



ARTICLE

Self-Triggered Consensus Filtering over Asynchronous Communication Sensor Networks

Huiwen Xue¹, Jiwei Wen^{1,*}, Akshya Kumar Swain² and Xiaoli Luan¹

¹Key Laboratory of Advanced Process Control for Light Industry (Ministry of Education), Jiangnan University, Wuxi, 214000, China

²Department of Electrical, Computer and Software Engineering, University of Auckland, Auckland, 0632, New Zealand

*Corresponding Author: Jiwei Wen. Email: wjw8143@jiangnan.edu.cn

Received: 05 November 2021 Accepted: 23 March 2022

ABSTRACT

In this paper, a self-triggered consensus filtering is developed for a class of discrete-time distributed filtering systems. Different from existing event-triggered filtering, the self-triggered one does not require to continuously judge the trigger condition at each sampling instant and can save computational burden while achieving good state estimation. The triggering policy is presented for pre-computing the next execution time for measurements according to the filter's own data and the latest released data of its neighbors at the current time. However, a challenging problem is that data will be asynchronously transmitted within the filtering network because each node self-triggers independently. Therefore, a co-design of the self-triggered policy and asynchronous distributed filter is developed to ensure consensus of the state estimates. Finally, a numerical example is given to illustrate the effectiveness of the consensus filtering approach.

KEYWORDS

Self-triggered policy; sensor networks; distributed consensus filtering

Nomenclature

\mathbb{R}^n	The n dimension Euclidean spaces
$\mathbb{R}^{n \times m}$	The set of all $n \times m$ real matrices
$diag_n \{A\}$	The block-diagonal matrix $diag \{A, A, \dots, A\}$
$diag_n \{A_i\}$	The block-diagonal matrix $diag \{A_1, A_2, \dots, A_n\}$
$vec_n \{x_i\}$	$[x_1, x_2, \dots, x_n]$
\otimes	Kronecker product for matrices
$\ x\ = \sqrt{(x^T x)}$	Euclidean norm of a column vector x
$\ell_2[0, \infty)$	The space of square-summable functions on $[0, \infty)$
*	The symmetric block of a symmetric matrix



1 Introduction

A wireless sensor network (WSN) is composed of a large number of sensor nodes distributed in a specific area. With the development of sensing, cloud computing, and wireless communication technologies, WSN has been successfully applied in a variety of practical environments, such as battlefield monitoring, target tracking, health monitoring, search and rescue operations after disasters, industrial automation, and so on [1,2]. However, noise ubiquitously exists in signal transmission and WSN environment, which often results in degradation of the filtering performance. Therefore, distributed filtering obtains an ever-increasing attention when estimating unavailable states through measured outputs and historical data [3–5]. Compared with traditional centralized filtering [6], each sensor node finalizes filtering according to its own and neighbors' data under a fixed interconnection topology within a distributed filtering network. Therefore, distributed filtering is robust to sensor failures and transmission constraints.

Usually, the sensor is powered by a lithium battery and has a limited capacity for memory. Therefore, it is of great significance to reduce communication and computation energy loss. Traditional time-triggered sampling requires signal transmission to be continuous or periodically updated. Although it is conducive to analysis and design, the rate of signal update is constant and quick, and may lead to waste use of limited communication resources. In addition, the narrow network bandwidth may lead to channel congestion, induced delay and data packet drop out [7,8], etc. The key to solving these problems is to reduce the transmission load on the premise of good filtering performance.

Therefore, event-triggered mechanisms (ETM) [9–11], which can greatly reduce the unnecessary data transmission and resource occupation, are developed with burgeoning research interests. In the past few years, the main progress of ETM for distributed filtering can be generally divided into four categories, i.e., triggering based on constant threshold [12–14], instantaneous measurement-/estimate-dependent threshold [15–17], released measurement-/estimate-dependent threshold [18–21] and dynamic event-triggering [22,23]. The first three categories can be summarized as static event-triggered mechanisms (SETM), which has a fixed scalar triggering threshold. In contrast, the fourth class is named as dynamic event-triggered mechanisms (DETM), which can adjust the triggering threshold. Aiming at distributed set-membership estimation for time-varying systems, a new DETM designed in [23] leads to larger average inter-event times and thus less totally released data packets.

It is worth noting that ETM are more effective in reducing transmission frequency compared to traditional time-triggered sampling at the cost of increased computational burden [24]. Specifically, in order to check whether the triggering conditions are met, the above ETM strategies embedded in the sensor nodes have to continuously make judgments at each sampling instant. For a large-scale WSN, there is numerous computational burden consumption. In order to overcome this shortcoming, a self-triggered policy was first proposed in [25] to optimize the allocation of computing burden and performance for real-time systems containing multiple control tasks. Since such a scheme pre-calculates the next update moment through the current samplings, the self-triggering is an active behavior. Up to now, the self-triggering policy is mostly employed in control systems [9,26–32] and it still remains open for solving filtering problems. The main motivation of this study is to shorten the gap between self-triggering theory and its applications to distributed filtering.

The main contributions of this paper are summarized as follows:

1. For a filtering network, a self-triggered policy is designed to save transmission energy and computational burden, especially in a large-scale WSN. Unlike the ETM, the self-triggered policy can predict the subsequent execution time in advance without checking the triggering

conditions at each sampling time. That is, the next triggering interval is calculated based on the latest transmission data, the latest state estimates of itself and neighboring nodes.

2. In filtering networks, since each filtering node is triggered independently and has its own triggering interval, this will lead to an asynchronous transmission phenomenon. Through the co-design of self-triggered policy and distributed consensus filtering, even when the WSN encounters asynchronous communication, the filtering system can maintain good state estimation.

2 Preliminaries and Problem Formulation

Consider a sensor network with n nodes to monitor the plant and estimate its states. The directed weighted graph $\mathcal{G} = (\mathcal{V}, \mathcal{E}, \mathcal{A})$ is used to model the network topology of interacting sensors, where $\mathcal{V} = \{1, 2, \dots, n\}$ denotes an index set of sensor nodes, $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$ represents the edge of paired sensor nodes and $\mathcal{A} = [a_{ij}]_{n \times n}$ ($a_{ij} \geq 0$) stands for the weighted adjacency matrix. The adjacency element $a_{ij} > 0 \Leftrightarrow (i, j) \in \mathcal{E}$ represents a positive weighting of the edge between two adjacent sensors, which implies that sensor i receives data from sensor j or sensor j transmits data to sensor i , otherwise, $a_{ij} = 0$ if no data link exists between sensor j to sensor i . In addition, we assume $a_{ii} = 1$ for all $i \in \mathcal{V}$ and (i, i) can be considered as an additional edge. The set of neighbors of node i including the node itself is denoted by $\mathcal{N}_i = \{j \in \mathcal{V} : (i, j) \in \mathcal{E}\}$. The Laplace matrix of the graph \mathcal{G} is defined as $\mathcal{L} = \mathcal{D} - \mathcal{A}$, where $\mathcal{D} = \text{diag}_n \{d_i\}$ with the diagonal element $d_i = \sum_{j \in \mathcal{N}_i} a_{ij}$.

