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Unique Solution of Integral Equations via Intuitionistic Extended Fuzzy b-Metric-Like Spaces

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ABSTRACT

In this manuscript, our goal is to introduce the notion of intuitionistic extended fuzzy b-metric-like spaces. We establish some fixed point theorems in this setting. Also, we plot some graphs of an example of obtained result for better understanding. We use the concepts of continuous triangular norms and continuous triangular conorms in an intuitionistic fuzzy metric-like space. Triangular norms are used to generalize with the probability distribution of triangle inequality in metric space conditions. Triangular conorms are known as dual operations of triangular norms. The obtained results boost the approaches of existing ones in the literature and are supported by some examples and applications.

KEYWORDS

Fixed point; extended fuzzy b-metric like space; intuitionistic extended fuzzy b-metric-like space; integral equation

1 Introduction

After being given the notion of fuzzy sets (FSs) by Zadeh [1], many researchers provided many generalizations. Schweizer et al. [2] introduced the notion of continuous t-norms. In this continuation, Kramosil et al. [3] introduced the approach of fuzzy metric spaces, while George et al. [4] introduced the concept of fuzzy metric spaces. Garbiec [5] gave the fuzzy interpretation of the Banach contraction principle in fuzzy metric spaces. Dey et al. [6] established an extension of Banach fixed point theorem in fuzzy metric space. Nadaban [7] introduced the notion of fuzzy b-metric spaces. Gregory et al. [8]

proved various fixed point theorems in fuzzy metric spaces. Bashir et al. [9] established several fixed point results of a generalized reversed F-contraction mapping and its application.

Recently, Harandi [10] initiated the concept of metric-like spaces, which generalized the notion of metric spaces in a nice way. Alghamdi et al. [11] used the concept of metric-like spaces and introduced the notion of b-metric-like spaces. In this sequel, Shukla et al. [12] generalized the concept of metric-like spaces and introduced fuzzy metric-like spaces and Javed et al. [13] introduced the concept of fuzzy b-metric-like spaces and prove some fixed point results.

Mehmood et al. [14] presented the notion of fuzzy extended b-metric spaces (FEBMSs) by replacing the coefficient $b \geq 1$ with a function $\alpha : \mathfrak{D} \times \mathfrak{D} \to [1, \infty)$. The approach of intuitionistic fuzzy metric spaces was tossed by Park et al. [15–18], Saleem et al. [19–28] proved several fixed theorems on intuitionistic fuzzy metric space. Sintunavarat et al. [29] established various fixed theorems for a generalized intuitionistic fuzzy contraction in intuitionistic fuzzy metric spaces. Saadati et al. [30] did amazing work in the sense of intuitionistic fuzzy topological spaces. Later, Konwar [31] presented the concept of an intuitionistic fuzzy b metric space (IFBMS). Mahmood et al. [32] did power aggregation operators and similarity measures based on improved intuitionistic hesitant fuzzy sets and their applications to multiple attribute decision making.

In this manuscript, we aim to introduce the concept of intuitionistic extended fuzzy b-metric-like space (IEFBMLS). In which, we generalize the concept of IFBMS by replacing the coefficient $b \geq 1$ with a function $\alpha : \mathfrak{D} \times \mathfrak{D} \to [1, \infty)$ in both triangular inequalities and we replace condition (III) of IFBMS, $M_b(\vartheta, \delta, \mathcal{T}) = 1 \;\; \Leftrightarrow \vartheta = \delta$ by $M_b(\vartheta, \delta, \mathcal{T}) = 1$ implies $\vartheta = \delta$ and similarly, we replace ‘‘$\Leftrightarrow$ by implies” in condition (VIII) of IFBMS. So, presented results in this manuuscript are more generalized in the existing literature. Also, we provide some fixed point (FP) results, non-trivial examples, an application to integral equations and application dynamic market equilibrium.

Main objectives of this manuscript are:

(a) To introduce the notion of intuitionistic extended fuzzy b-metric-like space.

(b) To enhance the literature of intuitionistic fuzzy fixed point theory.

(c) To plot some graphical structure of obtained result.

(d) To prove the existence and uniqueness of established results via integral equations.

(e) To provide an application dynamic market equilibrium.

2 Preliminaries

The following definitions are helpful in the sequel.

Definition 2.1 [15] A binary operation $*$: [0, 1] × [0, 1] → [0, 1] is called a continuous triangle norm (briefly CTN) if:

1. $\nu * \omega = \omega * \nu, \ \forall \ \nu, \omega \in [0, \ 1]$;
2. $*$ is continuous;
3. $\nu * 1 = \nu, \ \forall \ \nu \in [0, \ 1]$;
4. $(\nu * \omega) * \kappa = \nu * (\omega * \kappa), \ \forall \ \nu, \omega, \kappa \in [0, \ 1]$;
5. If $\nu \leq \kappa$ and $\omega \leq d$, with $\nu, \omega, \kappa, d \in [0, \ 1]$, then $\nu * \omega \leq \kappa * d$.

Definition 2.2 [15] A binary operation $\circ$: [0, 1] × [0, 1] → [0, 1] is called a continuous triangle conorm (briefly CTCN) if it meets the below assertions:

1. $\nu \circ \omega = \omega \circ \nu,\ \forall\ \nu, \omega \in [0,\ 1]$;
2. $\circ$ is continuous;
3. $\nu \circ 0 = 0,\ (\forall)\ \nu \in [0,\ 1]$;
4. $(\nu \circ \omega) \circ \kappa = \nu \circ (\omega \circ \kappa),\ \forall\ \nu, \omega, \kappa \in [0,\ 1]$;
5. If $\nu \leq \kappa$ and $\omega \leq d$, with $\nu, \omega, \kappa, d \in [0,\ 1]$, then $\nu \circ \omega \leq \kappa \circ d$.

Definition 2.3 [10] A mapping $P : \mathfrak{D} \times \mathfrak{D} \rightarrow [1, \infty)$, where $\mathfrak{D} \neq \emptyset$, fulfilling the below circumstances:

a. $P(\vartheta, \delta) = 0$ implies $\vartheta = \delta$;
b. $P(\vartheta, \delta) = P(\delta, \vartheta)$;
c. $P(\vartheta, \delta) \leq P(\vartheta, \beta) + P(\beta, \delta)$;

for all $\vartheta, \delta, \beta \in \mathfrak{D}$. Then P is called a metric-like and $(\mathfrak{D}, P)$ is named metric-like space.

Definition 2.4 [12] Take $\mathfrak{D} \neq \emptyset$. Let $*$ be a CTN and Q_b be a FS on $\mathfrak{D} \times \mathfrak{D} \times (0, \infty)$. A three tuple $(\mathfrak{D}, Q_b, *)$ is called fuzzy metric like space, if it verifies the following for all $\vartheta, \delta, \beta \in \mathfrak{D}$ and $\mathcal{T}, \mathcal{S} > 0$:

(F1) $Q_b(\vartheta, \delta, \mathcal{T}) > 0$;
(F2) $Q_b(\vartheta, \delta, \mathcal{T}) = 1$ implies $\vartheta = \delta$;
(F3) $Q_b(\vartheta, \delta, \mathcal{T}) = Q_b(\delta, \vartheta, \mathcal{T})$;
(F4) $Q_b(\vartheta, \beta, (\mathcal{T} + \mathcal{S})) \geq Q_b(\vartheta, \delta, \mathcal{T}) * Q_b(\delta, \beta, \mathcal{S})$;
(F5) $Q_b(\vartheta, \delta, \cdot) : (0, \infty) \rightarrow [0,\ 1]$ is continuous.

Definition 2.5 [14] A 4-tuple $(\mathfrak{D}, \Delta_\alpha, *, \alpha)$ is called an FEBMS if $\mathfrak{D}$ is a non-empty set, $\alpha : \mathfrak{D} \times \mathfrak{D} \rightarrow [1, \infty)$, $*$ is a CTN and Δ_α is a FS on $\mathfrak{D} \times \mathfrak{D} \times (0, \infty)$, so that for all $\vartheta, \delta, \beta \in \mathfrak{D}$ and $\mathcal{T}, \mathcal{S} > 0$:

Δ1) $\Delta_\alpha(\vartheta, \delta, 0) = 0$;
Δ2) $\Delta_\alpha(\vartheta, \delta, \mathcal{T}) = 1 \Leftrightarrow \vartheta = \delta$;
Δ3) $\Delta_\alpha(\vartheta, \delta, \mathcal{T}) = \Delta_\alpha(\delta, \vartheta, \mathcal{T})$;
Δ4) $\Delta_\alpha(\vartheta, \beta, \alpha(\vartheta, \beta)(\mathcal{T} + \mathcal{S})) \geq \Delta_\alpha(\vartheta, \delta, \mathcal{T}) * \Delta_\alpha(\delta, \beta, \mathcal{S})$;
Δ5) $\Delta_\alpha(\vartheta,\ \delta,\ \cdot) : (0, \infty) \rightarrow [0, 1]$ is continuous.

Definition 2.6 [31] Take $\mathfrak{D} \neq \emptyset$. Let $*$ be a CTN, $\circ$ be a CTCN, $b \geq 1$ and M_b, N_b be FSs on $\mathfrak{D} \times \mathfrak{D} \times (0, \infty)$. If $(\mathfrak{D}, M_b, N_b, *, \circ)$ verifies the following for all $\vartheta, \delta \in \mathfrak{D}$ and $\mathcal{S}, \mathcal{T} > 0$:

(I) $M_b(\vartheta, \delta, \mathcal{T}) + N_b(\vartheta, \delta, \mathcal{T}) \leq 1$;
(II) $M_b(\vartheta, \delta, \mathcal{T}) > 0$;
(III) $M_b(\vartheta, \delta, \mathcal{T}) = 1 \Leftrightarrow \vartheta = \delta$;
(IV) $M_b(\vartheta, \delta, \mathcal{T}) = M_b(\delta, \vartheta, \mathcal{T})$;
(V) $M_b(\vartheta, \beta, b(\mathcal{T} + \mathcal{S})) \geq M_b(\vartheta, \delta, \mathcal{T}) * M_b(\delta, \beta, \mathcal{S})$;
(VI) $M_b(\vartheta, \delta, \cdot)$ is a non-decreasing (ND) function of $\mathbb{R}^+$ and $\lim_{\mathcal{T} \rightarrow \infty} M_b(\vartheta, \delta, \mathcal{T}) = 1$;
(VII) $N_b(\vartheta, \delta, \mathcal{T}) > 0$;
(VIII) $N_b(\vartheta, \delta, \mathcal{T}) = 0 \Leftrightarrow \vartheta = \delta$;
(IX) $N_b(\vartheta, \delta, \mathcal{T}) = N_b(\delta, \vartheta, \mathcal{T})$;
(X) $N_b(\vartheta, \beta, b(\mathcal{T} + \mathcal{S})) \leq N_b(\vartheta, \delta, \mathcal{T}) \circ N_b(\delta, \beta, \mathcal{S})$;

(XI) $N_b(\vartheta, \delta, \cdot)$ is a non-increasing (NI) function of $\mathbb{R}^+$ and $\lim_{\mathcal{T}\to\infty} N_b(\vartheta, \delta, \mathcal{T}) = 0$,

then $(\mathfrak{D}, M_b, N_b, *, \circ)$ is an IFBMS.

3 Main Result

In this section, we introduce the notion of an IEFBMLS and prove some related FP results.

