak{F}_1,\mathfrak{F}_2,\theta\right)$ be an-other $NSBTS$. Let $\langle\not{f},\theta\rangle$: $(\tilde{\pi},\tau_1,\tau_2,\theta)\longrightarrow\left(\langle\tilde{Y}\rangle,\mathfrak{F}_1,\mathfrak{F}_2,\theta\right)$ be a $NS{*_b}$ open mapping such that it is onto. If the soft set $\left\{\left(\kappa_1{}^{\curlywedge}{}_{\langle\mathfrak{y}_1,\mathfrak{y}_2,\mathfrak{y}_3\rangle},\kappa_2{}^{\curlywedge/}{}_{\langle\mathfrak{y}_1{}',\mathfrak{y}_2{}',\mathfrak{y}_3{}'\rangle}\right):\not{f}\left(\left(\kappa_1{}^{\curlywedge}{}_{\langle\mathfrak{y}_1,\mathfrak{y}_2,\mathfrak{y}_3\rangle}\right)\right)=\not{f}\left(\left(\kappa_2{}^{\curlywedge/}{}_{\langle\mathfrak{y}_1{}',\mathfrak{y}_2{}',\mathfrak{y}_3{}'\rangle}\right)\right)\right\}$ is $NS{*_b}$ closed in $\langle\tilde{\pi},\tau_1,\tau_2,\theta\rangle\times\left(\langle\tilde{Y}\rangle,\mathfrak{F}_1,\mathfrak{F}_2,\theta\right)$, then $\langle\tilde{\pi},\tau_1,\tau_2,\theta\rangle$ will behave as $NSB{*_b}$ Hausdorff space.

Proof. Suppose $\not{f}\left(\kappa_1{}^{\curlywedge}{}_{\langle\mathfrak{y}_1,\mathfrak{y}_2,\mathfrak{y}_3\rangle}\right),\not{f}\left(\kappa_2{}^{\curlywedge/}{}_{\langle\mathfrak{y}_1{}',\mathfrak{y}_2{}',\mathfrak{y}_3{}'\rangle}\right)$ be two points of $\langle\tilde{Y}\rangle$ such that either $\not{f}\left(\kappa_1{}^{\curlywedge}{}_{\langle\mathfrak{y}_1,\mathfrak{y}_2,\mathfrak{y}_3\rangle}\right)\succ\not{f}\left(\kappa_2{}^{\curlywedge/}{}_{\langle\mathfrak{y}_1{}',\mathfrak{y}_2{}',\mathfrak{y}_3{}'\rangle}\right)$ or $\prec\not{f}\left(\kappa_1{}^{\curlywedge}{}_{\langle\mathfrak{y}_1,\mathfrak{y}_2,\mathfrak{y}_3\rangle}\right)\prec\not{f}\left(\kappa_2{}^{\curlywedge/}{}_{\langle\mathfrak{y}_1{}',\mathfrak{y}_2{}',\mathfrak{y}_3{}'\rangle}\right)$. Then $\left(\kappa_1{}^{\curlywedge}{}_{\langle\mathfrak{y}_1,\mathfrak{y}_2,\mathfrak{y}_3\rangle},\kappa_2{}^{\curlywedge/}{}_{\langle\mathfrak{y}_1{}',\mathfrak{y}_2{}',\mathfrak{y}_3{}'\rangle}\right)\notin\left\{\left(\kappa_2{}^{\curlywedge/}{}_{\langle\mathfrak{y}_1{}',\mathfrak{y}_2{}',\mathfrak{y}_3{}'\rangle}\right)\text{ with }\kappa_1{}^{\curlywedge}{}_{\langle\mathfrak{y}_1,\mathfrak{y}_2,\mathfrak{y}_3\rangle}\succ\left(\kappa_2{}^{\curlywedge/}{}_{\langle\mathfrak{y}_1{}',\mathfrak{y}_2{}',\mathfrak{y}_3{}'\rangle}\right):\not{f}\left(\kappa_1{}^{\curlywedge}{}_{\langle\mathfrak{y}_1,\mathfrak{y}_2,\mathfrak{y}_3\rangle}\right)\succ\not{f}\left(\kappa_2{}^{\curlywedge/}{}_{\langle\mathfrak{y}_1{}',\mathfrak{y}_2{}',\mathfrak{y}_3{}'\rangle}\right)\right\}$ or $\left(\kappa_1{}^{\curlywedge}{}_{\langle\mathfrak{y}_1,\mathfrak{y}_2,\mathfrak{y}_3\rangle},\kappa_2{}^{\curlywedge/}{}_{\langle\mathfrak{y}_1{}',\mathfrak{y}_2{}',\mathfrak{y}_3{}'\rangle}\right)\notin\left\{\kappa_1{}^{\curlywedge}{}_{\langle\mathfrak{y}_1,\mathfrak{y}_2,\mathfrak{y}_3\rangle}\text{ with }\kappa_1{}^{\curlywedge}{}_{\langle\mathfrak{y}_1,\mathfrak{y}_2,\mathfrak{y}_3\rangle}\prec\kappa_2{}^{\curlywedge/}{}_{\langle\mathfrak{y}_1{}',\mathfrak{y}_2{}',\mathfrak{y}_3{}'\rangle}:\not{f}\left(\kappa_1{}^{\curlywedge}{}_{\langle\mathfrak{y}_1,\mathfrak{y}_2,\mathfrak{y}_3\rangle}\right)\prec\not{f}\left(\kappa_2{}^{\curlywedge/}{}_{\langle\mathfrak{y}_1{}',\mathfrak{y}_2{}',\mathfrak{y}_3{}'\rangle}\right)\right\}$, that is $\left(\kappa_1{}^{\curlywedge}{}_{\langle\mathfrak{y}_1,\mathfrak{y}_2,\mathfrak{y}_3\rangle},\kappa_2{}^{\curlywedge/}{}_{\langle\mathfrak{y}_1{}',\mathfrak{y}_2{}',\mathfrak{y}_3{}'\rangle}\right)\in\left\{\left(\kappa_1{}^{\curlywedge}{}_{\langle\mathfrak{y}_1,\mathfrak{y}_2,\mathfrak{y}_3\rangle},\right.\right.$