Consider the plant described by a discrete-time linear system of the following form:

$$\begin{cases} x(k+1) = Ax(k) + B\omega(k) \\ z(k) = Mx(k) \end{cases} \quad (1)$$

where $x(k) \in \mathbb{R}^{n_x}$ is the system state; $z(k) \in \mathbb{R}^{n_z}$ is the output to be estimated; $\omega(k) \in \mathbb{R}^{n_\omega}$ is the external disturbance belonging to $l_2[0, \infty)$; A , B and M are known constant matrices with appropriate dimensions. The initial state $x(0)$ is an unknown vector.

For every sensor i ($i \in \mathcal{V}$), the model of sensor node i is in the form of

$$y_i(k) = C_i x(k) + D_i v_i(k) \quad (2)$$

where $y_i(k) \in \mathbb{R}^{n_y}$ is the measured output collected by node i , $v_i(k) \in \mathbb{R}^{n_v}$ is measurement noise belonging to $l_2[0, \infty)$, C_i and D_i are known constant matrices with appropriate dimensions.

In sensor networks, the data available for filter on the sensor node comes not only from the sensor i , but also from its neighbors. Considering the consensus problem, the whole distributed filtering network can be constructed as follows:

$$\begin{cases} \hat{x}_i(k+1) = A\hat{x}_i(k) + H_i(y_i(k) - C_i\hat{x}_i(k)) + G_i \sum_{j \in \mathcal{N}_i} (\hat{x}_i(m_{k_i}) - \hat{x}_j(m_{k_j})) \\ \hat{z}_i(k) = M\hat{x}_i(k) \end{cases} \quad (3)$$

where time sequence $\{m_{k_i} | m_{0_i}, m_{1_i}, \dots\}$ is used to represent the triggering instants of node i . Sampling moment k is in the time interval $[m_{k_i}, m_{(k+1)_i})$. m_{k_i} is the latest triggering time of the filter node i before the current time k . $\hat{x}_i(k) \in \mathbb{R}^{n_x}$ denotes the state estimation of the filter node i with the initial condition given by $\hat{x}_i(0)$. $\hat{z}_i(k) \in \mathbb{R}^{n_z}$ is the output estimation. H_i , G_i are filtering gains to be determined.

Assumption 1: We assume that sensors monitor the target plant at every sampling moment. Then we focus on reducing the communication frequency between sensors so as to achieve better resource efficiency.

The distributed self-triggered filtering system is illustrated in Fig. 1. For example, the sensor node i transmits the measurements $y_i(k)$ to the corresponding filter i . In the whole sensor network, each filter obtains the latest state estimation $\hat{x}_j(m_{k_j})$ from its neighbors and transmits its own filtering results $\hat{x}_i(m_{k_i})$ to other neighbors when meeting a well-defined condition. At each sampling instant, filter i calculates and updates its estimated state $\hat{x}_i(k)$. $\hat{x}_i(k)$ and its time-stamp k are integrated into a data packet $(k, \hat{x}_i(k))$. Similarly, the data packet $(m_{k_i}, \hat{x}_i(m_{k_i}))$ is considered as the latest released data when the filter i is triggered. Buffer i containing multiple units is driven by self-triggered policy, and has the capability of checking the time stamps of the newly arrived data packet $(m_{k_i}, \hat{x}_i(m_{k_i}))$ and discarding old data packets. In brief, the buffer can reserve the latest data packets until new data packets arrive.

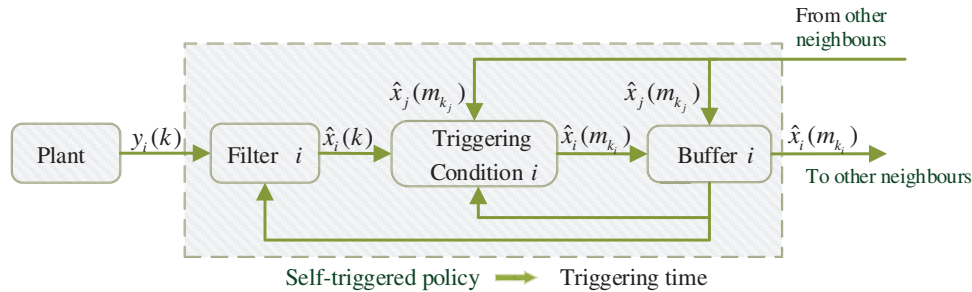


Figure 1: Block diagram of distributed self-triggered system

The self-triggered policy predicts the subsequent execution time and calculates the triggering interval by the latest data of each filter without a continuous judgment process. Therefore, the strategy proposed in this paper is beneficial to the energy saving of sensor networks with limited resources, as well as scant network channel bandwidth.

The self-triggered time-sequence diagram is illustrated in Fig. 2. The dash lines indicate that data is exchanged between nodes and the arrow points to the object of data transmission. The broadcasting and receiving of the latest released state estimation are determined by the filter node's own self-triggered policy. It is clear that each filter node has its own triggering time sequence, which causes different triggering intervals between each other. Moreover, for filter i , the time that data packets from neighbors arrive at buffer i may be asynchronous, since it is determined by different triggering conditions. Such an asynchronous transmission brings challenges to the design of consensus distributed filtering.