Definition 3.1 Let $\mathfrak{D} \neq \emptyset$, $*$ be a CTN, $\circ$ be a CTCN, $\phi : \mathfrak{D} \times \mathfrak{D} \to [1, \infty)$ be a mapping and M_ϕ, N_ϕ be FSs on $\mathfrak{D} \times \mathfrak{D} \times (0, \infty)$. If $(\mathfrak{D}, M_\phi, N_\phi, *, \circ)$ is such that for $\vartheta, \delta \in \mathfrak{D}$ and $\mathcal{S}, \mathcal{T} > 0$:

(i) $M_\phi(\vartheta, \delta, \mathcal{T}) + N_\phi(\vartheta, \delta, \mathcal{T}) \leq 1$;

(ii) $M_\phi(\vartheta, \delta, \mathcal{T}) > 0$;

(iii) $M_\phi(\vartheta, \delta, \mathcal{T}) = 1$ implies $\vartheta = \delta$;

(iv) $M_\phi(\vartheta, \delta, \mathcal{T}) = M_\phi(\delta, \vartheta, \mathcal{T})$;

(v) $M_\phi(\vartheta, \beta, \phi(\vartheta, \beta)(\mathcal{T} + \mathcal{S})) \geq M_\phi(\vartheta, \delta, \mathcal{T}) * M_\phi(\delta, \beta, \mathcal{S})$;

(vi) $M_\phi(\vartheta, \delta, \cdot)$ is a ND function of $\mathbb{R}^+$ and $\lim_{\mathcal{T}\to\infty} M_\phi(\vartheta, \delta, \mathcal{T}) = 1$;

(vii) $N_\phi(\vartheta, \delta, \mathcal{T}) > 0$;

(viii) $N_\phi(\vartheta, \delta, \mathcal{T}) = 0$ implies $\vartheta = \delta$;

(ix) $N_\phi(\vartheta, \delta, \mathcal{T}) = N_\phi(\delta, \vartheta, \mathcal{T})$;

(x) $N_\phi(\vartheta, \beta, \phi(\vartheta, \beta)(\mathcal{T} + \mathcal{S})) \leq N_\phi(\vartheta, \delta, \mathcal{T}) \circ N_\phi(\delta, \beta, \mathcal{S})$;

(xi) $N_\phi(\vartheta, \delta, \cdot)$ is a NI function of $\mathbb{R}^+$ and $\lim_{\mathcal{T}\to\infty} N_\phi(\vartheta, \delta, \mathcal{T}) = 0$,

then $(\mathfrak{D}, M_\phi, N_\phi, *, \circ)$ is an IEFBMLS.

Remark 3.2 In the above definition, the self distance in condition (iii) may not be equal to 1 and in condition (viii) the self distance may not be equal to 0. In triangular inequalities, we use $\phi : \mathfrak{D} \times \mathfrak{D} \to [1, \infty)$. So, this is cleared that IEFBMLS may not be an IFBMS but converse is true.

Example 3.3 Let $\mathfrak{D} = (0, \infty)$, define $M_\phi, N_\phi : \mathfrak{D} \times \mathfrak{D} \times (0, \infty) \to [0, 1]$ by

$$M_\phi(\vartheta, \delta, \mathcal{T}) = \frac{\mathcal{T}}{\mathcal{T} + \max\{\vartheta, \delta\}^2}, \quad N_\phi(\vartheta, \delta, \mathcal{T}) = \frac{\max\{\vartheta, \delta\}^2}{\mathcal{T} + \max\{\vartheta, \delta\}^2}$$

for all $\vartheta, \delta \in \mathfrak{D}$ and $\mathcal{T} > 0$. Define the CTN by: $\nu * \omega = \nu \cdot \omega$ and CTCN $''\circ''$ by $\nu \circ \omega = \max\{\nu, \omega\}$ and define $''\phi''$ by

$$\phi(\vartheta, \delta) = \begin{cases} 1 & \textit{if} \quad \vartheta = \delta \ \textit{or} \ \vartheta \in (0, 1), \\ 1 + \max\{\vartheta, \delta\} & \textit{if} \quad \textit{otherwise}. \end{cases}$$

Then $(\mathfrak{D}, M_\phi, N_\phi, *, \circ)$ is an IEFBMLS.

Example 3.4 Let $\mathfrak{D} = (0, \infty)$ and $\alpha : \mathfrak{D} \times \mathfrak{D} \to [1, \infty)$ be a function given by $\phi(\vartheta, \delta) = \vartheta + \delta + 1$. Define $M_\phi, N_\phi : \mathfrak{D} \times \mathfrak{D} \times (0, \infty) \to [0, 1]$ as

$$M_\phi(\vartheta, \delta, \mathcal{T}) = \frac{\mathcal{T}}{\mathcal{T} + \max\{\vartheta, \delta\}}$$

and

$$N_\phi(\vartheta, \delta, \mathcal{T}) = \frac{\max\{\vartheta, \delta\}}{\mathcal{T} + \max\{\vartheta, \delta\}}.$$

Then $(\mathfrak{D}, M_\phi, N_\phi, *, \circ)$ is an IEFBMLS with CTN $a * b = ab$ and CTCN $a \circ b = \max a, b$.

Remark 3.5 Above example also satisfied for CTN $a*b = \min\{a, b\}$ and CTCN $a\circ b = \max\{a, b\}$.

Example 3.6 Let $\mathfrak{D} = (0, \infty)$ and $\phi : \mathfrak{D} \times \mathfrak{D} \to [1, \infty)$ be a function given by $\phi(\vartheta, \delta) = \vartheta + \delta + 1$. Define $M_\phi, N_\phi : \mathfrak{D} \times \mathfrak{D} \times (0, \infty) \to [0, 1]$ as

$$M_\phi(\vartheta, \delta, \mathcal{T}) = \frac{\mathcal{T} + \min\{\vartheta, \delta\}}{\mathcal{T} + \max\{\vartheta, \delta\}}$$

and

$$N_\phi(\vartheta, \delta, \mathcal{T}) = 1 - \frac{\mathcal{T} + \min\{\vartheta, \delta\}}{\mathcal{T} + \max\{\vartheta, \delta\}}.$$

Then $(\mathfrak{D}, M_\phi, N_\phi, *, \circ)$ is an IEFBMLS with CTN $a * b = ab$ and CTCN $a \circ b = \max a, b$.

Proposition 3.7 Let $\mathfrak{D} = (0, \infty)$ and $\phi : \mathfrak{D} \times \mathfrak{D} \to [1, \infty)$ be a function given by $\phi(\vartheta, \delta) = 2(\vartheta + \delta + 1)$ Define $N,\ M$ as

$$M_\phi(\vartheta, \delta, \mathcal{T}) = e^{-\frac{\max\{\vartheta,\ \delta\}}{\mathcal{T}^n}},\ N_\phi(\vartheta, \delta, \mathcal{T}) = 1 - e^{-\frac{\max\{\vartheta,\ \delta\}}{\mathcal{T}^n}} \text{ for all } n \in \mathbb{N},\ \vartheta, \delta \in \mathfrak{D}, \mathcal{T} > 0.$$

Then $(\mathfrak{D}, M_\phi, N_\phi, *, \circ)$ is an IEFBMLS with CTN $a * b = ab$ and CTCN $a \circ b = \max a, b$.

Remark 3.8 The above proposition also satisfied for CTN $a * b = \min\{a, b\}$ and CTCN $a \circ b = \max\{a, b\}$.

Proposition 3.9 Let $\mathfrak{D} = [0, 1]$ and $\phi : \mathfrak{D} \times \mathfrak{D} \to [1, \infty)$ be a function given by $\phi(\vartheta, \delta) = 2(\vartheta + \delta + 1)$. Define $M_\phi,\ N_\phi$ as

$$M_\phi(\vartheta, \delta, \mathcal{T}) = \left[e^{\frac{\max\{\vartheta,\ \delta\}}{\mathcal{T}^n}}\right]^{-1},\ N_\phi(\vartheta, \delta, \mathcal{T}) = 1 - \left[e^{\frac{\max\{\vartheta,\ \delta\}}{\mathcal{T}^n}}\right]^{-1} \text{ for all } n \in \mathbb{N},\ \vartheta, \delta \in \mathfrak{D}, \mathcal{T} > 0.$$

Then $(\mathfrak{D}, M_\phi, N_\phi, *, \circ)$ is an IEFBMLS with CTN $a * b = ab$ and CTCN $a \circ b = \max a, b$.

Example 3.10 Let $\mathfrak{D} = (0, \infty)$, define $M_\phi, N_\phi : \mathfrak{D} \times \mathfrak{D} \times (0, \infty) \to [0, 1]$ by

$$M_\phi(\vartheta, \delta, \mathcal{T}) = \frac{\mathcal{T}}{\mathcal{T} + \max\{\vartheta,\ \delta\}},\ \ N_\phi(\vartheta, \delta, \mathcal{T}) = \frac{\max\{\vartheta,\ \delta\}}{\mathcal{T} + \max\{\vartheta,\ \delta\}}$$

for all $\vartheta, \delta \in \mathfrak{D}$ and $\mathcal{T} > 0$, define CTN $*$ by $\nu * \omega = \nu \cdot \omega$ and CTCN $\circ$ by $\nu \circ \omega = \max\{\nu, \omega\}$ and define ϕ by

$$\phi(\vartheta, \delta) = \begin{cases} 1 \quad \text{if} \quad \vartheta = \delta, \\ \dfrac{1 + \max\{\vartheta, \delta\}}{\min\{\vartheta, \delta\}} \quad \text{if } \vartheta \neq \delta \end{cases}$$

Then $(\mathfrak{D}, M_\phi, N_\phi, *, \circ)$ be an IEFBMLS.

Remark 3.11 In the above all examples self distance may not be equal to 1 and 0. In particular, assume an example 3.9, take $\vartheta = \delta$, then

$$M_\phi(\vartheta, \vartheta, \mathcal{T}) = \frac{\mathcal{T}}{\mathcal{T} + \max\{\vartheta,\ \vartheta\}} \neq 1,$$

$$N_\phi(\vartheta, \vartheta, \mathcal{T}) = \frac{\max\{\vartheta,\ \vartheta\}}{\mathcal{T} + \max\{\vartheta, \vartheta\}} \neq 0.$$

Remark 3.12 In the above Examples 3.3, 3.4, 3.6, 3.7, 3.10 and Proposition 3.9, it is easy to see that self-distance is not equal to 1 as in condition (iii) and the self-distance is not equal to 0 as in condition (viii) in Definition 3.1. So, the Examples 3.3, 3.4, 3.6, 3.7, 3.10 and Proposition 3.9 are becomes IEFBMLSs but not becomes IFBMSs.

Definition 3.13 Let $(\mathfrak{D}, M_\phi, N_\phi, *, \circ)$ be an IEFBMLS. Then

(a) A sequence $\{\vartheta_n\}$ in $\mathfrak{D}$ is named to be a G-Cauchy sequence (GCS) if and only if for all $q > 0$ and $\mathcal{T} > 0$, $\lim_{n\to\infty} M_\phi(\vartheta_n, \vartheta_{n+q}, \mathcal{T})$ and $\lim_{n\to\infty} N_\phi(\vartheta_n, \vartheta_{n+q}, \mathcal{T})$ exists and finite.

(b) A sequence $\{\vartheta_n\}$ in $\mathfrak{D}$ is named to be G-convergent (GC) to ϑ in $\mathfrak{D}$, if and only if for all $\mathcal{T} > 0$,

$$\lim_{n\to\infty} M_\phi(\vartheta_n, \vartheta, \mathcal{T}) = M_\phi(\vartheta, \vartheta, \mathcal{T}) \text{ and } \lim_{n\to\infty} N_\phi(\vartheta_n, \vartheta, \mathcal{T}) = N_\phi t(\vartheta, \vartheta, \mathcal{T}).$$

(c) An IEFBMS is named to be complete iff each GCS is convergent, i.e.,

$$\lim_{n\to\infty} M_\phi(\vartheta_n, \vartheta_{n+q}, \mathcal{T}) = \lim_{n\to\infty} M_\phi(\vartheta_n, \vartheta, \mathcal{T}) = M_\phi(\vartheta, \vartheta, \mathcal{T}),$$

$$\lim_{n\to\infty} N_\phi(\vartheta_n, \vartheta_{n+q}, \mathcal{T}) = \lim_{n\to\infty} N_\phi(\vartheta_n, \vartheta, \mathcal{T}) = N_\phi(\vartheta, \vartheta, \mathcal{T})$$

Now, we consider intuitionistic extended fuzzy like contractions.