$\kappa_2{}^{\curlywedge/}{}_{\langle \eta_1/,\eta_2/,\eta_3/\rangle})$ with $\kappa_1{}^{\curlywedge}{}_{\langle \eta_1,\eta_2,\eta_3\rangle} \succ \kappa_2{}^{\curlywedge/}{}_{\langle \eta_1/,\eta_2/,\eta_3/\rangle} : \not{f}\left(\kappa_1{}^{\curlywedge}{}_{\langle \eta_1,\eta_2,\eta_3\rangle}\right) \succ \not{f}\left(\kappa_2{}^{\curlywedge/}{}_{\langle \eta_1/,\eta_2/,\eta_3/\rangle}\right)\Big\}^c$ or $\left(\kappa_1{}^{\curlywedge}{}_{\langle \eta_1,\eta_2,\eta_3\rangle}, \kappa_2{}^{\curlywedge/}{}_{\langle \eta_1/,\eta_2/,\eta_3/\rangle}\right) \in \Big\{\left(\kappa_1{}^{\curlywedge}{}_{\langle \eta_1,\eta_2,\eta_3\rangle}, \kappa_2{}^{\curlywedge/}{}_{\langle \eta_1/,\eta_2/,\eta_3/\rangle}\right)$ with $\kappa_1{}^{\curlywedge}{}_{\langle \eta_1,\eta_2,\eta_3\rangle} \prec \kappa_2{}^{\curlywedge/}{}_{\langle \eta_1/,\eta_2/,\eta_3/\rangle} : \not{f}\left(\kappa_1{}^{\curlywedge}{}_{\langle \eta_1,\eta_2,\eta_3\rangle}\right) \prec \not{f}\left(\kappa_2{}^{\curlywedge/}{}_{\langle \eta_1/,\eta_2/,\eta_3/\rangle}\right)\Big\}^c$. Since, $\left(\kappa_1{}^{\curlywedge}{}_{\langle \eta_1,\eta_2,\eta_3\rangle}, \kappa_2{}^{\curlywedge/}{}_{\langle \eta_1/,\eta_2/,\eta_3/\rangle}\right) \in \Big\{\left(\kappa_1{}^{\curlywedge}{}_{\langle \eta_1,\eta_2,\eta_3\rangle}, \kappa_2{}^{\curlywedge/}{}_{\langle \eta_1/,\eta_2/,\eta_3/\rangle}\right)$ with $\kappa_1{}^{\curlywedge}{}_{\langle \eta_1,\eta_2,\eta_3\rangle} \succ \kappa_2{}^{\curlywedge/}{}_{\langle \eta_1/,\eta_2/,\eta_3/\rangle} : \not{f}\left(\kappa_1{}^{\curlywedge}{}_{\langle \eta_1,\eta_2,\eta_3\rangle}\right) \succ \not{f}\left(\kappa_2{}^{\curlywedge/}{}_{\langle \eta_1/,\eta_2/,\eta_3/\rangle}\right)\Big\}^c$ or $\left(\kappa_1{}^{\curlywedge}{}_{\langle \eta_1,\eta_2,\eta_3\rangle}, \kappa_2{}^{\curlywedge/}{}_{\langle \eta_1/,\eta_2/,\eta_3/\rangle}\right) \in \Big\{\left(\kappa_1{}^{\curlywedge}{}_{\langle \eta_1,\eta_2,\eta_3\rangle}, \kappa_2{}^{\curlywedge/}{}_{\langle \eta_1/,\eta_2/,\eta_3/\rangle}\right)$ with $\kappa_1{}^{\curlywedge}{}_{\langle \eta_1,\eta_2,\eta_3\rangle} \prec \kappa_2{}^{\curlywedge/}{}_{\langle \eta_1/,\eta_2/,\eta_3/\rangle} : \not{f}\left(\kappa_1{}^{\curlywedge}{}_{\langle \eta_1,\eta_2,\eta_3\rangle}\right) \prec \not{f}\left(\kappa_2{}^{\curlywedge/}{}_{\langle \eta_1/,\eta_2/,\eta_3/\rangle}\right)\Big\}^c$ is soft set in $\langle \tilde{\pi}, \tau_1, \tau_2, \theta\rangle \times \left\langle \langle \tilde{Y}\rangle, \mathfrak{F}_1, \mathfrak{F}_2, \theta\right\rangle$, then there exists $NS*_b$ open sets $\langle \mathcal{G}, \theta\rangle$ and $\langle \hbar, \theta\rangle$ in $\langle \tilde{\pi}, \tau_1, \tau_2, \theta\rangle$ such that $\left(\kappa_1{}^{\curlywedge}{}_{\langle \eta_1,\eta_2,\eta_3\rangle}, \kappa_2{}^{\curlywedge/}{}_{\langle \eta_1/,\eta_2/,\eta_3/\rangle}\right) \in \langle \mathcal{G}, \theta\rangle \times \langle \hbar, \theta\rangle \Subset \Big\{\left(\left(\kappa_1{}^{\curlywedge}{}_{\langle \eta_1,\eta_2,\eta_3\rangle}\right), \kappa_2{}^{\curlywedge/}{}_{\langle \eta_1/,\eta_2/,\eta_3/\rangle}\right)$ with $\kappa_1{}^{\curlywedge}{}_{\langle \eta_1,\eta_2,\eta_3\rangle} \succ \kappa_2{}^{\curlywedge/}{}_{\langle \eta_1/,\eta_2/,\eta_3/\rangle} : \not{f}\left(\left(\kappa_1{}^{\curlywedge}{}_{\langle \eta_1,\eta_2,\eta_3\rangle}\right)\right) \succ \not{f}\left(\kappa_2{}^{\curlywedge/}{}_{\langle \eta_1/,\eta_2/,\eta_3/\rangle}\right)\Big\}^c$ or $\left(\kappa_1{}^{\curlywedge}{}_{\langle \eta_1,\eta_2,\eta_3\rangle}, \kappa_2{}^{\curlywedge/}{}_{\langle \eta_1/,\eta_2/,\eta_3/\rangle}\right) \in \langle \mathcal{G}, \theta\rangle \times \langle \hbar, \theta\rangle \Subset \Big\{\left(\left(\kappa_1{}^{\curlywedge}{}_{\langle \eta_1,\eta_2,\eta_3\rangle}\right), \kappa_2{}^{\curlywedge/}{}_{\langle \eta_1/,\eta_2/,\eta_3/\rangle}\right)$ with $\kappa_1{}^{\curlywedge}{}_{\langle \eta_1,\eta_2,\eta_3\rangle} \prec \kappa_2{}^{\curlywedge/}{}_{\langle \eta_1/,\eta_2/,\eta_3/\rangle} : \not{f}\left(\kappa_1{}^{\curlywedge}{}_{\langle \eta_1,\eta_2,\eta_3\rangle}\right) \prec \not{f}\left(\kappa_2{}^{\curlywedge/}{}_{\langle \eta_1/,\eta_2/,\eta_3/\rangle}\right)\Big\}^c$. Then, since $\not{f}$ is $NS*_b$ open, $\not{f}(\langle \mathcal{G}, \theta\rangle)$ and $\not{f}(\langle \hbar, \theta\rangle)$ are $NS*_b$ open sets in $\left(\langle \tilde{Y}\rangle, \mathfrak{F}_1, \mathfrak{F}_2, \theta\right)$ containing $\not{f}\left(\kappa_1{}^{\curlywedge}{}_{\langle \eta_1,\eta_2,\eta_3\rangle}\right)$ and $\not{f}\left(\kappa_2{}^{\curlywedge/}{}_{\langle \eta_1/,\eta_2/,\eta_3/\rangle}\right)$ respectively, and $\not{f}(\langle \mathcal{G}, \theta\rangle) \,\tilde{\cap} \not{f}(\langle \hbar, \theta\rangle) = \widetilde{0_{\left(\langle\tilde{\pi}\rangle, \theta\right)}}$ otherwise $\not{f}(\langle \mathcal{G}, \theta\rangle) \times \not{f}(\langle \hbar, \theta\rangle)$ $\tilde{\cap} \Big\{\left(\left(\kappa_1{}^{\curlywedge}{}_{\langle \eta_1,\eta_2,\eta_3\rangle}\right), \right)$ with $\left(\kappa_1{}^{\curlywedge}{}_{\langle \eta_1,\eta_2,\eta_3\rangle}\right) \succ \kappa_2{}^{\curlywedge/}{}_{\langle \eta_1/,\eta_2/,\eta_3/\rangle} : \not{f}\left(\left(\kappa_1{}^{\curlywedge}{}_{\langle \eta_1,\eta_2,\eta_3\rangle}\right)\right) \succ \not{f}\left(\kappa_2{}^{\curlywedge/}{}_{\langle \eta_1/,\eta_2/,\eta_3/\rangle}\right)\Big\}$ or $\left(\left(\left(\kappa_1{}^{\curlywedge}{}_{\langle \eta_1,\eta_2,\eta_3\rangle}\right), \kappa_2{}^{\curlywedge/}{}_{\langle \eta_1/,\eta_2/,\eta_3/\rangle}\right)\right) \notin \Big\{\left(\left(\kappa_1{}^{\curlywedge}{}_{\langle \eta_1,\eta_2,\eta_3\rangle}\right), \kappa_2{}^{\curlywedge/}{}_{\langle \eta_1/,\eta_2/,\eta_3/\rangle}\right)$ with $\left(\kappa_1{}^{\curlywedge}{}_{\langle \eta_1,\eta_2,\eta_3\rangle}\right) \prec \kappa_2{}^{\curlywedge/}{}_{\langle \eta_1/,\eta_2/,\eta_3/\rangle} : \not{f}\left(\left(\kappa_1{}^{\curlywedge}{}_{\langle \eta_1,\eta_2,\eta_3\rangle}\right)\right) \prec \not{f}\left(\kappa_2{}^{\curlywedge/}{}_{\langle \eta_1/,\eta_2/,\eta_3/\rangle}\right)\Big\} = \widetilde{0_{\left(\langle\tilde{\pi}\rangle, \theta\right)}}$. It follows that $\left\langle \langle \tilde{Y}\rangle, \mathfrak{F}_1, \mathfrak{F}_2, \theta\right\rangle$ is $NSB*_b$ Hausdorff space.

Theorem 5.5. Let $\langle \tilde{\pi}, \tau_1, \tau_2, \theta\rangle$ be a *NSB* second countable space and let $\langle \not{f}, \theta\rangle$ be *NS* uncountable sub set of $\langle \tilde{\pi}, \tau_1, \tau_2, \theta\rangle$. Then, at least one point of $\langle \not{f}, \theta\rangle$ is a soft limit point of $\langle \not{f}, \theta\rangle$.

Proof. Let $\mathfrak{W} = \langle \mathcal{B}_1, \mathcal{B}_2, \mathcal{B}_3, \mathcal{B}_4, \ldots \mathcal{B}_n : n \in \mathbb{N}\rangle$ for $\langle \tilde{\pi}, \tau_1, \tau_2, \theta\rangle$.

Let, if possible, no point of $\langle \not{f}, \theta\rangle$ be a soft limit point of $\langle \not{f}, \theta\rangle$. Then, for each $\kappa_1{}^{\curlywedge}{}_{\langle \eta_1,\eta_2,\eta_3\rangle} \in \langle \not{f}, \theta\rangle$ there exists $NS*_b$ open set $\langle \rho, \theta\rangle_{\left(\kappa_1{}^{\curlywedge}{}_{\langle \eta_1,\eta_2,\eta_3\rangle}\right)}$ such that $\kappa_1{}^{\curlywedge}{}_{\langle \eta_1,\eta_2,\eta_3\rangle} \in \langle \rho, \theta\rangle_{\left(\kappa_1{}^{\curlywedge}{}_{\langle \eta_1,\eta_2,\eta_3\rangle}\right)}$ and $\langle \rho, \theta\rangle_{\left(\kappa_1{}^{\curlywedge}{}_{\langle \eta_1,\eta_2,\eta_3\rangle}\right)} \tilde{\cap} \langle \not{f}, \theta\rangle = \left\{\left(\kappa_1{}^{\curlywedge}{}_{\langle \eta_1,\eta_2,\eta_3\rangle}\right)\right\}$. Since $\mathfrak{W}$ is soft base there exists $\mathcal{B}_{n\left(\kappa_1{}^{\curlywedge}{}_{\langle \eta_1,\eta_2,\eta_3\rangle}\right)} \in \mathfrak{W}$ such that $\left(\kappa_1{}^{\curlywedge}{}_{\langle \eta_1,\eta_2,\eta_3\rangle}\right) \in \mathcal{B}_{n\left(\kappa_1{}^{\curlywedge}{}_{\langle \eta_1,\eta_2,\eta_3\rangle}\right)} \Subset \langle \rho, \theta\rangle_{\left(\kappa_1{}^{\curlywedge}{}_{\langle \eta_1,\eta_2,\eta_3\rangle}\right)}$. Therefore $\mathcal{B}_{n\left(\kappa_1{}^{\curlywedge}{}_{\langle \eta_1,\eta_2,\eta_3\rangle}\right)} \tilde{\cap} \langle \not{f}, \theta\rangle \Subset \langle \rho, \theta\rangle_{\left(\kappa_1{}^{\curlywedge}{}_{\langle \eta_1,\eta_2,\eta_3\rangle}\right)} \tilde{\cap} \langle \not{f}, \theta\rangle = \left\{\left(\kappa_1{}^{\curlywedge}{}_{\langle \eta_1,\eta_2,\eta_3\rangle}\right)\right\}$. More-over, if $\kappa_1{}^{\curlywedge}{}_{\langle \eta_1,\eta_2,\eta_3\rangle 1}$ and $\kappa_1{}^{\curlywedge}{}_{\langle \eta_1,\eta_2,\eta_3\rangle 2}$ be any two NS points such that $\kappa_1{}^{\curlywedge}{}_{\langle \eta_1,\eta_2,\eta_3\rangle 1} \neq \kappa_1{}^{\curlywedge}{}_{\langle \eta_1,\eta_2,\eta_3\rangle 2}$ which means either $\kappa_1{}^{\curlywedge}{}_{\langle \eta_1,\eta_2,\eta_3\rangle 1} \succ \kappa_1{}^{\curlywedge}{}_{\langle \eta_1,\eta_2,\eta_3\rangle 2}$ or $\kappa_1{}^{\curlywedge}{}_{\langle \eta_1,\eta_2,\eta_3\rangle 1} \prec \kappa_1{}^{\curlywedge}{}_{\langle \eta_1,\eta_2,\eta_3\rangle 2}$ then there exists $\mathcal{B}_{\mathrm{n}\left(\kappa_1{}^{\curlywedge}{}_{\langle \eta_1,\eta_2,\eta_3\rangle}\right)_1}$ and $\mathcal{B}_{\mathrm{n}\left(\kappa_1{}^{\curlywedge}{}_{\langle \eta_1,\eta_2,\eta_3\rangle}\right)_2}$ in $\mathfrak{W}$ such that $\mathcal{B}_{\mathrm{n}\left(\kappa_1{}^{\curlywedge}{}_{\langle \eta_1,\eta_2,\eta_3\rangle}\right)_1} \tilde{\cap} \langle \not{f}, \theta\rangle = \left\{\left(\kappa_1{}^{\curlywedge}{}_{\langle \eta_1,\eta_2,\eta_3\rangle}\right)_1\right\}$ and $\mathcal{B}_{\mathrm{n}\left(\kappa_1{}^{\curlywedge}{}_{\langle \eta_1,\eta_2,\eta_3\rangle}\right)_2} \tilde{\cap} \langle \not{f}, \theta\rangle = \left\{\left(\kappa_1{}^{\curlywedge}{}_{\langle \eta_1,\eta_2,\eta_3\rangle}\right)_2\right\}$. Now, $\left(\kappa_1{}^{\curlywedge}{}_{\langle \eta_1,\eta_2,\eta_3\rangle}\right)_1 \neq \left(\kappa_1{}^{\curlywedge}{}_{\langle \eta_1,\eta_2,\eta_3\rangle}\right)_2$ which guarantees that $\left\{\left(\kappa_1{}^{\curlywedge}{}_{\langle \eta_1,\eta_2,\eta_3\rangle}\right)_1\right\} \neq \left\{\left(\kappa_1{}^{\curlywedge}{}_{\langle \eta_1,\eta_2,\eta_3\rangle}\right)_2\right\}$