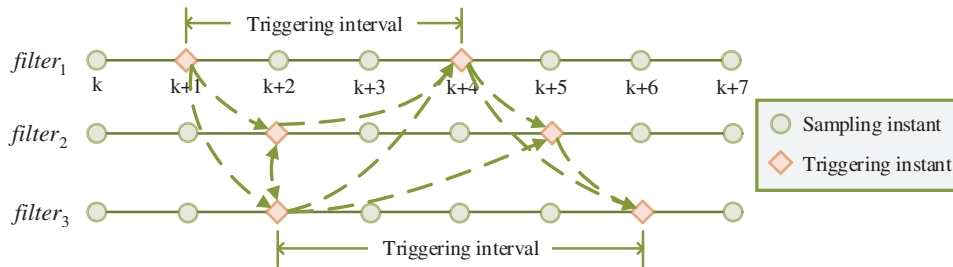


Figure 2: Self-triggered sequence diagram

For filter i , let's define a state estimation error $e_i(k) = x(k) - \hat{x}_i(k)$, an output estimation error $\tilde{z}_i(k) = z(k) - \hat{z}_i(k)$ and a state estimation update error $\tilde{e}_i(k) = \hat{x}_i(m_{k_i}) - \hat{x}_i(k)$, $k \in [m_{k_i}, m_{(k+1)_i})$. Then $\hat{x}_i(m_{k_i}) - \hat{x}_j(m_{k_j})$ can be expressed as $\tilde{e}_i(k) - \tilde{e}_j(k) - (e_i(k) - e_j(k))$. Combing (1)–(3), the estimation error dynamics can be rewritten as

$$\begin{cases} e_i(k+1) = (A - H_i C_i) e_i(k) + B w(k) - H_i D_i v_i(k) \\ \quad + \sum_{j \in \mathcal{N}_i} G_i (e_i(k) - e_j(k)) - \sum_{j \in \mathcal{N}_i} G_i (\tilde{e}_i(k) - \tilde{e}_j(k)) \\ \tilde{z}_i(k) = M e_i(k) \end{cases} \quad (4)$$

The topology of the sensor is determined by a given graph $\mathcal{G} = (\mathcal{V}, \mathcal{E}, \mathcal{A})$. For the sake of brevity, we denote

$$e(k) = \text{vec}_n^T \{e_i^T(k)\}, \tilde{z}(k) = \text{vec}_n^T \{\tilde{z}_i^T(k)\}, \tilde{e}(k) = \text{vec}_n^T \{\tilde{e}_i^T(k)\},$$

$$v(k) = \text{vec}_n^T \{v_i^T(k)\}, \bar{B} = \text{vec}_n^T \{B^T\}, \bar{M} = \text{diag}_n \{M\},$$

$$\bar{A} = \text{diag}_n \{A\}, \bar{C} = \text{diag}_n \{C_i\}, \bar{D} = \text{diag}_n \{D_i\}, \bar{G} = \text{diag}_n \{G_i\}, \bar{H} = \text{diag}_n \{H_i\}.$$

Then, the error dynamics governed by (4) can be rewritten in the following compact form:

$$\begin{cases} e(k) = \tilde{A} e(k) + \tilde{B} \tilde{e}(k) + \tilde{C} v(k) + \tilde{D} w(k) \\ \tilde{z}(k) = \tilde{M} e(k) \end{cases} \quad (5)$$

where $\tilde{A} = \bar{A} - \bar{H} \bar{C} + \bar{G} \bar{L}$, $\tilde{B} = -\bar{G} \bar{L}$, $\tilde{C} = -\bar{H} \bar{D}$, $\tilde{D} = \bar{B}$ and $\bar{L} = \mathcal{L} \otimes I_p$.

Definition 1 ([27]): Filters (3) are said to be a distributed self-triggered H_∞ consensus filtering system (1) if they meet the following conditions:

1) In the absence of system disturbance and measurement noise, the filtering error system (5) is exponentially stable, i.e., there exist positive constants η and $\alpha \in (0, 1)$ such that $\lim_{k \rightarrow \infty} \|e(k)\|^2 \leq \eta \alpha^k$, for all $k \geq 0$.

2) Under the condition of the system disturbance and measurement noises, the output filtering errors $\tilde{z}_i(k)$, $\forall i \in \mathcal{V}$ satisfy the following H_∞ performance:

$$\frac{1}{n} \sum_{i=1}^n \sum_{k=0}^{\infty} \|\tilde{z}_i(k)\|_2^2 \leq \gamma^2 \left(\frac{1}{n} \sum_{i=1}^n \sum_{k=0}^{\infty} \|v_i(k)\|_2^2 + \sum_{k=0}^{\infty} \|w(k)\|^2 + \frac{1}{n} \sum_{i=1}^n e_i^T(0) R_i e_i(0) \right) \quad (6)$$

where $\gamma > 0$ is the attenuation level and $R_i = R_i^T > 0$ are some given positive definite matrices.

3 Main Results

In this section, we first design a distributed self-triggered policy. Based on such a policy, sufficient conditions are given for the H_∞ consensus analysis of the filtering error system (4). Furthermore, a co-design method for self-triggered threshold parameters and distributed filter gains is presented.

3.1 Self-Triggered Policy

The self-triggered policy predicts that the subsequent execution time depends on the current sampling data and the estimated state updated by its neighbors. Based on this idea, we develop self-triggered distributed filtering. The next triggering instant is considered as follows:

$$m_{(k+1)_i} = m_{k_i} + M_{k_i} (\hat{x}_i(m_{k_i}), \hat{x}_j(m_{k_j}), y_i(k)). \quad (7)$$

The key to realizing the self-triggered policy is to obtain the triggering interval function $M_{k_i}(\cdot)$. We develop the following error-based self-triggered policy:

$$M_{k_i} = \inf \{ (m_{(k+1)_i} - m_{k_i}) \in \mathbb{N} \mid \tilde{e}_i^T(k) \Phi_i \tilde{e}_i(k) \geq \varepsilon_i \delta_i \Phi_i \delta_i \} \quad (8)$$

where $\delta_i = \sum_{j \in \mathcal{N}_i} a_{ij} (\hat{x}_i(m_{k_i}) - \hat{x}_j(m_{k_j}))$; $\varepsilon_i \in (0, 1]$ is the threshold parameter; $\Phi_i \in \mathbb{R}^{p \times p}$ is the weighting matrix to be determined.

Theorem 1: In order to formulate a suitable self-triggered policy for the distributed filtering system (5), the triggering interval function can be expressed as

$$M_{k_i} = m_{(k+1)_i} - m_{k_i} = \left\lfloor \ln \left(\frac{Q_1^i (S_i - U_i)}{Q_2^i} + 1 \right) / \ln(Q_1^i + 1) \right\rfloor \quad (9)$$

where $\lfloor \cdot \rfloor$ denotes rounding up to the nearest integer,

$$Q_1^i = \left\| \sqrt{\Phi_i} (I - A + H_i C_i) \sqrt{\Phi_i}^{-1} \right\|,$$

$$Q_2^i = \left\| \sqrt{\Phi_i} (I - A + H_i C_i) \hat{x}_i(m_{k_i}) \right\| + \left\| \sqrt{\Phi_i} G_i \sum_{j \in \mathcal{N}_i} (\hat{x}_i(m_{k_i}) - \hat{x}_j(m_{k_j})) \right\|,$$

$$Q_3^i = \left\| \sqrt{\Phi_i} H_i \right\|,$$

$$S_i = \left\| \sqrt{\varepsilon_i \Phi_i} \delta_i \right\|,$$

$$U_i = \sum_{\xi=0}^{k-m_{k_i}-1} (Q_1^i + 1)^\xi Q_3^i \|y(k-1-\xi)\|.$$

Proof: Combining relations (3) and (4), we can obtain

$$\begin{aligned} & \left\| \sqrt{\Phi_i} \tilde{e}_i(k+1) \right\| - \left\| \sqrt{\Phi_i} \tilde{e}_i(k) \right\| \\ & \leq \left\| \sqrt{\Phi_i} (\tilde{e}_i(k+1) - \tilde{e}_i(k)) \right\| \\ & = \left\| \sqrt{\Phi_i} (\hat{x}_i(k) - \hat{x}_i(k+1)) \right\| \\ & \leq \left\| \sqrt{\Phi_i} (I - A + H_i C_i) (\hat{x}_i(m_{k_i}) - \tilde{e}_i(k)) \right\| + \left\| \sqrt{\Phi_i} H_i y_i(k) \right\| + \left\| \sqrt{\Phi_i} G_i \sum_{j \in \mathcal{N}_i} (\hat{x}_i(m_{k_i}) - \hat{x}_j(m_{k_j})) \right\|. \\ & \leq Q_1^i \left\| \sqrt{\Phi_i} \tilde{e}_i(k) \right\| + Q_2^i + Q_3^i \|y_i(k)\|. \end{aligned} \quad (10)$$

Assume $Y_i(k) = \left\| \sqrt{\Phi_i} \tilde{e}_i(k) \right\|$. Then inequality (10) can be rewritten as

$$Y_i(k+1) - Y_i(k) \leq Q_1^i Y_i(k) + Q_2^i + Q_3^i \|y_i(k)\|. \quad (11)$$

From (11), we can obtain

$$\begin{aligned}
 Y_i(k) &\leq (1 + Q_1^i) Y_i(k-1) + Q_2^i + Q_3^i \|y_i(k-1)\| \\
 &\leq (1 + Q_1^i)^2 Y_i(k-2) + (1 + Q_1^i) Q_2^i + Q_2^i + (1 + Q_1^i) Q_3^i \|y_i(k-2)\| + Q_3^i \|y_i(k-1)\| \\
 &\quad \vdots \\
 &\leq (1 + Q_1^i)^{k-m_{k_i}} Y_i(m_{k_i}) \\
 &\quad + (1 + Q_1^i)^{k-m_{k_i}-1} Q_2^i + \dots + (1 + Q_1^i) Q_2^i + Q_2^i \\
 &\quad + (1 + Q_1^i)^{k-m_{k_i}-1} Q_3^i \|y_i(m_{k_i})\| + \dots + (1 + Q_1^i) Q_3^i \|y_i(k-2)\| + Q_3^i \|y_i(k-1)\|. \tag{12}
 \end{aligned}$$

Because $Y_i(m_{k_i}) = \|\sqrt{\Phi_i} \tilde{e}_i(m_{k_i})\| = 0$ and the right part of (12) is a geometric series, the above inequality could be rewritten as

$$Y_i(k) \leq \frac{(Q_1^i + 1)^{k-m_{k_i}} - 1}{Q_1^i} Q_2^i + \sum_{\xi=0}^{k-m_{k_i}-1} (Q_1^i + 1)^\xi Q_3^i \|y_i(k-1-\xi)\|. \tag{13}$$

According to self-triggered condition (8), if the filter node i is triggered, the following inequality is obtained

$$Y_i(k) \geq \left\| \sqrt{\varepsilon_i \Phi_i} \delta_i \right\|. \tag{14}$$

Combining inequality (13) and (14), we have

$$\left(1 - (Q_1^i + 1)^{k-m_{k_i}}\right) Q_2^i \geq (S_i - U_i) Q_1^i. \tag{15}$$

Through the simple mathematical manipulation of (15), triggering interval (9) is derived.

To guarantee the system performance, the current state estimation of the filter node i should be broadcasted to other neighbors immediately once the triggering conditions are satisfied. This means that the triggering interval M_{k_i} should be rounded up. The proof is completed.

3.2 H_∞ Consensus Performance Analysis

Lemma 1: Given a positive definite matrix $R_i = R_i^T > 0$ ($1 \leq i \leq n$), the filtering error system (4) is exponentially stable and satisfy H_∞ consensus performance under the self-triggered policy (9) and the initial condition $e_i(0)^T P_i e_i(0) \leq \gamma^2 e_i(0)^T R_i e_i(0)$, if there exist a positive scalar $\gamma > 0$ and real symmetric matrices $P_i > 0$, $\bar{\Phi}_i$ satisfying

$$\begin{bmatrix}
 \bar{\mathfrak{E}}_{11} & \bar{\mathfrak{E}}_{12} & 0 & 0 & (P\bar{A} - P\bar{H}\bar{C} + P\bar{G}\bar{L})^T \\
 * & \bar{\mathfrak{E}}_{22} & 0 & 0 & (-P\bar{G}\bar{L})^T \\
 * & * & -\gamma^2 I & 0 & (-P\bar{H}\bar{D})^T \\
 * & * & * & -n\gamma^2 I & P\bar{B}^T \\
 * & * & * & * & -P
 \end{bmatrix} \leq 0 \tag{16}$$

where $\Xi_{11} = -P + \bar{M}^T \bar{M} + \bar{\mathcal{L}}^T \bar{\varepsilon} \bar{\Phi} \bar{\mathcal{L}}$, $\Xi_{12} = -\bar{\mathcal{L}}^T \bar{\varepsilon} \bar{\Phi} \bar{\mathcal{L}}$, $\Xi_{22} = \bar{\mathcal{L}}^T \bar{\varepsilon} \bar{\Phi} \bar{\mathcal{L}} - \bar{\Phi}$, $\bar{\varepsilon}_i = \varepsilon_i \otimes I_p$, $\bar{\varepsilon} = \text{diag}_n \{\bar{\varepsilon}_i\}$, $\bar{\Phi}_i = \Phi_i \otimes I_p$, $\bar{\Phi} = \text{diag}_n \{\bar{\Phi}_i\}$, and $P = \text{diag}_n \{P_i\}$.

Proof: Consider a Lyapunov function

$$V(k) = e(k)^T P e(k). \tag{17}$$

The one-step time difference is

$$V(k+1) - V(k) = e^T(k+1) P e(k+1) - e^T(k) P e(k). \tag{18}$$

According to the self-triggered policy (9), the following inequality in augmented form is valid for $[m_{k_i}, m_{(k+1)_i}]$

$$\begin{bmatrix} e(k) \\ \tilde{e}(k) \end{bmatrix}^T \Psi \begin{bmatrix} e(k) \\ \tilde{e}(k) \end{bmatrix} \geq 0 \tag{19}$$

where Ψ is a real symmetric matrix, $\Psi_{11} = \bar{\mathcal{L}}^T \bar{\varepsilon} \bar{\Phi} \bar{\mathcal{L}}$, $\Psi_{12} = -\bar{\mathcal{L}}^T \bar{\varepsilon} \bar{\Phi} \bar{\mathcal{L}}$, $\Psi_{22} = \bar{\mathcal{L}}^T \bar{\varepsilon} \bar{\Phi} \bar{\mathcal{L}} - \bar{\Phi}$, and others are zero matrices.

Therefore, Eq. (18) can be rewritten into the following inequality

$$V(k+1) - V(k) + \|\tilde{z}(k)\|^2 - n\gamma^2 \|w(k)\|^2 - \gamma^2 \|v(k)\|^2 \leq \xi^T \rho^T P \rho \xi + \xi^T \Pi \xi \tag{20}$$

where $\xi^T = [e^T(k), \tilde{e}^T(k), v^T(k), w^T(k)]$, $\rho^T = [\tilde{A}^T(k), \tilde{B}^T(k), \tilde{C}^T(k), \tilde{D}^T(k)]$, $\Pi_{11} = -P + \bar{M}^T \bar{M} + \bar{\mathcal{L}}^T \bar{\varepsilon} \bar{\Phi} \bar{\mathcal{L}}$, $\Pi_{12} = -\bar{\mathcal{L}}^T \bar{\varepsilon} \bar{\Phi} \bar{\mathcal{L}}$, $\Pi_{22} = \bar{\mathcal{L}}^T \bar{\varepsilon} \bar{\Phi} \bar{\mathcal{L}} - \bar{\Phi}$, $\Pi_{33} = -\gamma^2 I$, $\Pi_{44} = -n\gamma^2 I$, and others are zero matrices.

We assume that

$$\Gamma \triangleq \rho^T P \rho + \Pi \leq 0. \tag{21}$$

By applying Schur complement lemma to (20), we obtain

$$\Gamma = \begin{bmatrix} \Pi & \Upsilon^T \\ * & -P \end{bmatrix} \leq 0 \tag{22}$$

where $\Upsilon = P \rho = [P \bar{A} - P \bar{H} \bar{C} + P \bar{G} \bar{\mathcal{L}}, -P \bar{G} \bar{\mathcal{L}}, -P \bar{H} \bar{D}, P \bar{B}]$.

First, we prove that the system (4) is exponentially stable when there is no disturbance and measurement noises. Denoting $\lambda_1 = \lambda_{\max}(\Gamma)$, obviously λ_1 is smaller than zero, which indicates that (20) satisfies

$$V(k+1) - V(k) + \|\tilde{z}(k)\|^2 - n\gamma^2 \|w(k)\|^2 - \gamma^2 \|v(k)\|^2 \leq \lambda_1 \|\xi(k)\|^2 \leq \lambda_1 \|e(k)\|^2 \tag{23}$$

Define $\lambda_2 = \lambda_{\max}(P) > 0$. Then given positive scalar μ , from inequality (23), we obtain

$$\frac{1}{\mu^{k+1}} v(k+1) - \frac{1}{\mu^{k+1}} v(k) \leq \frac{\lambda_1}{\mu^{k+1}} \|e(k)\|^2. \tag{24}$$

With some simple mathematical managements, we get

$$\frac{1}{\mu^{k+1}} v(k+1) - \frac{1}{\mu^k} v(k) \leq \frac{1}{\mu^k} \left[\frac{\lambda_1}{\mu^k} + \left(\frac{1}{\mu^k} - 1 \right) \lambda_2 \right] \|e(k)\|^2. \tag{25}$$

Since $\frac{1}{\mu^k} \left[\frac{\lambda_1}{\mu^k} + \left(\frac{1}{\mu^k} - 1 \right) \lambda_2 \right] = \lambda_1 < 0$, when $\mu = 1$, there exist a constant $\mu_0 \in (0, 1)$ such that $\frac{1}{\mu_0^k} \left[\frac{\lambda_1}{\mu_0^k} + \left(\frac{1}{\mu_0^k} - 1 \right) \lambda_2 \right] < 0$. By iteration of (25), we have

$$\|e(k)\|^2 < \frac{\lambda_2}{\lambda_3} \mu_0^k \|e(0)\|^2 \tag{26}$$

where $\lambda_3 = \lambda_{\min}(P) > 0$. When k approaches to infinity, we can obtain $\lim_{k \rightarrow \infty} \|e(k)\|^2 = 0$ which implies estimation error system (4) is exponentially stable.

Second, it follows from (20) and (21) that

$$V(k+1) - V(k) \leq n\gamma^2 \|w(k)\|^2 + \gamma^2 \|v(k)\|^2 - \|\tilde{z}(k)\|^2 \tag{27}$$

Summing up both sides of (27) from 0 to t with respect to k

$$\sum_{k=0}^t \|\tilde{z}(k)\|_2^2 \leq n\gamma^2 \sum_{k=0}^t \|w(k)\|^2 + \gamma^2 \sum_{k=0}^t \|v(k)\|_2^2 + V(0) - V(k+1). \tag{28}$$

Since $V(k+1) \geq 0$, letting t approaches to infinity, we can obtain

$$\sum_{k=0}^{\infty} \|\tilde{z}(k)\|_2^2 \leq n\gamma^2 \sum_{k=0}^{\infty} \|w(k)\|^2 + \gamma^2 \sum_{k=0}^{\infty} \|v(k)\|_2^2 + \gamma^2 e(0)^T Re(0). \tag{29}$$

Therefore, the filtering error system also satisfies the H_∞ filtering performance condition in Definition 1. The proof is completed.

3.3 Co-Design of Self-Triggered Policy and Asynchronous Distributed Filter

The above Theorem and Lemma will be employed to design distributed self-triggered consensus filtering network (3), which leads to the following theorem.

Theorem 2: Given a positive definite matrix $R_i = R_i^T > 0$ ($1 \leq i \leq n$), the distributed self-triggered filtering problem is solvable, if there exist a positive scalar $\gamma > 0$, real symmetric matrices $P_i = P_i^T > 0$, $\bar{\Phi}_i$, and real matrices $\mathcal{X}_i, \mathcal{Y}_i$ satisfying

initial conditions

$$\sum_{i=1}^n e_i(0)^T P_i e_i(0) \leq \gamma^2 \sum_{i=1}^n e_i(0)^T R_i e_i(0) \tag{30}$$

and the following set of linear matrix inequalities

$$\begin{bmatrix} \Xi_{11} & \Xi_{12} & 0 & 0 & (P\bar{A} - X\bar{C} + \mathcal{Y}\bar{\mathcal{L}})^T \\ * & \Xi_{22} & 0 & 0 & (-\mathcal{Y}\bar{\mathcal{L}})^T \\ * & * & -\gamma^2 I & 0 & (\mathcal{X}\bar{D})^T \\ * & * & * & -n\gamma^2 I & (P\bar{B})^T \\ * & * & * & * & -P \end{bmatrix} \leq 0 \tag{31}$$

where $\mathcal{X} = \text{diag}_n^i \{\mathcal{X}_i\}$, $\mathcal{Y} = \text{diag}_n^i \{\mathcal{Y}_i\}$.

Moreover, if the set of linear matrix inequalities (31) with (30) is feasible, the filter gains can be computed as

$$H_i = P_i^{-1} \mathcal{X}_i, G_i = P_i^{-1} \mathcal{Y}_i. \quad (32)$$

Proof: Denoting $\mathcal{X}_i = P_i H_i$, $\mathcal{Y}_i = P_i G_i$. Then we have

$$P\bar{H} = \mathcal{X}, P\bar{G} = \mathcal{Y}. \quad (33)$$

Substituting relations (33) into inequality (16), it can be seen that the inequality (16) in Lemma 1 results in inequality (31) with (30). The proof is completed.

Remark 1: The goal of the above co-design is to conquer the asynchronous communication caused by self-triggered policy and achieve good filtering performance. Through Theorem 2, the self-triggered threshold parameters Φ_i and the filter gain matrices H_i, G_i could be successfully solved together.

4 Numerical Example

In this section, a numerical example is given to demonstrate effectiveness of the above co-design.

The network topology is represented by directed weighted graph $\mathcal{G} = (\mathcal{V}, \mathcal{E}, \mathcal{A})$ with five nodes $\mathcal{V} = \{1, 2, 3, 4, 5\}$, set of edges $\mathcal{E} \subseteq \{(1, 1), (1, 2), (1, 3), (1, 4), (2, 2), (2, 3), (3, 2), (3, 3), (3, 4), (4, 1), (4, 4), (5, 3), (5, 5)\}$. The transmission paths between five nodes in Fig. 3 and adjacency matrix as follows:

$$A = \begin{bmatrix} 1 & 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 \end{bmatrix}.$$

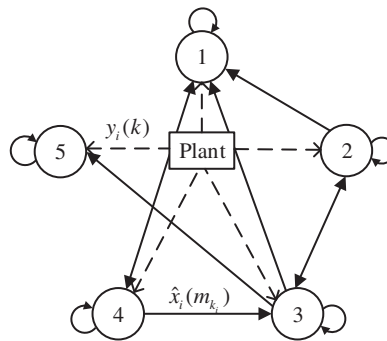


Figure 3: Schematic of distributed filtering over the WSN

The simulation is taken as 300-time units and each unit length is taken as 0.1. The system and sensor parameters are given as

$$A = \begin{bmatrix} 0.9 & -0.6 \\ 0.8 & 0.3 \end{bmatrix}, B = \begin{bmatrix} 0.5 \\ 1 \end{bmatrix}, M = \begin{bmatrix} 0.6 \\ 0.6 \end{bmatrix},$$

$$C_1 = [-0.2 \ 0.3], C_2 = [-0.3 \ 0.4], C_3 = [-0.2 \ 0.1],$$

$$C_4 = [-0.3 \ 0.2], C_5 = [0.2 \ -0.4], D_1 = D_2 = D_3 = D_4 = [1 \ 1].$$

The system noise and measurement noise are taken as $\omega(k) = e^{-2k}$ and $v_i(k) = [e^{-2k} \sin(k) \quad 2e^{-k} \cos(2k)]$, respectively. The initial state $x_0 = [0.2 \quad -0.1]^T$. The initial estimation of each node i are given as

$$\hat{x}_1(0) = [0.2 \quad -0.1]^T, \hat{x}_2(0) = [0.2 \quad -0.2]^T, \\ \hat{x}_3(0) = [-0.1 \quad -0.2]^T, \hat{x}_4(0) = [0.1 \quad -0.2]^T, \hat{x}_5(0) = [0.1 \quad -0.1]^T.$$

The self-triggered parameters of each node i are taken as $\varepsilon_1 = 0.7, \varepsilon_2 = 0.2, \varepsilon_3 = 0.6, \varepsilon_4 = 0.4, \varepsilon_5 = 0.3$. The positive definite matrices are given $R_1 = R_2 = R_3 = R_4 = R_5 = \text{diag}_2 \{0.7\}$.

The self-triggered matrices $\bar{\Phi}_i$ and the filter gain matrices H_i, G_i are obtained together by solving the linear matrix inequality conditions (30), (31) by the MATLAB software.

To further verify the strength of the self-triggered consensus filtering, a rectangular pulse $\omega_d(k)$ as an external disturbance is introduced into the system when $k \in [8, 10]$. The simulation results are shown in Figs. 4–6.

The output $z(k)$ and its estimation are depicted in Fig. 4. The filtering root means square error is given in Fig. 5. They show that the designed self-triggered consensus filter performs well in tracking the state of the target plant and the system has good robustness under the self-triggered policy.

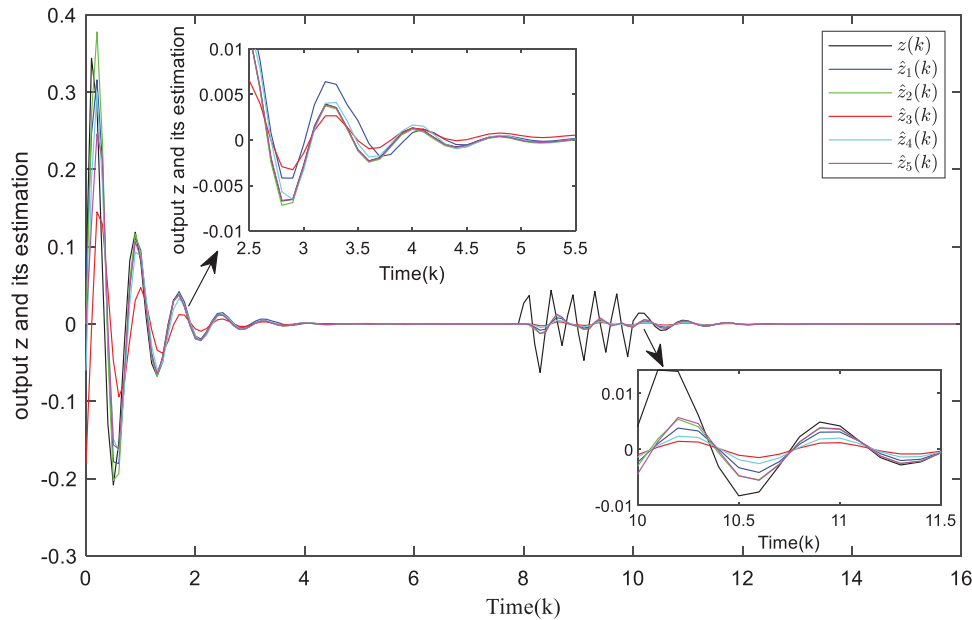


Figure 4: Plant output $z(k)$ and its estimation

Fig. 6 illustrates the self-triggered times of each filter. Under the periodic time-triggered mechanism, data transmits between the sensor and its neighbors at every sampling moment. It is clear that the self-triggered policy developed in this paper can effectively reduce the data transmission frequency within the network. In particular, in the first 6 time steps, filter 1-filter 5 is triggered less than 20 times under the self-triggered policy, while triggered 60 times under the time-triggered strategy. Therefore, the self-triggered policy developed in this paper can greatly save communication resources and computation burden of the filtering network.

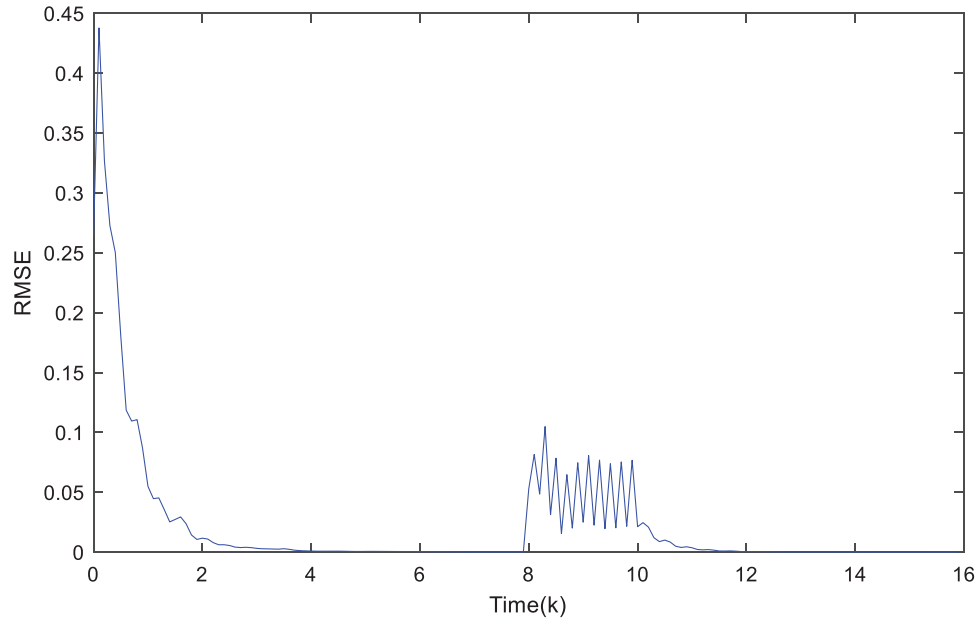


Figure 5: Filtering root mean square error

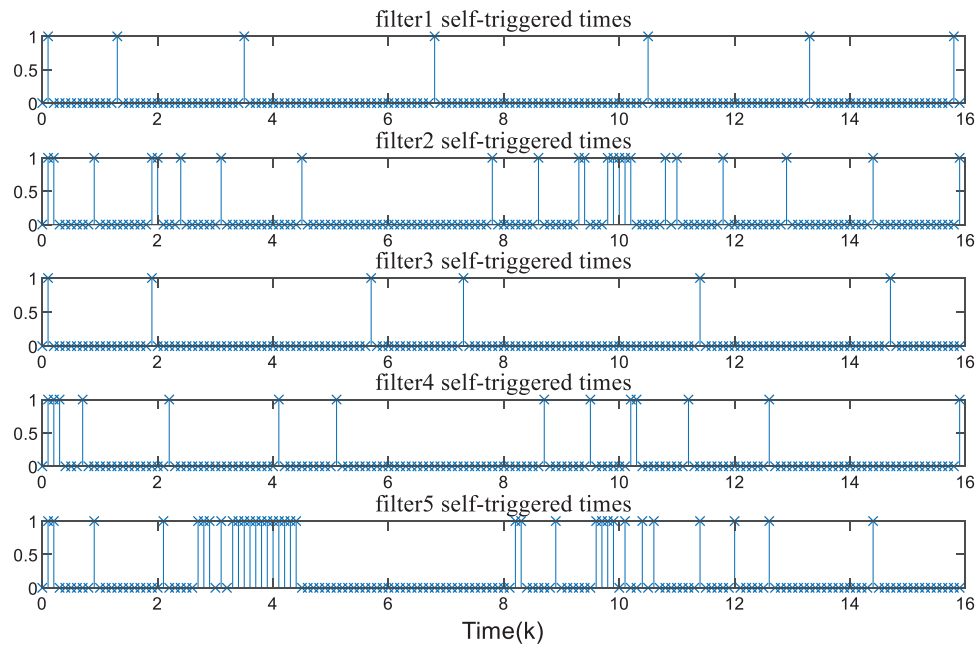


Figure 6: The self-triggered times of each filter

On the other hand, after the system is perturbed, the number of triggers intensively increases, and the triggers become gentle until the estimation becomes consensus during the time interval $k \in [8, 14]$. This also means that filter i should update data released from neighbors frequently when the filtering departs from consistency.

In summary, the simulation results illustrate that the self-triggered policy can save communication and computation burden while satisfying filtering performance.

5 Conclusion

The self-triggering policy is developed for a class of distributed filtering systems in this paper. This policy can actively predict the time when the following exchanged data will be updated. Through such a policy, the frequency of data exchange can be reduced, while communication resources can be saved within the filtering network. Compared with the existing event-triggered communication scheme, this means that it is not necessary to continuously judge trigger conditions, which saves the computing burden. The asynchronous transmission problem caused by each filter node's independent self-triggering can be solved by co-design. In order to make the research work close to the engineering practice, we will further consider transmission delay, packet loss, data conflicts, and other network induced phenomena.

Funding Statement: This work was jointly supported by the National Natural Science Foundation of China (Nos. 61991402, 62073154), the 111 Project (B12018), the Scientific Research Cooperation and High-Level Personnel Training Programs with New Zealand (1252011004200040).

Conflicts of Interest: The authors declare that they have no conflicts of interest to report regarding the present study.

References

1. Yang, X. S., Zhang, W. A., Yu, L., Xing, K. X. (2016). Multi-rate distributed fusion estimation for sensor network-based target tracking. *IEEE Sensor Journal*, 16(5), 1233–1242. DOI 10.1109/JSEN.2015.2497464.
2. La, H. M., Sheng, W. H. (2013). Distributed sensor fusion for scalar field mapping using mobile sensor networks. *IEEE Transactions on Cybernetics*, 43(2), 766–778. DOI 10.1109/TSMCB.2012.2215919.
3. Shen, B., Wang, Z. D., Hung, Y. S. (2010). Distributed H-infinity-consensus filtering in sensor networks with multiple missing measurements: The finite-horizon case. *Automatica*, 46(10), 7682–7688. DOI 10.1016/j.automatica.2010.06.025.
4. Ding, D., Wang, Z. D., Ho, D. W. C., Wei, G. L. (2017). Distributed recursive filtering for stochastic systems under uniform quantizations and deception attacks through sensor networks. *Automatica*, 78(1), 231–240. DOI 10.1016/j.automatica.2016.12.026.
5. Ma, L. F., Wang, Z. D., Alsaadi, F. E. (2019). Distributed filtering for nonlinear time-delay systems over sensor networks subject to multiplicative link noises and switching topology. *International Journal of Robust and Nonlinear Control*, 29(10), 2941–2959. DOI 10.1002/rnc.4535.
6. Liu, Y. R., Wang, Z. D., Liang, J. L., Liu, X. H. (2008). Synchronization and state estimation for discrete-time complex networks with distributed delays. *IEEE Transactions on Systems, Man, and Cybernetics, Part B (Cybernetics)*, 38(5), 1314–1325. DOI 10.1109/TSMCB.2008.925745.
7. Liu, Q. Y., Wang, Z. D., He, X., Zhou, D. H. (2015). Event-based distributed filtering with stochastic measurement fading. *IEEE Transactions on Industrial Informatics*, 11(6), 1643–1652. DOI 10.1109/TII.2015.2444355.
8. Zhang, X. M., Han, Q. L., Ge, X. H., Ding, D. R., Ding, L. et al. (2020). Networked control systems: A survey of trends and techniques. *IEEE/CAA Journal of Automatica Sinica*, 7(1), 1–17. DOI 10.1109/JAS.2019.1911651.
9. Wang, X., Su, H. S. (2019). Self-triggered leader-following consensus of multi-agent systems with input time delay. *Neurocomputing*, 330(9), 70–77. DOI 10.1016/j.neucom.2018.10.077.

10. Wang, M., Wang, Z. D., Chen, Y., Sheng, W. G. (2019). Event-based adaptive neural tracking control for discrete-time stochastic nonlinear systems: A triggering-threshold compensation strategy. *IEEE Transactions on Neural Networks and Learning Systems*, 31(6), 1968–1981. DOI 10.1109/TNNLS.2019.2927595.
11. Li, B., Wang, Z. D., Ma, L. F., Liu, H. J. (2019). Observer-based event-triggered control for nonlinear systems with mixed delays and disturbances: The input-to-state stability. *IEEE Transactions on Cybernetics*, 49(7), 2806–2819. DOI 10.1109/TCYB.2018.2837626.
12. Ma, L. F., Wang, Z. D., Lam, H. K., Kyriakoulis, N. (2017). Distributed event-based set-membership filtering for a class of nonlinear systems with sensor saturations over sensor networks. *IEEE Transactions on Cybernetics*, 47(11), 3772–3783. DOI 10.1109/TCYB.2016.2582081.
13. Su, H. S., Li, Z. H., Ye, Y. Y. (2017). Event-triggered Kalman-consensus filter for two-target tracking sensor networks. *ISA Transactions*, 71(1), 103–111. DOI 10.1016/j.isatra.2017.06.019.
14. Muehlebach, M., Trimpe, S. (2018). Distributed event-based state estimation for networked systems: An LMI approach. *IEEE Transactions on Automatic Control*, 63(1), 269–277. DOI 10.1109/TAC.2017.2726002.
15. Wang, L. C., Wang, Z. D., Han, Q. L., Wei, G. (2018). Event-based variance-constrained H-infinity filtering for stochastic parameter systems over sensor networks with successive missing measurements. *IEEE Transactions on Cybernetics*, 48(3), 1007–1017. DOI 10.1109/TCYB.2017.2671032.
16. Hu, J., Wang, Z. D., Liang, J. L., Dong, H. L. (2015). Event-triggered distributed state estimation with randomly occurring uncertainties and nonlinearities over sensor networks: A delay-fractioning approach. *Journal of the Franklin Institute*, 352(9), 3750–3763. DOI 10.1016/j.jfranklin.2014.12.006.
17. Gu, Z., Zhou, X. H., Zhang, T., Yang, F., Shen, M. Q. (2020). Event-triggered filter design for nonlinear cyber-physical systems subject to deception attacks. *ISA Transactions*, 104(3), 130–137. DOI 10.1016/j.isatra.2019.02.036.
18. Ge, X. H., Han, Q. L., Wang, Z. D. (2019). A threshold-parameter-dependent approach to designing distributed event-triggered H-infinity-consensus filters over sensor networks. *IEEE Transactions on Cybernetics*, 49(4), 1148–1159. DOI 10.1109/TCYB.2017.2789296.
19. Xiao, S. Y., Han, Q. L., Ge, X. H., Zhang, Y. J. (2020). Secure distributed finite-time filtering for positive systems over sensor networks under deception attacks. *IEEE Transactions on Cybernetics*, 50(3), 1220–1229. DOI 10.1109/TCYB.2019.2900478.
20. