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ARTICLE

On Single Valued Neutrosophic Regularity Spaces

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ABSTRACT

This article aims to present new terms of single-valued neutrosophic notions in the Šostak sense, known as single-valued neutrosophic regularity spaces. Concepts such as r-single-valued neutrosophic semi £-open, r-single-valued neutrosophic pre-£-open, r-single valued neutrosophic regular-£-open and r-single valued neutrosophic α£-open are defined and their properties are studied as well as the relationship between them. Moreover, we introduce the concept of r-single valued neutrosophic θ£-cluster point and r-single-valued neutrosophic γ£-cluster point, r-θ£-closed, and θ£-closure operators and study some of their properties. Also, we present and investigate the notions of r-single-valued neutrosophic θ£-connectedness and r-single valued neutrosophic δ£-connectedness and investigate relationship with single-valued neutrosophic almost £-regular. We compare all these forms of connectedness and investigate their properties in single-valued neutrosophic semiregular and single-valued neutrosophic almost regular in neutrosophic ideal topological spaces in Šostak sense. The usefulness of these concepts are incorporated to multiple attribute groups of comparison within the connectedness and separateness of θ£ and δ£.

KEYWORDS

Single valued neutrosophic θ£-closed; single valued neutrosophic θ£-separated; single valued neutrosophic δ£-separated; single-valued neutrosophic δ£-connected; single valued neutrosophic δ£-connected; single valued neutrosophic almost £-egular

1 Introduction

A neutrosophic set can be practical in addressing problems with indeterminate, imperfect, and inconsistent materials. The concept of neutrosophic set theory was introduced by Smarandache [1] as a new mathematical method that corresponds to the indeterminacy degree (uncertainty, etc.). Bakbak et al. [2] and Mishra et al. [3] applied the soft set theory successfully applied in several areas, such as the smoothness of functions, as well as architecture-based, neuro-linguistic programming. Wang et al. [4] proposed single-valued neutrosophic sets (SVNSs). Meanwhile, Kim et al. [5,6] inspected the single valued neutrosophic relations (SVNRs) and symmetric closure of SVNR, respectively. Recently, Saber et al. [7–9] introduced the concepts of single-valued neutrosophic ideal open local function and single-valued neutrosophic topological space. Many of their applications appear in the studies of Das et al. [10]. Alsharari et al. [11–13]. Riaz et al. [14]. Salama et al. [15–17]. Hur et al. [18,19]. Yang et al. [20]. El-Gayyar [21], AL-Nafee et al. [22]. Muhiuddin et al. [23,24] and Mukherjee et al. [25].

First, we define single-valued neutrosophic θ£-closed and single-valued neutrosophic δ£-closed sets as well as some of their core properties. We also present and explore the properties and characterizations of single valued neutrosophic operators namely θ£-closure ($\mathrm{CI}^{\theta£}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}$) and δ£-closure ($\mathrm{CI}^{\delta£}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}$) in the single valued neutrosophic ideal topological space $(\tilde{\mathcal{F}}, \tau^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}, £^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}})$. We then define the concept of single valued neutrosophic regularity spaces. Next, we study single-valued neutrosophic θ£-separated and single-valued neutrosophic δ£-separated with giving some definitions and theorems. Furthermore, we also introduce single-valued neutrosophic θ£-connected and single valued neutrosophic δ£-connected relying on the single valued neutrosophic θ£-closure and δ£-closure operators.

We define a fixed universe $\tilde{\mathcal{F}}$ to be a finite set of objects and ζ a closed unit interval [0, 1]. Additionally, we denote $\zeta^{\tilde{\mathcal{F}}}$ as the set of all single-valued neutrosophic subsets of $\tilde{\mathcal{F}}$.

2 Preliminaries

This section provides a complete survey, some previous studies, and concepts associated with this study.

Definition 1. [1] Let $\tilde{\mathcal{F}}$ be a non-empty set. A neutrosophic set (briefly, $\mathcal{NS}$) in $\tilde{\mathcal{F}}$ is an object having the form $\alpha_n = \{\langle \upsilon, \tilde{\varrho}_{\alpha_n}(\upsilon), \tilde{\sigma}_{\sigma_n}(\omega), \tilde{\varsigma}_{\alpha_n}(\upsilon)\rangle : \upsilon \in \tilde{\mathcal{F}}\}$ where

$$\tilde{\varrho}: \tilde{\mathcal{F}} \to \rfloor^{-}\mathbf{0}, \mathbf{1}^{+}\lfloor, \tilde{\sigma}: \tilde{\mathcal{F}} \to \rfloor^{-}\mathbf{0}, \mathbf{1}^{+}\lfloor, \tilde{\varsigma}: \tilde{\mathcal{F}} \to \rfloor^{-}\mathbf{0}, \mathbf{1}^{+}\lfloor \text{ and } {}^{-}\mathbf{0} \le \tilde{\varrho}_{\alpha_n}(\upsilon) + \tilde{\sigma}_{\alpha_n}(\upsilon) + \tilde{\varsigma}_{\alpha_n}(\upsilon) \le \mathbf{3}^{+} \quad (1)$$

Represent the degree of membership ($\tilde{\varrho}_{\alpha_n}$), the degree of indeterminacy ($\tilde{\sigma}_{\alpha_n}$), and the degree of non-membership ($\tilde{\varsigma}_{\alpha_n}$) respectively of any $\upsilon \in \tilde{\mathcal{F}}$ to the set α_n.

Definition 2. [4] Suppose that $\tilde{\mathcal{F}}$ is a universal set a space of points (objects), with a generic element in $\tilde{\mathcal{F}}$ denoted by υ. Then α_n is called a single valued neutrosophic set (briefly, $\mathcal{SVNS}$) in $\tilde{\mathcal{F}}$, if α_n has the form $\alpha_n = \{\langle \upsilon, \tilde{\varrho}_{\alpha_n}(\upsilon), \tilde{\sigma}_{\alpha_n}(\upsilon), \tilde{\varsigma}_{\alpha_n}(\upsilon)\rangle : \upsilon \in \tilde{\mathcal{F}}\}$. Now, $\tilde{\varrho}_{\alpha_n}, \tilde{\sigma}_{\sigma_n}, \tilde{\varsigma}_{\alpha_n}$ indicate the degree of non-membership, the degree of indeterminacy, and the degree of membership, respectively of any $\upsilon \in \tilde{\mathcal{F}}$ to the set α_n.

Definition 3. [4] Let $\alpha_n = \{\langle \upsilon, \tilde{\varrho}_{\alpha_n}(\upsilon), \tilde{\sigma}_{\sigma_n}(\upsilon), \tilde{\varsigma}_{\alpha_n}(\upsilon)\rangle : \upsilon \in \tilde{\mathcal{F}}\}$ be an SVNS on $\tilde{\mathcal{F}}$. The complement of the set α_n (briefly, α_n^c) defined as follows: $\tilde{\varrho}_{\alpha_n^c}(\upsilon) = \tilde{\varsigma}_{\alpha_n}(\upsilon), \tilde{\sigma}_{\alpha_n}(\upsilon) = [\tilde{\sigma}_{\alpha_n}]^c(\upsilon), \tilde{\varsigma}_{\alpha_n^c}(\upsilon) = \tilde{\varrho}_{\alpha_n}(\upsilon)$.

Definition 4. [26] Let $\tilde{\mathcal{F}}$ be a non-empty set and $\alpha_n, \varepsilon_n \in \zeta^{\tilde{\mathcal{F}}}$ be in the form: $\alpha_n = \{\langle \upsilon, \tilde{\varrho}_{\alpha_n}(\upsilon), \tilde{\sigma}_{\alpha_n}(\upsilon), \tilde{\varsigma}_{\alpha_n}(\upsilon)\rangle : \upsilon \in \tilde{\mathcal{F}}\}$ and $\varepsilon_n = \{\langle \upsilon, \tilde{\varrho}_{\varepsilon_n}(\upsilon), \tilde{\sigma}_{\varepsilon_n}(\upsilon), \tilde{\varsigma}_{\varepsilon_n}(\upsilon)\rangle : \upsilon \in \tilde{\mathcal{F}}\}$ on $\tilde{\mathcal{F}}$ then,

(a) $\alpha_n \subseteq \varepsilon_n$ for every $\upsilon \in \tilde{\mathcal{F}}$; $\tilde{\varrho}_{\alpha_n}(\upsilon) \le \tilde{\varrho}_{\varepsilon_n}(\upsilon), \tilde{\sigma}_{\alpha_n}(\upsilon) \ge \tilde{\sigma}_{\varepsilon_n}(\upsilon), \tilde{\varsigma}_{\alpha_n}(\upsilon) \ge \tilde{\varsigma}_{\varepsilon_n}(\upsilon)$.
(b) $\alpha_n = \varepsilon_n$ iff $\sigma_n \subseteq \varepsilon_n$ and $\sigma_n \supseteq \varepsilon_n$.
(c) $\tilde{0} = \langle 0, 1, 1\rangle$ and $\tilde{1} = \langle 1, 0, 0\rangle$.

Definition 5. **[20]** Let $\alpha_n, \varepsilon_n \in \zeta^{\tilde{\mathcal{F}}}$. Then,

(a) $\alpha_n \cap \varepsilon_n$ is an SVNS, if for every $\upsilon \in \tilde{\mathcal{F}}$,

$$\boldsymbol{\alpha_n \cap \varepsilon_n = \langle (\tilde{\varrho}_{\alpha_n} \cap \tilde{\varrho}_{\varepsilon_n})(\upsilon), (\tilde{\sigma}_{\alpha_n} \cup \tilde{\sigma}_{\varepsilon_n})(\upsilon), (\tilde{\varsigma}_{\alpha_n} \cup \tilde{\varsigma}_{\varepsilon_n})(\upsilon) \rangle}, \tag{2}$$

where, $(\tilde{\varrho}_{\alpha_n} \cap \tilde{\varrho}_{\varepsilon_n})(\upsilon) = \tilde{\varrho}_{\alpha_n}(\upsilon) \cap \tilde{\varrho}_{\varepsilon_n}(\upsilon)$ and $(\tilde{\varsigma}_{\alpha_n} \cup \tilde{\varsigma}_{\varepsilon_n})(\upsilon) = \tilde{\varsigma}_{\alpha_n}(\upsilon) \cup \tilde{\varsigma}_{\varepsilon_n}(\upsilon)$, for all $\upsilon \in \tilde{\mathcal{F}}$,

(b) $\alpha_n \cup \varepsilon_n$ is an SVNS, if for every $\upsilon \in \tilde{\mathcal{F}}$,

$$\boldsymbol{\alpha_n \cup \varepsilon_n = \langle (\tilde{\varrho}_{\alpha_n} \cup \tilde{\varrho}_{\varepsilon_n})(\upsilon), (\tilde{\sigma}_{\alpha_n} \cap \tilde{\sigma}_{\varepsilon_n})(\upsilon), (\tilde{\varsigma}_{\alpha_n} \cap \tilde{\varsigma}_{\varepsilon_n})(\upsilon) \rangle}. \tag{3}$$

Definition 6. **[15]** For an any arbitrary family $\{\alpha_n\}_{i \in j} \in \zeta^{\tilde{\mathcal{F}}}$ of SVNS the union and intersection are given by

(a) $\bigcap_{i \in j} [\alpha_n]_i = \langle \cap_{i \in j} \tilde{\varrho}_{[\alpha_n]_i}(\upsilon), \cup_{i \in j} \tilde{\sigma}_{[\alpha_n]_i}(\upsilon), \cup_{i \in j} \tilde{\varsigma}_{[\alpha_n]_i}(\upsilon) \rangle$,
(b) $\bigcup_{i \in j} [\alpha_n]_i = \langle \cup_{i \in j} \tilde{\varrho}_{[\alpha_n]_i}(\upsilon), \cap_{i \in j} \tilde{\sigma}_{[\alpha_n]_i}(\upsilon), \cap_{i \in j} \tilde{\varsigma}_{[\alpha_n]_i}(\upsilon) \rangle$.

Definition 7. **[21]** A single-valued neutrosophic topological spaces is an ordered $(\tilde{\mathcal{F}}, \tilde{\tau}^{\tilde{\varrho}}, \tilde{\tau}^{\tilde{\sigma}}, \tilde{\tau}^{\tilde{\varsigma}})$ where $\tilde{\tau}^{\tilde{\varrho}}, \tilde{\tau}^{\tilde{\sigma}}, \tilde{\tau}^{\tilde{\varsigma}} : \zeta^{\tilde{\mathcal{F}}} \to \zeta$ is a mapping satisfying the following axioms:

(SVNT1) $\tilde{\tau}^{\tilde{\varrho}}(\tilde{0}) = \tilde{\tau}^{\tilde{\varrho}}(\tilde{1}) = \tilde{\tau}^{\tilde{\sigma}}(\tilde{0}) = \tilde{\tau}^{\tilde{\sigma}}(\tilde{1}) = 0$ and $\tilde{\tau}^{\tilde{\varsigma}}(\tilde{0}) = \tilde{\tau}^{\tilde{\varsigma}}(\tilde{1}) = 1$.

(SVNT2) $\tilde{\tau}^{\tilde{\varrho}}(\alpha_n \cap \varepsilon_n) \geq \tilde{\tau}^{\tilde{\varrho}}(\alpha_n) \cap \tilde{\tau}^{\tilde{\varrho}}(\varepsilon_n), \tilde{\tau}^{\tilde{\sigma}}(\alpha_n \cap \varepsilon_n) \leq \tau^{\tilde{\sigma}}(\alpha_n) \cup \tilde{\tau}^{\tilde{\sigma}}(\varepsilon_n), \tilde{\tau}^{\tilde{\varsigma}}(\alpha_n \cap \varepsilon_n) \leq \tilde{\tau}^{\tilde{\varsigma}}(\alpha_n) \cup \tilde{\tau}^{\tilde{\varsigma}}(\varepsilon_n)$ for every, $\alpha_n, \varepsilon_n \in \zeta^{\tilde{\mathcal{F}}}$

(SVNT3) $\tilde{\tau}^{\tilde{\varrho}}(\cup_{j \in \Gamma}[\alpha_n]_j) \geq \cap_{j \in \Gamma} \tilde{\tau}^{\tilde{\varrho}}([\alpha_n]_j), \tilde{\tau}^{\tilde{\sigma}}(\cup_{i \in \Gamma}[\alpha_n]_j) \leq \cup_{j \in \Gamma} \tilde{\tau}^{\tilde{\sigma}}([\alpha_n]_j), \tilde{\tau}^{\tilde{\varsigma}}(\cup_{j \in \Gamma}[\alpha_n]_j) \leq \cup_{j \in \Gamma} \tilde{\tau}^{\tilde{\varsigma}}([\alpha_n]_j)$, for every $[\alpha_n]_j \in \zeta^{\tilde{\mathcal{F}}}$.

The quadruple $(\tilde{\mathcal{F}}, \tilde{\tau}^{\tilde{\varrho}}, \tilde{\tau}^{\tilde{\sigma}}, \tilde{\tau}^{\tilde{\varsigma}})$ is called a single-valued neutrosophic topological spaces (briefly, SVNT, for short). Occasionally write $\tau^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}$ for $(\tilde{\tau}^{\tilde{\varrho}}, \tilde{\tau}^{\tilde{\sigma}}, \tilde{\tau}^{\tilde{\varsigma}})$ and it will cause no ambiguity.

Definition 8. **[7]** Let $(\tilde{\mathcal{F}}, \tau^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}})$ be an **SVNTS.** Then, for every $\alpha_n \in \zeta^{\tilde{\mathcal{F}}}$ and $r \in \zeta_0$. Then the single valued neutrosophic closure and single valued neutrosophic interior of α_n are define by:

$$\mathbf{C}_{\tau^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}}(\boldsymbol{\alpha_n}, \boldsymbol{r}) = \bigcap \{\boldsymbol{\varepsilon_n} \in \boldsymbol{\zeta}^{\tilde{\mathcal{F}}} : \boldsymbol{\alpha_n} \leq \boldsymbol{\varepsilon_n}, \boldsymbol{\tau}^{\tilde{\varrho}}([\boldsymbol{\varepsilon_n}]^c) \geq \boldsymbol{r}, \boldsymbol{\tau}^{\tilde{\sigma}}([\boldsymbol{\varepsilon_n}]^c) \leq \mathbf{1} - \boldsymbol{r}, \boldsymbol{\tau}^{\tilde{\varsigma}}([\boldsymbol{\varepsilon_n}]^c) \leq \mathbf{1} - \boldsymbol{r}\} \tag{4}$$

$$\mathbf{int}_{\tau^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}}(\boldsymbol{\alpha_n}, \boldsymbol{r}) = \bigcup \{\boldsymbol{\varepsilon_n} \in \boldsymbol{\zeta}^{\tilde{\mathcal{F}}} : \boldsymbol{\alpha_n} \geq \boldsymbol{\varepsilon_n}, \boldsymbol{\tau}^{\tilde{\varrho}}(\boldsymbol{\varepsilon_n}) \geq \boldsymbol{r}, \boldsymbol{\tau}^{\tilde{\sigma}}(\boldsymbol{\varepsilon_n}) \leq \mathbf{1} - \boldsymbol{r}, \boldsymbol{\tau}^{\tilde{\varsigma}}(\boldsymbol{\varepsilon_n}) \leq \mathbf{1} - \boldsymbol{r}\} \tag{5}$$

Definition 9. [7] Let $(\tilde{\mathcal{F}})$ be a nonempty set and $\upsilon \in \tilde{\mathcal{F}}$, let $s \in (0,1]$, $t \in [0,1)$ and $k \in [0,1)$, then the single-valued neutrosophic point $x_{s,t,k}$ in $\tilde{\mathcal{F}}$ given by

$$x_{s,t,k}(\upsilon) = \begin{cases} (s,t,k), & \textit{if } x = \upsilon \\ (0,1,1), & \textit{otherwise}. \end{cases}$$

We define that, $x_{s,t,p} \in \alpha_n$ iff $s < \tilde{\varrho}_{\alpha_n}(\upsilon)$, $t \geq \tilde{\sigma}_{\alpha_n}(\upsilon)$ and $k \geq \tilde{\tilde{\varsigma}}_{\alpha_n}(\upsilon)$. We indicate the set of all single-valued neutrosophic points in $\tilde{\mathcal{F}}$ as $\mathrm{P}_{x_{s,t,k}}(\tilde{\mathcal{F}})$. A single-valued neutrosophic set α_n is said to be quasi-coincident with another single-valued neutrosophic set ε_n, denoted by $\alpha_n q \varepsilon_n$, if there exists an element $\upsilon \in \tilde{\mathcal{F}}$ such that $\tilde{\varrho}_{\alpha_n}(\upsilon) + \tilde{\varrho}_{\varepsilon_n}(\upsilon) > 1, \tilde{\sigma}_{\alpha_n}(\upsilon) + \tilde{\sigma}_{\varepsilon_n}(\upsilon) \leq 1, \tilde{\varsigma}_{\alpha_n}(\upsilon) + \tilde{\varsigma}_{\varepsilon_n}(\upsilon) \leq 1$.