Theorem 3.14 Let $(\mathfrak{D}, M_\phi, N_\phi, *, \circ)$ be a G-complete IEFBMLS (with the function $\phi : \mathfrak{D} \times \mathfrak{D} \to [1, \infty)$) and suppose that

$$\lim_{\mathcal{T}\to\infty} M_\phi(\vartheta, \delta, \mathcal{T}) = 1 \text{ and } \lim_{\mathcal{T}\to\infty} N_\phi(\vartheta, \delta, \mathcal{T}) = 0 \tag{1}$$

for all $\vartheta, \delta \in \mathfrak{D}$ and $\mathcal{T} > 0$. Let f : $\mathfrak{D} \to \mathfrak{D}$ be a mapping satisfying

$$M_\phi(f\vartheta, f\delta, k\mathcal{T}) \geq M_\phi(\vartheta, \delta, \mathcal{T}) \text{ and } N_\phi(f\vartheta, f\delta, k\mathcal{T}) \leq N_\phi(\vartheta, \delta, \mathcal{T}) \tag{2}$$

for all $\vartheta, \delta \in \mathfrak{D}$ and $\mathcal{T} > 0$ with $0 < k < 1$. Further, suppose that for an arbitrary $\vartheta_0 \in \mathfrak{D}$ and $n, q \in \mathbb{N}$, we have

$$\phi(\vartheta_n, \vartheta_{n+q}) < \frac{1}{k},$$

where $\vartheta_n = f^n\vartheta_0 = f\nu_{n-1}$. Then f has a unique FP.

Proof: Let ϑ_0 be a random element in $\mathfrak{D}$ and consider $\vartheta_n = f^n\vartheta_0 = f\nu_{n-1}$, $n \in \mathbb{N}$. By using (2) for all $\mathcal{T} > 0$, we have

$$\begin{aligned} M_\phi(\vartheta_n, \vartheta_{n+1}, k\mathcal{T}) = M_\phi(f\vartheta_{n-1}, f\vartheta_n, k\mathcal{T}) \geq M_\phi(\vartheta_{n-1}, \vartheta_n, \mathcal{T}) \geq M_\phi\left(\vartheta_{n-2}, \vartheta_{n-1}, \frac{\mathcal{T}}{k}\right) \\ \geq M_\phi\left(\vartheta_{n-3}, \vartheta_{n-2}, \frac{\mathcal{T}}{k^2}\right) \geq \cdots \geq M_\phi\left(\vartheta_0, \vartheta_1, \frac{\mathcal{T}}{k^n}\right) \end{aligned}$$

and

$$\begin{aligned} N_\phi(\vartheta_n, \vartheta_{n+1}, k\mathcal{T}) = N_\phi(f\vartheta_{n-1}, f\vartheta_n, k\mathcal{T}) \leq N_\phi(\vartheta_{n-1}, \vartheta_n, \mathcal{T}) \leq N_\phi\left(\vartheta_{n-2}, \vartheta_{n-1}, \frac{\mathcal{T}}{k}\right) \\ \leq N_\phi\left(\vartheta_{n-3}, \vartheta_{n-2}, \frac{\mathcal{T}}{k^2}\right) \leq \cdots \leq N_\phi\left(\vartheta_0, \vartheta_1, \frac{\mathcal{T}}{k^n}\right). \end{aligned}$$

We obtain

$$M_\phi(\vartheta_n,\vartheta_{n+1},k\mathcal{T}) \geq M_\phi\left(\vartheta_0,\vartheta_1,\frac{\mathcal{T}}{k^n}\right) \text{ and } N_\phi(\vartheta_n,\vartheta_{n+1},k\mathcal{T}) \leq N_\phi\left(\vartheta_0,\vartheta_1,\frac{\mathcal{T}}{k^n}\right) \tag{3}$$

for any $q \in \mathbb{N}$, $\mathcal{T} = \frac{q\mathcal{T}}{\mathcal{T}} = \frac{\mathcal{T}}{q} + \frac{\mathcal{T}}{q} + \cdots + \frac{\mathcal{T}}{q}$ and using (v) and (x), we deduce

$$\begin{aligned}
M_\phi\left(\vartheta_n,\vartheta_{n+q},\mathcal{T}\right) \geq\ & M_\phi\left(\vartheta_n,\vartheta_{n+1},\frac{\mathcal{T}}{q\left(\phi\left(\vartheta_n,\vartheta_{n+q}\right)\right)}\right)\\
& * M_\phi\left(\vartheta_{n+1},\vartheta_{n+2},\frac{\mathcal{T}}{q\left(\phi\left(\vartheta_n,\vartheta_{n+q}\right)\phi\left(\vartheta_{n+1},\vartheta_{n+q}\right)\right)}\right)\\
& * M_\phi\left(\vartheta_{n+2},\vartheta_{n+3},\frac{\mathcal{T}}{q\left(\phi\left(\vartheta_n,\vartheta_{n+q}\right)\phi\left(\vartheta_{n+1},\vartheta_{n+q}\right)\phi\left(\vartheta_{n+2},\vartheta_{n+q}\right)\right)}\right) * \cdots *\\
& M_\phi\left(\vartheta_{n+q-1},\vartheta_{n+q},\frac{\mathcal{T}}{q\left(\phi\left(\vartheta_n,\vartheta_{n+q}\right)\phi\left(\vartheta_{n+1},\vartheta_{n+q}\right)\phi\left(\vartheta_{n+2},\vartheta_{n+q}\right)\cdots\phi\left(\vartheta_{n+q-1},\vartheta_{n+q}\right)\right)}\right)
\end{aligned}$$

and

$$\begin{aligned}
N_\phi\left(\vartheta_n,\vartheta_{n+q},\mathcal{T}\right) \leq\ & N_\phi\left(\vartheta_n,\vartheta_{n+1},\frac{\mathcal{T}}{q\left(\phi\left(\vartheta_n,\vartheta_{n+q}\right)\right)}\right) \circ N_\phi\left(\vartheta_{n+1},\vartheta_{n+2},\frac{\mathcal{T}}{q\left(\phi\left(\vartheta_n,\vartheta_{n+q}\right)\phi\left(\vartheta_{n+1},\vartheta_{n+q}\right)\right)}\right)\\
& \circ N_\phi\left(\vartheta_{n+2},\vartheta_{n+3},\frac{\mathcal{T}}{q\left(\phi\left(\vartheta_n,\vartheta_{n+q}\right)\phi\left(\vartheta_{n+1},\vartheta_{n+q}\right)\phi\left(\vartheta_{n+2},\vartheta_{n+q}\right)\right)}\right) \circ \cdots \circ\\
& N_\phi\left(\vartheta_{n+q-1},\vartheta_{n+q},\frac{\mathcal{T}}{q\left(\phi\left(\vartheta_n,\vartheta_{n+q}\right)\phi\left(\vartheta_{n+1},\vartheta_{n+q}\right)\phi\left(\vartheta_{n+2},\vartheta_{n+q}\right)\cdots\phi\left(\vartheta_{n+q-1},\vartheta_{n+q}\right)\right)}\right).
\end{aligned}$$

Using (3) (v) and (x), one writes

$$\begin{aligned}
M_\phi\left(\vartheta_n,\vartheta_{n+q},\mathcal{T}\right) \geq\ & M_\phi\left(\vartheta_0,\vartheta_1,\frac{\mathcal{T}}{q\left(\phi\left(\vartheta_n,\vartheta_{n+q}\right)\right)k^n}\right) * M_\phi\left(\vartheta_0,\vartheta_1,\frac{\mathcal{T}}{q\left(\phi\left(\vartheta_n,\vartheta_{n+q}\right)\phi\left(\vartheta_{n+1},\vartheta_{n+q}\right)\right)k^{n+1}}\right)\\
& * M_\phi\left(\vartheta_0,\vartheta_1,\frac{\mathcal{T}}{q\left(\phi\left(\vartheta_n,\vartheta_{n+q}\right)\phi\left(\vartheta_{n+1},\vartheta_{n+q}\right)\phi\left(\vartheta_{n+2},\vartheta_{n+q}\right)\right)k^{n+2}}\right) * \cdots *\\
& M_\phi\left(\vartheta_0,\vartheta_1,\frac{\mathcal{T}}{q\left(\phi\left(\vartheta_n,\vartheta_{n+q}\right)\phi\left(\vartheta_{n+1},\vartheta_{n+q}\right)\phi\left(\vartheta_{n+2},\vartheta_{n+q}\right)\cdots\phi\left(\vartheta_{n+q-1},\vartheta_{n+q}\right)\right)k^{n+q-1}}\right)
\end{aligned}$$

and

$$\begin{aligned}
N_\phi\left(\vartheta_n,\vartheta_{n+q},\mathcal{T}\right) \leq\ & N_\phi\left(\vartheta_0,\vartheta_1,\frac{\mathcal{T}}{q\left(\phi\left(\vartheta_n,\vartheta_{n+q}\right)\right)k^n}\right) \circ N_\phi\left(\vartheta_0,\vartheta_1,\frac{\mathcal{T}}{q\left(\phi\left(\vartheta_n,\vartheta_{n+q}\right)\phi\left(\vartheta_{n+1},\vartheta_{n+q}\right)\right)k^{n+2}}\right)\\
& \circ N_\phi\left(\vartheta_0,\vartheta_1,\frac{\mathcal{T}}{q\left(\phi\left(\vartheta_n,\vartheta_{n+q}\right)\phi\left(\vartheta_{n+1},\vartheta_{n+q}\right)\phi\left(\vartheta_{n+2},\vartheta_{n+q}\right)\right)k^{n+3}}\right) \circ \cdots \circ
\end{aligned}$$

$$N_\phi\left(\vartheta_0, \vartheta_1, \frac{\mathcal{T}}{q\left(\phi\left(\vartheta_n, \vartheta_{n+q}\right)\phi\left(\vartheta_{n+1}, \vartheta_{n+q}\right)\phi\left(\vartheta_{n+2}, \vartheta_{n+q}\right)\cdots\phi\left(\vartheta_{n+q-1}, \vartheta_{n+q}\right)\right)k^{n+q-1}}\right).$$

We obtain for all $n, q \in \mathbb{N}$, $\phi\left(\vartheta_n, \vartheta_{n+q}\right)k < 1$, where $0 < k < 1$. Therefore, from (vi), (xi), (1) and letting $n \to \infty$,

$$\lim_{n\to\infty} M_\phi\left(\vartheta_n, \vartheta_{n+q}, \mathcal{T}\right) = 1 * 1 * \cdots * = 1$$

and

$$\lim_{n\to\infty} N_\phi\left(\vartheta_n, \vartheta_{n+q}, \mathcal{T}\right) = 0 \circ 0 \circ \cdots \circ 0 = 0.$$

That is, $\{\vartheta_n\}$ is a GCS. Since $\left(\mathfrak{D}, M_\phi, N_\phi, *, \circ\right)$ is a G-complete IEFBMLS, there ϑ in $\mathfrak{D}$ so that

$$\lim_{n\to\infty} M_\phi\left(\vartheta_n, \vartheta_{n+q}, \mathcal{T}\right) = \lim_{n\to\infty} M_\phi\left(\vartheta_n, \vartheta, \mathcal{T}\right) = M_\phi(\vartheta, \vartheta, \mathcal{T}),$$

$$\lim_{n\to\infty} N_\phi\left(\vartheta_n, \vartheta_{n+q}, \mathcal{T}\right) = \lim_{n\to\infty} N_\phi\left(\vartheta_n, \vartheta, \mathcal{T}\right) = N_\phi\left(\vartheta, \vartheta, \mathcal{T}\right)$$

Now, using (v), (x) and (1), we obtain

$$\begin{aligned} M_\phi\left(f\vartheta, \vartheta, \mathcal{T}\right) &\geq M_\phi\left(f\nu, f\vartheta_n, \frac{\mathcal{T}}{2\left(\phi\left(f\vartheta, \vartheta\right)\right)}\right) * M_\phi\left(f\vartheta_n, \nu, \frac{\mathcal{T}}{2\left(\phi\left(f\vartheta, \vartheta\right)\right)}\right) \\ &\geq M_\phi\left(\nu, \vartheta_n, \frac{\mathcal{T}}{2\left(\phi\left(f\vartheta, \vartheta\right)\right)k}\right) * M_\phi\left(\vartheta_{n+1}, \vartheta_n, \frac{\mathcal{T}}{2\left(\phi\left(f\vartheta, \vartheta\right)\right)}\right) \to 1 * 1 = 1 \text{ as } n \to \infty, \end{aligned}$$

and

$$\begin{aligned} N_\phi\left(f\vartheta, \vartheta, \mathcal{T}\right) &\leq N_\phi\left(f\nu, f\vartheta_n, \frac{\mathcal{T}}{2\left(\phi\left(f\vartheta, \vartheta\right)\right)}\right) \circ N_\phi\left(f\vartheta_n, \nu, \frac{\mathcal{T}}{2\left(\phi\left(f\vartheta, \vartheta\right)\right)}\right) \\ &\leq N_\phi\left(\nu, \vartheta_n, \frac{\mathcal{T}}{2\left(\phi\left(f\vartheta, \vartheta\right)\right)k}\right) \circ N_\phi\left(\vartheta_{n+1}, \vartheta_n, \frac{\mathcal{T}}{2\left(\phi\left(f\vartheta, \vartheta\right)\right)}\right) \to 0 \circ 0 = 0 \text{ as } n \to \infty. \end{aligned}$$

This implies that $f\vartheta = \vartheta$. To prove the uniqueness, suppose that $fc = c$ for some $c \in \mathfrak{D}$, then

$$\begin{aligned} 1 &\geq M_\phi\left(c, \vartheta, \mathcal{T}\right) = M_\phi\left(fc, f\vartheta, \mathcal{T}\right) \geq M_\phi\left(c, \vartheta, \frac{\mathcal{T}}{k}\right) = M_\phi\left(fc, f\vartheta, \frac{\mathcal{T}}{k}\right) \\ &\geq M_\phi\left(c, \vartheta, \frac{\mathcal{T}}{k^2}\right) \geq \cdots \geq M_\phi\left(c, \vartheta, \frac{\mathcal{T}}{k^n}\right) \to 1 \text{ as } n \to \infty, \end{aligned}$$

and

$$\begin{aligned} 0 \leq N_\phi\left(c, \vartheta, \mathcal{T}\right) = N_\phi\left(fc, f\vartheta, \mathcal{T}\right) &\leq N_\phi\left(c, \vartheta, \frac{\mathcal{T}}{k}\right) = N_\phi\left(fc, f\vartheta, \frac{\mathcal{T}}{k}\right) \\ &\leq N_\phi\left(c, \vartheta, \frac{\mathcal{T}}{k^2}\right) \leq \cdots \leq N_\phi\left(c, \vartheta, \frac{\mathcal{T}}{k^n}\right) \to 0 \text{ as } n \to \infty. \end{aligned}$$

By using (iii) and (viii), we get $\vartheta = c$.

Example 3.15 Let $\mathfrak{D} = [0, 1]$ and define $M_\phi, N_\phi : \mathfrak{D} \times \mathfrak{D} \times (0, \infty) \to [0, 1]$ as

$$M_\phi\left(\vartheta, \delta, \mathcal{T}\right) = \frac{\mathcal{T}}{\mathcal{T} + \max\{\vartheta, \delta\}}$$

and

$$N_\phi(\vartheta, \delta, \mathcal{T}) = 1 - \frac{\mathcal{T}}{\mathcal{T} + \max\{\vartheta, \delta\}}$$

with CTN $*$ such that $a * b = a \cdot b$ and CTCN $\circ$ such that $a \circ b = \max\{a, b\}$. Define $\phi : \mathfrak{D} \times \mathfrak{D} \to [1, \infty)$ as

$$\phi(\vartheta, \delta) = \begin{cases} 1 + \vartheta & if \quad \vartheta > \delta; \\ 1 + \delta & if \quad \delta > \vartheta; \\ 1 + \vartheta + \delta & if \quad otherwise. \end{cases}$$

Then clearly $(\mathfrak{D}, M_\phi, N_\phi, *, \circ)$ is a complete IEFBMLS. Now, define f : $\mathfrak{D} \times \mathfrak{D} \to \mathfrak{D}$ as

$$f(\vartheta) = \frac{\vartheta}{6}.$$

Let $k \in \left[\frac{1}{2}, 1\right)$, then we have the following:

$$M_\phi(f\vartheta, f\delta, k\mathcal{T}) = M_\phi\left(\frac{\vartheta}{6}, \frac{\delta}{6}, k\mathcal{T}\right) = \frac{k\mathcal{T}}{k\mathcal{T} + \max\left\{\frac{\vartheta}{6}, \frac{\delta}{6}\right\}} = \frac{\mathcal{T}}{\mathcal{T} + \max\left\{\frac{\vartheta}{k6}, \frac{\delta}{k6}\right\}} \geq \frac{\mathcal{T}}{\mathcal{T} + \max\{\vartheta, \delta\}}$$

$$= M_\phi(\vartheta, \delta, \mathcal{T})$$

and

$$N_\phi(f\vartheta, f\delta, k\mathcal{T}) = N_\phi\left(\frac{\vartheta}{6}, \frac{\delta}{6}, k\mathcal{T}\right) = 1 - \frac{k\mathcal{T}}{k\mathcal{T} + \max\left\{\frac{\vartheta}{6}, \frac{\delta}{6}\right\}} = 1 - \frac{\mathcal{T}}{\mathcal{T} + \max\left\{\frac{\vartheta}{k6}, \frac{\delta}{k6}\right\}}$$

$$\leq 1 - \frac{\mathcal{T}}{\mathcal{T} + \max\{\vartheta, \delta\}} = N_\phi(\vartheta, \delta, \mathcal{T}).$$

Observe that all the conditions of Theorem 3.14 are satisfied and 0 is a unique fixed point, i.e., $f(0) = 0$.

Now, we use the Example 3.15 to show the graphical view of contraction mapping and a unique fixed point. Below, in Fig. 1, we show the graphical view of $M\varphi$ (f ϑ, f δ, kT) = $M\varphi$ (ϑ, δ, T). Table 1 shows the values of $M\varphi$ (f ϑ, f δ, kT) and Table 2 shows the values of $M\varphi$ (ϑ, δ, T). In Fig. 2, we show the graphical view of $N\varphi$ (f ϑ, f δ, kT) = $N\varphi$ (ϑ, δ, T). Table 3 shows the values of $N\varphi$ (f ϑ, f δ, kT) and Table 4 shows the values of $N\varphi$ (ϑ, δ, T). In Fig. 3, we show the view of unique fixed point.

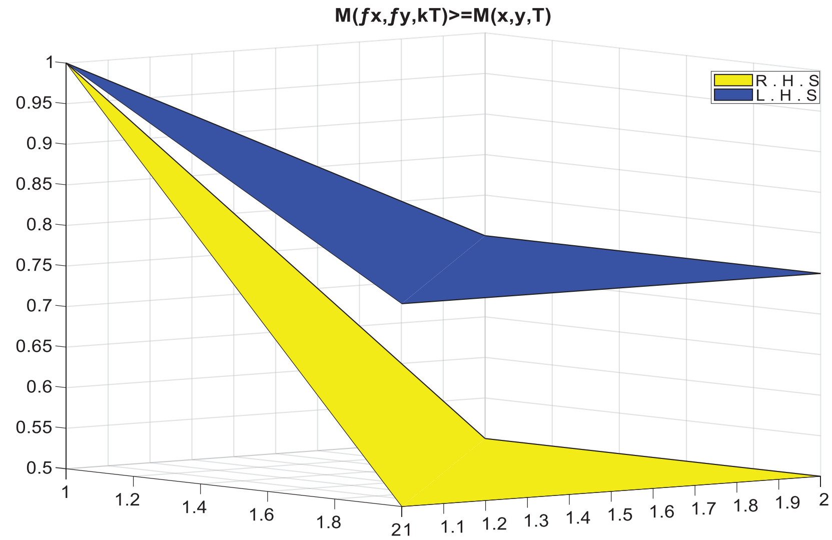

Figure 1: Variation of L.H.S. = $M_\phi\left(f\vartheta, f\delta, k\mathcal{T}\right)$ with R.H.S. = $M_\phi\left(\vartheta, \delta, \mathcal{T}\right)$ of an Example 3.15 for $\mathcal{T} = 1$ and $k \in \left[\frac{1}{2}, 1\right)$.

Table 1: The matrix of values of L.H.S. = $M_\phi(f\vartheta, f\delta, k\mathcal{T})$, in which first row represents the values of δ and first column represents the values of ϑ

	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
0	1	0.9677	0.9375	0.9090	0.8823	0.8571	0.8333	0.8108	0.7894	0.7692	0.7500
0.1	0.9677	0.9677	0.9375	0.9090	0.8823	0.8571	0.8333	0.8108	0.7894	0.7692	0.7500
0.2	0.9375	0.9375	0.9375	0.9090	0.8823	0.8571	0.8333	0.8108	0.7894	0.7692	0.7500
0.3	0.9090	0.9090	0.9090	0.9090	0.8823	0.8571	0.8333	0.8108	0.7894	0.7692	0.7500
0.4	0.8823	0.8823	0.8823	0.8823	0.8823	0.8571	0.8333	0.8108	0.7894	0.7692	0.7500
0.5	0.8571	0.8571	0.8571	0.8571	0.8571	0.8571	0.8333	0.8108	0.7894	0.7692	0.7500
0.6	0.8333	0.8333	0.8333	0.8333	0.8333	0.8333	0.8333	0.8108	0.7894	0.7692	0.7500
0.7	0.8108	0.8108	0.8108	0.8108	0.8108	0.8108	0.8108	0.8108	0.7894	0.7692	0.7500
0.8	0.7894	0.7894	0.7894	0.7894	0.7894	0.7894	0.7894	0.7894	0.7894	0.7692	0.7500
0.9	0.7692	0.7692	0.7692	0.7692	0.7692	0.7692	0.7692	0.7692	0.7692	0.7692	0.7500
1	0.7500	0.7500	0.7500	0.7500	0.7500	0.7500	0.7500	0.7500	0.7500	0.7500	0.7500

Table 2: The matrix of values of R.H.S. = $M_\phi(\vartheta, \delta, \mathcal{T})$, in which first row represents the values of δ and first column represents the values of ϑ

	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
0	1	0.9090	0.8333	0.7692	0.7142	0.6666	0.6250	0.5882	0.5555	0.5263	0.5000
0.1	0.9090	0.9090	0.8333	0.7692	0.7142	0.6666	0.6250	0.5882	0.5555	0.5263	0.5000
0.2	0.8333	0.8333	0.8333	0.7692	0.7142	0.6666	0.6250	0.5882	0.5555	0.5263	0.5000

(Continued)

Table 2 (continued)

	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
0.3	0.7692	0.7692	0.7692	0.7692	0.7142	0.6666	0.6250	0.5882	0.5555	0.5263	0.5000
0.4	0.7142	0.7142	0.7142	0.7142	0.7142	0.6666	0.6250	0.5882	0.5555	0.5263	0.5000
0.5	0.6666	0.6666	0.6666	0.6666	0.6666	0.6666	0.6250	0.5882	0.5555	0.5263	0.5000
0.6	0.6250	0.6250	0.6250	0.6250	0.6250	0.6250	0.6250	0.5882	0.5555	0.5263	0.5000
0.7	0.5882	0.5882	0.5882	0.5882	0.5882	0.5882	0.5882	0.5882	0.5555	0.5263	0.5000
0.8	0.5555	0.5555	0.5555	0.5555	0.5555	0.5555	0.5555	0.5555	0.5555	0.5263	0.5000
0.9	0.5263	0.5263	0.5263	0.5263	0.5263	0.5263	0.5263	0.5263	0.5263	0.5263	0.5000
1	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000

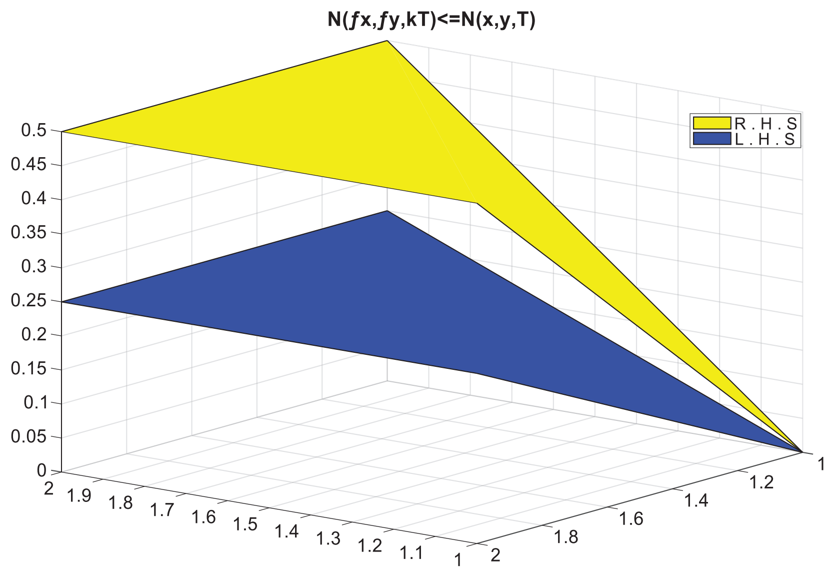

Figure 2: Variation of L.H.S. $= N_\phi\,(f\vartheta, f\delta, k\mathcal{T})$ with R.H.S. $= N_\phi\,(\vartheta, \delta, \mathcal{T})$ of an Example 3.15 for $\mathcal{T} = 1$ and $k \in \left[\frac{1}{2}, 1\right)$.

Table 3: The matrix of values of R.H.S. $= N_\phi(\vartheta, \delta, \mathcal{T})$, in which first row represents the values of δ and first column represents the values of ϑ

	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
0	0	0.0909	0.1666	0.2307	0.2857	0.3333	0.3750	0.4117	0.4444	0.4736	0.5000
0.1	0.0909	0.0909	0.1666	0.2307	0.2857	0.3333	0.3750	0.4117	0.4444	0.4736	0.5000
0.2	0.1666	0.1666	0.1666	0.2307	0.2857	0.3333	0.3750	0.4117	0.4444	0.4736	0.5000
0.3	0.2307	0.2307	0.2307	0.2307	0.2857	0.3333	0.3750	0.4117	0.4444	0.4736	0.5000
0.4	0.2857	0.2857	0.2857	0.2857	0.2857	0.3333	0.3750	0.4117	0.4444	0.4736	0.5000
0.5	0.3333	0.3333	0.3333	0.3333	0.3333	0.3333	0.3750	0.4117	0.4444	0.4736	0.5000
0.6	0.3750	0.3750	0.3750	0.3750	0.3750	0.3750	0.3750	0.4117	0.4444	0.4736	0.5000

(Continued)

Table 3 (continued)

	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
0.7	0.4117	0.4117	0.4117	0.4117	0.4117	0.4117	0.4117	0.4117	0.4444	0.4736	0.5000
0.8	0.4444	0.4444	0.4444	0.4444	0.4444	0.4444	0.4444	0.4444	0.4444	0.4736	0.5000
0.9	0.4736	0.4736	0.4736	0.4736	0.4736	0.4736	0.4736	0.4736	0.4736	0.4736	0.5000
1	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000

Table 4: The matrix of values of L.H.S. $= N_{\phi}(f\vartheta, f\delta, k\mathcal{T})$, in which first row represents the values of δ and first column represents the values of ϑ

	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
0	0	0.0322	0.0625	0.0909	0.1176	0.1428	0.1666	0.1891	0.2105	0.2307	0.2500
0.1	0.0322	0.0322	0.0625	0.0909	0.1176	0.1428	0.1666	0.1891	0.2105	0.2307	0.2500
0.2	0.0625	0.0625	0.0625	0.0909	0.1176	0.1428	0.1666	0.1891	0.2105	0.2307	0.2500
0.3	0.0909	0.0909	0.0909	0.0909	0.1176	0.1428	0.1666	0.1891	0.2105	0.2307	0.2500
0.4	0.1176	0.1176	0.1176	0.1176	0.1176	0.1428	0.1666	0.1891	0.2105	0.2307	0.2500
0.5	0.1428	0.1428	0.1428	0.1428	0.1428	0.1428	0.1666	0.1891	0.2105	0.2307	0.2500
0.6	0.1666	0.1666	0.1666	0.1666	0.1666	0.1666	0.1666	0.1891	0.2105	0.2307	0.2500
0.7	0.1891	0.1891	0.1891	0.1891	0.1891	0.1891	0.1891	0.1891	0.2105	0.2307	0.2500
0.8	0.2105	0.2105	0.2105	0.2105	0.2105	0.2105	0.2105	0.2105	0.2105	0.2307	0.2500
0.9	0.2307	0.2307	0.2307	0.2307	0.2307	0.2307	0.2307	0.2307	0.2307	0.2307	0.2500
1	0.2500	0.2500	0.2500	0.2500	0.2500	0.2500	0.2500	0.2500	0.2500	0.2500	0.2500

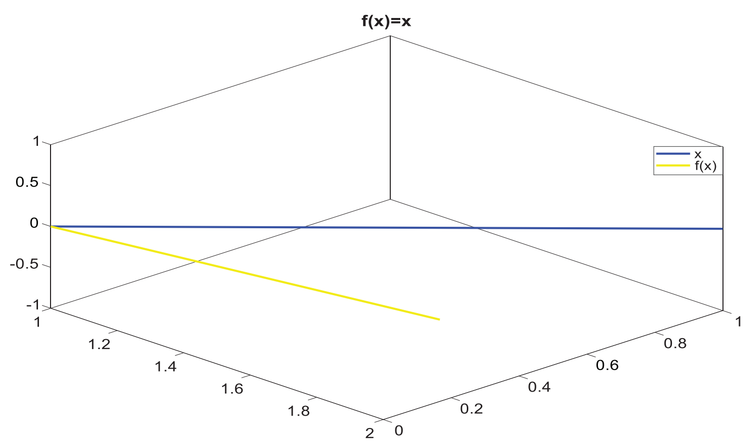

Figure 3: Graph of $f\vartheta = \vartheta$, we can see that both lines intersect each other at 0. This shows that 0 is a unique fixed point

Definition 3.16 Let $(\mathfrak{D}, M_\phi, N_\phi, *, \circ)$ be an IEFBMLS. A map f : $\mathfrak{D} \to \mathfrak{D}$ is an intuitionistic extended fuzzy b-like contraction mapping if there exists $0 < k < 1$ such that

$$\frac{1}{M_\phi(f\vartheta, f\delta, \mathcal{T})} - 1 \leq k\left[\frac{1}{M_\phi(\vartheta, \delta, \mathcal{T})} - 1\right] \tag{4}$$

and

$$N_\phi(f\vartheta, f\delta, \mathcal{T}) \leq kN_\phi(\vartheta, \delta, \mathcal{T}), \tag{5}$$

for all $\vartheta, \delta \in \mathfrak{D}$ and $\mathcal{T} > 0$.

Now, we prove the following theorem related to above contraction mapping.

Theorem 3.17 Let $(\mathfrak{D}, M_\phi, N_\phi, *, \circ)$ be a G-complete IEFBMLS with $\phi : \mathfrak{D} \times \mathfrak{D} \to [1, \infty)$. Suppose that

$$\lim_{\mathcal{T}\to\infty} M_\phi(\vartheta, \delta, \mathcal{T}) = 1 \text{ and } \lim_{\mathcal{T}\to\infty} N_\phi(\vartheta, \delta, \mathcal{T}) = 0 \tag{6}$$

for all $\vartheta, \delta \in \mathfrak{D}$ and $\mathcal{T} > 0$. Let f : $\mathfrak{D} \to \mathfrak{D}$ be an intuitionistic extended fuzzy b-like contraction mapping. Further, suppose that for an arbitrary $\vartheta_0 \in \mathfrak{D}$, and $n, q \in \mathbb{N}$, we have $\vartheta_n = f^n\vartheta_0 = f\nu_{n-1}$. Then f has a unique FP.

Proof: Let ϑ_0 be in $\mathfrak{D}$. Take $\vartheta_n = f^n\vartheta_0 = f\nu_{n-1}$, $n \in \mathbb{N}$. By using (4) and (5) for all $\mathcal{T} > 0$, $n > q$, we have

$$\frac{1}{M_\phi(\vartheta_n, \vartheta_{n+1}, \mathcal{T})} - 1 = \frac{1}{M_\phi(f\vartheta_{n-1}, \vartheta_n, \mathcal{T})} - 1$$
$$\leq k\left[\frac{1}{M_\phi(\vartheta_{n-1}, \vartheta_n, \mathcal{T})} - 1\right] = \frac{k}{M_\phi(\vartheta_{n-1}, \vartheta_n, \mathcal{T})} - k.$$

That is

$$\frac{1}{M_\phi(\vartheta_n, \vartheta_{n+1}, \mathcal{T})} \leq \frac{k}{M_\phi(\vartheta_{n-1}, \vartheta_n, \mathcal{T})} + (1-k) \leq \frac{k^2}{M_\phi(\vartheta_{n-2}, \vartheta_{n-1}, \mathcal{T})} + k(1-k) + (1-k).$$

Continuing in this way, we get

$$\frac{1}{M_\phi(\vartheta_n, \vartheta_{n+1}, \mathcal{T})} \leq \frac{k^n}{M_\phi(\vartheta_0, \vartheta_1, \mathcal{T})} + k^{n-1}(1-k) + k^{n-2}(1-k) + \cdots + k(1-k) + (1-k)$$
$$\leq \frac{k^n}{M_\phi(\vartheta_0, \vartheta_1, \mathcal{T})} + \left(k^{n-1} + k^{n-2} + \cdots + 1\right)(1-k) \leq \frac{k^n}{M_\phi(\vartheta_0, \vartheta_1, \mathcal{T})} + (1-k^n).$$

We obtain

$$\frac{1}{\frac{k^n}{M_\phi(\vartheta_0, \vartheta_1, \mathcal{T})} + (1-k^n)} \leq M_\phi(\vartheta_n, \vartheta_{n+1}, \mathcal{T}) \tag{7}$$

and

$$N_\phi(\vartheta_n, \vartheta_{n+1}, \mathcal{T}) = N_\phi(f\vartheta_{n-1}, \vartheta_n, \mathcal{T}) \leq kN_\phi(\vartheta_{n-1}, \vartheta_n, \mathcal{T}) = N_\phi(f\vartheta_{n-2}, \vartheta_{n-1}, \mathcal{T}) \tag{8}$$
$$\leq k^2N_\phi(\vartheta_{n-2}, \vartheta_{n-1}, \mathcal{T}) \leq \cdots \leq k^nN_\phi(\vartheta_0, \vartheta_1, \mathcal{T})$$

for all $q \in \mathbb{N}$, $\mathcal{T} = \frac{q\mathcal{T}}{\mathcal{T}} = \frac{\mathcal{T}}{q} + \frac{\mathcal{T}}{q} + \cdots + \frac{\mathcal{T}}{q}$. Using (v) and (x), we deduce

$$M_\phi\left(\vartheta_n, \vartheta_{n+q}, \mathcal{T}\right) \geq M_\phi\left(\vartheta_n, \vartheta_{n+1}, \frac{\mathcal{T}}{q\left(\phi\left(\vartheta_n, \vartheta_{n+q}\right)\right)}\right) * M_\phi\left(\vartheta_{n+1}, \vartheta_{n+2}, \frac{\mathcal{T}}{q\left(\phi\left(\vartheta_n, \vartheta_{n+q}\right)\phi\left(\vartheta_{n+1}, \vartheta_{n+q}\right)\right)}\right)$$
$$* M_\phi\left(\vartheta_{n+2}, \vartheta_{n+3}, \frac{\mathcal{T}}{q\left(\phi\left(\vartheta_n, \vartheta_{n+q}\right)\phi\left(\vartheta_{n+1}, \vartheta_{n+q}\right)\phi\left(\vartheta_{n+2}, \vartheta_{n+q}\right)\right)}\right) * \cdots *$$
$$M_\phi\left(\vartheta_{n+q-1}, \vartheta_{n+q}, \frac{\mathcal{T}}{q\left(\phi\left(\vartheta_n, \vartheta_{n+q}\right)\phi\left(\vartheta_{n+1}, \vartheta_{n+q}\right)\phi\left(\vartheta_{n+2}, \vartheta_{n+q}\right)\cdots\phi\left(\vartheta_{n+q-1}, \vartheta_{n+q}\right)\right)}\right)$$

and

$$N_\phi\left(\vartheta_n, \vartheta_{n+q}, \mathcal{T}\right) \leq N_\phi\left(\vartheta_n, \vartheta_{n+1}, \frac{\mathcal{T}}{q\left(\phi\left(\vartheta_n, \vartheta_{n+q}\right)\right)}\right) \circ N_\phi\left(\vartheta_{n+1}, \vartheta_{n+2}, \frac{\mathcal{T}}{q\left(\phi\left(\vartheta_n, \vartheta_{n+q}\right)\phi\left(\vartheta_{n+1}, \vartheta_{n+q}\right)\right)}\right)$$
$$\circ N_\phi\left(\vartheta_{n+2}, \vartheta_{n+3}, \frac{\mathcal{T}}{q\left(\phi\left(\vartheta_n, \vartheta_{n+q}\right)\phi\left(\vartheta_{n+1}, \vartheta_{n+q}\right)\phi\left(\vartheta_{n+2}, \vartheta_{n+q}\right)\right)}\right) \circ \cdots \circ$$
$$N_\phi\left(\vartheta_{n+q-1}, \vartheta_{n+q}, \frac{\mathcal{T}}{q\left(\phi\left(\vartheta_n, \vartheta_{n+q}\right)\phi\left(\vartheta_{n+1}, \vartheta_{n+q}\right)\phi\left(\vartheta_{n+2}, \vartheta_{n+q}\right)\cdots\phi\left(\vartheta_{n+q-1}, \vartheta_{n+q}\right)\right)}\right).$$

Using (3), (v) and (x), we deduce

$$M_\phi\left(\vartheta_n, \vartheta_{n+q}, \mathcal{T}\right) \geq \frac{1}{\frac{k^n}{M_\phi\left(\vartheta_0, \vartheta_1, \frac{\mathcal{T}}{q\left(\phi\left(\vartheta_n, \vartheta_{n+q}\right)\right)}\right)} + (1 - k^n)} * \frac{1}{\frac{k^{n+1}}{M_\phi\left(\vartheta_0, \vartheta_1, \frac{\mathcal{T}}{q\left(\phi\left(\vartheta_n, \vartheta_{n+q}\right)\phi\left(\vartheta_{n+1}, \vartheta_{n+q}\right)\right)}\right)} + (1 - k^{n+1})}$$
$$* \frac{1}{\frac{k^{n+2}}{M_\phi\left(\vartheta_0, \vartheta_1, \frac{\mathcal{T}}{q\left(\phi\left(\vartheta_n, \vartheta_{n+q}\right)\phi\left(\vartheta_{n+1}, \vartheta_{n+q}\right)\phi\left(\vartheta_{n+2}, \vartheta_{n+q}\right)\right)}\right)} + (1 - k^{n+2})} * \cdots *$$
$$\frac{1}{\frac{k^{n+q-1}}{M_\phi\left(\vartheta_0, \vartheta_1, \frac{\mathcal{T}}{q\left(\phi\left(\vartheta_n, \vartheta_{n+q}\right)\phi\left(\vartheta_{n+1}, \vartheta_{n+q}\right)\phi\left(\vartheta_{n+2}, \vartheta_{n+q}\right)\cdots\phi\left(\vartheta_{n+q-1}, \vartheta_{n+q}\right)\right)}\right)} + (1 - k^{n+q-1})}$$

and

$$N_\phi\left(\vartheta_n, \vartheta_{n+q}, \mathcal{T}\right) \leq k^n N_\phi\left(\vartheta_0, \vartheta_1, \frac{\mathcal{T}}{q\left(\phi\left(\vartheta_n, \vartheta_{n+q}\right)\right)}\right) \circ k^{n+1} N_\phi\left(\vartheta_0, \vartheta_1, \frac{\mathcal{T}}{q\left(\phi\left(\vartheta_n, \vartheta_{n+q}\right)\phi\left(\vartheta_{n+1}, \vartheta_{n+q}\right)\right)}\right)$$
$$\circ k^{n+2} N_\phi\left(\vartheta_0, \vartheta_1, \frac{\mathcal{T}}{q\left(\phi\left(\vartheta_n, \vartheta_{n+q}\right)\phi\left(\vartheta_{n+1}, \vartheta_{n+q}\right)\phi\left(\vartheta_{n+2}, \vartheta_{n+q}\right)\right)}\right) \circ \cdots \circ$$
$$k^{n+q-1} N_\phi\left(\vartheta_0, \vartheta_1, \frac{\mathcal{T}}{q\left(\phi\left(\vartheta_n, \vartheta_{n+q}\right)\phi\left(\vartheta_{n+1}, \vartheta_{n+q}\right)\phi\left(\vartheta_{n+2}, \vartheta_{n+q}\right)\cdots\phi\left(\vartheta_{n+q-1}, \vartheta_{n+q}\right)\right)}\right).$$

Consequently, for all $n, q \in \mathbb{N}$, we obtain $\phi\left(\vartheta_n, \vartheta_{n+q}\right) k < 1$, where $0 < k < 1$. Therefore, from (vi), (xi), (1) and for $n \to \infty$,

$$\lim_{n\to\infty} M_\phi\left(\vartheta_n, \vartheta_{n+q}, \mathcal{T}\right) = 1 * 1 * \cdots * = 1$$

and

$$\lim_{n\to\infty} N_\phi\left(\vartheta_n, \vartheta_{n+q}, \mathcal{T}\right) = 0 \circ 0 \circ \cdots \circ 0 = 0,$$

i.e., $\{\vartheta_n\}$ is a GCS. Since $\left(\mathfrak{D}, M_\phi, N_\phi, *, \circ\right)$ is a G-complete IEFBMLS, there exists

$$\lim_{n\to\infty} M_\phi\left(\vartheta_n, \vartheta_{n+q}, \mathcal{T}\right) = \lim_{n\to\infty} M_\phi\left(\vartheta_n, \vartheta, \mathcal{T}\right) = M_\phi(\vartheta, \vartheta, \mathcal{T}),$$

$$\lim_{n\to\infty} N_\phi\left(\vartheta_n, \vartheta_{n+q}, \mathcal{T}\right) = \lim_{n\to\infty} N_\phi\left(\vartheta_n, \vartheta, \mathcal{T}\right) = N_\phi(\vartheta, \vartheta, \mathcal{T}).$$

Now, we investigate that ϑ is a FP of f. Using (v), (x) and (1), we obtain

$$\frac{1}{M_\phi\left(f\vartheta_n, f\vartheta, \mathcal{T}\right)} - 1 \leq k\left[\frac{1}{M_\phi\left(\vartheta_n, \vartheta, \mathcal{T}\right)} - 1\right] = \frac{k}{M_\phi\left(\vartheta_n, \vartheta, \mathcal{T}\right)} - k.$$

That is,

$$\frac{1}{\frac{k}{M_\phi(\vartheta_n, \vartheta, \mathcal{T})} + (1-k)} \leq M_\phi(f\vartheta_n, f\vartheta, \mathcal{T}).$$

It implies that

$$\begin{aligned} M_\phi\left(\vartheta, f\vartheta, \mathcal{T}\right) &\geq M_\phi\left(\vartheta, \vartheta_{n+1}, \frac{\mathcal{T}}{2\phi\left(\vartheta, f\vartheta\right)}\right) * M_\phi\left(f\vartheta_n, f\vartheta, \frac{\mathcal{T}}{2\phi\left(\vartheta, f\vartheta\right)}\right) \\ &\geq M_\phi\left(\vartheta_n, \vartheta_{n+1}, \frac{\mathcal{T}}{2\phi\left(\vartheta, f\vartheta\right)}\right) * \frac{1}{\frac{k}{M_\phi\left(\vartheta_n, \vartheta, \frac{\mathcal{T}}{2\phi(\vartheta, f\vartheta)}\right)} + (1-k)} \to 1 * 1 = 1 \text{ as } n \to \infty \end{aligned}$$

and

$$\begin{aligned} N_\phi\left(\vartheta, f\vartheta, \mathcal{T}\right) &\leq M_\phi\left(\vartheta, \vartheta_{n+1}, \frac{\mathcal{T}}{2\phi\left(\vartheta, f\vartheta\right)}\right) \circ N_\phi\left(f\vartheta_n, f\vartheta, \frac{\mathcal{T}}{2\phi\left(\vartheta, f\vartheta\right)}\right) \\ &\leq N_\phi\left(\vartheta_n, \vartheta_{n+1}, \frac{\mathcal{T}}{2\phi\left(\vartheta, f\vartheta\right)}\right) \circ kN_\phi\left(\vartheta_n, \vartheta, \frac{\mathcal{T}}{2\phi\left(\vartheta, f\vartheta\right)}\right) \to 0 \circ 0 = 0 \text{ as } n \to \infty. \end{aligned}$$

This yields that $f\vartheta = \vartheta$, a FP. Now, we show the uniqueness. Suppose $fc = c$ for some $c \in \mathfrak{D}$, then

$$\begin{aligned} &\frac{1}{M_\phi\left(\vartheta, c, \mathcal{T}\right)} - 1 = \frac{1}{M_\phi\left(f\vartheta, fc, \mathcal{T}\right)} - 1 \\ &\leq k\left[\frac{1}{M_\phi\left(\vartheta, c, \mathcal{T}\right)} - 1\right] < \frac{1}{M_\phi\left(\vartheta, c, \mathcal{T}\right)} - 1 \end{aligned}$$

which is a contradiction. Also,

$$N_\phi\left(\vartheta, c, \mathcal{T}\right) = N_\phi\left(f\vartheta, fc, \mathcal{T}\right) \leq kN_\phi\left(\vartheta, c, \mathcal{T}\right) < N_\phi(\vartheta, c, \mathcal{T}).$$

Again, it is a contradiction. Therefore, we must have $M_\phi\left(\vartheta, c, \mathcal{T}\right) = 1$ and $N_\phi\left(\vartheta, c, \mathcal{T}\right) = 0$, hence $\vartheta = c$.

Example 3.18 Let $\mathfrak{D} = [0, 1]$. Define ϕ by

$$\phi(\vartheta, \delta) = \begin{cases} 1 & if \quad \vartheta = \delta, \\ 1 + \max\{\vartheta, \delta\} & if \ \vartheta \neq \delta \end{cases}$$

Also take

$$M_\phi(\vartheta, \delta, \mathcal{T}) = e^{-\frac{\max\{\vartheta,\delta\}^2}{\mathcal{T}}} \text{ and } N_\phi(\vartheta, \delta, \mathcal{T}) = 1 - e^{-\frac{\max\{\vartheta,\delta\}^2}{\mathcal{T}}}$$

with $\nu * \omega = \nu.\omega$ and $\nu \circ \omega = \max\{\nu, \omega\}$. Then $(\mathfrak{D}, M_\phi, N_\phi, *, \circ)$ is a G-complete IEFBMLS. Define f : $\mathfrak{D} \to \mathfrak{D}$ by

$$f(\vartheta) = \begin{cases} 0 & if \ \vartheta \in \left[0, \frac{1}{2}\right], \\ \frac{\vartheta}{9} & if \ \vartheta \in \left(\frac{1}{2}, 1\right] \end{cases}$$

Then we have four cases:

1. If $\vartheta, \delta \in \left[0, \frac{1}{2}\right]$, then $f\vartheta = f\delta = 0$;
2. If $\vartheta \in \left[0, \frac{1}{2}\right]$ and $\delta \in \left(\frac{1}{2}, 1\right]$, then $f\vartheta = 0$ and $f\delta = \frac{\delta}{9}$;
3. If $\delta \in \left[0, \frac{1}{2}\right]$ and $\vartheta \in \left(\frac{1}{2}, 1\right]$, then $f\delta = 0$ and $f\vartheta = \frac{\delta}{9}$;
4. If $\vartheta, \delta \in \left(\frac{1}{2}, 1\right]$, then $f\vartheta = \frac{\delta}{9}$ and $f\delta = \frac{\delta}{9}$;

In all $1 - 4$ cases,

$$M_\phi(f\vartheta, f\delta, k\mathcal{T}) \geq M_\phi(\vartheta, \delta, \mathcal{T}) \text{ and } N_\phi(f\vartheta, f\delta, k\mathcal{T}) \leq N_\phi(\vartheta, \delta, \mathcal{T})$$

are satisfied for $k \in \left[\frac{1}{2}, 1\right)$, and also

$$\frac{1}{M_\phi(f\vartheta, f\delta, \mathcal{T})} - 1 \leq k\left[\frac{1}{M_\phi(\vartheta, \delta, \mathcal{T})} - 1\right] \text{ and } N_\phi(f\vartheta, f\delta, \mathcal{T}) \leq kN_\phi(\vartheta, \delta, \mathcal{T})$$

Satisfied for $k \in \left[\frac{1}{2}, 1\right)$.

Observe that all circumstances of Theorems 3.14 and 3.17 are fulfilled, and 0 is a unique FP of f.

Example 3.19 Let $\mathfrak{D} = [0, 1]$ and $\phi : \mathfrak{D} \times \mathfrak{D} \to [1, \infty)$ be a function given by $\phi(\vartheta, \delta) = \vartheta + \delta + 1$. Define $M_\phi, N_\phi : \mathfrak{D} \times \mathfrak{D} \times (0, \infty) \to [0, 1]$ as

$$M_\phi(\vartheta, \delta, \mathcal{T}) = \begin{cases} 1 & if \ \vartheta = \delta \\ k & if \ \vartheta \neq \delta, \end{cases}$$

$$N_\phi(\vartheta, \delta, \mathcal{T}) = \begin{cases} 0 & if \quad \vartheta = \delta \\ 1 - k & if \ \vartheta \neq \delta \end{cases}$$

for all $k \in \mathfrak{D}$. Then $(\mathfrak{D}, M_\phi, N_\phi, *, \circ)$ is an G-complete IEFBMLS with CTN $a * b = ab$ and CTCN $a \circ b = \max\{a, b\}$. Define f : $\mathfrak{D} \to \mathfrak{D}$ by

$$f(\vartheta) = \frac{1 - 2^{-\vartheta}}{3}.$$

Then,

$M_\phi(f\vartheta, f\delta, k\mathcal{T}) \geq M_\phi(\vartheta, \delta, \mathcal{T})$ and $N_\phi(f\vartheta, f\delta, k\mathcal{T}) \leq N_\phi(\vartheta, \delta, \mathcal{T})$

are satisfied for $k \in \left[\frac{1}{2}, 1\right)$, and also

$$\frac{1}{M_\phi(f\vartheta, f\delta, \mathcal{T})} - 1 \leq k\left[\frac{1}{M_\phi(\vartheta, \delta, \mathcal{T})} - 1\right]$$

and

$$N_\phi(f\vartheta, f\delta, \mathcal{T}) \leq kN_\phi(\vartheta, \delta, \mathcal{T})$$

satisfied for $k \in \left[\frac{1}{2}, 1\right)$.

Observe that all circumstances of Theorems 3.14 and 3.17 are fulfilled, and 0 is a unique FP of f.

4 Application to Fuzzy Fredholm Integral Equations

Let $\mathfrak{D} = C([e, g], \mathbb{R})$ be the set of all continuous real valued functions defined on the interval $[e, g]$.

Now, we let the fuzzy integral equation

$$\vartheta(l) = f(j) + \beta \int_e^g F(l, j)\,\vartheta(l)\,dj \text{ for } l, j \in [e, g] \tag{9}$$

where $\beta > 0$, $f(j)$ is a fuzzy function of $j \in [e, g]$ and $F \in \mathfrak{D}$. Define M_ϕ and N_ϕ by

$$M_\phi(\vartheta(l), \delta(l), \mathcal{T}) = \sup_{l\in[e,g]} \frac{\mathcal{T}}{\mathcal{T} + \max\{\vartheta(l), \delta(l)\}^2}$$

and

$$N_\phi(\vartheta(l), \delta(l), \mathcal{T}) = 1 - \sup_{l\in[e,g]} \frac{\mathcal{T}}{\mathcal{T} + \max\{\vartheta(l), \delta(l)\}^2}$$

for all $\vartheta, \delta \in \mathfrak{D}$ and $\mathcal{T} > 0$, with the CTN and CTCN defined by $\nu * \omega = \nu.\omega$ and $\nu \circ \omega = \max \nu, \omega$. Define $\phi : \mathfrak{D} \times \mathfrak{D} \to [1, \infty)$ by

$$\phi(\vartheta, \delta) = \begin{cases} 1 & \textit{if} \quad \vartheta = \delta; \\ 1 + \max\{\vartheta, \delta\} & \textit{if otherwise}. \end{cases}$$

Then $(\mathfrak{D}, M_\phi, N_\phi, *, \circ)$ be a G-complete IEFBMLS.

Assume that

$\max\{F(l, j)\vartheta(l), F(l, j)\delta(l)\} \leq \max\{\vartheta(l), \delta(l)\}$ for $\vartheta, \delta \in \mathfrak{D}, k \in (0, 1)$ and $\forall\, l, j \in [e, g]$. Also consider $\left(\beta \int_e^g dj\right)^2 \leq k < 1$. then fuzzy integral equation in Eq. (9) has a unique solution.

Proof: Define f : $\mathfrak{D} \to \mathfrak{D}$ by

$$f\vartheta(l) = f(j) + \beta \int_e^g F(l, j)\,e(l)\,dj \text{ for all } l, j \in [e, g]$$

Scrutinize that survival of an FP of the operator f is come to the survival of solution of the fuzzy integral equation.

Now for all $\vartheta, \ \delta \in \mathfrak{D}$, we obtain

$$M_\phi\left(f\vartheta\left(l\right), f\delta\left(l\right), k\,\mathcal{T}\right) = \sup_{l\in[e,\,g]} \frac{k\mathcal{T}}{k\mathcal{T} + \max\left\{f\vartheta\left(l\right), f\delta\left(l\right)\right\}^2}$$

$$= \sup_{l\in[e,\,g]} \frac{k\mathcal{T}}{k\mathcal{T} + \max\left\{f\left(j\right) + \beta\int_e^g F\left(l,\ j\right)e\left(l\right)dj, f\left(j\right) + \beta\int_e^g F\left(l,\ j\right)e\left(l\right)dj\right\}^2}$$

$$= \sup_{l\in[e,\,g]} \frac{k\mathcal{T}}{k\mathcal{T} + \max\left\{\beta\int_e^g F\left(l,\ j\right)e\left(l\right)dj, \beta\int_e^g F\left(l,\ j\right)e\left(l\right)dj\right\}^2}$$

$$= \sup_{l\in[e,\,g]} \frac{k\mathcal{T}}{k\mathcal{T} + \max\left\{F\left(l,\ j\right)\vartheta\left(l\right), F\left(l,\ j\right)\delta\left(l\right)\right\}^2\left(\beta\int_e^g dj\right)^2}$$

$$\geq \sup_{l\in[e,\,g]} \frac{\mathcal{T}}{\mathcal{T} + \max\left\{\vartheta\left(l\right),\ \delta\left(l\right)\right\}^2}$$

$$\geq M_\phi(\vartheta\left(l\right),\ \delta\left(l\right),\ \mathcal{T}).$$

$$N_\phi\left(f\vartheta\left(l\right), f\delta\left(l\right), k\,\mathcal{T}\right) = 1 - \sup_{l\in[e,\,g]} \frac{k\mathcal{T}}{k\mathcal{T} + \max\left\{f\vartheta\left(l\right), f\delta\left(l\right)\right\}^2}$$

$$= 1 - \sup_{l\in[e,\,g]} \frac{k\mathcal{T}}{k\mathcal{T} + \max\left\{f\left(j\right) + \beta\int_e^g F\left(l,\ j\right)e\left(l\right)dj, f\left(j\right) + \beta\int_e^g F\left(l,\ j\right)e\left(l\right)dj\right\}^2}$$

$$= 1 - \sup_{l\in[e,\,g]} \frac{k\mathcal{T}}{k\mathcal{T} + \max\left\{\beta\int_e^g F\left(l,\ j\right)e\left(l\right)dj, \beta\int_e^g F\left(l,\ j\right)e\left(l\right)dj\right\}^2}$$

$$= 1 - \sup_{l\in[e,\,g]} \frac{k\mathcal{T}}{k\mathcal{T} + \max\left\{F\left(l,\ j\right)\vartheta\left(l\right), F\left(l,\ j\right)\delta\left(l\right)\right\}^2\left(\beta\int_e^g dj\right)^2}$$

$$\leq \sup_{l\in[e,\,g]} \frac{\mathcal{T}}{\mathcal{T} + \max\left\{\vartheta\left(l\right),\ \delta\left(l\right)\right\}^2}$$

$$\leq N_\phi(\vartheta\left(l\right),\ \delta\left(l\right),\ \mathcal{T}).$$

Therefore, all the conditions of Theorem 3.11 are fulfilled. Hence operator f has a unique FP. This implies that fuzzy integral Eq. (9) has a unique solution.

Corollary 3.1 Let $\left(\mathfrak{D}, M_\phi, N_\phi, *, \circ\right)$ be a G-complete IFBMS. Define f : $\mathfrak{D} \to \mathfrak{D}$ be

$f\vartheta\left(l\right) = f\left(j\right) + \beta\int_e^g F\left(l,\ j\right)e\left(l\right)dj$ for all $l,\ j \in [e,\ g]$.

Suppose the below conditions meet:

I. $\max\left\{F\left(l,\ j\right)\vartheta\left(l\right), F\left(l,\ j\right)\delta\left(l\right)\right\} \leq \max\left\{\vartheta\left(l\right),\ \delta\left(l\right)\right\}$ for $\vartheta,\ \delta \in \mathfrak{D}$, $k \in (0,\ 1)$ and $\forall\ l,\ j \in [e,\ g]$,

II. $\left(\beta\int_e^g dj\right)^2 \leq k < 1$

Then integral Eq. (9) has a solution.

We can prove easily by follow the above proof.

5 Application to Dynamic Market Equilibrium

In real business cycle models, economy is always in its long run equilibrium but in Keynesian business cycle theory the economy could be above or below the long-term potential, full employment GDP. While the real business cycle model seeks to overcome the distinction between the long run growth model and the real business cycle. Now we show how our established result can be used to find the unique solution to an integral equation in dynamic market equilibrium economics.

Let us denote the supply $\mathcal{Q}_\beta$ and demand $\mathcal{Q}_d$, in many markets, current prices and pricing trends (whether prices are rising or dropping and whether they are rising or falling at an increasing or decreasing rate) have an impact. The economist, therefore, wants to know what the current price is $\mathcal{P}(\mathcal{T})$, by using the first derivative $\frac{d\mathcal{P}(\mathcal{T})}{d\mathcal{T}}$, and the second derivative $\frac{d^2\mathcal{P}(\mathcal{T})}{d\mathcal{T}^2}$. Assume that,

$$\mathcal{Q}_\beta = \sigma_1 + \upsilon_1\mathcal{P}(\mathcal{T}) + \alpha_1\frac{d\mathcal{P}(\mathcal{T})}{d\mathcal{T}} + \omega_1\frac{d^2\mathcal{P}(\mathcal{T})}{d\mathcal{T}^2}$$

$$\mathcal{Q}_d = \sigma_2 + \upsilon_2\mathcal{P}(\mathcal{T}) + \alpha_2\frac{d\mathcal{P}(\mathcal{T})}{d\mathcal{T}} + \omega_2\frac{d^2\mathcal{P}(\mathcal{T})}{d\mathcal{T}^2}.$$

$\sigma_1, \sigma_2, \upsilon_1, \upsilon_2, \alpha_1$ and α_2 are constants. If pricing clears the market at each point in time, comment on the dynamic stability of the market. In equilibrium, $\mathcal{Q}_\beta = \mathcal{Q}_d$. So,

$$\sigma_1 + \upsilon_1\mathcal{P}(\mathcal{T}) + \alpha_1\frac{d\mathcal{P}(\mathcal{T})}{d\mathcal{T}} + \omega_1\frac{d^2\mathcal{P}(\mathcal{T})}{d\mathcal{T}^2} = \sigma_2 + \upsilon_2\mathcal{P}(\mathcal{T}) + \alpha_2\frac{d\mathcal{P}(\mathcal{T})}{d\mathcal{T}} + \omega_2\frac{d^2\mathcal{P}(\mathcal{T})}{d\mathcal{T}^2}.$$

since

$$(\omega_1 - \omega_2)\frac{d^2\mathcal{P}(\mathcal{T})}{d\mathcal{T}^2} + (\alpha_1 - \alpha_2)\, d\frac{d\mathcal{P}(\mathcal{T})}{d\mathcal{T}} + (\upsilon_1 - \upsilon_2)\mathcal{P}(\mathcal{T}) = -(\sigma_1 - \sigma_2)$$

Letting $\omega = \omega_1 - \omega_2$, $\alpha = \alpha_1 - \alpha_2$, $\upsilon = \upsilon_1 - \upsilon_2$ and $\sigma = \sigma_1 - \sigma_2$ in above, we have

$$\omega\frac{d^2\mathcal{P}(\mathcal{T})}{d\mathcal{T}^2} + \alpha\frac{d\mathcal{P}(\mathcal{T})}{d\mathcal{T}} + \upsilon\mathcal{P}(\mathcal{T}) = -\sigma.$$

Dividing through by ω, $\mathcal{P}(\mathcal{T})$ is governed by the following initial value problem:

$$\begin{cases} \mathcal{P}'' + \frac{\alpha}{\omega}\mathcal{P}'' + \frac{\upsilon}{\omega}P(\mathcal{T}) = -\frac{\sigma}{\omega} \\ P(0) = 0 \\ \mathcal{P}'(0) = 0, \end{cases} \tag{10}$$

where $\frac{\alpha^2}{\omega} = \frac{4\upsilon}{\omega}$ and $\frac{\upsilon}{\alpha} = \mu$ is a continuous function. It is easy to show that the problem (10) is equivalent to the integral equation

$$\mathcal{P}(\mathcal{T}) = \int_0^T \xi(\mathcal{T}, r) f(\mathcal{T}, r, \mathcal{P}(r))\, dr.$$

where $\xi(\mathcal{T}, r)$ is Green's function given by

$$\xi(\mathcal{T}, r) = \begin{cases} r\alpha\frac{\mu}{2}^{(\mathcal{T}-r)} & \text{if } 0 \le r \le \mathcal{T} \le T \\ T\alpha\frac{\mu}{2}^{(r-\mathcal{T})} & \text{if } 0 \le \mathcal{T} \le r \le \mathcal{T} \le T. \end{cases}$$

We will show the existence of a solution to the integral equation

$$\mathcal{P}(\mathcal{T}) = \int_0^T G(\mathcal{T}, r, \mathcal{P}(r))dr. \tag{11}$$

Let $\mathfrak{D} = C([0, T])$ set of real continuous functions defined on $[0, T]$ for $\mathcal{T} > 0$, we define

$$M_\phi(\vartheta, \omega, \mathcal{T}) = \sup_{\mathcal{T}\epsilon[0,T]} \frac{\min\{\vartheta, \omega\} + \mathcal{T}}{\max\{\vartheta, \omega\} + \mathcal{T}}$$

and

$$N_\phi(\vartheta, \omega, \mathcal{T}) = 1 - \sup_{\mathcal{T}\epsilon[0,T]} \frac{\min\{\vartheta, \omega\} + \mathcal{T}}{\max\{\vartheta, \omega\} + \mathcal{T}}$$

for all $\vartheta, \omega \epsilon \mathfrak{D}$ with the CTN $'*'$ such that $a * b = a \cdot b$ and CTCN $\circ$ such that $a \circ b = \max\{a, b\}$. Define $\phi : \mathfrak{D} \times \mathfrak{D} \to [1, \infty)$. As

$$\mathcal{Q}(\omega, d) = 1 + \omega + d.$$

It is easy to show that $(\mathfrak{D}, M_\phi, N_\phi, *, \circ)$ is a complete IEFBMLS and $f : \mathfrak{D} \to \mathfrak{D}$ defined by

$$\mathrm{f}\mathcal{P}(\mathcal{T}) = \int_0^T G(\mathcal{T}, r, \mathcal{P}(r)\, dr.$$

Theorem 4.1 Consider Eq. (11) and suppose that

(i) $G : [0, T] \times [0, T] \to \mathbb{R}^+$ is continuous function,

(ii) There exist a continuous function $\xi : [0, T] \times [0, T] \to \mathbb{R}^+$ such that $\sup_{\mathcal{T}\in[0,T]} \int_0^T \xi(\mathcal{T}, r)\, dr \geq 1$;

(iii) $\max\{G(\mathcal{T}, r, \vartheta(r)) - G(\mathcal{T}, r, \omega(r))\} \geq \xi(\mathcal{T}, r) \max\{\vartheta(r), \omega(r)\}$ and $\min\{G(\mathcal{T}, r, \vartheta(r)) - G(\mathcal{T}, r, \omega(r))\} \geq \xi(\mathcal{T}, r) \min \vartheta(r), \omega(r)$.

Then, the integral Eq. (11) has a unique solution.

Proof: For $\vartheta, \omega \in \mathfrak{D}$, by using of assumptions (i) to (iii), we have

$$\begin{aligned}
M_\phi(f\vartheta, f\omega, k\mathcal{T}) &= \sup_{\mathcal{T}\epsilon[0,T]} \frac{\min\left\{\int_0^T G(\mathcal{T}, r, \vartheta(r))\, dr, \int_0^T G(\mathcal{T}, r, \omega(r))\, dr\right\} + k\mathcal{T}}{\max\{\int_0^T G(\mathcal{T}, r, \vartheta(r))\, dr, \int_0^T G(\mathcal{T}, r, \omega(r))\, dr\} + k\mathcal{T}} \\
&= \sup_{\mathcal{T}\epsilon[0,T]} \frac{\int_0^T \min\{G(\mathcal{T}, r, \vartheta(r)), G(\mathcal{T}, r, \omega(r))\}\, dr + k\mathcal{T}}{\int_0^T \max\{G(\mathcal{T}, r, \vartheta(r)), G(\mathcal{T}, r, \omega(r))\}\, dr + k\mathcal{T}} \\
&\geq \sup_{\mathcal{T}\epsilon[0,T]} \frac{\int_0^T \xi(\mathcal{T}, r) \min\{\vartheta(r), \omega(r)\}\, dr + k\mathcal{T}}{\int_0^T \xi(\mathcal{T}, r) \max\{\vartheta(r), \omega(r)\}\, dr + k\mathcal{T}} \\
&\geq \sup_{\mathcal{T}\epsilon[0,T]} \frac{\min\{\vartheta(r), \omega(r)\} \int_0^T \xi(\mathcal{T}, r)\, dr + k\mathcal{T}}{\max\{\vartheta(r), \omega(r)\} \int_0^T \xi(\mathcal{T}, r)\, dr + k\mathcal{T}}
\end{aligned}$$

$$\geq \left(\frac{\min\{\vartheta(r),\ \omega(r)\}+\mathcal{T}}{\max\{\vartheta(r),\ \omega(r)\}+\mathcal{T}}\right)=M_{\phi}(\vartheta,\omega,\mathcal{T})$$

and

$$N_{\phi}(f\vartheta,f\omega,k\mathcal{T})=1-\sup_{\mathcal{T}\epsilon[0,T]}\frac{\min\left\{\int_0^T G(\mathcal{T},r,\vartheta(r))\,dr,\int_0^T G(\mathcal{T},r,\omega(r))\,dr\right\}+k\mathcal{T}}{\max\{\int_0^T G(\mathcal{T},r,\vartheta(r))\,dr,\int_0^T G(\mathcal{T},r,\omega(r))\,dr\}+k\mathcal{T}}$$

$$=1-\sup_{\mathcal{T}\epsilon[0,T]}\frac{\int_0^T\min\{G(\mathcal{T},r,\vartheta(r)),G(\mathcal{T},r,\omega(r))\}\,dr+k\mathcal{T}}{\int_0^T\max\{G(\mathcal{T},r,\vartheta(r)),G(\mathcal{T},r,\omega(r))\}\,dr+k\mathcal{T}}$$

$$\leq 1-\sup_{\mathcal{T}\epsilon[0,T]}\frac{\int_0^T\xi(\mathcal{T},r)\min\{\vartheta(r),\omega(r)\}\,dr+k\mathcal{T}}{\int_0^T\xi(\mathcal{T},r)\max\{\vartheta(r),\omega(r)\}\,dr+k\mathcal{T}}$$

$$\leq 1-\sup_{\mathcal{T}\epsilon[0,T]}\frac{\min\{\vartheta(r),\omega(r)\}\int_0^T\xi(\mathcal{T},r)\,dr+k\mathcal{T}}{\max\{\vartheta(r),\omega(r)\}\int_0^T\xi(\mathcal{T},r)\,dr+k\mathcal{T}}$$

$$\leq 1-\left(\frac{\min\{\vartheta(r),\ \omega(r)\}+\mathcal{T}}{\max\{\vartheta(r),\ \omega(r)\}+\mathcal{T}}\right)=N_{\phi}(\vartheta,\omega,\mathcal{T}).$$

Thus $M_{\phi}(f\vartheta,\ f\omega,\ k\mathcal{T})\geq M_{\phi}(\vartheta,\omega\ ,\mathcal{T})$ and $N_{\phi}(f\vartheta,\ f\omega,\ k\mathcal{T})\leq N_{\phi}(\vartheta,\omega,\mathcal{T})$ for all $\vartheta,\ \omega\in\mathfrak{D}$, and all conditions of Theorem 3.14 are satisfied. Therefore, Eq. (11) has a unique fixed point.

6 Conclusion

Herein, we introduced the notion of intuitionistic extended fuzzy b-metric-like spaces and some new types of fixed point theorems in this new setting. Moreover, we provided non-trivial examples and plotted some graphs to demonstrate the viability of the proposed methods. We provided an application of the obtained results in a dynamic equilibrium market. We have supplemented this work with applications demonstrating how the built method outperforms those found in the literature. Since our structure is more general than the class of fuzzy b-metric like space and intuitionistic fuzzy b-metric space, our results and notions expand and generalize several previously published results. This work can easily extend to the structure of neutrosophic extended b-metric-like spaces, controlled intuitionistic fuzzy b-metric-like spaces, and many other structures.

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