which implies that $\mathcal{B}_{n_{\left(\kappa_1{}^{\wedge}{}_{\langle\mathfrak{y}_1,\mathfrak{y}_2,\mathfrak{y}_3\rangle}\right)_1}} \tilde{\cap}\langle \mathscr{f},\theta\rangle \neq \mathcal{B}_{n_{\left(\kappa_1{}^{\wedge}{}_{\langle\mathfrak{y}_1,\mathfrak{y}_2,\mathfrak{y}_3\rangle}\right)_2}} \tilde{\cap}\langle \mathscr{f},\theta\rangle$ which implies $\mathcal{B}_{n_{\left(\kappa_1{}^{\wedge}{}_{\langle\mathfrak{y}_1,\mathfrak{y}_2,\mathfrak{y}_3\rangle}\right)_1}} \neq \mathcal{B}_{n_{\left(\kappa_1{}^{\wedge}{}_{\langle\mathfrak{y}_1,\mathfrak{y}_2,\mathfrak{y}_3\rangle}\right)_2}}$. Thus, there exists a one to one soft correspondence of $\langle \mathscr{f},\theta\rangle$ on to $\left\{\mathcal{B}_{n_{\left(\kappa_1{}^{\wedge}{}_{\langle\mathfrak{y}_1,\mathfrak{y}_2,\mathfrak{y}_3\rangle}\right)}} : \left(\kappa_1{}^{\wedge}{}_{\langle\mathfrak{y}_1,\mathfrak{y}_2,\mathfrak{y}_3\rangle}\right) \in \langle \mathscr{f},\theta\rangle\right\}$. Now, $\langle \mathscr{f},\theta\rangle$ being *NS* uncountable, it follows that $\left\{\mathcal{B}_{n_{\left(\kappa_1{}^{\wedge}{}_{\langle\mathfrak{y}_1,\mathfrak{y}_2,\mathfrak{y}_3\rangle}\right)}} : \left(\kappa_1{}^{\wedge}{}_{\langle\mathfrak{y}_1,\mathfrak{y}_2,\mathfrak{y}_3\rangle}\right) \in \langle \mathscr{f},\theta\rangle\right\}$ is NS uncountable. But, this is purely a contradiction.

Theorem 5.6. Let $\langle\tilde{\pi},\tau_1,\tau_2,\theta\rangle$ and $\left\langle\langle\tilde{Y}\rangle,\mathfrak{F}_1,\mathfrak{F}_2,\theta\right\rangle$ be two *NSBTS* and suppose $\langle\mathfrak{f},\theta\rangle$ be a *NS* continuous function such that $\langle\mathfrak{f},\theta\rangle : \langle\tilde{\pi},\tau_1,\tau_2,\theta\rangle \longrightarrow \left\langle\langle\tilde{Y}\rangle,\mathfrak{F}_1,\mathfrak{F}_2,\theta\right\rangle$ is *NS* continuous function and let $\langle\mathcal{L},\theta\rangle \Subset \langle\tilde{\pi},\tau_1,\tau_2,\theta\rangle$ supposes the $B.V.P.$ then safely $\mathfrak{f}(\langle\mathcal{L},\theta\rangle)$ has the $B.V.P.$

Proof: Suppose $\langle\mathcal{L},\theta\rangle$ be an infinite *NS* sub-set of $\langle\mathfrak{f},\theta\rangle$, so that $\langle\mathcal{L},\theta\rangle$ contains an enumerable *NS* set $\left\langle\left(\kappa_1{}^{\wedge}{}_{\langle\mathfrak{y}_1,\mathfrak{y}_2,\mathfrak{y}_3\rangle}\right)_n : n\in N\right\rangle$ then there exists enumerable *NS* set $\left\langle\left(\kappa_2{}^{\wedge/}{}_{\langle\mathfrak{y}_1{}',\mathfrak{y}_2{}',\mathfrak{y}_3{}'\rangle}\right)_n : n\in N\right\rangle \Subset \langle\mathcal{L},\theta\rangle$ s.t. $\mathfrak{f}\left(\left(\kappa_2{}^{\wedge/}{}_{\langle\mathfrak{y}_1{}',\mathfrak{y}_2{}',\mathfrak{y}_3{}'\rangle}\right)_n\right) = \left(\kappa_1{}^{\wedge}{}_{\langle\mathfrak{y}_1,\mathfrak{y}_2,\mathfrak{y}_3\rangle}\right)_n$. $\langle\mathcal{L},\theta\rangle$ has $B.V.P$ implies that every infinite soft subset of $\langle\mathcal{L},\theta\rangle$ supposes soft accumulation point belonging to $\langle\mathcal{L},\theta\rangle$ this implies that $\left\langle\left(\kappa_2{}^{\wedge/}{}_{\langle\mathfrak{y}_1{}',\mathfrak{y}_2{}',\mathfrak{y}_3{}'\rangle}\right)_n : n\in N\right\rangle$ has soft neutrosophic limit poit, say, $\left(\kappa_2{}^{\wedge/}{}_{\langle\mathfrak{y}_1{}',\mathfrak{y}_2{}',\mathfrak{y}_3{}'\rangle}\right)_0 \in \langle\mathcal{L},\theta\rangle$ implies that the limit of soft sequence $\left\langle\left(\kappa_2{}^{\wedge/}{}_{\langle\mathfrak{y}_1{}',\mathfrak{y}_2{}',\mathfrak{y}_3{}'\rangle}\right)_n : n\in N\right\rangle$ is $\left(\kappa_2{}^{\wedge/}{}_{\langle\mathfrak{y}_1{}',\mathfrak{y}_2{}',\mathfrak{y}_3{}'\rangle}\right)_0 \in \langle\mathcal{L},\theta\rangle \implies \left(\kappa_2{}^{\wedge/}{}_{\langle\mathfrak{y}_1{}',\mathfrak{y}_2{}',\mathfrak{y}_3{}'\rangle}\right)_n \longrightarrow \left(\kappa_2{}^{\wedge/}{}_{\langle\mathfrak{y}_1{}',\mathfrak{y}_2{}',\mathfrak{y}_3{}'\rangle}\right)_0 \in \langle\mathcal{L},\theta\rangle$. f is soft continuous implies that it is soft continuous. Furthermore $\left(\kappa_2{}^{\wedge/}{}_{\langle\mathfrak{y}_1{}',\mathfrak{y}_2{}',\mathfrak{y}_3{}'\rangle}\right)_n \longrightarrow \left(\kappa_2{}^{\wedge/}{}_{\langle\mathfrak{y}_1{}',\mathfrak{y}_2{}',\mathfrak{y}_3{}'\rangle}\right)_0 \in \langle\mathcal{L},\theta\rangle \implies \mathfrak{f}\left(\left(\left(\kappa_2{}^{\wedge/}{}_{\langle\mathfrak{y}_1{}',\mathfrak{y}_2{}',\mathfrak{y}_3{}'\rangle}\right)_n\right)\right) \longrightarrow \mathfrak{f}\left(\left(\left(\kappa_2{}^{\wedge/}{}_{\langle\mathfrak{y}_1{}',\mathfrak{y}_2{}',\mathfrak{y}_3{}'\rangle}\right)_0\right)\right) \in \mathfrak{f}(\langle\mathcal{L},\theta\rangle) \implies \left(\kappa_1{}^{\wedge}{}_{\langle\mathfrak{y}_1,\mathfrak{y}_2,\mathfrak{y}_3\rangle}\right)_n \longrightarrow \mathfrak{f}\left(\left(\kappa_2{}^{\wedge/}{}_{\langle\mathfrak{y}_1{}',\mathfrak{y}_2{}',\mathfrak{y}_3{}'\rangle}\right)_0\right) \in \mathfrak{f}(\langle\mathcal{L},\theta\rangle)$ implies that limit of a soft sequence $\left\langle\left(\kappa_1{}^{\wedge}{}_{\langle\mathfrak{y}_1,\mathfrak{y}_2,\mathfrak{y}_3\rangle}\right)_n : n\in N\right\rangle$ is $\mathfrak{f}\left(\left(\kappa_2{}^{\wedge/}{}_{\langle\mathfrak{y}_1{}',\mathfrak{y}_2{}',\mathfrak{y}_3{}'\rangle}\right)_0\right) \in \mathfrak{f}(\langle\mathcal{L},\theta\rangle)$ implies that limit of a soft sequence $\left\langle\left(\kappa_1{}^{\wedge}{}_{\langle\mathfrak{y}_1,\mathfrak{y}_2,\mathfrak{y}_3\rangle}\right)_n : n\in N\right\rangle$ is $\mathfrak{f}(\langle\mathfrak{y}_1{}',\mathfrak{y}_2{}',\mathfrak{y}_3{}'\rangle) \in \langle\mathfrak{f},\theta\rangle(\langle\mathcal{L},\theta\rangle)$. Finally we have shown that there exists infinite soft subset $\left\langle\left(\kappa_1{}^{\wedge}{}_{\langle\mathfrak{y}_1,\mathfrak{y}_2,\mathfrak{y}_3\rangle}\right)_n : n\in N\right\rangle$ of $\mathfrak{f}(\langle\mathcal{L},\theta\rangle)$ containing a limit point $\mathfrak{f}\left(\left(\kappa_2{}^{\wedge/}{}_{\langle\mathfrak{y}_1{}',\mathfrak{y}_2{}',\mathfrak{y}_3{}'\rangle}\right)_0\right) \in \mathfrak{f}(\langle\mathcal{L},\theta\rangle)$. This guarantees that $\mathfrak{f}(\langle\mathcal{L},\theta\rangle)$ has $B.V.P.$

Theorem 5.7. Let $\langle\tilde{\pi},\tau_1,\tau_2,\theta\rangle$ *NSBTS* and let $\left\langle\left(\kappa_1{}^{\wedge}\widetilde{{}_{\langle\mathfrak{y}_1,\mathfrak{y}_2,\mathfrak{y}_3\rangle}}\right)_n\right\rangle$ be a *NS* sequence in $\langle\tilde{\pi},\tau_1,\tau_2,\theta\rangle$ such that it converges to a point $\left(\kappa_1{}^{\wedge}{}_{\langle\mathfrak{y}_1,\mathfrak{y}_2,\mathfrak{y}_3\rangle}\right)_0$ then the soft set $\langle \mathscr{g},\theta\rangle$ consisting of the points $\left(\kappa_1{}^{\wedge}{}_{\langle\mathfrak{y}_1,\mathfrak{y}_2,\mathfrak{y}_3\rangle}\right)_{n_0}$ and $\left(\kappa_1{}^{\wedge}{}_{\langle\mathfrak{y}_1,\mathfrak{y}_2,\mathfrak{y}_3\rangle}\right)_n (n=1,2,3,\ldots)$ is soft *NSB* compact.

Proof. Given $\langle\tilde{\pi},\tau_1,\tau_2,\theta\rangle$ *NSBTS* and let $\left\langle\left(\kappa_1{}^{\wedge}\widetilde{{}_{\langle\mathfrak{y}_1,\mathfrak{y}_2,\mathfrak{y}_3\rangle}}\right)_n\right\rangle$ be a *NS* sequence in $\langle\tilde{\pi},\tau_1,\tau_2,\theta\rangle$ such that it converges to a point $\left(\kappa_1{}^{\wedge}{}_{\langle\mathfrak{y}_1,\mathfrak{y}_2,\mathfrak{y}_3\rangle}\right)_{n_0}$ that is $\left(\kappa_1{}^{\wedge}\widetilde{{}_{\langle\mathfrak{y}_1,\mathfrak{y}_2,\mathfrak{y}_3\rangle}}\right)_n \longrightarrow \left(\kappa_1{}^{\wedge}{}_{\langle\mathfrak{y}_1,\mathfrak{y}_2,\mathfrak{y}_3\rangle}\right)_{n_0} \in \tilde{\pi}$.

Let $\langle \mathscr{g},\theta\rangle = \begin{bmatrix} \kappa_1{}^{\curlywedge}{}_{\widetilde{\langle\mathfrak{y}_1,\mathfrak{y}_2,\mathfrak{y}_3\rangle}}, \left(\kappa_1{}^{\curlywedge}{}_{\widetilde{\langle\mathfrak{y}_1,\mathfrak{y}_2,\mathfrak{y}_3\rangle}}\right)_2, \left(\kappa_1{}^{\curlywedge}{}_{\widetilde{\langle\mathfrak{y}_1,\mathfrak{y}_2,\mathfrak{y}_3\rangle}}\right)_3, \\ \left(\kappa_1{}^{\curlywedge}{}_{\widetilde{\langle\mathfrak{y}_1,\mathfrak{y}_2,\mathfrak{y}_3\rangle}}\right)_4, \left(\kappa_1{}^{\curlywedge}{}_{\widetilde{\langle\mathfrak{y}_1,\mathfrak{y}_2,\mathfrak{y}_3\rangle}}\right)_5, \\ \left(\kappa_1{}^{\curlywedge}{}_{\langle\mathfrak{y}_1,\mathfrak{y}_2,\mathfrak{y}_3\rangle}\right)_6, \left(\kappa_1{}^{\curlywedge}{}_{\langle\mathfrak{y}_1,\mathfrak{y}_2,\mathfrak{y}_3\rangle}\right)_7, \ldots \end{bmatrix}$. Let $\{\langle\mathfrak{S},\theta\rangle_\alpha : \alpha\in\Delta\}$ be $NS{*_b}$ open covering of $\langle \mathscr{g},\theta\rangle$ so that $\langle \mathscr{g},\theta\rangle \Subset \tilde{\cup}\{\langle\mathfrak{S},\theta\rangle_\alpha : \alpha\in\Delta\}, \left(\kappa_1{}^{\curlywedge}{}_{\langle\mathfrak{y}_1,\mathfrak{y}_2,\mathfrak{y}_3\rangle}\right)_{n_0} \in \langle \mathscr{g},\theta\rangle$ implies that there exists $\alpha_0 \in \Delta$ such that $\left(\kappa_1{}^{\curlywedge}{}_{\langle\mathfrak{y}_1,\mathfrak{y}_2,\mathfrak{y}_3\rangle}\right)_{n_0} \in \langle\mathfrak{S},\theta\rangle_{\alpha_0}$. According to the definition of soft convergence, $\left(\kappa_1{}^{\curlywedge}{}_{\langle\mathfrak{y}_1,\mathfrak{y}_2,\mathfrak{y}_3\rangle}\right)_{n_0} \in \langle\mathfrak{S},\theta\rangle_{\alpha_0} \in (\tilde{\pi},\tau_1,\tau_2,\theta)$ implies thatthere exists $n_0 \in N\,s.t.\, n \geq n_0$ and $\left(\kappa_1{}^{\curlywedge}{}_{\langle\mathfrak{y}_1,\mathfrak{y}_2,\mathfrak{y}_3\rangle}\right)_n \in \langle\mathfrak{S},\theta\rangle_{\alpha_0}$. Evidently, $\langle\mathfrak{S},\theta\rangle_{\alpha_0}$ contains the points $\left(\kappa_1{}^{\curlywedge}{}_{\langle\mathfrak{y}_1,\mathfrak{y}_2,\mathfrak{y}_3\rangle}\right)_{n_0}, \left(\kappa_1{}^{\curlywedge}{}_{\langle\mathfrak{y}_1,\mathfrak{y}_2,\mathfrak{y}_3\rangle}\right)_{n_{0+1}}$, $\left(\kappa_1{}^{\curlywedge}{}_{\langle\mathfrak{y}_1,\mathfrak{y}_2,\mathfrak{y}_3\rangle}\right)_{n_{0+2}}$, $\left(\kappa_1{}^{\curlywedge}{}_{\langle\mathfrak{y}_1,\mathfrak{y}_2,\mathfrak{y}_3\rangle}\right)_{n_{0+3}}, \left(\kappa_1{}^{\curlywedge}{}_{\langle\mathfrak{y}_1,\mathfrak{y}_2,\mathfrak{y}_3\rangle}\right), \ldots \left(\kappa_1{}^{\curlywedge}{}_{\langle\mathfrak{y}_1,\mathfrak{y}_2,\mathfrak{y}_3\rangle}\right)_{n_{0+n}}, \ldots$ Look carefully at the points and train them in a way as, $\left(\kappa_1{}^{\curlywedge}{}_{\langle\mathfrak{y}_1,\mathfrak{y}_2,\mathfrak{y}_3\rangle}\right)_1, \left(\kappa_1{}^{\curlywedge}{}_{\langle\mathfrak{y}_1,\mathfrak{y}_2,\mathfrak{y}_3\rangle}\right)_2, \left(\kappa_1{}^{\curlywedge}{}_{\langle\mathfrak{y}_1,\mathfrak{y}_2,\mathfrak{y}_3\rangle}\right)_3$, $\left(\kappa_1{}^{\curlywedge}{}_{\langle\mathfrak{y}_1,\mathfrak{y}_2,\mathfrak{y}_3\rangle}\right)_4, \ldots$ $\left(\kappa_1{}^{\curlywedge}{}_{\langle\mathfrak{y}_1,\mathfrak{y}_2,\mathfrak{y}_3\rangle}\right)_n, \ldots$ generating a finite soft set. Let $1 \leq n_{0-1}$. Then $\left(\kappa_1{}^{\curlywedge}{}_{\langle\mathfrak{y}_1,\mathfrak{y}_2,\mathfrak{y}_3\rangle}\right)_i \in \langle \mathscr{g},\theta\rangle$. For this $i, \left(\kappa_1{}^{\curlywedge}{}_{\langle\mathfrak{y}_1,\mathfrak{y}_2,\mathfrak{y}_3\rangle}\right)_i \in \langle \mathscr{g},\theta\rangle$. Hence there exists $\alpha_i \in \Delta$ such that $\left(\kappa_1{}^{\curlywedge}{}_{\langle\mathfrak{y}_1,\mathfrak{y}_2,\mathfrak{y}_3\rangle}\right)_i \in \langle\mathfrak{S},\theta\rangle_{\alpha_i}$. Evidently $\langle \mathscr{g},\theta\rangle \Subset \widetilde{\bigcup}_{r=0}^{n_{0-1}} \langle\mathfrak{S},\theta\rangle_{\alpha_i}$. This shows that $\left\{\langle\mathfrak{S},\theta\rangle_{\alpha_i} : 0 \leq n_{0-1}\right\}$ is $NS{*_b}$ open cover of $\langle \mathscr{g},\theta\rangle$. Thus an arbitrary $NS{*_b}$ open cover $\{\langle\mathfrak{S},\theta\rangle_\alpha : \alpha\in\Delta\}$ of $\langle \mathscr{g},\theta\rangle$ is reducible to a finite NS cub-cover $\left\{\langle\mathfrak{S},\theta\rangle_{\alpha i} : i = 0,1,2,3,\ldots n_{0-1}\right\}$, it follows that $\langle \mathscr{g},\theta\rangle$ is soft $NSB{*_b}$ compact.