Definition 10. [7] A mapping $£^{\tilde{\varrho}}, £^{\tilde{\sigma}}, £^{\tilde{\varsigma}}: \zeta^{\tilde{\mathcal{F}}} \to \zeta$ is called single-valued neutrosophic ideal (SVNI) on $\tilde{\mathcal{F}}$ if, it satisfies the following conditions:

(£₁) $£^{\tilde{\varrho}}(\tilde{0}) = 1$ and $£^{\tilde{\sigma}}(\tilde{0}) = £^{\tilde{\varsigma}}(\tilde{0}) = 0$.

(£₂) If $\sigma_n \leq \gamma_n$, then $£^{\tilde{\varrho}}(\varepsilon_n) \leq £^{\tilde{\varrho}}(\alpha_n)$, $£^{\tilde{\sigma}}(\varepsilon_n) \geq £^{\tilde{\sigma}}(\alpha_n)$ and $£^{\tilde{\varsigma}}(\varepsilon_n) \geq £^{\tilde{\varsigma}}(\alpha_n)$, for $\varepsilon_n, \alpha_n \in \zeta^{\tilde{\mathcal{F}}}$.

(£₃) $£^{\tilde{\varrho}}(\alpha_n \cup \varepsilon_n) \geq £^{\tilde{\varrho}}(\alpha_n) \cap £^{\tilde{\varrho}}(\varepsilon_n)$, $£^{\tilde{\sigma}}(\alpha_n \cup \varepsilon_n) \leq £^{\tilde{\sigma}}(\alpha_n) \cup £^{\tilde{\sigma}}(\varepsilon_n)$ and $£^{\tilde{\varsigma}}(\alpha_n \cup \varepsilon_n) \leq £^{\tilde{\varsigma}}(\alpha_n) \cup £^{\tilde{\varsigma}}(\varepsilon_n)$, for $\alpha_n, \varepsilon_n \in \zeta^{\tilde{\mathcal{F}}}$.

The tribal $(\tilde{\mathcal{F}}, \tau^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}, £^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}})$ is called a single valued neutrosophic ideal topological space in Šostak sense (briefly, SVNITS).

Definition 11. [7] Let $(\tilde{\mathcal{F}}, \tau^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}, £^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}})$ be an SVNITS for each $\alpha_n \in \zeta^{\tilde{\mathcal{F}}}$. Then, the single valued neutrosophic ideal open local function $[\alpha_n]_r^{\odot}(\tau^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}, £^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}})$ of α_n is the union of all single-valued neutrosophic points $x_{s,t,k}$ such that if $\varepsilon_n \in Q_{\tau^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}}(x_{s,t,k}, r)$ and $£^{\tilde{\varrho}}(\omega_n) \geq r$, $£^{\tilde{\sigma}}(\omega_n) \leq 1-r$, $£^{\tilde{\varsigma}}(\omega_n) \leq 1-r$, then there is at least one $\upsilon \in \tilde{\mathcal{F}}$ for which

$$\tilde{\varrho}_{\alpha_n}(\upsilon) + \tilde{\varrho}_{\varepsilon_n}(\upsilon) - 1 > \tilde{\varrho}_{\omega_n}(\upsilon), \quad \tilde{\sigma}_{\alpha_n}(\upsilon) + \tilde{\sigma}_{\varepsilon_n}(\upsilon) - 1 \leq \tilde{\sigma}_{\omega_n}(\upsilon), \quad \tilde{\varsigma}_{\alpha_n}(\upsilon) + \tilde{\varsigma}_{\varepsilon_n}(\upsilon) - 1 \leq \tilde{\varsigma}_{\omega_n}(\upsilon) \quad (6)$$

Occasionally, we will write $[\alpha_n]_r^{\odot}$ for $[\alpha_n]_r^{\odot}(\tau^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}, £^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}})$ herein to avoid ambiguity.

Remark 1. [7] Let $(\tilde{\mathcal{F}}, \tau^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}, £^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}})$ be an SVNITS and $\alpha_n \in \zeta^{\tilde{\mathcal{F}}}$. Hence, we can write

$$\mathbf{CI}^{\odot}_{\tau^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}}(\alpha_n, r) = \alpha_n \cup [\alpha_n]_r^{\odot}, \quad \mathbf{int}^{\odot}_{\tau^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}}(\alpha_n, r) = \alpha_n \cap [(\alpha_n^c)_r^{\odot}]^c \quad (7)$$

Clearly, $\mathrm{CI}^{\odot}_{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}$ is a single-valued neutrosophic closure operator and $(\tau^{\tilde{\varrho}\odot}(£), \tau^{\tilde{\sigma}\odot}(£), \tau^{\tilde{\varsigma}\odot}(£))$ is the single-valued neutrosophic topology generated by $\mathrm{CI}^{\odot}_{\tau^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}}$, i.e., $\tau^{\odot}(\mathcal{J})(\alpha_n) = \bigcup\{r | \mathrm{CI}^{\odot}_{\tilde{\tau}^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}}(\alpha_n^c, r) = \alpha_n^c\}$.

Theorem 1. [7] Let $\{[\alpha_n]_i\}_{i \in J} \subset \zeta^{\tilde{\mathcal{F}}}$ be a family of single-valued neutrosophic sets on $\tilde{\mathcal{F}}$ and $(\tilde{\mathcal{F}}, \tilde{\tau}^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}, £^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}})$ be a SVNITS. Then,

(a) $(\cup([\alpha_n]_i)_r^{\odot} : i \in J) \leq (\cup[\alpha_n]_i : i \in j)_r^{\odot}$,
(b) $(\cap([\alpha_n]_i) : i \in j)_r^{\odot} \geq (\cap([\alpha_n]_i)_r^{\odot} : i \in J)$.

Theorem 2. [7] Let $(\tilde{\mathcal{F}}, \tau^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}, £^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}})$ be an SVNITS and $r \in \zeta, \alpha_n, \varepsilon_n \in \zeta^{\tilde{\mathcal{F}}}$. Then,

(a) $\mathrm{int}^{\odot}_{\tilde{\tau}^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}}(\alpha_n \vee \varepsilon_n, r) \leq \mathrm{int}^{\odot}_{\tilde{\tau}^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}}(\alpha_n, r) \vee \mathrm{int}^{\odot}_{\tilde{\tau}^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}}(\varepsilon_n, r)$,

(b) $\mathrm{int}_{\tilde{\tau}^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}}(\alpha_n, r) \leq \mathrm{int}^{\odot}_{\tilde{\tau}^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}}(\alpha_n, r) \leq \alpha_n \leq \mathrm{CI}^{\odot}_{\tilde{\tau}^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}}(\alpha_n, r) \leq \mathrm{C}_{\tilde{\tau}^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}}(\alpha_n, r)$,

(c) $\mathrm{CI}^{\odot}_{\tilde{\tau}^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}}([\alpha_n]^c, r) = [\mathrm{int}^{\odot}_{\tilde{\tau}^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}}(\alpha_n, r)]^c$,

(d) $[\mathrm{CI}^{\odot}_{\tilde{\tau}^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}}(\alpha_n, r)]^c = \mathrm{int}^{\odot}_{\tilde{\tau}^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}}([\alpha_n]^c, r)$,

(e) $\mathrm{int}^{\odot}_{\tilde{\tau}^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}}(\alpha_n \wedge \varepsilon_n, r) = \mathrm{int}^{\odot}_{\tilde{\tau}^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}}(\alpha_n, r) \wedge \mathrm{int}^{\odot}_{\tilde{\tau}^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}}(\varepsilon_n, r)$.

Definition 12. [8] Let $(\tilde{\mathcal{F}}, \tau^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}})$ be an SVNITS. For every $\alpha_n, \varepsilon_n, \omega_n \in \zeta^{\tilde{\mathcal{F}}}$, α_n and ε_n are called r-single-valued neutrosophic separated if for $r \in \zeta_0$,

$$\mathbf{CI}_{\tau^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}}(\alpha_n, r) \cap \varepsilon_n = \mathbf{CI}_{\tau^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}}(\varepsilon_n, r) \cap \alpha_n = \tilde{0} \quad (8)$$

An $\mathcal{SVNS}$, ω_n is called r-single-valued neutrosophic connected if r-$\mathcal{SVNSEP}$ $\alpha_n, \varepsilon_n \in \zeta^{\tilde{\mathcal{F}}} - \{\tilde{0}\}$ such that $\omega_n = \alpha_n \cup \varepsilon_n$ does not exist. A $\mathcal{SVNS}$ α_n is said to be r-single-valued neutrosophic connected if it is r-single-valued neutrosophic connected for any $r \in \zeta_0$. A $(\tilde{\mathcal{F}}, \tau^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}})$ is said to be r-single-valued neutrosophic connected if $\tilde{1}$ is r-single-valued neutrosophic connected.

3 Single Valued Neutrosophic δ£-Cluster Point and Single Valued Neutrosophic θ£-Cluster Point

In this section, we introduce the r-single-valued neutrosophic δ£-cluster point (abbreviated $SVN\delta$£-cluster point) and r-single-valued neutrosophic £-closed set (abbreviated **SVN£C**). Furthermore, we analyze the single-valued neutrosophic δ£-closure operator (δ£-closure operator for brevity) and single-valued neutrosophic θ£-closure operator (θ£-closure operator for brevity).

Definition 13. Let $(\tilde{\mathcal{F}}, \tau^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}, £^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}})$ be an SVNITS and $\alpha_n \in \zeta^{\tilde{\mathcal{F}}}$, $r \in \zeta_0$. Then,

(a) α_n is said to be r-single valued neutrosophic £-open (briefly, r-$SVN£O$), if and only if $\alpha_n \leq \text{int}_{\tilde{\tau}\tilde{\tau}^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}}([\alpha_n]_r^{\odot}, r)$,
(b) α_n is said to be r-single valued neutrosophic semi-£-open (briefly, r-$SVNS£O$) if and only if $\alpha_n \leq \text{CI}^{\odot}_{\tilde{\tau}^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}}(\text{int}_{\tilde{\tau}^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}}([\alpha_n]_r^{\odot}, r), r)$,
(c) α_n is called r-single valued neutrosophic pre-£-open (briefly, r-$SVNP£O$) if and only if $\alpha_n \leq \text{int}_{\tilde{\tau}^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}}(\text{CI}_{\tilde{\tau}^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}}([\alpha_n]_r^{\odot}, r), r)$,
(d) α_n is called r-single valued neutrosophic regular-£-open (briefly, r-$SVNR£O$) if and only if $\alpha_n = \text{int}_{\tilde{\tau}^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}}(\text{CI}^{\odot}_{\tilde{\tau}^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}}([\alpha_n]_r^{\odot}, r), r)$,
(e) α_n is said to be r-single valued neutrosophic α£-open (briefly, r-$SVN\alpha£O$) if and only if $\alpha_n \leq \text{int}_{\tilde{\tau}^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}}(\text{CI}^{\odot}_{\tilde{\tau}^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}}(\text{int}_{\tilde{\tau}^{\tilde{\varrho}}}\left([\alpha_n]_r^{\odot}, r\right), r\right)$,
(f) α_n is said to be r-single valued neutrosophic $\star$-open set (briefly, r-$SVN \star$ O) if and only if $\alpha_n = \text{CI}^{\odot}_{\tilde{\tau}^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}}(\alpha_n, r)$.

The complement of an r−SVN£O (resp, r-SVNS£O, r-SVNP£O, r-SVNR£O, r-SVNα£O, r-SVN⋆O) is said to be an r−SVN£C (resp, r-SVNS£C, r-SVNP£C, r-SVNR£C, r-SVNα£C, r-SVN⋆C) respectively.

Remark 2. r-single valued neutrosophic open set ($\text{r} - \mathcal{SVN}$O) and r-SVN£O are independent notions as shown by the following example.

Example 1. Let $\tilde{\mathcal{F}} = \{a, b, c\}$ be a set. Define $\varepsilon_n, \pi_n, \omega_n \in \zeta^{\tilde{\mathcal{F}}}$ as follows:

$\varepsilon_n = \langle (0.3, 0.3, 0.3), (0.3, 0.3, 0.3), (0.3, 0.3, 0.3) \rangle$; $\quad \pi_n = \langle (0.4, 0.4, 0.4), (0.4, 0.4, 0.4), (0.4, 0.4, 0.4) \rangle$,

$\omega_n = \langle (0.5, 0.5, 0.5), (0.2, 0.2, 0.2), (0.1, 0.1, 0.1) \rangle$.

We define an SVNITS$(\tau^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}, £^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}})$ on $\tilde{\mathcal{F}}$ as follows: for each $\alpha_n \in \zeta^{\tilde{\mathcal{F}}}$,

$$\tilde{\tau}^{\tilde{\varrho}}(\alpha_n) = \begin{cases} 1, & \textit{if } \alpha_n = \left\{\tilde{0}, \tilde{1}\right\}, \\ \frac{2}{3}, & \textit{if } \alpha_n = \{\varepsilon_n, \pi_n\}, \\ 0, & \textit{otherwise}, \end{cases} \qquad £^{\tilde{\varrho}}(\alpha_n) = \begin{cases} 1, & \textit{if } \alpha_n = \tilde{0}, \\ \frac{2}{3}, & \textit{if } 0 < \alpha_n \leq \omega_n \\ 0, & \textit{otherwise}, \end{cases}$$

$$\tilde{\tau}^{\tilde{\sigma}}(\alpha_n)=\begin{cases}0, & \mathit{if}\ \alpha_n=\left\{\tilde{0},\tilde{1}\right\},\\ \frac{1}{3}, & \mathit{if}\ \alpha_n=\{\varepsilon_n,\pi_n\},\\ 1, & \mathit{otherwise},\end{cases}\quad £^{\tilde{\sigma}}(\alpha_n)=\begin{cases}0, & \mathit{if}\ \alpha_n=\tilde{0},\\ \frac{1}{3}, & \mathit{if}\ 0<\alpha_n\le\omega_n,\\ 1, & \mathit{otherwise},\end{cases}$$

$$\tilde{\tau}^{\tilde{\varsigma}}(\alpha_n)=\begin{cases}0, & \mathit{if}\ \alpha_n=\left\{\tilde{0},\tilde{1}\right\},\\ \frac{1}{3}, & \mathit{if}\ \alpha_n=\{\varepsilon_n,\pi_n\},\\ 1, & \mathit{otherwise},\end{cases}\quad £^{\tilde{\varsigma}}(\alpha_n)=\begin{cases}0, & \mathit{if}\ \alpha_n=\tilde{0},\\ \frac{1}{3}, & \mathit{if}\ 0<\alpha_n\le\omega_n,\\ 1, & \mathit{otherwise}.\end{cases}$$

Based on $\varepsilon_n=\langle(0.3,0.3,0.3),(0.3,0.3,0.3),(0.3,0.3,0.3)\rangle$, it's clear that, $\frac{2}{3}-\mathcal{SVN}$O is set because $\tau^{\tilde{\varrho}}(\langle(0.3,0.3,0.3),(0.3,0.3,0.3),(0.3,0.3,0.3)\rangle)\ge\frac{2}{3},\tau^{\tilde{\sigma}}(\langle(0.3,0.3,0.3),(0.3,0.3,0.3),(0.3,0.3,0.3)\rangle)\le\frac{1}{3},\tau^{\tilde{\varsigma}}(\langle(0.3,0.3,0.3),(0.3,0.3,0.3),(0.3,0.3,0.3)\rangle)\le\frac{1}{3}$.

However ε_{n} is not an r-SVN£O set, and for that, we must prove that $\varepsilon_n\nleq\mathrm{int}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}([\varepsilon_n]^{\odot}_{\frac{2}{3}},\frac{2}{3})$. So, we must first obtain $[\varepsilon_n]^{\odot}_{\frac{2}{3}}$. Based on Eq. (11), $\tilde{1},\varepsilon_n,\pi_n\in Q_{\tau\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(x_{s,t,k},\frac{2}{3})$ and $£^{\tilde{\varrho}}(\langle(0.5,0.5,0.5),(0.2,0.2,0.2),(0.1,0.1,0.1)\rangle)\ge\frac{2}{3}$, $£^{\tilde{\sigma}}(\langle(0.5,0.5,0.5),(0.2,0.2,0.2),(0.1,0.1,0.1)\rangle)\le\frac{1}{3}$, $£^{\tilde{\varsigma}}(\langle(0.5,0.5,0.5),(0.2,0.2,0.2),(0.1,0.1,0.1)\rangle)\le\frac{1}{3}$,
such that by using Eqs. (2), (3) and (6) we obtain,

$$\tilde{\varrho}_{\varepsilon_n}(\upsilon)+\tilde{\varrho}_{\tilde{1}}(\nu)-1>\tilde{\varrho}_{\omega_n}(\upsilon),\tilde{\sigma}_{\varepsilon_n}(\upsilon)+\tilde{\sigma}_{\tilde{1}}(\upsilon)-1\le\tilde{\sigma}_{\omega_n}(\upsilon),\tilde{\varsigma}_{\varepsilon_n}(\upsilon)+\tilde{\varsigma}_{\tilde{1}}(\upsilon)-1\le\tilde{\varsigma}_{\omega_n}(\upsilon).$$

$$(0.3,0.3,0.3)(\upsilon)+(1,1,1)(\nu)-1\ngtr(0.5,0.5,0.5)(\upsilon),$$

$$(0.3,0.3,0.3)(\upsilon)+(0,0,0)(\upsilon)-1\le(0.2,0.2,0.2)(\upsilon),$$

$$(0.3,0.3,0.3)(\upsilon)+(0,0,0)(\upsilon)-1\le(0.1,0.1,0.1)(\upsilon),$$

$$\tilde{\varrho}_{\varepsilon_n}(\upsilon)+\tilde{\varrho}_{\pi_n}(\nu)-1>\tilde{\varrho}_{\omega_n}(\upsilon),\tilde{\sigma}_{\varepsilon_n}(\upsilon)+\tilde{\sigma}_{\pi_n}(\upsilon)-1\le\tilde{\sigma}_{\omega_n}(\upsilon),\tilde{\varsigma}_{\varepsilon_n}(\upsilon)+\tilde{\varsigma}_{\pi_n}(\upsilon)-1\le\tilde{\varsigma}_{\omega_n}(\upsilon).$$

$$(0.3,0.3,0.3)(\upsilon)+(0.4,0.4,0.4)(\nu)-1\ngtr(0.5,0.5,0.5)(\upsilon),$$

$$(0.3,0.3,0.3)(\upsilon)+(0.4,0.4,0.4)(\upsilon)-1\le(0.2,0.2,0.2)(\upsilon),$$

$$(0.3,0.3,0.3)(\upsilon)+(0.4,0.4,0.4)(\upsilon)-1\le(0.1,0.1,0.1)(\upsilon)$$

$$\tilde{\varrho}_{\varepsilon_n}(\upsilon)+\tilde{\varrho}_{\varepsilon_n}(\nu)-1>\tilde{\varrho}_{\omega_n}(\upsilon),\tilde{\sigma}_{\varepsilon_n}(\upsilon)+\tilde{\sigma}_{\varepsilon_n}(\upsilon)-1\le\tilde{\sigma}_{\omega_n}(\upsilon),\tilde{\varsigma}_{\varepsilon_n}(\upsilon)+\tilde{\varsigma}_{\varepsilon_n}(\upsilon)-1\le\tilde{\varsigma}_{\omega_n}(\upsilon).$$

$$(0.3,0.3,0.3)(\upsilon)+(0.3,0.3,0.3)(\nu)-1\ngtr(0.5,0.5,0.5)(\upsilon),$$

$$(0.3,0.3,0.3)(\upsilon)+(0.3,0.3,0.3)(\upsilon)-1\le(0.2,0.2,0.2)(\upsilon),$$

$$(0.3,0.3,0.3)(\upsilon)+(0.3,0.3,0.3)(\upsilon)-1\le(0.1,0.1,0.1)(\upsilon)$$

Therefore, $[\varepsilon_n]^{\odot}_{\frac{2}{3}}=\tilde{0}$. Subsequently, using Eq. (7) we obtain $\mathrm{int}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}([\varepsilon_n]^{\odot}_{\frac{2}{3}},\frac{2}{3})=int_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\tilde{0},\frac{2}{3})=\tilde{0}$, which implies that

$$\langle(0.3,0.3,0.3),(0.3,0.3,0.3),(0.3,0.3,0.3)\rangle=\varepsilon_n\nleq\mathrm{int}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}\left([\varepsilon_n]^{\odot}_{\frac{2}{3}},\frac{2}{3}\right)=\tilde{0}.$$

Hence, ε_n is not an r-SVN£O set.

Definition 14. Let $(\tilde{\mathcal{F}}, \tau^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}, £^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}})$ be an SVNITS, $\alpha_n \in \zeta^{\tilde{\mathcal{F}}}$, $x_{s,t,k} \in \mathrm{P}_{s,t,k}(\tilde{\mathcal{F}})$ and $r \in \zeta_0$. Then,

(a) α_n is an r-single valued neutrosophic $Q_{\tau^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}}$-neighborhood of $x_{s,t,k}$ if $x_{s,t,k} q \alpha_n$ with $\tau^{\tilde{\varrho}}(\alpha_n) \geq r, \tau^{\tilde{\sigma}}(\alpha_n) \leq 1-r, \tau^{\tilde{\varsigma}}(\alpha_n) \leq 1-r$;
(b) $x_{s,t,k}$ is an r-single valued neutrosophic θ£-cluster point (r-δ£-*cluster point*) of α_n if for every $\varepsilon_n \in Q_{\tilde{\tau}^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}}(x_{s,t,k}, r)$, we have $\alpha_n q \mathrm{int}_{\tilde{\tau}^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}}(\mathrm{CI}^{\odot}_{\tilde{\tau}^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}}(\varepsilon_n, r), r)$;
(c) δ£-closure operator is the mapping of $\mathrm{CI}^{\delta £}_{\tilde{\tau}^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}} : \zeta^{\tilde{\mathcal{F}}} \times \zeta_0 \to \zeta^{\tilde{\mathcal{F}}}$ defined as

$$\mathrm{CI}^{\delta £}_{\tilde{\tau}^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}}(\alpha_n, r) = \cup \left\{ x_{s,t,k} \in \mathrm{P}_{s,t,k}\left(\tilde{\mathcal{F}}\right) : x_{s,t,k} \ \textit{is}\ r\text{−}\delta£\text{−}\textit{cluster point of}\ \alpha_n \right\}.$$

Definition 15. Let $(\tilde{\mathcal{F}}, \tau^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}, £^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}})$ be an SVNITS, $\alpha_n \in \zeta^{\tilde{\mathcal{F}}}$, $x_{s,t,k} \in \mathrm{P}_{s,t,k}(\tilde{\mathcal{F}})$ and $r \in \zeta_0$. Then,

(a) α_n is called r-Single valued neutrosophic $\mathfrak{R}^{£}_{\tau^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}}$-neighborhood of $x_{s,t,k}$ if $x_{s,t,k} q \alpha_n$ and αn is *r-SVNRIO*. We denote $\mathfrak{R}^{£}_{\tau^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}} = \left\{ \alpha_n \in \zeta^{\tilde{\mathcal{F}}} | x_{s,t,k} q \alpha_n, \alpha_n \ \textit{is}\ \mathrm{r-SVNRIO} \right\}$,
(b) $x_{s,t,k}$ is called r-single valued neutrosophic θ£-cluster point (r-θ£-*cluster point*) of α_n if for any $\varepsilon_n \in Q_{\tilde{\tau}^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}}\left(x_{s,t,k}, r\right)$, we have $\alpha_n q \mathrm{CI}^{\odot}_{\tilde{\tau}^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}}(\varepsilon_n, r)$,
(c) θ£-closure operator is mapping $\mathrm{CI}^{\theta £}_{\tilde{\tau}^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}} : \zeta^{\tilde{\mathcal{F}}} \times \zeta_0 \to \zeta^{\tilde{\mathcal{F}}}$ defined as

$$\boldsymbol{\mathrm{CI}^{\theta £}_{\tilde{\tau}^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}}(\alpha_n, r) = \cup \left\{ x_{s,t,k} \in P_{s,t,k}\left(\tilde{\mathcal{F}}\right) : x_{s,t,k} \ \textit{is}\ r\text{−}\theta£\text{−}\textit{cluster point of}\ \alpha_n \right\}} \tag{9}$$

Example 2. Let $\tilde{\mathcal{F}} = \{a, b, c\}$ be a set. Define $\varepsilon_n, \pi_n \in \zeta^{\tilde{\mathcal{F}}}$ as follows:
$\varepsilon_n = \langle (0.4, 0.4, 0.4), (0.4, 0.4, 0.4), (0.4, 0.4, 0.4) \rangle ; \pi_n = \langle (0.2, 0.2, 0.2), (0.2, 0.2, 0.2), (0.2, 0.2, 0.2) \rangle$.

We define an SVNITS$(\tau^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}, £^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}})$ on $\tilde{\mathcal{F}}$ as follows: for each $\alpha_n \in \zeta^{\tilde{\mathcal{F}}}$,

$$\tilde{\tau}^{\tilde{\varrho}}(\alpha_n) = \begin{cases} 1, & \textit{if}\ \alpha_n = \tilde{0}, \\ 1, & \textit{if}\ \alpha_n = \tilde{1}, \\ \frac{2}{3}, & \textit{if}\ \alpha_n = \varepsilon_n, \\ 0, & \textit{otherwise}, \end{cases} \qquad £^{\tilde{\varrho}}(\alpha_n) = \begin{cases} 1, & \textit{if}\ \alpha_n = \tilde{0}, \\ \frac{1}{3}, & \textit{if}\ \pi_n = \varepsilon_n \\ \frac{2}{3}, & \textit{if}\ 0 < \alpha_n < \pi_n \\ 0, & \textit{otherwise}, \end{cases}$$

$$\tilde{\tau}^{\tilde{\sigma}}(\alpha_n) = \begin{cases} 0, & \textit{if}\ \alpha_n = \tilde{0}, \\ 0, & \textit{if}\ \alpha_n = \tilde{1}, \\ \frac{1}{3}, & \textit{if}\ \alpha_n = \varepsilon_n, \\ 1, & \textit{otherwise}, \end{cases} \qquad £^{\tilde{\sigma}}(\alpha_n) = \begin{cases} 0, & \textit{if}\ \alpha_n = \tilde{0}, \\ \frac{2}{3}, & \textit{if}\ \pi_n = \varepsilon_n \\ \frac{1}{3}, & \textit{if}\ 0 < \alpha_n < \pi_n \\ 1, & \textit{otherwise}, \end{cases}$$

$$\tilde{\tau}^{\tilde{\varsigma}}(\alpha_n)=\begin{cases}0, & if\ \ \alpha_n=\tilde{0},\\ 0, & if\ \ \alpha_n=\tilde{1},\\ \dfrac{1}{3}, & if\ \ \alpha_n=\varepsilon_n,\\ 1, & otherwise,\end{cases}\qquad £^{\tilde{\varsigma}}(\alpha_n)=\begin{cases}0, & if\ \ \alpha_n=\tilde{0},\\ \dfrac{2}{3}, & if\ \ \pi_n=\varepsilon_n\\ \dfrac{1}{3}, & if\ \ 0<\alpha_n<\pi_n\\ 1, & otherwise,\end{cases}$$

From using (9) we get, we obtain

$$\mathrm{CI}^{\theta £}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\alpha_n,r)=\begin{cases}\tilde{0}, & if\ \ \alpha_n=\tilde{0},\\ \varepsilon_n^c, & if\ \ \tilde{0}\neq\alpha_n\leq\varepsilon_n^c, r\leq\dfrac{1}{3}, 1-r\geq\dfrac{2}{3},\\ 1, & otherwise.\end{cases}$$

Theorem 3. Let $(\tilde{\mathcal{F}},\tau^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}},£^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}})$ be an SVNITS, $r\in\zeta_0$ and $\alpha_n,\varepsilon_n\in\zeta^{\tilde{\mathcal{F}}}$. Then the following properties are holds:

(a) $\alpha_n\leq\mathrm{CI}^{\delta £}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\alpha_n,r)$,

(b) If $\alpha_n\leq\varepsilon_n$, then $\mathrm{CI}^{\delta £}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\alpha_n,r)\leq\mathrm{CI}^{\delta £}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\varepsilon_n,r)$,

(c) $\mathrm{int}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\mathrm{CI}^{\odot}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\alpha_n,r),r)$ is r-*SVNRIO*,

(d) $\mathrm{CI}^{\delta £}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\alpha_n,r)=\cap\{\varepsilon_n\in\zeta^{\tilde{\mathcal{F}}}|\alpha_n\leq\varepsilon_n,\varepsilon_n$ is r-*SVNRIC*$\}$,

(e) $\mathrm{CI}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\alpha_n,r)\leq\mathrm{CI}^{\delta £}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\alpha_n,r)$.

Proof. (a) and (b) are easily proved from (9).

(c) Let $\varepsilon_n\in\zeta^{\tilde{\mathcal{F}}}$ and $\varepsilon_n=\mathrm{int}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\mathrm{CI}^{\odot}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\alpha_n,r),r)$. Then, we have

$$\begin{aligned}\mathrm{int}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}\left(\mathrm{CI}^{\odot}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\alpha_n,r),r\right)&=\mathrm{int}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}\left(\mathrm{CI}^{\odot}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}\left(\mathrm{int}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}\left(\mathrm{CI}^{\odot}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\alpha_n,r),r\right),r\right),r\right)\\ &\leq\mathrm{int}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\mathrm{CI}^{\odot}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\mathrm{CI}^{\odot}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\alpha_n,r),r),r)\\ &=\mathrm{int}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}\left(\mathrm{CI}^{\odot}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\alpha_n,r),r\right)=\varepsilon_n.\end{aligned}$$

Since $\varepsilon_n=\mathrm{int}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\varepsilon_n,r)\leq\mathrm{int}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\mathrm{CI}^{\odot}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\varepsilon_n,r),r)$, we have $\mathrm{int}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\mathrm{CI}^{\odot}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\varepsilon_n,r),r)=\varepsilon_n$.

(d) Based on $\mathcal{P}=\cap\{\varepsilon_n\in\zeta^{\tilde{\mathcal{F}}}|\alpha_n\leq\varepsilon_n,\varepsilon_n$ is *r-SVNRIC*$\}$, let $\mathrm{CI}^{\delta £}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\alpha_n,r)\ngeq\mathcal{P}$; therefore, $\upsilon\in\tilde{\mathcal{F}}$ and $s\in(0,1]$, $t\in[0,1),k\in[0,1)]$ exist such that

$$\left.\begin{aligned}&\tilde{\varrho}_{\mathrm{CI}^{\delta £}_{\tilde{\tau}\tilde{\varrho}}(\alpha_n,r)}(\upsilon)<s<\tilde{\varrho}_{\mathcal{P}}(\upsilon)\\ &\tilde{\sigma}_{CI^{\delta £}_{\tilde{\tau}\tilde{\sigma}}(\alpha_n,r)}(\upsilon)\geq t\geq\tilde{\sigma}_{\mathcal{P}}(\upsilon)\\ &\tilde{\varsigma}_{CI^{\delta £}_{\tilde{\tau}\tilde{\varsigma}}(\alpha_n,r)}(\upsilon)\geq k\geq\tilde{\varsigma}_{\mathcal{P}}(\upsilon)\end{aligned}\right\}\tag{10}$$

Therefore, $x_{s,t,k}$ is not an *r-δ£-cluster point* of α_n. As such, $\varepsilon_n \in Q_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}\left(x_{s,t,k}, r\right)$ and $\alpha_n \leq [\mathrm{int}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\varepsilon_n, r)]^c$. Consequently, $\alpha_n \leq [\mathrm{int}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\mathrm{CI}^{\odot}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\varepsilon_n, r), r)]^c = \mathrm{Cl}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\mathrm{int}^{\odot}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}([\varepsilon_n]^c, r), r)$.

Since $\mathrm{Cl}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\mathrm{int}^{\odot}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}([\varepsilon_n]^c, r), r)$ is *r-SVNRIC*, we have $\tilde{\varrho}_{\mathcal{P}}(\upsilon) \leq \tilde{\varrho}_{\mathrm{Cl}_{\tilde{\tau}\tilde{\varrho}}(\mathrm{int}^{\odot}_{\tilde{\tau}\tilde{\varrho}}([\varepsilon_n]^c,r),r)}(\upsilon) < s, \tilde{\sigma}_{\mathcal{P}}(\upsilon) \geq \tilde{\sigma}_{Cl_{\tilde{\tau}\tilde{\sigma}}(int^{\odot}_{\tilde{\tau}\tilde{\sigma}}([\varepsilon_n]^c,r),r)}(\upsilon) > t$ and $\tilde{\varsigma}_{\mathcal{P}}(\upsilon) \geq \tilde{\varsigma}_{Cl_{\tilde{\tau}\tilde{\varsigma}}(int^{\odot}_{\tilde{\tau}\tilde{\varsigma}}([\varepsilon_n]^c,r),r)}(\upsilon) > k$. This is a contradiction to Eq. (10). Therefore, $\mathrm{CI}^{\delta£}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\alpha_n, r) \geq \mathcal{P}$.

Meanwhile, by setting $\mathrm{CI}^{\delta£}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\alpha_n, r) \nleq \mathcal{P}$, then an *r-δ£-cluster point* of $y_{s_1,t_1,k_1} \in P_{s,t,k}(\tilde{\mathcal{F}})$ of α_n exists such that

$$\left.\begin{array}{l} \tilde{\varrho}_{\mathrm{CI}^{\delta£}_{\tilde{\tau}\tilde{\varrho}}(\alpha_n,r)}(y) > s_1 > \tilde{\varrho}_{\mathcal{P}}(y) \\ \tilde{\sigma}_{CI^{\delta£}_{\tilde{\tau}\tilde{\sigma}}(\alpha_n,r)}(y) \leq t_1 \leq \tilde{\sigma}_{\mathcal{P}}(y) \\ \tilde{\varsigma}_{CI^{\delta£}_{\tilde{\tau}\tilde{\varsigma}}(\alpha_n,r)}(y) \leq k_1 \leq \tilde{\varsigma}_{\mathcal{P}}(y) \end{array}\right\} \tag{11}$$

Owing to $\mathcal{P}$, there exists *r-SVNRIC* $\varepsilon_n \in \zeta^{\tilde{\mathcal{F}}}$ *with* $\alpha_n \leq \varepsilon_n$ such that $\tilde{\varrho}_{\mathrm{CI}^{\delta£}_{\tilde{\tau}\tilde{\varrho}}(\alpha_n,r)}(y) > s_1 > \tilde{\varrho}_{\varepsilon_n} \geq \tilde{\varrho}_{\mathcal{P}}(y)$, $\tilde{\sigma}_{CI^{\delta£}_{\tilde{\tau}\tilde{\sigma}}(\alpha_n,r)}(y) \leq t_1 \leq \tilde{\varrho}_{\varepsilon_n} \leq \tilde{\sigma}_{\mathcal{P}}(y)$ and $\tilde{\varsigma}_{CI^{\delta£}_{\tilde{\tau}\tilde{\varsigma}}(\alpha_n,r)}(y) \leq k_1 \leq \tilde{\varrho}_{\varepsilon_n} \leq \tilde{\varsigma}_{\mathcal{P}}(y)$. Therefore, $[\varepsilon_n]^c \in Q_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(y_{s_1,t_1}, k_1)$. So, $\alpha_n \leq \varepsilon_n = [\mathrm{int}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\mathrm{Cl}^{\odot}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}([\varepsilon_n]^c, r), r)]^c$. Hence, $\alpha_n \bar{q} \mathrm{int}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\mathrm{Cl}^{\odot}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}([\varepsilon_n]^c, r), r)$.

Additionally, y_{s_1,t_1,k_1} is not an *r-δ£-cluster point* of α_n, that is, $\tilde{\varrho}_{\mathrm{CI}^{\delta£}_{\tilde{\tau}\tilde{\varrho}}(\alpha_n,r)}(y) < s_1, \tilde{\sigma}_{CI^{\delta£}_{\tilde{\tau}\tilde{\sigma}}(\alpha_n,r)}(y) \geq t_1, \tilde{\varsigma}_{CI^{\delta£}_{\tilde{\tau}\tilde{\varsigma}}(\alpha_n,r)}(y) \geq k_1$. This is a contradiction to Eq. (11). Therefore, $\mathrm{CI}^{\delta£}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\alpha_n, r) \leq \mathcal{P}$,

(e) Suppose that $\mathrm{Cl}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\alpha_n, r) \nleq \mathrm{CI}^{\delta£}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\alpha_n, r)$; therefore, $\upsilon \in \tilde{\mathcal{F}}$ and $[s \in (0,1], t \in [0,1), k \in [0,1)]$ exist such that

$$\left.\begin{array}{l} \tilde{\varrho}_{\mathrm{CI}_{\tilde{\tau}\tilde{\varrho}}(\alpha_n,r)}(\upsilon) > s > \tilde{\varrho}_{\mathrm{CI}^{\delta£}_{\tilde{\tau}\tilde{\varrho}}(\alpha_n,r)}(\upsilon) \\ \tilde{\sigma}_{CI_{\tilde{\tau}\tilde{\sigma}}(\alpha_n,r)}(\upsilon) \leq t \leq \tilde{\sigma}_{CI^{\delta£}_{\tilde{\tau}\tilde{\sigma}}(\alpha_n,r)}(\upsilon) \\ \tilde{\varsigma}_{CI_{\tilde{\tau}\tilde{\sigma}}(\alpha_n,r)}(\upsilon) \leq k \leq \tilde{\varsigma}_{CI^{\delta£}_{\tilde{\tau}\tilde{\varsigma}}(\alpha_n,r)}(\upsilon), \end{array}\right\} \tag{12}$$

Since, $\tilde{\varrho}_{\mathrm{CI}_{\tilde{\tau}\tilde{\varrho}}(\alpha_n,r)}(\upsilon) < s, \tilde{\sigma}_{\mathrm{CI}_{\tilde{\tau}\tilde{\sigma}}(\alpha_n,r)}(\upsilon) \geq t, \tilde{\varsigma}_{\mathrm{CI}_{\tilde{\tau}\tilde{\sigma}}(\alpha_n,r)}(\upsilon) \geq k$, we have $x_{s,t,k}$ not *r-δ£-cluster point* of α_n. Therefore, there exists $\varepsilon_n \in Q_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(x_{s,t,k}, r)$ and $\alpha_n \leq [\mathrm{int}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\mathrm{Cl}^{\odot}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\varepsilon_n, r), r)]^c$. Hence, $\tilde{\varrho}_{\mathrm{Cl}_{\tilde{\tau}\tilde{\varrho}}(\alpha_n,r)}(\upsilon) \leq \tilde{\varrho}_{[\mathrm{int}_{\tilde{\tau}\tilde{\varrho}}(\mathrm{Cl}^{\odot}_{\tilde{\tau}\tilde{\varrho}}(\varepsilon_n,r),r)]^c}(\upsilon) < s$, $\tilde{\sigma}_{Cl_{\tilde{\tau}\tilde{\sigma}}(\alpha_n,r)}(\upsilon) \leq \tilde{\varrho}_{[int_{\tilde{\tau}\tilde{\sigma}}(Cl^{\odot}_{\tilde{\tau}\tilde{\sigma}}(\varepsilon_n,r),r)]^c}(\upsilon) \geq t$ and $\tilde{\varsigma}_{Cl_{\tilde{\tau}\tilde{\varsigma}}(\alpha_n,r)}(\upsilon) \leq \tilde{\varrho}_{[int_{\tilde{\tau}\tilde{\varsigma}}(Cl^{\odot}_{\tilde{\tau}\tilde{\varsigma}}(\varepsilon_n,r),r)]^c}(\upsilon) \geq k$. It is a contradiction for Eq. (12). Thus $\mathrm{Cl}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\alpha_n, r) \leq \mathrm{CI}^{\delta£}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\alpha_n, r)$.

Theorem 4. Let $(\tilde{\mathcal{F}}, \tau^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}, £^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}})$ be an SVNITS, for each $r \in \zeta_0$ and $\alpha_n, \varepsilon_n \in \zeta^{\tilde{\mathcal{F}}}$. Then the following properties hold:

(a) $\alpha_n \leq \mathrm{CI}^{\theta£}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\alpha_n, r)$,

(b) If $\alpha_n \leq \varepsilon_n$, then $\mathrm{CI}^{\theta£}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\alpha_n, r) \leq \mathrm{CI}^{\theta£}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\varepsilon_n, r)$,

(c) $\mathrm{CI}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\alpha_n, r) \leq \cup\{x_{s,t,k} \in P_{s,t,k}(\tilde{\mathcal{F}}) | x_{s,t,k}$ is r-δ£-cluster point of $\alpha_n\}$,

(d) $\mathrm{CI}^{\theta £}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\alpha_n, r) = \cap\{\varepsilon_n \in \zeta^{\tilde{\mathcal{F}}} | \alpha_n \leq \mathrm{int}^{\odot}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\varepsilon_n, r),\ \tau^{\tilde{\varrho}}([\varepsilon_n]^c) \geq r,\ \tau^{\tilde{\sigma}}([\varepsilon_n]^c) \leq 1-r,\ \tau^{\tilde{\varsigma}}([\varepsilon_n]^c) \leq 1-r\}$,

(e) $\mathrm{CI}^{\delta £}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\alpha_n, r) = \cap\{\varepsilon_n \in \zeta^{\tilde{\mathcal{F}}} | \alpha_n \leq \varepsilon_n, \varepsilon_n$ is r-δ£-cluster point of $\alpha_n\}$

(f) $x_{s,t,k}$ is r-θ£-cluster point of α_n iff $x_{s,t,k} \in \mathrm{CI}^{\theta £}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\alpha_n, r)$,

(g) $x_{s,t,k}$ is r-δ£-cluster point of α_n iff $x_{s,t,k} \in \mathrm{CI}^{\delta £}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\alpha_n, r)$,

(h) If $\alpha_n = \mathrm{Cl}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\mathrm{intl}^{\odot}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\alpha_n, r), r)$, then $\mathrm{CI}^{\delta £}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\alpha_n, r) = \alpha_n$,

(i) $\alpha_n \leq \mathrm{CI}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\alpha_n, r) \leq \mathrm{CI}^{\delta £}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\alpha_n, r) \leq \mathrm{CI}^{\theta £}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\alpha_n, r)$,

(j) $W(\alpha_n \vee \varepsilon_n, r) = W(\alpha_n, r) \vee W(\varepsilon_n, r)$ for each $W = \left\{CI^{\theta £}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}, CI^{\delta £}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}\right\}$,

(k) $\mathrm{CI}^{\delta £}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\mathrm{CI}^{\delta £}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\alpha_n, r), r) = \mathrm{CI}^{\delta £}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\alpha_n, r)$.

Proof. (a) and (b) are easily proved from Definition 14.

(c) Set $\mathcal{P} = \cup\{x_{s,t,k} \in P_{s,t,k}(\tilde{\mathcal{F}}) | x_{s,t,k}$ as an r-δ£-cluster point of $\alpha_n\}$. Suppose that $\mathrm{CI}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\alpha_n, r) \nleq \mathcal{P}$. Then there exists $\upsilon \in \tilde{\mathcal{F}}$, and $[s \in (0,1],\ t \in [0,1),\ k \in [0,1)]$ such that

$$\left.\begin{array}{l}\tilde{\varrho}_{\mathrm{CI}_{\tilde{\tau}\tilde{\varrho}}(\alpha_n, r)}(\upsilon) > s > \tilde{\varrho}_{\mathcal{P}}(\upsilon) \\ \tilde{\sigma}_{CI_{\tilde{\tau}\tilde{\sigma}}(\alpha_n, r)}(\upsilon) \leq t \leq \tilde{\sigma}_{\mathcal{P}}(\upsilon) \\ \tilde{\varsigma}_{CI_{\tilde{\tau}\tilde{\varsigma}}(\alpha_n, r)}(\upsilon) \leq k \leq \tilde{\varsigma}_{\mathcal{P}}(\upsilon)\end{array}\right\} \tag{13}$$

Consequently, $x_{s,t,k}$ is not r-δ£-cluster point of α_n. So, there exists $\varepsilon_n \in Q_{\tau\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(x_{s,t,k}, r)$ and

$\alpha_n \leq \left[\mathrm{int}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}\left(\mathrm{CI}^{\odot}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\varepsilon_n, r), r\right)\right]^c \leq [\varepsilon_n]^c$

Based on Eq. (4), $\tilde{\varrho}_{\mathrm{CI}_{\tilde{\tau}\tilde{\varrho}}(\alpha_n, r)}(\upsilon) \leq \tilde{\varrho}_{[\varepsilon_n]^c}(\upsilon) < s, \tilde{\sigma}_{CI_{\tilde{\tau}\tilde{\sigma}}(\alpha_n, r)}(\upsilon) \geq$
$\tilde{\sigma}_{[\varepsilon_n]^c}(\upsilon) \geq t$ and $\tilde{\varsigma}_{CI_{\tilde{\tau}\tilde{\varsigma}}(\alpha_n, r)}(\upsilon) \geq \tilde{\varsigma}_{[\varepsilon_n]^c}(\upsilon) \geq k$.

It is a contradiction for Eq. (13). Thus $\mathrm{CI}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\alpha_n, r) \leq \mathcal{P}$.

(d) $\gamma = \cap\{\varepsilon_n \in \zeta^{\tilde{\mathcal{F}}} | \alpha_n \leq \mathrm{int}^{\odot}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\varepsilon_n, r),\ \tau^{\tilde{\varrho}}([\varepsilon_n]^c) \geq r,\ \tau^{\tilde{\sigma}}([\varepsilon_n]^c) \leq 1-r,\ \tau^{\tilde{\varsigma}}([\varepsilon_n]^c) \leq 1-r\}$.

Suppose that $\mathrm{CI}^{\theta £}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\alpha_n, r) \ngeq \gamma$, then there exists $\upsilon \in \tilde{\mathcal{F}}$ and $[s \in (0,1],\ t \in [0,1),\ k \in [0,1)]$ such that

$$\left.\begin{array}{l}\tilde{\varrho}_{\mathrm{CI}^{\theta £}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\alpha_n, r)}(\upsilon) < s \leq \tilde{\varrho}_{\gamma}(\upsilon) \\ \tilde{\sigma}_{CI^{\theta £}_{\tilde{\tau}\tilde{\sigma}}(\alpha_n, r)}(\upsilon) > t \geq \tilde{\sigma}_{\gamma}(\upsilon) \\ \tilde{\varsigma}_{CI^{\theta £}_{\tilde{\tau}\tilde{\varsigma}}(\alpha_n, r)}(\upsilon) > k \geq \tilde{\varsigma}_{\gamma}(\upsilon)\end{array}\right\} \tag{14}$$

Consequently, $x_{s,t,k}$ is not r-θ£-cluster point of α_n. So, there exists $\varepsilon_n \in Q_{\tau\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(x_{s,t,k}, r)$, $\alpha_n \leq [(\mathrm{Cl}^{\odot}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\varepsilon_n, r), r)]^c$. Thus, $\alpha_n \leq [(\mathrm{Cl}^{\odot}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\varepsilon_n, r), r)]^c = (\mathrm{int}^{\odot}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}([\varepsilon_n]^c, r), r)$, $\tau^{\tilde{\varrho}}(\varepsilon_n) \geq r$, $\tau^{\tilde{\sigma}}(\varepsilon_n) \leq 1-r$, $\tau^{\tilde{\varsigma}}(\varepsilon_n) \leq 1-r\}$. Hence, $\tilde{\varrho}_{\gamma}(\upsilon) \leq \tilde{\varrho}_{[\varepsilon_n]^c}(\upsilon) < s, \tilde{\sigma}_{\gamma}(\upsilon) \leq \tilde{\sigma}_{[\varepsilon_n]^c}(\upsilon) < t, \tilde{\varsigma}_{\gamma}(\upsilon) \leq \tilde{\varsigma}_{[\varepsilon_n]^c}(\upsilon) < k$.

It is a contradiction to Eq. (14). Thus $\mathrm{CI}^{\theta £}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\alpha_n, r) \geq \gamma$.

Suppose that $\mathrm{CI}^{\theta \pounds}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\alpha_n, r) \nleq \gamma$, then there exists r-θ£-cluster point of α_n. $y_{s_1,t_1,k_1} \in P_{s,t,k}(\tilde{\mathcal{F}})$ of α_n, such that

$$\left.\begin{aligned} &\tilde{\varrho}_{\mathrm{CI}^{\theta \pounds}_{\tilde{\tau}\tilde{\varrho}}(\alpha_n, r)}(y) > s_1 > \tilde{\varrho}_{\gamma}(y) \\ &\tilde{\sigma}_{CI^{\theta \pounds}_{\tilde{\tau}\tilde{\sigma}}(\alpha_n, r)}(y) < t_1 \leq \tilde{\sigma}_{\gamma}(y) \\ &\tilde{\varsigma}_{CI^{\theta \pounds}_{\tilde{\tau}\tilde{\varsigma}}(\alpha_n, r)}(y) < k_1 \leq \tilde{\varsigma}_{\gamma}(y) \end{aligned}\right\} \tag{15}$$

By the definition of γ, there exists $\varepsilon_n \in \zeta^{\tilde{\mathcal{F}}}$ with $\tau^{\tilde{\varrho}}(\varepsilon_n) \geq r$, $\tau^{\tilde{\sigma}}(\varepsilon_n) \leq 1 - r, \tau^{\tilde{\varsigma}}(\varepsilon_n) \leq 1 - r$ and $\alpha_n \leq \mathrm{int}^{\odot}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\varepsilon_n, r)$, s.t $\tilde{\varrho}_{\mathrm{CI}^{\theta \pounds}_{\tilde{\tau}\tilde{\varrho}}(\alpha_n, r)}(y) > s_1 > \tilde{\varrho}_{\varepsilon_n}(y) \geq \tilde{\varrho}_{\gamma}(y), \tilde{\sigma}_{CI^{\theta \pounds}_{\tilde{\tau}\tilde{\sigma}}(\alpha_n, r)}(y) < t_1 \leq \tilde{\sigma}_{\varepsilon_n}(y) \leq \tilde{\sigma}_{\gamma}(y)$ and $\tilde{\varsigma}_{CI^{\theta \pounds}_{\tilde{\tau}\tilde{\varsigma}}(\alpha_n, r)}(y) < k_1 \leq \tilde{\varsigma}_{\varepsilon_n}(y) \leq \tilde{\varsigma}_{\gamma}(y)$. Additionally, $[\varepsilon_n]^c \in Q_{\tau\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(y_{s_1,t_1,k_1}, r)$. $\alpha_n \leq \mathrm{int}^{\odot}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\varepsilon_n, r) = [\mathrm{Cl}^{\odot}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}([\varepsilon_n]^c, r)]^c$, implies $\alpha_n \overline{q} \mathrm{Cl}^{\odot}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}([\varepsilon_n]^c, r)$. Hence y_{s_1,t_1,k_1} is not an r-θ£-cluster point of α_n. It is a contradiction for Eq. (15). Thus $\mathrm{CI}^{\theta \pounds}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\alpha_n, r) \leq \gamma$.

(e) Similar results are shown in (c) and (d).

(f) ($\Rightarrow$), clear.

($\Leftarrow$) Suppose that $x_{s,t,k}$ is not an r-θ£-cluster point of α_n. There exists $\varepsilon_n \in Q_{\tau\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(x_{s,t,k}, r)$ such that $\mathrm{Cl}^{\odot}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\varepsilon_n, r) \leq \alpha_n$. Thus $\alpha_n \leq [\mathrm{Cl}^{\odot}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\varepsilon_n, r)]^c = \mathrm{Cl}^{\odot}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}([\varepsilon_n]^c, r)$. By (d), $\tilde{\varrho}_{\mathrm{CI}^{\theta \pounds}_{\tilde{\tau}\tilde{\varrho}}(\alpha_n, r)}(\upsilon) \leq \tilde{\varrho}_{[\varepsilon_n]^c}(\upsilon) < s, \tilde{\sigma}_{CI^{\theta \pounds}_{\tilde{\tau}\tilde{\sigma}}(\alpha_n, r)}(\upsilon) \geq \tilde{\sigma}_{[\varepsilon_n]^c}(\upsilon) > t$ and $\tilde{\varsigma}_{CI^{\theta \pounds}_{\tilde{\tau}\tilde{\varsigma}}(\alpha_n, r)}(\upsilon) \geq \tilde{\varsigma}_{[\varepsilon_n]^c}(\upsilon) > t$. Hence $x_{s,t,k} \notin \mathrm{CI}^{\theta \pounds}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\alpha_n, r)$.

(g) is similarly proved as in (f).

(h) The validity of this axiom is obvious from Theorem 3 (4).

(i) Based on Theorem 3(e), we show that $\mathrm{CI}^{\delta \pounds}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\alpha_n, r) \leq \mathrm{CI}^{\theta \pounds}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\alpha_n, r)$. Suppose that $\mathrm{CI}^{\delta \pounds}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\alpha_n, r) \nleq \mathrm{CI}^{\theta \pounds}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\alpha_n, r)$, then there exists $\upsilon \in \zeta$ and $[s \in (0, 1], t \in [0, 1), k \in [0, 1)]$ such that

$$\left.\begin{aligned} &\tilde{\varrho}_{\mathrm{CI}^{\delta \pounds}_{\tilde{\tau}\tilde{\varrho}}(\alpha_n, r)}(\upsilon) > s > \tilde{\varrho}_{\mathrm{CI}^{\theta \pounds}_{\tilde{\tau}\tilde{\varrho}}(\alpha_n, r)}(\upsilon) \\ &\tilde{\sigma}_{CI^{\delta \pounds}_{\tilde{\tau}\tilde{\sigma}}(\alpha_n, r)}(\upsilon) > t \geq \tilde{\sigma}_{CI^{\theta \pounds}_{\tilde{\tau}\tilde{\sigma}}(\alpha_n, r)}(\upsilon) \\ &\tilde{\varsigma}_{CI^{\delta \pounds}_{\tilde{\varsigma}}(\alpha_n, r)}(\upsilon) > k \geq \tilde{\varsigma}_{CI^{\theta \pounds}_{\tilde{\tau}\tilde{\varsigma}}(\alpha_n, r)}(\upsilon) \end{aligned}\right\} \tag{16}$$

Since $\tilde{\varrho}_{\mathrm{CI}^{\theta \pounds}_{\tilde{\tau}\tilde{\varrho}}(\alpha_n, r)}(\upsilon) < s, \tilde{\sigma}_{\mathrm{CI}^{\theta \pounds}_{\tilde{\tau}\tilde{\sigma}}(\alpha_n, r)}(\upsilon) \leq t$ and $\tilde{\varsigma}_{\mathrm{CI}^{\theta \pounds}_{\tilde{\tau}\tilde{\varsigma}}(\alpha_n, r)}(\upsilon) \leq k$, then we have $x_{s,t.k}$ is not r-θ£-cluster point of α_n So, there exists $\varepsilon_n \in Q_{\tau\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(y_{s_1,t_1,k_1}, r)$, $\alpha_n \leq [\mathrm{Cl}^{\odot}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\varepsilon_n, r)]^c$, implies $A\overline{q}\mathrm{int}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\mathrm{Cl}^{\odot}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\varepsilon_n, r), r)$. Hence, $x_{s,t,k}$ is not r-δ£-cluster point of α_n, by (7), we can get than, $\tilde{\varrho}_{\mathrm{CI}^{\delta \pounds}_{\tilde{\tau}\tilde{\varrho}}(\alpha_n, r)}(\upsilon) < s, \sigma_{\mathrm{CI}^{\delta \pounds}_{\tilde{\tau}\tilde{\sigma}}(\alpha_n, r)}(\upsilon) \geq t, \tilde{\varsigma}_{\mathrm{CI}^{\delta \pounds}_{\tilde{\tau}\tilde{\varsigma}}(\alpha_n, r)}(\upsilon) \geq k$. It is a contradiction for Eq. (16). Thus, $\mathrm{CI}^{\delta \pounds}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\alpha_n, r) \leq \mathrm{CI}^{\theta \pounds}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\alpha_n, r)$.

(j) Let $\mathrm{CI}^{\delta\pounds}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\varepsilon_n, r) \vee \mathrm{CI}^{\delta\pounds}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\alpha_n, r) \not\geq \mathrm{CI}^{\delta\pounds}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\alpha_n \vee \varepsilon_n, r)$. Then there exists $\upsilon \in \tilde{\mathcal{F}}$ such that

$$\left.\begin{array}{l} \tilde{\varrho}_{\mathrm{CI}^{\delta\pounds}_{\tilde{\tau}\tilde{\varrho}}(\varepsilon_n,r)}(\upsilon) \vee \tilde{\varrho}_{\mathrm{CI}^{\delta\pounds}_{\tilde{\tau}\tilde{\varrho}}(\alpha_n,r)}(\upsilon) < s < \tilde{\varrho}_{\mathrm{CI}^{\delta\pounds}_{\tilde{\tau}\tilde{\varrho}}(\alpha_n \vee \varepsilon_n,r)}(\upsilon) \\ \tilde{\sigma}_{CI^{\delta\pounds}_{\tilde{\tau}\tilde{\sigma}}(\varepsilon_n,r)}(\upsilon) \vee \tilde{\sigma}_{CI^{\delta\pounds}_{\tilde{\tau}\tilde{\sigma}}(\alpha_n,r)}(\upsilon) > t > \tilde{\sigma}_{CI^{\delta\pounds}_{\tilde{\tau}\tilde{\varrho}}(\alpha_n \vee \varepsilon_n,r)}(\upsilon) \\ \tilde{\varsigma}_{CI^{\delta\pounds}_{\tilde{\tau}\tilde{\varsigma}}(\varepsilon_n,r)}(\upsilon) \vee \tilde{\varsigma}_{CI^{\delta\pounds}_{\tilde{\tau}\tilde{\sigma}}(\alpha_n,r)}(\upsilon) > t > \tilde{\varsigma}_{CI^{\delta\pounds}_{\tilde{\tau}\tilde{\varsigma}}(\alpha_n \vee \varepsilon_n,r)}(\upsilon) \end{array}\right\} \tag{17}$$

Since $\tilde{\varrho}_{\mathrm{CI}^{\delta\pounds}_{\tilde{\tau}\tilde{\varrho}}(\alpha_n,r)}(\upsilon) < s, \tilde{\sigma}_{CI^{\delta\pounds}_{\tilde{\tau}\tilde{\sigma}}(\alpha_n,r)}(\upsilon) > t,\ \tilde{\varsigma}_{CI^{\delta\pounds}_{\tilde{\tau}\tilde{\varsigma}}(\alpha_n,r)}(\upsilon) > k$ and $\tilde{\varrho}_{\mathrm{CI}^{\delta\pounds}_{\tilde{\tau}\tilde{\varrho}}(\varepsilon_n,r)}(\upsilon) < s, \tilde{\sigma}_{CI^{\delta\pounds}_{\tilde{\tau}\tilde{\sigma}}(\varepsilon_n,r)}(\upsilon) > t,\ \tilde{\varsigma}_{CI^{\delta\pounds}_{\tilde{\tau}\tilde{\varsigma}}(\varepsilon_n,r)}(\upsilon) > k$. We obtain, $x_{s,t,k}$ is not r-$\delta\pounds$-cluster point of α_n and ε_n So, there exists $[\alpha_\mathrm{n}]_1, [\varepsilon_n]_1 \in Q_{\tau\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(x_{s,t,k}, r)$, and $\alpha_n \leq [\mathrm{int}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\mathrm{Cl}^{\odot}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}([\alpha_n]_1, r), r)]^c, \varepsilon_n \leq [int_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(Cl^{\odot}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}([\varepsilon_n]_1, r), r)]^c$. Thus, $[\alpha_n]_1 \wedge [\varepsilon_n]_1 \in Q_{\tau\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}\left(x_{s,t,k}, r\right)$.

Using Eqs. (4) and (5) we obtain,

$$\begin{aligned} \alpha_n \vee \varepsilon_n &\leq \left[\mathrm{int}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}\left(\mathrm{Cl}^{\odot}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}\left([\alpha_n]_1, r\right), r\right) \wedge int_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}\left(Cl^{\odot}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}\left([\varepsilon_n]_1, r\right), r\right)\right]^c \\ &\leq [\mathrm{int}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\mathrm{Cl}^{\odot}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}([\alpha_n]_1, r) \wedge \mathrm{Cl}^{\odot}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}([\varepsilon_n]_1, r), r)]^c \\ &\leq \left[\mathrm{int}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}\left(\mathrm{Cl}^{\odot}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}\left([\alpha_n]_1 \wedge [\varepsilon_n]_1, r\right), r\right)\right]^c. \end{aligned}$$

Therefore, $\alpha_n \vee \varepsilon_n \overline{q}\ \mathrm{int}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\mathrm{Cl}^{\odot}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}([\alpha_n]_1 \wedge [\varepsilon_n]_1, r), r)$. Hence, $x_{s,t,k}$ is not r-$\delta\pounds$-cluster point of $\alpha_n \vee \varepsilon_n$, by (g), $\tilde{\varrho}_{\mathrm{Cl}^{\odot}_{\tilde{\tau}\tilde{\varrho}}(\alpha_n \vee \varepsilon_n, r)}(\upsilon) < s, \tilde{\sigma}_{Cl^{\odot}_{\tilde{\tau}\tilde{\sigma}}\alpha_n \vee \varepsilon_n(,r)}(\upsilon) > t$ and $\tilde{\varsigma}_{Cl^{\odot}_{\tilde{\tau}\tilde{\varsigma}}(\alpha_n \vee \varepsilon_n, r)}(\upsilon) > k$. It is a contradiction for Eq. (17), and hence, $\mathrm{CI}^{\delta\pounds}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\alpha_n \vee \varepsilon_n, r) \leq \mathrm{CI}^{\delta\pounds}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\varepsilon_n, r) \vee \mathrm{CI}^{\delta\pounds}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\alpha_n, r)$.

Meanwhile, $\alpha_n \vee \varepsilon_n \geq \alpha_n$ and $\alpha_n \vee \varepsilon_n \geq \varepsilon_n$. Hence $\mathrm{CI}^{\delta\pounds}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\alpha_n \vee \varepsilon_n, r) \geq \mathrm{CI}^{\delta\pounds}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\varepsilon_n, r) \vee \mathrm{CI}^{\delta\pounds}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\alpha_n, r)$. Therefore, $\mathrm{CI}^{\delta\pounds}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\varepsilon_n, r) \vee \mathrm{CI}^{\delta\pounds}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\alpha_n, r) = \mathrm{CI}^{\delta\pounds}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\alpha_n \vee \varepsilon_n, r)$.

(k) Since $\alpha_n \leq \mathrm{CI}^{\delta\pounds}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\alpha_n, r)$, we have $\mathrm{CI}^{\delta\pounds}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\alpha_n, r) \leq \mathrm{CI}^{\delta\pounds}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\mathrm{CI}^{\delta\pounds}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\alpha_n, r), r)$. On the other hand, suppose that $\mathrm{CI}^{\delta\pounds}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\alpha_n, r) \not\geq \mathrm{CI}^{\delta\pounds}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\mathrm{CI}^{\delta\pounds}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\alpha_n, r), r)$. Then there exists $\upsilon \in \tilde{\mathcal{F}}$ and $[s \in (0, 1], t \in [0, 1), k \in [0, 1)]$ such that

$$\left.\begin{array}{l} \tilde{\varrho}_{\mathrm{CI}^{\delta\pounds}_{\tilde{\tau}\tilde{\varrho}}(\alpha_n,r)}(\upsilon) < s < \tilde{\varrho}_{\mathrm{CI}^{\delta\pounds}_{\tilde{\tau}\tilde{\varrho}}\left(\mathrm{CI}^{\delta\pounds}_{\tilde{\tau}\tilde{\varrho}}(\alpha_n,r),r\right)}(\upsilon) \\ \tilde{\sigma}_{CI^{\delta\pounds}_{\tilde{\tau}\tilde{\sigma}}(\alpha_n,r)}(\upsilon) > t \geq \tilde{\sigma}_{CI^{\delta\pounds}_{\tilde{\tau}\tilde{\varrho}}\left(CI^{\delta\pounds}_{\tilde{\tau}\tilde{\sigma}}(\alpha_n,r),r\right)}(\upsilon) \\ \tilde{\varsigma}_{CI^{\delta\pounds}_{\tilde{\tau}\tilde{\varsigma}}(\alpha_n,r)}(\upsilon) > k \geq \tilde{\varsigma}_{CI^{\delta\pounds}_{\tilde{\tau}\tilde{\varrho}}(CI^{\delta\pounds}_{\tilde{\tau}\tilde{\varsigma}}(\alpha_n,r),r)}(\upsilon) \end{array}\right\} \tag{18}$$

Since $\tilde{\varrho}_{\mathrm{CI}^{\delta\pounds}_{\tilde{\tau}\tilde{\varrho}}(\alpha_n,r)}(\upsilon) < s, \tilde{\sigma}_{\mathrm{CI}^{\delta\pounds}_{\tilde{\tau}\tilde{\sigma}}(\alpha_n,r)}(\upsilon) > t, \tilde{\varsigma}_{\mathrm{CI}^{\delta\pounds}_{\tilde{\tau}\tilde{\varsigma}}(\alpha_n,r)}(\upsilon) > k$, we have $x_{s,t,k}$ is not an r-$\delta\pounds$-cluster point of α_n. So, there exists $\varepsilon_n \in Q_{\tau\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(x_{s,t,k}, r)$ such that $\alpha_n \leq [\mathrm{int}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\mathrm{Cl}^{\odot}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\varepsilon_{n,r),r})]^c = \mathrm{Cl}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\mathrm{int}^{\odot}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\varepsilon_n, r, r)$, since, $\mathrm{Cl}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\mathrm{int}^{\odot}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\varepsilon_n, r, r)$ is r−SVNRIC. Then by Theorem 3(d), $\mathrm{CI}^{\delta\pounds}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\alpha_n, r) \leq \mathrm{Cl}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\mathrm{int}^{\odot}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\varepsilon_n, r, r)$.

Similarly, $\mathrm{CI}^{\delta\pounds}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\mathrm{CI}^{\delta\pounds}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\alpha_n, r), r) \leq \mathrm{CI}^{\delta\pounds}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\mathrm{Cl}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\mathrm{int}^{\odot}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\varepsilon_n, r), r), r) = \mathrm{Cl}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\mathrm{int}^{\odot}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\varepsilon_n, r), r)$. Hence,

$\mathrm{CI}^{\delta\pounds}_{\tilde{\tau}^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}}(\mathrm{CI}^{\delta\pounds}_{\tilde{\tau}^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}}(\alpha_n, r), r) \leq \mathrm{Cl}_{\tilde{\tau}^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}}(\mathrm{int}^{\odot}_{\tilde{\tau}^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}}(\varepsilon_n, r), r) < x_{s,t,k}$. It is a contradiction for Eq. (18).

Theorem 5. Let $(\tilde{\mathcal{F}}, \tau^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}, \pounds^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}})$ be an SVNITS, for $r \in \zeta_0$ and $\alpha_n, \varepsilon_n \in \zeta^{\tilde{\mathcal{F}}}$. Then the following properties hold:

(a) α_n is r-*SVNPIC* iff $\mathrm{CI}_{\tilde{\tau}^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}}(\alpha_n, r) = \mathrm{CI}^{\delta\pounds}_{\tilde{\tau}^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}}(\alpha_n, r)$,

(b) α_n is r-*SVNSIC* iff $\mathrm{CI}_{\tilde{\tau}^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}}(\alpha_n, r) = \mathrm{CI}^{\delta\pounds}_{\tilde{\tau}^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}}(\alpha_n, r)$,

(c) α_n is r-$SVN\alpha IO$ iff $\mathrm{CI}_{\tilde{\tau}^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}}(\alpha_n, r) = \mathrm{CI}^{\delta\pounds}_{\tilde{\tau}^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}}(\alpha_n, r) = \mathrm{CI}^{\theta\pounds}_{\tilde{\tau}^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}}(\alpha_n, r)$.

Proof. (a) Let α_n be an *r-SVNPIC*. Then $\alpha_n \leq \mathrm{CI}_{\tilde{\tau}^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}}(\alpha_n, r)$, and by Theorem 3 (3) and (4), we have

$$\mathrm{CI}^{\delta\pounds}_{\tilde{\tau}^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}}(\alpha_n, r) \leq \mathrm{CI}^{\delta\pounds}_{\tilde{\tau}^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}}\left(\mathrm{CI}_{\tilde{\tau}^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}}\left(\mathrm{int}^{\odot}_{\tilde{\tau}^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}}(\alpha_n, r), r\right), r\right) = \mathrm{Cl}_{\tilde{\tau}^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}}\left(\mathrm{int}^{\odot}_{\tilde{\tau}^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}}(\alpha_n, r), r\right) \leq \mathrm{Cl}_{\tilde{\tau}^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}}(\alpha_n, r)$$
$$\leq \mathrm{CI}^{\delta\pounds}_{\tilde{\tau}^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}}(\alpha_n, r).$$

Conversely, suppose that there exist $\upsilon \in \tilde{\mathcal{F}}$ and $[s \in (0,1],\ t \in [0,1),\ k \in [0,1)]$ such that $\tilde{\varrho}_{\mathrm{CI}^{\delta\pounds}_{\tilde{\tau}^{\tilde{\varrho}}}(\alpha_n,r)}(\upsilon) > s > \tilde{\varrho}_{\mathrm{Cl}_{\tilde{\tau}^{\tilde{\varrho}}}(\alpha_n,r)}(\upsilon), \tilde{\sigma}_{CI^{\delta\pounds}_{\tilde{\tau}^{\tilde{\sigma}}}(\alpha_n,r)}(\upsilon) < t \leq \tilde{\sigma}_{Cl_{\tilde{\tau}^{\tilde{\sigma}}}(\alpha_n,r)}(\upsilon)$ and $\tilde{\varsigma}_{CI^{\delta\pounds}_{\tilde{\tau}^{\tilde{\varsigma}}}(\alpha_n,r)}(\upsilon) < k \leq \tilde{\varsigma}_{Cl_{\tilde{\tau}^{\tilde{\varsigma}}}(\alpha_n,r)}(\upsilon)$. Then $x_{s,t,k}$ is not r-δ-cluster point of α_n. So, there exists $\varepsilon_n \in Q_{\tau^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}}(x_{s,t,k}, r)$, with $\alpha_n \leq [\varepsilon_n]^c$ Since $x_{s,t,k}$ is r-$\delta\pounds$-cluster point of α_n, for $\varepsilon_n \in Q_{\tau^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}}(x_{s,t,k}, r)$, we have $\mathrm{int}_{\tilde{\tau}^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}}(\mathrm{Cl}^{\odot}_{\tilde{\tau}^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}}(\varepsilon_n, r), r) q \alpha_n$. Since,

$$\mathrm{int}_{\tilde{\tau}^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}}\left(\mathrm{Cl}^{\odot}_{\tilde{\tau}^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}}(\varepsilon_n, r), r\right) \leq \mathrm{int}_{\tilde{\tau}^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}}\left(\mathrm{Cl}^{\odot}_{\tilde{\tau}^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}}([\alpha_n]^c, r), r\right),$$

we obtain, $\alpha_n \geq [\mathrm{int}_{\tilde{\tau}^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}}(\mathrm{Cl}^{\odot}_{\tilde{\tau}^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}}(\varepsilon_n, r), r)]^c \geq [\mathrm{int}_{\tilde{\tau}^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}}(\mathrm{Cl}^{\odot}_{\tilde{\tau}^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}}([\alpha_n]^c, r), r)]^c = \mathrm{Cl}_{\tilde{\tau}^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}}(\mathrm{int}^{\odot}_{\tilde{\tau}^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}}([\alpha_n], r), r)$.

Hence, α_n is not *r-SVNIC* set.

(b) Let α_n is an *r-SVNSIC* set. Then, $\alpha_n \leq \mathrm{int}^{\odot}_{\tilde{\tau}^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}}(\mathrm{Cl}_{\tilde{\tau}^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}}([\alpha_n]^c, r), r)$ and$\tau^{\tilde{\varrho}}([\mathrm{Cl}_{\tilde{\tau}^{\tilde{\varrho}}}([\alpha_n, r)]^c \geq r, \tau^{\tilde{\sigma}}([\mathrm{Cl}_{\tilde{\tau}^{\tilde{\sigma}}}([\alpha_n, r)]^c \leq r, \tau^{\tilde{\varsigma}}([\mathrm{Cl}_{\tilde{\tau}^{\tilde{\varsigma}}}([\alpha_n, r)]^c \leq r$. By Theorem 4(d), we have $\mathrm{CI}^{\theta\pounds}_{\tilde{\tau}^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}}(\alpha_n, r) \leq \mathrm{CI}_{\tilde{\tau}^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}}(\alpha_n, r)$,

Conversely, suppose that there exist $\alpha_n \in \zeta^{\tilde{\mathcal{F}}}, r \in \zeta_0, \upsilon \in \tilde{\mathcal{F}}$ and $[s \in (0,1], t \in [0,1),\ k \in [0,1)]$ such that $\tilde{\varrho}_{\mathrm{CI}^{\theta\pounds}_{\tilde{\tau}^{\tilde{\varrho}}}(\alpha_n,r)}(\upsilon) > t > \tilde{\varrho}_{\mathrm{Cl}_{\tilde{\tau}^{\tilde{\varrho}}}(\alpha_n,r)}(\upsilon), \tilde{\sigma}_{CI^{\theta\pounds}_{\tilde{\tau}^{\tilde{\sigma}}}(\alpha_n,r)}(\upsilon) < t \leq \tilde{\sigma}_{Cl_{\tilde{\tau}^{\tilde{\sigma}}}(\alpha_n,r)}(\upsilon)$ and $\tilde{\varsigma}_{CI^{\theta\pounds}_{\tilde{\tau}^{\tilde{\varsigma}}}(\alpha_n,r)}(\upsilon) < t \leq \tilde{\varsigma}_{Cl_{\tilde{\tau}^{\tilde{\varsigma}}}(\alpha_n,r)}(\upsilon)$. Then $[\mathrm{Cl}_{\tilde{\tau}^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}}(\alpha_n, r]^c) = \mathrm{int}_{\tilde{\tau}^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}}([\alpha_n]^c, r) \in Q_{\tau^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}}(x_{s,t,k}, r)$ Since $x_{s,t,k}$ is r-$\theta\pounds$-cluster point of α_n, we have $\mathrm{Cl}^{\odot}_{\tilde{\tau}^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}}(\mathrm{int}_{\tilde{\tau}^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}}([\alpha_n]^c, r), r) q \alpha_n$. It implies $\alpha_n \nleq [\mathrm{Cl}^{\odot}_{\tilde{\tau}^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}}(\mathrm{int}_{\tilde{\tau}^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}}([\alpha_n]^c, r), r)]^c = \mathrm{int}^{\odot}_{\tilde{\tau}^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}}(\mathrm{Cl}_{\tilde{\tau}^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}}(\alpha_n, r), r)$. Thus, α_n is not an *r-SVNSIC*.

(c) Similar results are shown in (a) and (b).

4 *r*-δ£-Closed and *r*-θ£-Closed

In this section, we firstly introduce and analyze the r-δ£-closed and r-θ£-closed of an SVNITS$(\tilde{\mathcal{F}}, \tau^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}, £^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}})$. Subsequently, we define and analyze the single-valued neutrosophic £-regular and the single-valued neutrosophic almost £-regular of $\tilde{\mathcal{F}}$. The findings have resulted in many theorems.

Definition 16. Let $(\tilde{\mathcal{F}}, \tau^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}, £^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}})$ be an SVNITS. For $r \in \zeta_0$ and $\alpha_n, \varepsilon_n \in \zeta^{\tilde{\mathcal{F}}}$. Therefore,

(a) α_n is said to be r-δ£-closed ($[\alpha_n]_{\delta £}$) [resp. r-θ£-closed $[\alpha_n]_{\theta £}$] iff $\mathrm{CI}^{\delta £}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\alpha_n, r) = \alpha_n$ (resp. $\mathrm{CI}^{\theta £}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\alpha_n, r) = \alpha_n$). We define

$$\Delta^{\delta £}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\boldsymbol{\alpha_n}, \boldsymbol{r}) = \cap\{\boldsymbol{\varepsilon_n} | \boldsymbol{\alpha_n} \leq \boldsymbol{\varepsilon_n}, \boldsymbol{\varepsilon_n} = \mathbf{CI}^{\delta £}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\boldsymbol{\varepsilon_n}, \boldsymbol{r})\} \tag{19}$$

$$\Theta^{\theta £}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\boldsymbol{\alpha_n}, \boldsymbol{r}) = \cap\{\boldsymbol{\varepsilon_n} | \boldsymbol{\alpha_n} \leq \boldsymbol{\varepsilon_n}, \boldsymbol{\varepsilon_n} = \mathbf{CI}^{\theta £}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\boldsymbol{\varepsilon_n}, \boldsymbol{r})\} \tag{20}$$

(b) The complement of r-δ£-closed (resp. r-θ£-closed) set is called r-δ£-open (resp. r-θ£-open).

Theorem 6. Let $(\tilde{\mathcal{F}}, \tau^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}, £^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}})$ be an SVNITS. For $r \in \zeta_0$ and $\alpha_n \in \zeta^{\tilde{\mathcal{F}}}$. Then the following properties are holds:

(c). $\Delta^{\delta £}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\alpha_n, r) = \mathrm{CI}^{\delta £}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\alpha_n, r)$,

(d). $\Delta^{\delta £}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\alpha_n, r)$ is r-δ£-closed,

(e). $\Theta^{\theta £}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\alpha_n, r) = \mathrm{CI}^{\theta £}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\Theta^{\delta £}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\alpha_n, r), r)$,

(f). $\Theta^{\theta £}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\alpha_n, r)$ is r-θ£-closed,

(g). $\mathrm{CI}^{\theta £}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\alpha_n, r) \leq \Theta^{\theta £}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\alpha_n, r)$.

Proof. (1) Based on Theorem 4(i,j), $\alpha_n \leq \mathrm{CI}^{\delta £}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\alpha_n, r) = \mathrm{CI}^{\delta £}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\mathrm{CI}^{\delta £}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\alpha_n, r), r)$, which implies $\Delta^{\delta £}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\alpha_n, r) \leq \mathrm{CI}^{\delta £}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\alpha_n, r)$. Suppose that $\Delta^{\delta £}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\alpha_n, r) \ngeq \mathrm{CI}^{\delta £}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\alpha_n, r)$. Then there exist $\upsilon \in \tilde{\mathcal{F}}$ and $[s \in (0,1], t \in [0,1),\ k \in [0,1)]$ such that $\tilde{\varrho}_{\Delta^{\delta £}_{\tilde{\tau}\tilde{\varrho}}(\alpha_n, r)}(\upsilon) < s < \tilde{\varrho}_{\mathrm{CI}^{\delta £}_{\tilde{\tau}\tilde{\varrho}}(\alpha_n, r)}(\upsilon), \tilde{\sigma}_{\Delta^{\delta £}_{\tilde{\tau}\tilde{\sigma}}(\alpha_n, r)}(\upsilon) > t > \tilde{\sigma}_{CI^{\delta £}_{\tilde{\tau}\tilde{\sigma}}(\alpha_n, r)}(\upsilon)$ and $\tilde{\varsigma}_{\Delta^{\delta £}_{\tilde{\tau}\tilde{\varsigma}}(\alpha_n, r)}(\upsilon) > k > \tilde{\varsigma}_{CI^{\delta £}_{\tilde{\tau}\tilde{\varsigma}}(\alpha_n, r)}(\upsilon)$. Based on Eq. (19), there exist $\varepsilon_n \in \zeta^{\tilde{\mathcal{F}}}$ and $\alpha_n \leq \varepsilon_n = \mathrm{CI}^{\delta £}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\varepsilon_n, r)$ such that $\tilde{\varrho}_{\Delta^{\delta £}_{\tilde{\tau}\tilde{\varrho}}(\alpha_n, r)}(\upsilon) \leq \tilde{\varrho}_{\varepsilon_n}(\upsilon) < s < \tilde{\varrho}_{\mathrm{CI}^{\delta £}_{\tilde{\tau}\tilde{\varrho}}(\alpha_n, r)}(\upsilon), \tilde{\sigma}_{\Delta^{\delta £}_{\tilde{\tau}\tilde{\sigma}}(\alpha_n, r)}(\upsilon) \geq \tilde{\varrho}_{\varepsilon_n}(\upsilon) > t > \tilde{\sigma}_{CI^{\delta £}_{\tilde{\tau}\tilde{\sigma}}(\alpha_n, r)}(\upsilon)$ and $\tilde{\varsigma}_{\Delta^{\delta £}_{\tilde{\tau}\tilde{\varsigma}}(\alpha_n, r)}(\upsilon) \geq \tilde{\varrho}_{\varepsilon_n}(\upsilon) > k > \tilde{\varsigma}_{CI^{\delta £}_{\tilde{\tau}\tilde{\varsigma}}(\alpha_n, r)}(\upsilon)$.

Meanwhile, $\mathrm{CI}^{\delta £}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\alpha_n, r) \leq \mathrm{CI}^{\delta £}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\varepsilon_n, r) = \varepsilon_n$, which is a contradiction. Hence, $\Delta^{\delta £}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\alpha_n, r) \geq \mathrm{CI}^{\delta £}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\alpha_n, r)$.

(b) is similar to Theorem 4 (k).

(c) Let $\alpha_n \leq [\varepsilon_n]_i = \mathrm{CI}^{\theta £}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}([\varepsilon_n]_i, r)$. Therefore, $\underset{i \in \Gamma}{\wedge}[\varepsilon_n]_i \leq \mathrm{CI}^{\theta £}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\underset{i \in \Gamma}{\wedge}[\varepsilon_n]_i, r) \leq \mathrm{CI}^{\theta £}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}([\varepsilon_n]_i, r) = [\varepsilon_n]_i$. Consequently, $\wedge_{i \in \Gamma}[\varepsilon_n]_i \leq C_{\theta \mathcal{J}\tau}(\wedge_{i \in \Gamma}[\varepsilon_n]_i, r)$. Hence, $\Theta^{\theta £}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\alpha_n, r) = \mathrm{CI}^{\theta £}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\Theta^{\theta £}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\alpha_n, r), r)$.

(d) It is directly obtained from (c).

(e) Since $\alpha_n \le \Theta^{\theta £}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\alpha_n, r)$, by (c) and Eq. (19), $\mathrm{CI}^{\theta £}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\alpha_n, r) \le \mathrm{CI}^{\theta £}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\Theta^{\theta £}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\alpha_n, r), r) = \Theta^{\theta £}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\alpha_n, r)$.

Definition 17. Let $(\tilde{\mathcal{F}}, \tau^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}, £^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}})$ be an SVNITS, $\alpha_n, \varepsilon_n \in \zeta^{\tilde{\mathcal{F}}}$, and $r \in \zeta_0$. Then $\tilde{\tilde{\mathcal{F}}}$ is called,

(a) single valued neutrosophic £-regular (SVN£-regular) if for any $\alpha_n \in Q_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(x_{s,t,k}, r)$, there exists $\varepsilon_n \in Q_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(x_{s,t,k}, r)$ such that $\mathrm{Cl}^{\odot}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\varepsilon_n, r) \le \alpha_n$,

(b) single valued neutrosophic almost £-regular ($SVNA$£-regular), if for any $\alpha_n \in \mathfrak{R}^{£}_{\tau\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(x_{s,t,k}, r)$, then there exists $\varepsilon_n \in \mathfrak{R}^{£}_{\tau\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(x_{s,t,k}, r)$ such that $\mathrm{Cl}^{\odot}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\varepsilon_n, r) \le \alpha_n$.

Theorem 7. Let $(\tilde{\mathcal{F}}, \tau^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}, £^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}})$ be an SVNITS, $\alpha_n, \varepsilon_n \in \zeta^{\tilde{\mathcal{F}}}$ and $r \in \zeta_0$. Then the following statements are equivalent:

(a) $(\tilde{\mathcal{F}}, \tau^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}, £^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}})$ is called SVN£-regular,

(b) For each $x_{s,t,k} \in P_{s,t,k}\left(\tilde{\mathcal{F}}\right)$ and $\alpha_n \in Q_{\tau\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}\left(x_{s,t,k}, r\right)$, there exists $\varepsilon_n \in \mathfrak{R}^{£}_{\tau\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}\left(x_{s,t,k}, r\right)$ such that $\mathrm{Cl}^{\odot}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\varepsilon_n, r) \le \mathrm{int}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}\left(\mathrm{Cl}^{\odot}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\alpha_n, r), r\right)$,

(c) For each $x_{s,t,k} \in P_{s,t,k}\left(\tilde{\mathcal{F}}\right)$ and each $\alpha_n \in Q_{\tau\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}\left(x_{s,t,k}, r\right)$, there exists $\varepsilon_n \in Q_{\tau\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}\left(x_{s,t,k}, r\right)$ such that $\mathrm{Cl}^{\odot}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\varepsilon_n, r) \le \mathrm{int}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}\left(\mathrm{Cl}^{\odot}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\alpha_n, r), r\right)$,

(d) For each $x_{s,t,k} \in P_{s,t,k}(\tilde{\mathcal{F}})$ and r-$SVNRIC$ set $\omega_n \in \zeta^{\tilde{\mathcal{F}}}$ with $x_{s,t,k} \notin \omega_n$, there exists $\varepsilon_n \in Q_{\tau\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(x_{s,t,k}, r)$ and α_n is r-$SVN\star$-open set such that $\omega_n \le \alpha_n$ and $\mathrm{Cl}^{\odot}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\alpha_n, r)\overline{q}\mathrm{Cl}^{\odot}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\varepsilon_n, r)$,

(e) For each $x_{s,t,k} \in P_{s,t,k}\left(\tilde{\mathcal{F}}\right)$ and r-$SVNRIC$ set $\omega_n \in \zeta^{\tilde{\mathcal{F}}}$ with $x_{s,t,k} \notin \omega_n$, there exists $\varepsilon_n \in Q_{\tau\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}\left(x_{s,t,k}, r\right)$ and α_n is r-$SVN\star$-open set such that $\omega_n \le \alpha_n$ and $\mathrm{Cl}^{\odot}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\varepsilon_n, r)\,\overline{q}\alpha_n$,

(f) For each r-$SVNRIO$ set $\alpha_n \in \zeta^{\tilde{\mathcal{F}}}$ with $\omega_n q \alpha_n$, there exists r-$SVNRIO$ set $\varepsilon_n \in \zeta^{\tilde{\mathcal{F}}}$ such that $\omega_n q \varepsilon_n \le \mathrm{Cl}^{\odot}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\varepsilon_n, r) \le \alpha_n$.

(g) For each r-$SVNRIC$ set $\alpha_n \in \zeta^{\tilde{\mathcal{F}}}$ with $\omega_n \not\le \alpha_n$, there exists r-$SVNRIO$ set $\varepsilon_n \in \zeta^{\tilde{\mathcal{F}}}$ and is r-$SVN\star$-open set $\pi_n \in \zeta^{\tilde{\mathcal{F}}}$ such that $\omega_n q \varepsilon_n$, $\alpha_n \le \pi_n$ and $\varepsilon_n \overline{q} \pi_n$.

Proof. The proof of (a)⇒(b) and (b)⇒(c) are clear.

(c)⇒(a): $x_{s,t,k} \in P_{s,t,k}(\tilde{\mathcal{F}})$ and $\alpha_n \in \mathfrak{R}^{£}_{\tau\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(x_{s,t,k}, r)$. Then, by (c), there exists $\varepsilon_n \in Q_{\tau\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(x_{s,t,k}, r)$ such that $\mathrm{Cl}^{\odot}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\varepsilon_n, r) \le \mathrm{int}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}((\mathrm{Cl}^{\odot}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\alpha_n, r), r) = \alpha_n$. since, $\varepsilon_n \in Q_{\tau\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(x_{s,t,k}, r)$ we have $\mathrm{int}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\mathrm{Cl}^{\odot}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\varepsilon_n, r), r) \in \mathfrak{R}^{£}_{\tau\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(x_{s,t,k}, r)$.

Moreover, since, $\omega_n = \mathrm{int}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\mathrm{Cl}^{\odot}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\varepsilon_n, r), r) \le \mathrm{Cl}^{\odot}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\varepsilon_n, r)$, we have $\mathrm{Cl}^{\odot}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\omega_n, r) \le \mathrm{Cl}^{\odot}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\varepsilon_n, r)$, and hence $x_{s,t,k} q \omega_n \le \mathrm{Cl}^{\odot}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\omega_n, r) \le \mathrm{Cl}^{\odot}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\varepsilon_n, r) \le \alpha_n$ where $\omega_n \in \mathfrak{R}^{£}_{\tau\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(x_{s,t,k}, r)$.

(c)⇒(d): Let ω_n be an r-$SVNRIC$ set in $\tilde{\mathcal{F}}$ and $x_t \in \mathrm{P}_{s,t,k}(\tilde{\mathcal{F}})$ with $x_{s,t,k} \notin \omega_n$. Then $x_{s,t,k} q [\omega_n]^c$ and $[\omega_n]^c \in \mathfrak{R}^{£}_{\tau\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(x_{s,t,k}, r) \subset Q_{\tau\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(x_{s,t,k}, r)$. By (c), there exists $\pi_n \in Q_{\tau\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(x_{s,t,k}, r)$ such that

$$\mathrm{Cl}^{\odot}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\pi_n, r) \le \mathrm{int}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}\left(\mathrm{Cl}^{\odot}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}\left([\omega_n]^c, r\right), r\right) = [\omega_n]^c.$$

Next, $x_{s,t,k} q \mathrm{int}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}\left(\mathrm{Cl}^{\odot}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\pi_n, r), r\right)$, then $\mathrm{int}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}\left(\mathrm{Cl}^{\odot}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\pi_n, r), r\right) \in Q_{\tau^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}}\left(x_{s,t,k}, r\right)$, and hence by hypothesis, there exists $\varepsilon_n \in Q_{\tau^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}}(x_{s,t,k}, r)$ such that $\mathrm{Cl}^{\odot}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\varepsilon_n, r) \leq \mathrm{int}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\mathrm{Cl}^{\odot}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\pi_n, r), r)$. Then, $\omega_n \leq [\mathrm{Cl}^{\odot}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\pi_n, r), r)]^c$. Put $\alpha_n = [\mathrm{Cl}^{\odot}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\pi_n, r), r)]^c$ then α_n is *r-SVN⋆O* set. Hence

$$\mathrm{Cl}^{\odot}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\alpha_n, r) \leq \left[\mathrm{int}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}\left(\mathrm{Cl}^{\odot}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\pi_n, r), r\right)\right]^c \leq \left[\mathrm{Cl}^{\odot}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\varepsilon_n, r)\right].$$

Therefore, $\mathrm{Cl}^{\odot}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\varepsilon_n, r)\overline{q}\mathrm{Cl}^{\odot}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\alpha_n, r)$.

(d)⇒(e): It is trivial.

(e)⇒(f): Suppose that α_n is an *r-SVNRIO* set with $\omega_n q \alpha_n$, then $\omega_n \not\leq [\alpha_n]^c$. Hence there exists $x_{s,t,k} \in \mathrm{P}_{s,t,k}(\tilde{\mathcal{F}})$ such that $x_{s,t,k} \in \omega_n$ and $\omega_n \not\leq [\alpha_n]^c$ where $[\alpha_n]^c$ is *r-SVNRIC* set. By (e), there exists $\varepsilon_n \in Q_{\tau^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}}(x_{s,t,k}, r)$ and $\pi_n \in \zeta^{\tilde{\mathcal{F}}}$ is *r-SVN⋆O* set such that $[\alpha_n]^c \leq \pi_n$ and $\mathrm{Cl}^{\odot}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\varepsilon_n, r)\overline{q}\pi_n$. From $\varepsilon_n \in Q_{\tau^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}}(x_{s,t,k}, r)$ we have $x_{s,t,k} q \varepsilon_n \leq \mathrm{int}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\mathrm{Cl}^{\odot}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\varepsilon_n, r), r)$.

By setting $[\varepsilon_n]_1 = \mathrm{int}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\mathrm{Cl}^{\odot}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\varepsilon_n, r), r)$, we have $\omega_n q [\varepsilon_n]_1$ and $[\varepsilon_n]_1$ is *r-SVNRIO* set such that

$$\omega_n q [\varepsilon_n]_1 \leq \mathrm{Cl}^{\odot}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}\left([\varepsilon_n]_1, r\right) \leq \mathrm{Cl}^{\odot}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\varepsilon_n, r) \leq \underline{1} - \pi_n \leq \alpha_n$$

(f)⇒(g): Let α_n be an *r-SVNRIC* set $\alpha_n \in \zeta^{\tilde{\mathcal{F}}}$ with $\omega_n \not\leq \alpha_n$. Therefore, $\omega_n q [\alpha_n]^c$ and hence by, then there exists an *r-SVNRIO* set $\varepsilon_n \in \zeta^{\tilde{\mathcal{F}}}$ such that $\omega_n q \varepsilon_n \leq \mathrm{Cl}^{\odot}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\varepsilon_n, r) \leq [\alpha_n]^c$. Then, ε_n is an *r-SVNRIO* set and $[\mathrm{Cl}^{\odot}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\varepsilon_n, r)]^c$ is an *r-SVN⋆O* set such that $\omega_n q \varepsilon_n$, $\alpha_n \leq [\mathrm{Cl}^{\odot}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\varepsilon_n, r)]^c$ and $\varepsilon_n \overline{q} [\mathrm{Cl}^{\odot}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\varepsilon_n, r)]^c$.

(g)⇒(a): Let $\alpha_n \in \mathfrak{R}^{\pounds}_{\tau^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}}(x_{s,t,k}, r)$ Then $x_{s,t,k} \not\leq [\alpha_n]^c$ and $[\alpha_n]^c$ is an *r-SVNRIC* set. By (g), there exist *r-SVNRIO* set $\varepsilon_n \in \zeta^{\tilde{\mathcal{F}}}$ and it is *r-SVN⋆O* set $\pi_n \in \zeta^{\tilde{\mathcal{F}}}$ such that $x_{s,t,k} q \varepsilon_n$, $[\alpha_n]^c \leq \pi_n$ and $\varepsilon_n \overline{q} \pi_n$. Then, $\varepsilon_n \in \mathfrak{R}^{\pounds}_{\tau^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}}(x_{s,t,k}, r)$. Since, π_n is *r-SVN⋆O* set, $\mathrm{Cl}^{\odot}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\varepsilon_n, r)\overline{q}\pi_n$. Therefore, $x_{s,t,k} q \varepsilon_n \leq \mathrm{Cl}^{\odot}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\varepsilon_n, r) \leq [\pi_n]^c \leq \alpha_n$. Hence $(\tilde{\mathcal{F}}, \tau^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}, \pounds^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}})$ is *SVN*£-regular.

Theorem 8. Let $(\tilde{\mathcal{F}}, \tau^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}, \pounds^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}})$ be an SVNITS, $\alpha_n \in \zeta^{\tilde{\mathcal{F}}}$ and $r \in \zeta_0$. Then the following statements are equivalent:

(a) $(\tilde{\mathcal{F}}, \tau^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}, \pounds^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}})$ is called *SVN*£-regular,

(b) For each $x_{s,t,k} \in P_{s,t,k}(\tilde{\mathcal{F}})$, $\alpha_n \in \zeta^{\tilde{\mathcal{F}}}$ with $\tau^{\tilde{\varrho}}([\alpha_n]^c) \geq r, \tau^{\tilde{\sigma}}([\alpha_n]^c) \leq 1 - r, \tau^{\tilde{\varsigma}}([\alpha_n]^c) \leq 1 - r$, and $x_{s,t,k} \notin \alpha_n$, there exists $\varepsilon_n \in \zeta^{\tilde{\mathcal{F}}}$ with ε_n is r-*SVN⋆O* such that $x_{s,t,k} \notin \mathrm{Cl}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\varepsilon_n, r)$ and $\alpha_n \leq \varepsilon_n$,

(c) For each $x_{s,t,k} \in P_{s,t,k}(\tilde{\mathcal{F}})$, $\alpha_n \in \zeta^{\tilde{\mathcal{F}}}$ with $\tau^{\tilde{\varrho}}([\alpha_n]^c) \geq r$, $\tau^{\tilde{\sigma}}([\alpha_n]^c) \leq 1 - r$, $\tau^{\tilde{\varsigma}}([\alpha_n]^c) \leq 1 - r$, and $x_{s,t,k} \notin \alpha_n$, there exists, $\varepsilon_n \in Q_{\tau^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}}(x_{s,t,k}, r)$ and $\pi_n \in \zeta^{\tilde{\mathcal{F}}}$ with π_n is r-*SVN⋆O* such that $\alpha_n \leq \varepsilon_n$ and $\varepsilon_n \overline{q} \pi_n$,

(d) For each $\omega_n, \alpha_n \in \zeta^{\tilde{\mathcal{F}}}$ with $\tau^{\tilde{\varrho}}([\alpha_n]^c) \geq r$, $\tau^{\tilde{\sigma}}([\alpha_n]^c) \leq 1 - r$, $\tau^{\tilde{\varsigma}}([\alpha_n]^c) \leq 1 - r$, and $\omega_n \not\leq \alpha_n$, then there exists $\varepsilon_n \in Q_{\tau^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}}(x_{s,t,k}, r)$ and $\varepsilon_n, \pi_n \in \zeta^{\tilde{\mathcal{F}}}$ with $\tau^{\tilde{\varrho}}(\varepsilon_n) \geq r, \tau^{\tilde{\sigma}}(\varepsilon_n) \leq 1 - r, \tau^{\tilde{\varsigma}}(\varepsilon_n) \leq 1 - r$ and π_n is r-*SVN⋆O* sets such that $\omega_n q \varepsilon_n$, $\alpha_n \leq \pi_n$ and $\varepsilon_n \overline{q} \pi_n$.

Proof. Similar to the proof of Theorem 7.

Theorem 9. An SVNITS $(\tilde{\mathcal{F}}, \tau^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}, £^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}})$ is $SVNA£$-regular iff for each $\alpha_n \in \zeta^{\tilde{\mathcal{F}}}$ and $r \in \zeta_0, \mathrm{CI}^{\delta£}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\alpha_n, r) = \mathrm{CI}^{\theta£}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\alpha_n, r)$.

Proof. From Theorem 4(i), we only show that $\mathrm{CI}^{\delta£}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\alpha_n, r) \geq \mathrm{CI}^{\theta£}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\alpha_n, r)$.

Suppose that $\mathrm{CI}^{\delta£}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\alpha_n, r) \ngeq \mathrm{CI}^{\theta£}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\alpha_n, r)$. Then there exist $\upsilon \in \tilde{\mathcal{F}}$ and $[s \in (0,1], t \in [0,1), k \in [0,1)]$ such that

$$\left.\begin{array}{l} \tilde{\varrho}_{\mathrm{CI}^{\delta£}_{\tilde{\tau}\tilde{\varrho}}(\alpha_n,r)}(\upsilon) < s < \tilde{\varrho}_{\mathrm{CI}^{\theta£}_{\tilde{\tau}\tilde{\varrho}}(\alpha_n,r)}(\upsilon) \\ \tilde{\sigma}_{CI^{\delta£}_{\tilde{\tau}\tilde{\sigma}}(\alpha_n,r)}(\upsilon) > t > \tilde{\sigma}_{CI^{\theta£}_{\tilde{\tau}\tilde{\sigma}}(\alpha_n,r)}(\upsilon) \\ \tilde{\varsigma}_{CI^{\delta£}_{\tilde{\tau}\tilde{\varsigma}}(\alpha_n,r)}(\upsilon) > k > \tilde{\varsigma}_{CI^{\theta£}_{\tilde{\tau}\tilde{\varsigma}}(\alpha_n,r)}(\upsilon) \end{array}\right\} \tag{21}$$

Because $\tilde{\varrho}_{\mathrm{CI}^{\delta£}_{\tilde{\tau}\tilde{\varrho}}(\alpha_n,r)}(\upsilon) < s, \tilde{\sigma}_{\mathrm{CI}^{\delta£}_{\tilde{\tau}\tilde{\sigma}}(\alpha_n,r)}(\upsilon) > t, \tilde{\varsigma}_{\mathrm{CI}^{\delta£}_{\tilde{\tau}\tilde{\varsigma}}(\alpha_n,r)}(\upsilon) > k$, and $x_{s,t,k}$ is not an r-$\delta£$-cluster point of α_n. So, there exists $\varepsilon_n \in Q_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(x_{s,t,k}, r)$ with $\alpha_n \leq [\mathrm{int}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\mathrm{Cl}^{\odot}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\varepsilon_n, r), r)]^c$ Since $\varepsilon_n \in Q_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(x_{s,t,k}, r)$ we have $\mathrm{int}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\mathrm{Cl}^{\odot}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\varepsilon_n, r), r) \in \mathfrak{R}^{£}_{\tau\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(x_{s,t,k}, r)$. By $SVNA£$-regularity of $\tilde{\mathcal{F}}$, there exists $\omega_n \in \mathfrak{R}^{£}_{\tau\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(x_{s,t,k}, r)$ such that $\mathrm{Cl}^{\odot}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\omega_n, r), r) \leq \mathrm{int}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\mathrm{Cl}^{\odot}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\varepsilon_n, r), r)$. Thus,

$$\alpha_n \leq \left[\mathrm{int}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}\left(\mathrm{Cl}^{\odot}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\varepsilon_n, r), r\right)\right]^c \leq \left[\mathrm{Cl}^{\odot}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\omega_n, r)\right]^c = \mathrm{int}^{\odot}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}\left([\omega_n]^c, r\right),$$

and $\tau^{\tilde{\varrho}}(\omega_n) \geq r, \tau^{\tilde{\sigma}}(\omega_n) \leq 1 - r, \tau^{\tilde{\varsigma}}(\omega_n) \leq 1 - r$. By Theorem 4(d), $\tilde{\varrho}_{\mathrm{CI}^{\theta£}_{\tilde{\tau}\tilde{\varrho}}(\alpha_n)}(\upsilon) \leq \tilde{\varrho}_{[\omega_n]^c}(\upsilon) < s, \tilde{\sigma}_{CI^{\theta£}_{\tilde{\tau}\tilde{\sigma}}(\alpha_n,r)}(\upsilon) \geq \tilde{\sigma}_{[\omega_n]^c}(\upsilon) > t$ and $\tilde{\varsigma}_{CI^{\theta£}_{\tilde{\tau}\tilde{\varsigma}}(\alpha_n,r)}(\upsilon) \geq \tilde{\varsigma}_{[\omega_n]^c}(\upsilon) > k$. It is a contradiction for Eq. (21).

Conversely, let $\alpha_n \in \mathfrak{R}^{£}_{\tau\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(x_{s,t,k}, r) \subset Q_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(x_{s,t,k})$. Then by Theorem 4(h), $s > \tilde{\varrho}_{[\alpha_n]^n}(\upsilon) = \tilde{\varrho}_{\mathrm{CI}^{\delta£}_{\tilde{\tau}\tilde{\varrho}}([\alpha_n]^c,r)}(\upsilon), s > \tilde{\varrho}_{[\alpha_n]^n}(\upsilon) = \tilde{\varrho}_{\mathrm{CI}^{\delta£}_{\tilde{\tau}\tilde{\varrho}}([\alpha_n]^c,r)}(\upsilon)$ and $k < \tilde{\sigma}_{[\alpha_n]^n}(\upsilon) = \tilde{\sigma}_{CI^{\delta£}_{\tilde{\tau}\tilde{\sigma}}([\alpha_n]^c,r)}(\upsilon)$. Since, $\mathrm{CI}^{\delta£}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}([\alpha_n]^c, r) = \mathrm{CI}^{\theta£}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}([\alpha_n]^c, r)$, $x_{s,t,k}$ is not an r-$\theta\mathcal{J}$-cluster point of $[\alpha_n]^c$. Then there exists $\varepsilon_n \in Q_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(x_{s,t,k}, r)$ such that $[\alpha_n]^c \overline{q} \mathrm{Cl}^{\odot}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\varepsilon_n, r)$ implies $\mathrm{Cl}^{\odot}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\varepsilon_n, r) \leq \alpha_n = \mathrm{int}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\mathrm{Cl}^{\odot}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\alpha_n, r), r)$ and by Theorem 7(c), $(\tilde{\mathcal{F}}, \tau^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}, £^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}})$ is $SVNA£$-regular.

Theorem 10. An SVNITS$(\tilde{\mathcal{F}}, \tau^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}, £^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}})$ is $SVNA£$-regular iff for each r-$SVNRIC$ set $\alpha_n \in \zeta^{\tilde{\mathcal{F}}}$ and $\in \zeta_0, \mathrm{CI}^{\theta£}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\alpha_n, r) = \alpha_n$.

Proof. The proof is similar to Theorem 9; additionally, r-$SVNRIC$ set is r-$\delta£$-closed.

Conversely, let α_n be any r-FRIC set with $x_t \notin \alpha_n$. Then, $x_t \notin \mathrm{CI}^{\theta£}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\alpha_n, r)$ and hence, x_t is not r-$\theta£$-cluster point of α_n so, there there exists $\varepsilon_n \in Q_{\tau\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(x_{s,t,k}, r)$ such that $\alpha_n \overline{q} \mathrm{Cl}^{\odot}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\varepsilon_n, r)$. Thus, $\alpha_n \leq [\mathrm{Cl}^{\odot}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\varepsilon_n, r)]^c = \omega_n$ and ω_n is r-$SVN\star O$ implies $\omega_n \overline{q} \mathrm{Cl}^{\odot}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\varepsilon_n, r)$. Hence, by Theorem 4(e), $(\tilde{\mathcal{F}}, \tau^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}, £^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}})$ is SVNA£-regular.

Lemma 1. If $\alpha_n, \varepsilon_n \in \zeta^{\tilde{\mathcal{F}}}$, $r \in \zeta_0$ such that $\alpha_n \overline{q} \varepsilon_n$ where ε_n is r-$\delta£$-open, then $\mathrm{CI}^{\delta£}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\alpha_n, r) \overline{q} \varepsilon_n$.

Proof. Let $\alpha_n \overline{q} \varepsilon_n$ where ε_n is r-δ£-open. Then, $\alpha_n \leq [\varepsilon_n]^c = \mathrm{CI}^{\delta £}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}([\varepsilon_n]^c$, by Theorem 4(k), $\mathrm{CI}^{\delta £}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\alpha_n, r) \leq \mathrm{CI}^{\delta £}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\mathrm{CI}^{\delta £}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}([\varepsilon_n]^c, r), r) = \mathrm{CI}^{\delta £}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}([\varepsilon_n]^c, r) = [\varepsilon_n]^c$. Hence, $\mathrm{CI}^{\delta £}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\boldsymbol{\mathcal{A}}, r) \overline{q} \varepsilon_n$.

Lemma 2. Let $(\tilde{\mathcal{F}}, \tau^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}, £^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}})$ be an SVNITS and $\alpha_n \in \zeta^{(\tilde{\mathcal{F}}}$ is δ£-open iff for each $x_{x,t,k} \in Q_{\tau^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}}(x_{s,t,k}, r)$ with $x_{s,t,k} q \alpha_n$, there exists r-$SVNRIO$ set $\varepsilon_n \in \zeta^{\tilde{\mathcal{F}}}$ such that $x_{x,t,k} q \varepsilon_n \leq \alpha_n$.

Proof. Let $x_{x,t,k} \in \mathrm{P}_{s,t,k}((\tilde{\mathcal{F}})$ with $x_{x,t,k} q \alpha_n$ Then $x_{x,t,k} \notin \alpha_n]^c$. Since α_n is an r-δ£-open set, $x_{x,t,k} \notin [\alpha_n]^c = \mathrm{CI}^{\delta £}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}([\alpha_n]^c, r)$. Thus, $x_{x,t,k}$ is not r-δ£-cluster point of $[\alpha_n]^c$. So, there exists $\omega_n \in Q_{\tau^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}}(x_{s,t,k}, r)$ such that $[\alpha_n]^c \overline{q} \mathrm{CI}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\mathrm{Cl}^{\odot}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\omega_n, r), r)$. Put $\varepsilon_n = \mathrm{int}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\mathrm{Cl}^{\odot}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\omega_n, r), r)$, so, ε_n is an r-$SVNRIO$ set with $x_{x,t,k} q \varepsilon_n \leq \alpha_n$.

Conversely, let $[\alpha_n]^c \neq \mathrm{CI}^{\delta £}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}([\alpha_n]^c, r)$, then there exist $\upsilon \in \tilde{\mathcal{F}}$ and $s, t, k \in \zeta_0$ such that

$$\left.\begin{array}{l} \tilde{\varrho}_{[\alpha_n]^c}(\upsilon) < s < \tilde{\varrho}_{\mathrm{CI}^{\delta £}_{\tilde{\tau}\tilde{\varrho}}([\alpha_n]^c, r)}(\upsilon) \\ \tilde{\sigma}_{[\alpha_n]^c}(\upsilon) > t > \tilde{\sigma}_{CI^{\delta £}_{\tilde{\tau}\tilde{\sigma}}([\alpha_n]^c, r)}(\upsilon) \\ \tilde{\varsigma}_{[\alpha_n]^c}(\upsilon) > k > \tilde{\varsigma}_{CI^{\delta £}_{\tilde{\tau}\tilde{\varsigma}}([\alpha_n]^c, r)}(\upsilon). \end{array}\right\} \tag{22}$$

Because of $x_{x,t,k} q \alpha_n$, then there exists an r-$SVNRIO$ set ε_n such that $x_{x,t,k} q \varepsilon_n \leq \alpha_n$. This implies $[\alpha_n]^c \leq [\varepsilon_n]^n = \mathrm{Cl}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\mathrm{int}^{\odot}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}([\varepsilon_n]^n, r), r)$. By Theorem 3(d), we have $\tilde{\varrho}_{\mathrm{CI}^{\delta £}_{\tilde{\tau}\tilde{\varrho}}([\alpha_n]^c, r)}(\upsilon) \tilde{\varrho}_{\leq([\varepsilon_n]^n)}(\upsilon) < s, \tilde{\sigma}_{CI^{\delta £}_{\tilde{\tau}\tilde{\sigma}}([\alpha_n]^c, r)}(\upsilon) \tilde{\sigma}_{\leq([\varepsilon_n]^n)}(\upsilon) > t$ and $\tilde{\varsigma}_{CI^{\delta £}_{\tilde{\tau}\tilde{\varsigma}}([\alpha_n]^c, r)}(\upsilon) \tilde{\varsigma}_{\leq([\varepsilon_n]^n)}(\upsilon) > k$. It is a contradiction for Eq. (22). Hence, $[\alpha_n]^c = \mathrm{CI}^{\delta £}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}([\alpha_n]^c, r)$, i.e., α_n is an r-δ£-open set.

Lemma 3. If $\tau^{\tilde{\varrho}}(\alpha_n) \geq r, \tau^{\tilde{\sigma}}(\alpha_n) \leq 1 - r, \tau^{\tilde{\varsigma}}(\alpha_n) \leq 1 - r$, then $\mathrm{CI}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\alpha_n, r) = \mathrm{CI}^{\delta £}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\alpha_n, r)$.

Proof. Follows easily by virtue of Theorem 4.

Theorem 11. Let $(\tilde{\mathcal{F}}, \tau^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}, £^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}})$ be an SVNITS. Then the following statements are equivalent:

(a) $(\tilde{\mathcal{F}}, \tau^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}, £^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}})$ is $SVNA$£-regular,

(b) For each $r - \delta$£-open set $\alpha_n \in \zeta^{\tilde{\mathcal{F}}}$ and each $x_{x,t,k} \in \mathrm{P}_{s,t,k}(\tilde{\mathcal{F}})$ with $x_{s,t,k} q \boldsymbol{\mathcal{A}}$, there exists r-δ£-open set $\varepsilon_n \in \zeta^{\tilde{\mathcal{F}}}$ such that $x_{x,t,k} q \varepsilon_n \leq \mathrm{Cl}^{\odot}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\varepsilon_n, r) \leq \alpha_n$.

Proof. (a)⇒(b): Let α_n be r-fuzzy $\delta\mathcal{J}$-open set such each $x_{s,t,k} q \alpha_n$. Then by Lemma 3, there exists an r-$SVNRIO$ set $\pi_n \in \zeta^{\tilde{\mathcal{F}}}$ such that $x_{s,t,k} q \pi_n \leq \alpha_n$. By $SVNA$£-regularity of X, there exists an r-**FRIO** set ε_n (which is also r-δ£-open such that $x_{s,t,k} q \varepsilon_n \leq \mathrm{Cl}^{\odot}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\varepsilon_n, r) \leq \pi_n \leq \alpha_n$.

Therefore, (b) (a) is clear.

5 Single Valued Neutrosophic θ£-Connected

The aim of this section is to introduce the r-single-valued neutrosophic θ£-separated and r-single-valued neutrosophic δ£-separated. Moreover, we introduce r-single-valued neutrosophic θ£-connected and r-single valued neutrosophic δ£-connected related to the r-single valued neutrosophic operator θ and δ defined on the set $\tilde{\mathcal{F}}$.

Definition 18. Let $(\tilde{\mathcal{F}}, \tau^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}, £^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}})$ be an SVNITS. For $r \in \zeta_0$ and $\alpha_n, \varepsilon_n \in \zeta^{\tilde{\mathcal{F}}}$. Then,

(a) Two non-null SVNSs $\alpha_n, \varepsilon_n \in \zeta^{\tilde{\mathcal{F}}}$ are said to be r-single-valued neutrosophic θ£-separated if $\alpha_n \overline{q} [\varepsilon_n]_{\theta £}$ and $\varepsilon_n \overline{q} [\alpha_n]_{\theta £}$,

(b) Two non-null SVNSs $\alpha_n, \varepsilon_n \in \zeta^{\tilde{\mathcal{F}}}$ are said to be r-single-valued neutrosophic δ£-separated if $\alpha_n \overline{q} [\varepsilon_n]_{\delta £}$ and $\varepsilon_n \overline{q} [\alpha_n]_{\delta £}$,

Remark 2. For any two non-null SVNSs$\alpha_n, \varepsilon_n \in \zeta^{\tilde{\mathcal{F}}}$, and by Eq. (8). The following implications hold: r-single-valued neutrosophic θ£-separated $\Rightarrow$ r-single-valued neutrosophic δ£-separated $\Rightarrow$ r-single-valued neutrosophic separated.

The following example shows that the concept of r-single-valued neutrosophic δ£-separated is weaker than that of r-single-valued neutrosophic θ£-separated.

Example 3. Let $\tilde{\mathcal{F}} = \{a, b, c\}$ be a set. Define $[\varepsilon_n]_1, [\varepsilon_n]_2 \in \zeta^{\tilde{\mathcal{F}}}$ as follows:

$$[\varepsilon_n]_1 = \langle (1,1,0), (1,1,0), (1,1,0) \rangle ; [\varepsilon_n]_2 = \langle (0,0,1), (0,0,1), (0,0,1) \rangle .$$

We define an SVNITS$(\tau^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}, £^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}})$ on $\tilde{\mathcal{F}}$ as follows: for each $\alpha_n \in \zeta^{\tilde{\mathcal{F}}}$,

$$\tilde{\tau}^{\tilde{\varrho}}(\alpha_n) = \begin{cases} 1, & if \ \alpha_n = \tilde{0}, \\ 1, & if \ \alpha_n = \tilde{1}, \\ \frac{1}{3}, & if \ \alpha_n = [\varepsilon_n]_1, \\ \frac{1}{2}, & if \ \alpha_n = [\varepsilon_n]_2, \\ 0, & otherwise, \end{cases} \quad £^{\tilde{\varrho}}(\alpha_n) = \begin{cases} 1, & if \ \alpha_n = \tilde{0}, \\ 0, & otherwise, \end{cases}$$

$$\tilde{\tau}^{\tilde{\sigma}}(\alpha_n) = \begin{cases} 0, & if \ \alpha_n = \tilde{0}, \\ 0, & if \ \alpha_n = \tilde{1}, \\ \frac{2}{3}, & if \ \alpha_n = [\varepsilon_n]_1, \\ \frac{1}{2}, & if \ \alpha_n = [\varepsilon_n]_2, \\ 1, & otherwise \end{cases} \quad £^{\tilde{\sigma}}(\alpha_n) = \begin{cases} 0, & if \ \alpha_n = \tilde{1}, \\ 1, & otherwise, \end{cases}$$

$$\tilde{\tau}^{\tilde{\varsigma}}(\alpha_n) = \begin{cases} 0, & if \alpha_n = \tilde{0}, \\ 0, & if\ \alpha_n = \tilde{0}, \\ \frac{2}{3}, & if\ \alpha_n = [\varepsilon_n]_1, \\ \frac{1}{2}, & if\ \alpha_n = [\varepsilon_n]_2, \\ 1, & otherwise, \end{cases} \quad £^{\tilde{\varsigma}}(\alpha_n) = \begin{cases} 0, & if\ \alpha_n = \tilde{1}, \\ 1, & otherwise. \end{cases}$$

Therefore, we obtain

$$\mathrm{CI}^{\theta £}_{\tilde{\tau}\tilde{\varsigma}}(\alpha_n, r) = \begin{cases} \tilde{0}, & if\ \alpha_n = \tilde{0}, r \in \zeta_0, \\ \boldsymbol{\mathcal{E}}_2^c, & if\ \alpha_n \leq [\varepsilon_n]_1, r \leq \frac{1}{2}, 1 - r \geq \frac{1}{2}, \\ \boldsymbol{\mathcal{E}}_1^c, & if\ \alpha_n \leq [\varepsilon_n]_2, r \leq \frac{1}{3}, 1 - r \geq \frac{2}{3}, \\ \tilde{0}, & otherwise. \end{cases}$$

If $r \leq \frac{1}{3}$ and $1 - r \geq \frac{2}{3}$, then $[\varepsilon_n]_2^c$ and $[\varepsilon_n]_2$ are not r-single-valued neutrosophic $\theta £$-separated for $r \leq \frac{1}{3}$ and $1 - r \geq \frac{2}{3}$. If $r > \frac{1}{3}$ and $1 - r < \frac{2}{3}$, we have $[\varepsilon_n]_2^c$ and $[\varepsilon_n]_2$ are r-single-valued neutrosophic separated.

Theorem 12. Let $(\tilde{\mathcal{F}}, \tau^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}, £^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}})$ be an SVNITS. For $r \in \zeta_0$ and $\alpha_n, \varepsilon_n \in \zeta^{\tilde{\mathcal{F}}}$.

(a) If α_n and ε_n are single-valued neutrosophic $\theta £$-separated, and $[\alpha_n]_1, [\varepsilon_n]_1 \in \zeta^{\tilde{\mathcal{F}}}$ such that $[\alpha_n]_1 \leq \alpha_n [\varepsilon_n]_1 \leq \varepsilon_n$, then $[\alpha_n]_1$ and $[\varepsilon_n]_1$ are also single-valued neutrosophic $\theta £$-separated,
(b) If $\alpha_n \overline{q} \varepsilon_n$ either both are r-$\theta £$-open or r-$\delta £$-closed, then α_n and ε_n are single-valued neutrosophic $\theta £$-separated,
(c) If α_n and ε_n either both are r-$\theta £$-open or r-$\delta £$-closed and if $[\omega_n]_1 = \alpha_n \cap [\varepsilon_n]^c$ and $\omega_2 = \varepsilon_n \cap [\alpha_n]^c$, then $[\omega_n]_1$ and $[\omega_n]_1$ are single-valued neutrosophic $\theta £$-separated.

Proof. (a) Since $[\alpha_n]_1 \leq \alpha_n$ we have $[[\alpha_n]_1]_{\theta £} \leq [\alpha_n]_{\theta £}$. Then, $\varepsilon_n \leq [\alpha_n]_{\theta £} \Rightarrow [\varepsilon_n]_1 \leq [\alpha_n]_{\theta £} \Rightarrow [\varepsilon_n]_1 \leq [[\alpha_n]_1]_{\theta £}$. Similarly $[\alpha_n]_1 \leq [[\varepsilon_n]_1]_{\theta £}$. Hence $[\alpha_n]_1$ and $[\varepsilon_n]_1$ are single-valued neutrosophic $\theta £$-separated.

(b) When α_n and ε_n are r-$\delta £$-closed, then $\alpha_n = [\alpha_n]_{\theta £}$ and $\varepsilon_n = [\varepsilon_n]_{\theta £}$. Since $\alpha_n \overline{q} \varepsilon_n$ we have $[\alpha_n]_{\theta £} \overline{q} \varepsilon_n$ and $[\varepsilon_n]_{\theta £} \overline{q} \alpha_n$.

When α_n and ε_n are r-$\theta £$-open, $[\alpha_n]^c$ and $[\varepsilon_n]^c$ are r-$\theta £$-closed. Then $\alpha_n \overline{q} \varepsilon_n \Rightarrow \alpha_n \leq [\varepsilon_n]^c \Rightarrow [\alpha_n]_{\theta £} \leq [[\varepsilon_n]^c]_{\theta £} = [\varepsilon_n]^c \Rightarrow [\alpha_n]_{\theta £} \overline{q} \varepsilon_n$. Similarly, $[\varepsilon_n]_{\theta £} \overline{q} \alpha_n$. Hence α_n and ε_n are single-valued neutrosophic $\theta £$-separated.

(c) When α_n and ε_n are r-$\theta £$-open, $[\alpha_n]^c$ and $[\varepsilon_n]^c$ are r-$\theta £$-closed. Since $[\omega_n]_1 \leq [\varepsilon_n]^c$, $[[\omega_n]_1]_{\theta £} \leq [[\varepsilon_n]^c]_{\theta £} = [\varepsilon_n]^c$ and so $[[\omega_n]_1]_{\theta £} \overline{q} \varepsilon_n$. Thus $[\omega_n]_2 \overline{q} [[\omega_n]_1]_{\theta £}$. Similarly, $[\omega_n]_1 \overline{q} [[\omega_n]_2]_{\theta £}$. Hence $[\omega_n]_1$ and $[\omega_n]_1$ are single-valued neutrosophic $\theta £$-separated.

When α_n and ε_n are r-θ£-closed, $\alpha_n = [\alpha_n]_{\theta £}$ and $\varepsilon_n = [\varepsilon_n]_{\theta £}$. Since $[\omega_n]_1 \leq [\varepsilon_n]^c$, $[\varepsilon_n]_{\theta £}\overline{q}[\omega_n]_1$ and hence $[[\omega_n]_2]_{\theta £}\overline{q}[\omega_n]_1$. Similarly, $[[\omega_n]_1]_{\theta £}\overline{q}[\omega_n]_2$. Hence $[\omega_n]_1$ and $[\omega_n]_1$ are single-valued neutrosophic θ£-separated.

Theorem 13. Two non-null $\alpha_n, \varepsilon_n \in \zeta^{\tilde{\mathcal{F}}}$ are single-valued neutrosophic θ£-separated if and only if there exist two r-θ£-open sets ω_n and π_n such that $\alpha_n \leq \omega_n$, $\varepsilon_n \leq \pi_n$, $\alpha_n\overline{q}\pi_n$ and $\varepsilon_n\overline{q}\omega_n$.

***Proof*.** Let α_n and ε_n be single-valued neutrosophic θ£-separated. Putting $\pi_n = [[\alpha_n]_{\theta £}]^c$ and $\omega_n = [[\varepsilon_n]_{\theta £}]^c$, then ω_n and π_n are r-θ£-open such that $\alpha_n \leq \omega_n$, $\varepsilon_n \leq \pi_n$, $\alpha_n\overline{q}\pi_n$ and $\varepsilon_n\overline{q}\omega_n$.

Conversely, let ω_n and π_n be r-θ£-open sets such that $\alpha_n \leq \omega_n$, $\varepsilon_n \leq \pi_n$, $\alpha_n\overline{q}\pi_n$ and $\varepsilon_n\overline{q}\omega_n$. Since $[\pi_n]^c$ and $[\omega_n]^c$ are r-θ£-closed, we have $[\alpha_n]_{\theta £} \leq [\pi_n]^c \leq [\varepsilon_n]^c$ and $[\varepsilon_n]_{\theta £} \leq [\omega_n]^c \leq [\alpha_n]^c$. Thus $[\alpha_n]_{\theta £}\overline{q}\varepsilon_n$ and $[\varepsilon_n]_{\theta £}\overline{q}\alpha_n$. Hence α_n and ε_n are single-valued neutrosophic θ£-separated.

Definition 19. An SVNS which cannot be expressed as the union of two single-valued neutrosophic θ£-separated is said to be single-valued neutrosophic θ£-connected.

Definition 20. An SVNS α_n in a SVNITS $(\tilde{\mathcal{F}}, \tau^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}, £^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}})$ is said to be single-valued neutrosophic δ£-connected if α_n cannot be expressed as the union of two single-valued neutrosophic δ£-separated.

For an SVNS α_n in a SVNITS$(\tilde{\mathcal{F}}, \tau^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}, £^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}})$, the following implications hold: single-valued neutrosophic connected $\Rightarrow$ single-valued neutrosophic δ£-connected $\Rightarrow$ single-valued neutrosophic θ£-connected. If $\tau^{\tilde{\varrho}}(\alpha_n) \geq r, \tau^{\tilde{\sigma}}(\alpha_n) \leq 1-r, \tau^{\tilde{\varsigma}}(\alpha_n) \leq 1-r$, then these three properties are equivalent.

Theorem 14. Let α_n be a non-null single-valued neutrosophic θ£-connected in a SVNITS $(\tilde{\mathcal{F}}, \tau^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}, £^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}})$. If α_n is contained in the union of two single-valued neutrosophic θ£-separated ε_n and ω_n, then exactly one of the following conditions (a) or (b) holds:

(a) $\alpha_n \leq \varepsilon_n$ and $\alpha_n \cap \omega_n = \tilde{0}$,
(b) $\alpha_n \leq \omega_n$ and $\alpha_n \cap \varepsilon_n = \tilde{0}$.

***Proof*.** We first note that when $\alpha_n \cap \omega_n = \tilde{0}$, then $\alpha_n \leq \varepsilon_n$, since $\alpha_n \leq \varepsilon_n \cup \omega_n$. Similarly, when $\alpha_n \cap \varepsilon_n = \tilde{0}$, we have $\alpha_n \leq \omega_n$. Since $\alpha_n \leq \varepsilon_n \cup \omega_n$, both $\alpha_n \cap \varepsilon_n = \tilde{0}$ and $\alpha_n \cap \omega_n = \tilde{0}$ cannot hold simultaneously. Again, if $\alpha_n \cap \varepsilon_n \neq \tilde{0}$ and $\alpha_n \cap \omega_n \neq \tilde{0}$, then, by Theorem 12 (1), $\alpha_n \cap \omega_n$ and $\alpha_n \cap \varepsilon_n$ are single-valued neutrosophic θ£-separated such that $\alpha_n = (\alpha_n \cap \varepsilon_n) \cup (\alpha_n \cap \omega_n)$, contradicting the single-valued neutrosophic θ£-connectedness of α_n. Hence, exactly one of the conditions (1) or (2) above must hold.

Theorem 15. Let $\{[\alpha_n]_j | j \in J\}$ be a collection of single-valued neutrosophic θ£-connected in $(\tilde{\mathcal{F}}, \tau^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}, £^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}})$. If there exists $i \in J$ such that $[\alpha_n]_j \cap [\alpha_n]_i \neq \tilde{0}$ for each $j \in J$, then $\alpha_n = \cup\{[\alpha_n]_j | j \in J\}$ is single-valued neutrosophic θ£-connected.

***Proof*.** Suppose that α_n is not single-valued neutrosophic θ£-connected. Then there exist single-valued neutrosophic θ£-separated ε_n and ω_n such that $\alpha_n = \varepsilon_n \cap \omega_n$. By Theorem 14, we have either (a) $[\alpha_n]_j \leq \varepsilon_n$ with $[\alpha_n]_j \cap \omega_n = \tilde{0}$ or (b) $[\alpha_n]_j \leq \omega_n$ with $[\alpha_n]_j \cap \varepsilon_n = \tilde{0}$ for each $j \in J$. Similarly, either (a$'$)$[\alpha_n]_i \leq \varepsilon_n$ with $[\alpha_n]_i \cap \omega_n = \tilde{0}$ or (b$'$)$[\alpha_n]_i \leq \omega_n$ with $[\alpha_n]_i \cap \varepsilon_n = \tilde{0}$ for each $i \in J$. We may assume, without loss of generality, that $[\alpha_n]_j$ is non-null for each $j \in J$, and hence exactly one of the conditions (a) and (b), and exactly one of (a$'$) and (b$'$) will hold.

Since $[\alpha_n]_j \cap [\alpha_n]_i \neq \tilde{0}$ for each $j \in J$, the conditions (a) and (b$'$) cannot happen, and similarly (b) and (1$'$) cannot hold simultaneously. If (a) and (a$'$) hold, then $[\alpha_n]_j \leq \varepsilon_n$ with $[\alpha_n]_j \cap \omega_n = \tilde{0}$.

Then $\alpha_n \leq \varepsilon_n$ with $\alpha_n \cap \omega_n = \tilde{0}$ and thus $\omega_n = \tilde{0}$ a contradiction. Similarly, if (b) and (b′) hold, then we have $\varepsilon_n = \tilde{0}$ again a contradiction.

Lemma 4. An SVNITS$(\tilde{\mathcal{F}}, \tau^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}, £^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}})$ is SVNA£-regular iff $[\alpha_n]_{\delta £} = [\alpha_n]_{\theta £}$ for every $\alpha_n \in \zeta^{\tilde{\mathcal{F}}}$.

Proof. Obvious.

Theorem 16. Let $(\tilde{\mathcal{F}}, \tau^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}, £^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}})$ be an SVNITS, $\alpha_n \in \zeta^{\tilde{\mathcal{F}}}$, $r \in \zeta_0$. If $(\tilde{\mathcal{F}}, \tau^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}, £^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}})$ is SVNA£-regular and α_n is single-valued neutrosophic θ£-connected set, then α_n is single-valued neutrosophic δ£-connected set.

Proof. Follows easily by virtue of Lemma 4.

Corollary 1. For a $\alpha_n \in \zeta^{\tilde{\mathcal{F}}}$ of SVNA£-regular space $(\tilde{\mathcal{F}}, \tau^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}, £^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}})$, the following are equivalent:

(a) α_n is r-single-valued neutrosophic connected,
(b) α_n is r-single-valued neutrosophic δ£-connected,
(c) α_n is r-single-valued neutrosophic θ£-connected.

Proof. Follows easily by virtue of Theorem 16.

6 Conclusion

The neutrosophic set theory has been established and applied extensively to many problems involving uncertainties. Herein, we provided clear definitions of single-valued neutrosophic operators $\mathrm{CI}^{\theta £}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}$ and $\mathrm{CI}^{\delta £}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}$ created from an SVNI topological space $\left(\tilde{\mathcal{F}}, \tau^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}, £^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}\right)$ and we established that $\mathrm{CI}^{\delta £}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\alpha_n, r) = \mathrm{CI}_{\tilde{\tau}\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}(\alpha_n, r)$ when $£^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}} = £_0^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}$. In addition, we presented the idea of r-single-valued neutrosophic θ£-connectedness based on a single-valued neutrosophic ideal $£^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}$ which has kindred with a preceding r-single-valued neutrosophic connectedness and the relationships among them are inspected. Moreover, we introduced an r-single-valued neutrosophic δ£-connectedness connected to a single-valued neutrosophic δ on the set $\tilde{\mathcal{F}}$ and analyzed some of their properties. This study not only provides a hypothetical basis for additional requests in neutrosophic topology, but also for the expansion of other methodical aspects.

Discussion for further works:

The current concept can be extended by

- Investigating neutrosophic metric topological spaces;
- Investigating the products of connected and Hausdorff spaces for $(\tilde{\mathcal{F}}, \tau^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}}, £^{\tilde{\varrho}\tilde{\sigma}\tilde{\varsigma}})$.

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