Theorem 5.8. If $(\tilde{\pi},\tau_1,\tau_2,\theta)\,NSBTS$ such that it has the characteristics of $NS{*_b}$ sequentially compactness. Then $\langle\tilde{\pi},\tau_1,\tau_2,\theta\rangle$ is safely $NSB{*_b}$ countably compact.

Proof. Let $\langle\tilde{\pi},\tau_1,\tau_2,\theta\rangle\,NSBTS$ and let $\langle\rho,\theta\rangle$ be finite soft sub-set of $\langle\tilde{\pi},\tau_1,\tau_2,\theta\rangle$. Let $\begin{bmatrix} \left(\kappa_1{}^{\curlywedge}{}_{\widetilde{\langle\mathfrak{y}_1,\mathfrak{y}_2,\mathfrak{y}_3\rangle}}\right)_1, \left(\kappa_1{}^{\curlywedge}{}_{\widetilde{\langle\mathfrak{y}_1,\mathfrak{y}_2,\mathfrak{y}_3\rangle}}\right)_2, \left(\kappa_1{}^{\curlywedge}{}_{\widetilde{\langle\mathfrak{y}_1,\mathfrak{y}_2,\mathfrak{y}_3\rangle}}\right)_3, \\ \left(\kappa_1{}^{\curlywedge}{}_{\widetilde{\langle\mathfrak{y}_1,\mathfrak{y}_2,\mathfrak{y}_3\rangle}}\right)_4, \left(\kappa_1{}^{\curlywedge}{}_{\widetilde{\langle\mathfrak{y}_1,\mathfrak{y}_2,\mathfrak{y}_3\rangle}}\right)_5, \\ \left(\kappa_1{}^{\curlywedge}{}_{\widetilde{\langle\mathfrak{y}_1,\mathfrak{y}_2,\mathfrak{y}_3\rangle}}\right)_6, \left(\kappa_1{}^{\curlywedge}{}_{\widetilde{\langle\mathfrak{y}_1,\mathfrak{y}_2,\mathfrak{y}_3\rangle}}\right)_7, \ldots \end{bmatrix}$ be a soft sequence of soft points of $\langle\rho,\theta\rangle$. Then, $\langle\rho,\theta\rangle$ being finite, at least one of the elements in $\langle\rho,\theta\rangle$ say $\left(\kappa_1{}^{\curlywedge}{}_{\widetilde{\langle\mathfrak{y}_1,\mathfrak{y}_2,\mathfrak{y}_3\rangle}}\right)_0$ must be duplicated an in-finite number of times in the NS sequence. Hence, $\begin{bmatrix} \left(\kappa_1{}^{\curlywedge}{}_{\widetilde{\langle\mathfrak{y}_1,\mathfrak{y}_2,\mathfrak{y}_3\rangle}}\right)_0, \left(\kappa_1{}^{\curlywedge}{}_{\widetilde{\langle\mathfrak{y}_1,\mathfrak{y}_2,\mathfrak{y}_3\rangle}}\right)_0, \\ \left(\kappa_1{}^{\curlywedge}{}_{\widetilde{\langle\mathfrak{y}_1,\mathfrak{y}_2,\mathfrak{y}_3\rangle}}\right)_0, \left(\kappa_1{}^{\curlywedge}{}_{\widetilde{\langle\mathfrak{y}_1,\mathfrak{y}_2,\mathfrak{y}_3\rangle}}\right)_0, \left(\kappa_1{}^{\curlywedge}{}_{\langle\mathfrak{y}_1,\mathfrak{y}_2,\mathfrak{y}_3\rangle}\right)_0, \\ \left(\kappa_1{}^{\curlywedge}{}_{\langle\mathfrak{y}_1,\mathfrak{y}_2,\mathfrak{y}_3\rangle}\right)_0, \left(\kappa_1{}^{\curlywedge}{}_{\langle\mathfrak{y}_1,\mathfrak{y}_2,\mathfrak{y}_3\rangle}\right)_0, \ldots \end{bmatrix}$ is soft sub-sequence of $\left\langle \left(\kappa_1{}^{\curlywedge}{}_{\widetilde{\langle\mathfrak{y}_1,\mathfrak{y}_2,\mathfrak{y}_3\rangle}}\right)_n \right\rangle$ such that it is soft constant sequence and repeatedly constructed by single soft number $\left(\kappa_1{}^{\curlywedge}{}_{\widetilde{\langle\mathfrak{y}_1,\mathfrak{y}_2,\mathfrak{y}_3\rangle}}\right)_0$ and we know that a soft constant sequence converges on its self. So it converges to $\left(\kappa_1{}^{\curlywedge}{}_{\widetilde{\langle\mathfrak{y}_1,\mathfrak{y}_2,\mathfrak{y}_3\rangle}}\right)_0$ which belongs to $\langle\rho,\theta\rangle$. Hence, $\langle\rho,\theta\rangle$ is soft sequentially $NSB{*_b}$ compact.

Theorem 5.9. Let $\langle\tilde{\pi},\tau_1,\tau_2,\theta\rangle\,NSBTS$ and $\left(\langle\tilde{Y}\rangle,\mathfrak{F}_1,\mathfrak{F}_2,\theta\right)$ be another $NSBTS$. Let $\langle \not{f},\theta\rangle$ be a soft continuous mapping of a soft neutrosophic sequentially compact $NS{*_b}$ space $\langle\tilde{\pi},\tau_1,\tau_2,\theta\rangle$ into $\left(\langle\tilde{Y}\rangle,\mathfrak{F}_1,\mathfrak{F}_2,\theta\right)$, then $\langle \not{f},\theta\rangle\langle\tilde{\pi},\tau_1,\tau_2,\theta\rangle$ is $NSB{*_b}$ sequentially compact.

Proof. Given $\langle\tilde{\pi},\tau_1,\tau_2,\theta\rangle$ *NSBTS* and $\left(\langle\tilde{Y}\rangle,\mathfrak{F}_1,\mathfrak{F}_2,\theta\right)$ be another *NSBTS*. Let $\langle\not{f},\theta\rangle$ be a soft continuous mapping of a *NSB* sequentially compact space $(\tilde{\pi},\tau_1,\tau_2,\theta)$ into $\left(\langle\tilde{Y}\rangle,\mathfrak{F}_1,\mathfrak{F}_2,\theta\right)$ then we have to prove $\langle\not{f},\theta\rangle\langle\tilde{\pi},\tau_1,\tau_2,\theta\rangle$ *NS* sequentially. For this we proceed as. Let

$$\left\langle\begin{bmatrix}\left(\kappa_2{}^{\curlywedge/}\widetilde{\langle\mathfrak{y}_1{}',\mathfrak{y}_2{}',\mathfrak{y}_3{}'\rangle}\right)_1,\left(\kappa_2{}^{\curlywedge/}\widetilde{\langle\mathfrak{y}_1{}',\mathfrak{y}_2{}',\mathfrak{y}_3{}'\rangle}\right)_2,\\ ,\left(\kappa_2{}^{\curlywedge/}\widetilde{\langle\mathfrak{y}_1{}',\mathfrak{y}_2{}',\mathfrak{y}_3{}'\rangle}\right)_5,\left(\kappa_2{}^{\curlywedge/}\widetilde{\langle\mathfrak{y}_1{}',\mathfrak{y}_2{}',\mathfrak{y}_3{}'\rangle}\right)_6,\\ \left(\kappa_2{}^{\curlywedge/}\widetilde{\langle\mathfrak{y}_1{}',\mathfrak{y}_2{}',\mathfrak{y}_3{}'\rangle}\right)_7,\ldots\left(\kappa_2{}^{\curlywedge/}\widetilde{\langle\mathfrak{y}_1{}',\mathfrak{y}_2{}',\mathfrak{y}_3{}'\rangle}\right)_n,\ldots\end{bmatrix}\right\rangle$$

be a soft sequence of neutrosophic soft points in $\langle\not{f},\theta\rangle(\tilde{\pi},\tau_1,\tau_2,\theta)$, Then for each $n\in N$ there exists

$$\left\langle\begin{matrix}\left(\kappa_1{}^{\curlywedge}\widetilde{\langle\mathfrak{y}_1,\mathfrak{y}_2,\mathfrak{y}_3\rangle}\right)_1,\left(\kappa_1{}^{\curlywedge}\widetilde{\langle\mathfrak{y}_1,\mathfrak{y}_2,\mathfrak{y}_3\rangle}\right)_2,\\ \left(\kappa_1{}^{\curlywedge}\widetilde{\langle\mathfrak{y}_1,\mathfrak{y}_2,\mathfrak{y}_3\rangle}\right)_4,\left(\kappa_1{}^{\curlywedge}\widetilde{\langle\mathfrak{y}_1,\mathfrak{y}_2,\mathfrak{y}_3\rangle}\right)_5,\\ \left(\kappa_1{}^{\curlywedge}\widetilde{\langle\mathfrak{y}_1,\mathfrak{y}_2,\mathfrak{y}_3\rangle}\right)_7,\ldots\left(\kappa_1{}^{\curlywedge}\widetilde{\langle\mathfrak{y}_1,\mathfrak{y}_2,\mathfrak{y}_3\rangle}\right)_n,\ldots\in(\tilde{\pi},\tau_1,\tau_2,\theta)\end{matrix}\right\rangle$$

such that

$$\langle\not{f},\theta\rangle\left[\begin{pmatrix}\left(\kappa_1{}^{\curlywedge}\widetilde{\langle\mathfrak{y}_1,\mathfrak{y}_2,\mathfrak{y}_3\rangle}\right)_1,\left(\kappa_1{}^{\curlywedge}\widetilde{\langle\mathfrak{y}_1,\mathfrak{y}_2,\mathfrak{y}_3\rangle}\right)_2,\\ \left(\kappa_1{}^{\curlywedge}\widetilde{\langle\mathfrak{y}_1,\mathfrak{y}_2,\mathfrak{y}_3\rangle}\right)_3,\\ \left(\kappa_1{}^{\curlywedge}\widetilde{\langle\mathfrak{y}_1,\mathfrak{y}_2,\mathfrak{y}_3\rangle}\right)_7,\ldots\left(\kappa_1{}^{\curlywedge}\widetilde{\langle\mathfrak{y}_1,\mathfrak{y}_2,\mathfrak{y}_3\rangle}\right)_n,\ldots\end{pmatrix}\right]=\left\langle\begin{bmatrix}\left(\kappa_2{}^{\curlywedge/}\widetilde{\langle\mathfrak{y}_1{}',\mathfrak{y}_2{}',\mathfrak{y}_3{}'\rangle}\right)_1,\left(\kappa_2{}^{\curlywedge/}\widetilde{\langle\mathfrak{y}_1{}',\mathfrak{y}_2{}',\mathfrak{y}_3{}'\rangle}\right)_2,\\ \left(\kappa_2{}^{\curlywedge/}\widetilde{\langle\mathfrak{y}_1{}',\mathfrak{y}_2{}',\mathfrak{y}_3{}'\rangle}\right)_3,\kappa_2{}^{\curlywedge/}\widetilde{\langle\mathfrak{y}_1{}',\mathfrak{y}_2{}',\mathfrak{y}_3{}'\rangle}_4,\\ \left(\kappa_2{}^{\curlywedge/}\widetilde{\langle\mathfrak{y}_1{}',\mathfrak{y}_2{}',\mathfrak{y}_3{}'\rangle}\right)_6,\left(\kappa_2{}^{\curlywedge/}\widetilde{\langle\mathfrak{y}_1{}',\mathfrak{y}_2{}',\mathfrak{y}_3{}'\rangle}\right)_7,\\ \ldots\left(\kappa_2{}^{\curlywedge/}\widetilde{\langle\mathfrak{y}_1{}',\mathfrak{y}_2{}',\mathfrak{y}_3{}'\rangle}\right)_n,\ldots\end{bmatrix}\right\rangle.$$

Thus we obtain a soft sequence

$$\begin{bmatrix}\left(\kappa_1{}^{\curlywedge}\widetilde{\langle\mathfrak{y}_1,\mathfrak{y}_2,\mathfrak{y}_3\rangle}\right)_1,\left(\kappa_1{}^{\curlywedge}\widetilde{\langle\mathfrak{y}_1,\mathfrak{y}_2,\mathfrak{y}_3\rangle}\right)_2,\\ \left(\kappa_1{}^{\curlywedge}\widetilde{\langle\mathfrak{y}_1,\mathfrak{y}_2,\mathfrak{y}_3\rangle}\right)_3,\left(\kappa_1{}^{\curlywedge}\widetilde{\langle\mathfrak{y}_1,\mathfrak{y}_2,\mathfrak{y}_3\rangle}\right)_4,\\ \left(\kappa_1{}^{\curlywedge}\widetilde{\langle\mathfrak{y}_1,\mathfrak{y}_2,\mathfrak{y}_3\rangle}\right)_5,\left(\kappa_1{}^{\curlywedge}\widetilde{\langle\mathfrak{y}_1,\mathfrak{y}_2,\mathfrak{y}_3\rangle}\right)_6,\\ \left(\kappa_1{}^{\curlywedge}\widetilde{\langle\mathfrak{y}_1,\mathfrak{y}_2,\mathfrak{y}_3\rangle}\right)_7,\ldots\left(\kappa_1{}^{\curlywedge}\widetilde{\langle\mathfrak{y}_1,\mathfrak{y}_2,\mathfrak{y}_3\rangle}\right)_n,\ldots\end{bmatrix}$$

in $\langle\tilde{\pi},\tau_1,\tau_2,\theta\rangle$. But $\langle\tilde{\pi},\tau_1,\tau_2,\theta\rangle$ being soft sequentially $NSB{*_b}$ compact, there is a *NS* sub-sequence $\left\langle\left(\kappa_1{}^{\curlywedge}\widetilde{\langle\mathfrak{y}_1,\mathfrak{y}_2,\mathfrak{y}_3\rangle}\right)_{n_i}\right\rangle$ of

$\left\langle\left(\kappa_1{}^{\curlywedge}\widetilde{\langle\mathfrak{y}_1,\mathfrak{y}_2,\mathfrak{y}_3\rangle}\right)_n\right\rangle$ such that $\left\langle\left(\kappa_1{}^{\curlywedge}\widetilde{\langle\mathfrak{y}_1,\mathfrak{y}_2,\mathfrak{y}_3\rangle}\right)_{n_i}\right\rangle\longrightarrow\left(\kappa_1{}^{\curlywedge}\widetilde{\langle\mathfrak{y}_1,\mathfrak{y}_2,\mathfrak{y}_3\rangle}\right)\in\langle\tilde{\pi},\tau_1,\tau_2,\theta\rangle$. So, by $NS{*_b}$ contiuity of $\langle\not{f},\theta\rangle$, $\left\langle\left(\kappa_1{}^{\curlywedge}\widetilde{\langle\mathfrak{y}_1,\mathfrak{y}_2,\mathfrak{y}_3\rangle}\right)_{n_i}\right\rangle\longrightarrow\left(\kappa_1{}^{\curlywedge}\widetilde{\langle\mathfrak{y}_1,\mathfrak{y}_2,\mathfrak{y}_3\rangle}\theta\right)$ implies that $\langle\not{f},\theta\rangle\left(\left\langle\left(\kappa_1{}^{\curlywedge}\widetilde{\langle\mathfrak{y}_1,\mathfrak{y}_2,\mathfrak{y}_3\rangle}\right)_{n_i}\right\rangle\right)\longrightarrow$ $\langle\not{f},\theta\rangle\left(\left\langle\left(\kappa_1{}^{\curlywedge}\widetilde{\langle\mathfrak{y}_1,\mathfrak{y}_2,\mathfrak{y}_3\rangle}\right)_n\right\rangle\right)\in\langle\not{f},\theta\rangle\langle\tilde{\pi},\tau_1,\tau_2,\theta\rangle$. Thus, $\langle\not{f},\theta\rangle\left(\left\langle\left(\kappa_2{}^{\curlywedge/}\widetilde{\langle\mathfrak{y}_1{}',\mathfrak{y}_2{}',\mathfrak{y}_3{}'\rangle}\right)_{n_i}\right\rangle\right)$ is a soft sub-sequence of $\left\langle\begin{bmatrix} \left(\kappa_2{}^{\curlywedge/}\widetilde{\langle \mathfrak{y}_1{}^{/},\mathfrak{y}_2{}^{/},\mathfrak{y}_3{}^{/}\rangle}\right)_1, \left(\kappa_2{}^{\curlywedge/}\widetilde{\langle \mathfrak{y}_1{}^{/},\mathfrak{y}_2{}^{/},\mathfrak{y}_3{}^{/}\rangle}\right)_2, \\ \left(\kappa_2{}^{\curlywedge/}\langle \mathfrak{y}_1{}^{/},\mathfrak{y}_2{}^{/},\mathfrak{y}_3{}^{/}\rangle\right)_3, \left(\kappa_2{}^{\curlywedge/}\langle \mathfrak{y}_1{}^{/},\mathfrak{y}_2{}^{/},\mathfrak{y}_3{}^{/}\rangle\right)_5, \\ \left(\kappa_2{}^{\curlywedge/}\langle \mathfrak{y}_1{}^{/},\mathfrak{y}_2{}^{/},\mathfrak{y}_3{}^{/}\rangle\right)_5, \left(\kappa_2{}^{\curlywedge/}\langle \mathfrak{y}_1{}^{/},\mathfrak{y}_2{}^{/},\mathfrak{y}_3{}^{/}\rangle\right)_6, \\ \left(\kappa_2{}^{\curlywedge/}\langle \mathfrak{y}_1{}^{/},\mathfrak{y}_2{}^{/},\mathfrak{y}_3{}^{/}\rangle\right)_7, \dots \left(\kappa_2{}^{\curlywedge/}\langle \mathfrak{y}_1{}^{/},\mathfrak{y}_2{}^{/},\mathfrak{y}_3{}^{/}\rangle\right)_n, \dots \end{bmatrix}\right\rangle$ converges to $(\langle \not{f},\theta\rangle)(\widetilde{\kappa_1})$ in $\langle \not{f},\theta\rangle$ $\langle \tilde{\pi},\tau_1,\tau_2,\theta\rangle$. Hence, $\langle \not{f},\theta\rangle\langle \tilde{\pi},\tau_1,\tau_2,\theta\rangle$ is $NS{*_b}$ sequentially compact.

Theorem 5.10. Let $\langle \tilde{\pi},\tau_1,\tau_2,\theta\rangle$ *NSBTS* and suppose $\langle \not{f},\theta\rangle, \langle g,\theta\rangle$ be two *NS* continuous function on a *NS* BTS $\langle \tilde{\pi},\tau_1,\tau_2,\theta\rangle$ in to *NSBTS* $\left\langle \langle \tilde{Y}\rangle, \mathfrak{F}_1, \mathfrak{F}_2, \theta\right\rangle$ which is $NSB{*_b}$ Hausdorff. Then, soft set $\left\{\left(\kappa_1{}^{\curlywedge}\langle \mathfrak{y}_1,\mathfrak{y}_2,\mathfrak{y}_3\rangle\right) \in \tilde{\pi} : (\not{f})\left(\left(\kappa_1{}^{\curlywedge}\langle \mathfrak{y}_1,\mathfrak{y}_2,\mathfrak{y}_3\rangle\right)\right) = (g)\left(\left(\kappa_1{}^{\curlywedge}\langle \mathfrak{y}_1,\mathfrak{y}_2,\mathfrak{y}_3\rangle\right)\right)\right\}$ is $NS{*_b}$ closed of $\langle \tilde{\pi},\tau_1,\tau_2,\theta\rangle$.

Proof: Let If $\left\{\left(\kappa_1{}^{\curlywedge}\langle \mathfrak{y}_1,\mathfrak{y}_2,\mathfrak{y}_3\rangle\right) \in \tilde{\pi} : (\not{f})\left(\left(\kappa_1{}^{\curlywedge}\langle \mathfrak{y}_1,\mathfrak{y}_2,\mathfrak{y}_3\rangle\right)\right) = (g)\left(\left(\kappa_1{}^{\curlywedge}\langle \mathfrak{y}_1,\mathfrak{y}_2,\mathfrak{y}_3\rangle\right)\right)\right\}$ is a NS set of function. If $\left\{\left(\kappa_1{}^{\curlywedge}\langle \mathfrak{y}_1,\mathfrak{y}_2,\mathfrak{y}_3\rangle\right) \in \tilde{\pi} : (\not{f})\left(\left(\kappa_1{}^{\curlywedge}\langle \mathfrak{y}_1,\mathfrak{y}_2,\mathfrak{y}_3\rangle\right)\right) = (g)(\kappa_1)\right\}^c = \tilde{\varnothing}$, it is clearly $NS{*_b}$ open and therefore, $\left\{\left(\kappa_1{}^{\curlywedge}\langle \mathfrak{y}_1,\mathfrak{y}_2,\mathfrak{y}_3\rangle\right) \in \tilde{\pi} : (\not{f})\left(\left(\kappa_1{}^{\curlywedge}\langle \mathfrak{y}_1,\mathfrak{y}_2,\mathfrak{y}_3\rangle\right)\right) = (g)\left(\left(\kappa_1{}^{\curlywedge}\langle \mathfrak{y}_1,\mathfrak{y}_2,\mathfrak{y}_3\rangle\right)\right)\right\}$ is ${*_b}$ closed, that is nothing is proved in this case. Let us consider the case when $\left\{\left(\kappa_1{}^{\curlywedge}\langle \mathfrak{y}_1,\mathfrak{y}_2,\mathfrak{y}_3\rangle\right) \in \tilde{\pi} : (\not{f})\left(\left(\kappa_1{}^{\curlywedge}\langle \mathfrak{y}_1,\mathfrak{y}_2,\mathfrak{y}_3\rangle\right)\right) = (g)\left(\left(\kappa_1{}^{\curlywedge}\langle \mathfrak{y}_1,\mathfrak{y}_2,\mathfrak{y}_3\rangle\right)\right)\right\}^c \neq \left(\kappa_1{}^{\curlywedge}\langle \mathfrak{y}_1,\mathfrak{y}_2,\mathfrak{y}_3\rangle\right)$ and let $\rho \in \left\{\left(\kappa_1{}^{\curlywedge}\langle \mathfrak{y}_1,\mathfrak{y}_2,\mathfrak{y}_3\rangle\right) \in \tilde{\pi} : (\not{f})\left(\left(\kappa_1{}^{\curlywedge}\langle \mathfrak{y}_1,\mathfrak{y}_2,\mathfrak{y}_3\rangle\right)\right) = (g)\left(\left(\kappa_1{}^{\curlywedge}\langle \mathfrak{y}_1,\mathfrak{y}_2,\mathfrak{y}_3\rangle\right)\right)\right\}^c$. Then ρ does not belong $\left\{\left(\kappa_1{}^{\curlywedge}\langle \mathfrak{y}_1,\mathfrak{y}_2,\mathfrak{y}_3\rangle\right) \in \tilde{\pi} : (\not{f})\left(\left(\kappa_1{}^{\curlywedge}\langle \mathfrak{y}_1,\mathfrak{y}_2,\mathfrak{y}_3\rangle\right)\right) = (g)(\kappa_1)\right\}$. Result in $(\not{f})(\rho) \neq (g)(\rho)$. Now, $\left(\langle \tilde{Y}\rangle, \mathfrak{F}_1, \mathfrak{F}_2, \theta\right)$ being $NSB{*_b}$ Hausdorff space so there exists $NS{*_b}$ open sets $\langle g,\theta\rangle$, $\langle \mathfrak{H},\theta\rangle$ of $(\not{f})(\rho)$ and $(g)(\rho)$ respectively such that $\langle g,\theta\rangle$ and $\langle \mathfrak{H},\theta\rangle$ such that these *NS* sets such that the possibility of one rules out the possibility of other. By soft continuity of $\langle \not{f},\theta\rangle, \langle g,\theta\rangle$, $\langle \not{f},\theta\rangle^{-1}$ as well as $\langle g,\theta\rangle^{-1}$ is $NS{*_b}$ open nhd of ρ and therefore, so is $\langle \not{f},\theta\rangle^{-1} \tilde{\cap} \langle g,\theta\rangle^{-1}$ is contained in $\left\{\left(\kappa_1{}^{\curlywedge}\langle \mathfrak{y}_1,\mathfrak{y}_2,\mathfrak{y}_3\rangle\right) \in \tilde{\pi} : (\not{f})\left(\left(\kappa_1{}^{\curlywedge}\langle \mathfrak{y}_1,\mathfrak{y}_2,\mathfrak{y}_3\rangle\right)\right) = (g)\left(\left(\kappa_1{}^{\curlywedge}\langle \mathfrak{y}_1,\mathfrak{y}_2,\mathfrak{y}_3\rangle\right)\right)\right\}$, for, $\left(\kappa_1{}^{\curlywedge}\langle \mathfrak{y}_1,\mathfrak{y}_2,\mathfrak{y}_3\rangle\right) \in (\langle \not{f},\theta\rangle^{-1} \tilde{\cap} \langle g,\theta\rangle^{-1}) \Longrightarrow (\not{f})\left(\left(\kappa_1{}^{\curlywedge}\langle \mathfrak{y}_1,\mathfrak{y}_2,\mathfrak{y}_3\rangle\right)\right) \in \langle g,\theta\rangle$ and $(g)((\not{f})\left(\left(\kappa_1{}^{\curlywedge}\langle \mathfrak{y}_1,\mathfrak{y}_2,\mathfrak{y}_3\rangle\right)\right) \neq (g)\left(\left(\kappa_1{}^{\curlywedge}\langle \mathfrak{y}_1,\mathfrak{y}_2,\mathfrak{y}_3\rangle\right)\right)$ because $\langle g,\theta\rangle$ and $\langle \mathfrak{H},\theta\rangle$ are mutually exclusive. This implies that κ_1 does not belong to $\left\{\left(\kappa_1{}^{\curlywedge}\langle \mathfrak{y}_1,\mathfrak{y}_2,\mathfrak{y}_3\rangle\right) \in \tilde{\pi} : (\not{f})\left(\left(\kappa_1{}^{\curlywedge}\langle \mathfrak{y}_1,\mathfrak{y}_2,\mathfrak{y}_3\rangle\right)\right) = (g)\left(\left(\kappa_1{}^{\curlywedge}\langle \mathfrak{y}_1,\mathfrak{y}_2,\mathfrak{y}_3\rangle\right)\right)\right\}$. Therefore, $\rho \in (\not{f})^{-1}(\langle g,\theta\rangle) \tilde{\cap} (g)^{-1}(\langle g,\theta\rangle) \Subset \left\{\kappa_1{}^{\curlywedge}\langle \mathfrak{y}_1,\mathfrak{y}_2,\mathfrak{y}_3\rangle \in \tilde{\pi} : (\not{f})\left(\kappa_1{}^{\curlywedge}\langle \mathfrak{y}_1,\mathfrak{y}_2,\mathfrak{y}_3\rangle\right) = (g)\left(\kappa_1{}^{\curlywedge}\langle \mathfrak{y}_1,\mathfrak{y}_2,\mathfrak{y}_3\rangle\right)\right\}^c$. This shows that $\left\{\kappa_1{}^{\curlywedge}\langle \mathfrak{y}_1,\mathfrak{y}_2,\mathfrak{y}_3\rangle \in \tilde{\pi} : (\not{f})\left(\kappa_1{}^{\curlywedge}\langle \mathfrak{y}_1,\mathfrak{y}_2,\mathfrak{y}_3\rangle\right) = (g)\left(\kappa_1{}^{\curlywedge}\langle \mathfrak{y}_1,\mathfrak{y}_2,\mathfrak{y}_3\rangle\right)\right\}^c$ is neighborhood of each of its points. So, $\left\{\kappa_1{}^{\curlywedge}\langle \mathfrak{y}_1,\mathfrak{y}_2,\mathfrak{y}_3\rangle \in \tilde{\pi} : (\not{f})\left(\kappa_1{}^{\curlywedge}\langle \mathfrak{y}_1,\mathfrak{y}_2,\mathfrak{y}_3\rangle\right) = (g)\left(\kappa_1{}^{\curlywedge}\langle \mathfrak{y}_1,\mathfrak{y}_2,\mathfrak{y}_3\rangle\right)\right\}^c$ $NS{*_b}$ open and hence $\left\{\kappa_1{}^{\curlywedge}\langle \mathfrak{y}_1,\mathfrak{y}_2,\mathfrak{y}_3\rangle \in \tilde{\pi} : (\not{f})\left(\kappa_1{}^{\curlywedge}\langle \mathfrak{y}_1,\mathfrak{y}_2,\mathfrak{y}_3\rangle\right) = (g)(\kappa_1{}^{\curlywedge}\langle \mathfrak{y}_1,\mathfrak{y}_2,\mathfrak{y}_3\rangle)\right\}$ is $NS{*_b}$ closed.

Theorem 5.11. Let $\langle \tilde{\pi},\tau_1,\tau_2,\theta\rangle$ *NSBTS* such that it is $NSB{*_b}$ Hausdorff space and let $(\not{f})$ be soft continuous function of $(\tilde{\pi},\tau_1,\tau_2,\theta)$ into itself. Then, the *NS* set of fixed points under $(\not{f})$ is a $NSB{*_b}$ closed set.

Proof. Let $\delta = \{(\not{f})\left(\left(\kappa_1{}^{\curlywedge}\langle \mathfrak{y}_1,\mathfrak{y}_2,\mathfrak{y}_3\rangle\right)\right) = \left(\kappa_1{}^{\curlywedge}\langle \mathfrak{y}_1,\mathfrak{y}_2,\mathfrak{y}_3\rangle\right)\}$. If $\delta^c = \tilde{\varnothing}$, Then is $NS{*_b}$ open and therefore $\left\{(\not{f})\left(\left(\kappa_1{}^{\curlywedge}\langle \mathfrak{y}_1,\mathfrak{y}_2,\mathfrak{y}_3\rangle\right)\right) = \left(\kappa_1{}^{\curlywedge}\langle \mathfrak{y}_1,\mathfrak{y}_2,\mathfrak{y}_3\rangle\right)\right\}$ $NS{*_b}$ closed. So, let $\{(\not{f})\left(\left(\kappa_1{}^{\curlywedge}\langle \mathfrak{y}_1,\mathfrak{y}_2,\mathfrak{y}_3\rangle\right)\right) = \left(\kappa_1{}^{\curlywedge}\langle \mathfrak{y}_1,\mathfrak{y}_2,\mathfrak{y}_3\rangle\right)\}^c \neq \tilde{\varnothing}$ and let $\kappa_2{}^{\curlywedge/}\langle \mathfrak{y}_1{}^{/},\mathfrak{y}_2{}^{/},\mathfrak{y}_3{}^{/}\rangle \in \{(\not{f})\left(\left(\kappa_1{}^{\curlywedge}\langle \mathfrak{y}_1,\mathfrak{y}_2,\mathfrak{y}_3\rangle\right)\right) = \left(\kappa_1{}^{\curlywedge}\langle \mathfrak{y}_1,\mathfrak{y}_2,\mathfrak{y}_3\rangle\right)\}^c$. Then,

${\kappa_2}^{\curlywedge/}{}_{\langle\mathfrak{y}_1/,\mathfrak{y}_2/,\mathfrak{y}_3/\rangle}$ does not belong to $\{(\not{f})\left({\kappa_1}^{\curlywedge}{}_{\langle\mathfrak{y}_1,\mathfrak{y}_2,\mathfrak{y}_3\rangle}\right)={\kappa_1}^{\curlywedge}{}_{\langle\mathfrak{y}_1,\mathfrak{y}_2,\mathfrak{y}_3\rangle}\}$ and therefore $(\not{f})\left({\kappa_2}^{\curlywedge/}{}_{\langle\mathfrak{y}_1/,\mathfrak{y}_2/,\mathfrak{y}_3/\rangle}\right)$ $\neq {\kappa_2}^{\curlywedge/}{}_{\langle\mathfrak{y}_1/,\mathfrak{y}_2/,\mathfrak{y}_3/\rangle}$. Now, ${\kappa_2}^{\curlywedge/}{}_{\langle\mathfrak{y}_1/,\mathfrak{y}_2/,\mathfrak{y}_3/\rangle}$, $(\not{f})\left({\kappa_2}^{\curlywedge/}{}_{\langle\mathfrak{y}_1/,\mathfrak{y}_2/,\mathfrak{y}_3/\rangle}\right)$ being two distinct points of the $NSB{*_b}$ Hausdorff space $\langle\tilde{\pi},\tau_1,\tau_2,\theta\rangle$, so there exists $NS{*_b}$ open sets $\langle\mathscr{g},\theta\rangle$, $\langle\mathfrak{H},\theta\rangle$ such that ${\kappa_2}^{\curlywedge/}{}_{\langle\mathfrak{y}_1/,\mathfrak{y}_2/,\mathfrak{y}_3/\rangle} \in \langle\mathscr{g},\theta\rangle, (\not{f})\left({\kappa_2}^{\curlywedge/}{}_{\langle\mathfrak{y}_1/,\mathfrak{y}_2/,\mathfrak{y}_3/\rangle}\right) \in \langle\mathfrak{H},\theta\rangle$ and $\langle\mathscr{g},\theta\rangle, \langle\mathfrak{H},\theta\rangle$ are disjoint. Also, by the NS continuity of $(\not{f})$, $(\not{f})^{-1}(\langle H,\theta\rangle$ is $NS{*_b}$ open set containing $\mathscr{y}$. We pretend that $\langle\mathscr{g},\theta\rangle\tilde{\cap}(\not{f})^{-1}(\langle\mathfrak{H},\theta\rangle)\Subset\{(\not{f})\left(\left({\kappa_1}^{\curlywedge}{}_{\langle\mathfrak{y}_1,\mathfrak{y}_2,\mathfrak{y}_3\rangle}\right)\right)=\left({\kappa_1}^{\curlywedge}{}_{\langle\mathfrak{y}_1,\mathfrak{y}_2,\mathfrak{y}_3\rangle}\right)\}^c$. Since $\mu\in\langle\mathscr{g},\theta\rangle\tilde{\cap}(\not{f})^{-1}(\langle\mathfrak{H},\theta\rangle)\Longrightarrow\mu\in$ $\langle\mathscr{g},\theta\rangle, \mu\in(\not{f})^{-1}\Longrightarrow\mu\in\langle\mathscr{g},\theta\rangle, (\not{f})(\mu)\in\langle\mathfrak{H},\theta\rangle\Longrightarrow\mu\neq(\not{f})(\mu)$. As $\langle\mathscr{g},\theta\rangle\tilde{\cap}\langle\mathfrak{H},\theta\rangle=\tilde{\varnothing}$ implies that μ does not belong to $\{(\not{f})\left({\kappa_1}^{\curlywedge}{}_{\langle\mathfrak{y}_1,\mathfrak{y}_2,\mathfrak{y}_3\rangle}\right)=\left({\kappa_1}^{\curlywedge}{}_{\langle\mathfrak{y}_1,\mathfrak{y}_2,\mathfrak{y}_3\rangle}\right)\}\Longrightarrow\mu\in\{(\not{f})\left({\kappa_1}^{\curlywedge}{}_{\langle\mathfrak{y}_1,\mathfrak{y}_2,\mathfrak{y}_3\rangle}\right)={\kappa_1}^{\curlywedge}{}_{\langle\mathfrak{y}_1,\mathfrak{y}_2,\mathfrak{y}_3\rangle}\}^c$. Therefore, ${\kappa_2}^{\curlywedge/}{}_{\langle\mathfrak{y}_1/,\mathfrak{y}_2/,\mathfrak{y}_3/\rangle} \in \langle\mathscr{g},\theta\rangle\tilde{\cap}(\not{f})^{-1}(\langle\mathfrak{H},\theta\rangle) \Subset \{(\not{f})\left({\kappa_1}^{\curlywedge}{}_{\langle\mathfrak{y}_1,\mathfrak{y}_2,\mathfrak{y}_3\rangle}\right) = \left({\kappa_1}^{\curlywedge}{}_{\langle\mathfrak{y}_1,\mathfrak{y}_2,\mathfrak{y}_3\rangle}\right)\}^c$. Thus, $\{(\not{f})\left(\left({\kappa_1}^{\curlywedge}{}_{\langle\mathfrak{y}_1,\mathfrak{y}_2,\mathfrak{y}_3\rangle}\right)\right) = \left({\kappa_1}^{\curlywedge}{}_{\langle\mathfrak{y}_1,\mathfrak{y}_2,\mathfrak{y}_3\rangle}\right)\}^c$ is the NS neighbourhood of each of its points. So, $\{(\not{f})\left(\left({\kappa_1}^{\curlywedge}{}_{\langle\mathfrak{y}_1,\mathfrak{y}_2,\mathfrak{y}_3\rangle}\right)\right)=\left({\kappa_1}^{\curlywedge}{}_{\langle\mathfrak{y}_1,\mathfrak{y}_2,\mathfrak{y}_3\rangle}\right)\}^c$ is $NS{*_b}$ open and hence $\{(\not{f})\left(\left({\kappa_1}^{\curlywedge}{}_{\langle\mathfrak{y}_1,\mathfrak{y}_2,\mathfrak{y}_3\rangle}\right)\right)=\left({\kappa_1}^{\curlywedge}{}_{\langle\mathfrak{y}_1,\mathfrak{y}_2,\mathfrak{y}_3\rangle}\right)\}$ is $NSB{*_b}$ closed.

6 Conclusion

Neutrosophic soft topology (NST) is extension of vague soft topology (VST). Vague soft topology gives two types of information. One is true and the second one is false. It does not give information about the indeterminacy (doubtful) case. Neutrosophic soft topology is dominant over vague soft topology because it supposes all the information that is true, false and indeterminacy at the same time. Neutrosophic soft topology has a narrow domain as compared to neutrosophic soft bi-topology (NSBT). In our work, we regenerated some structures in neutrosophic soft bi-topological spaces with a new definition that is $*_b$ open sets concerning soft points. We worked with the operations given in references [14–16] which are entirely different from those in references [13,17]. Then unlike [18,19], neutrosophic soft bi-topological structure is reconstructed. In future, we would try to see the validity of the given structures in neutrosophic soft tri-topological structures.

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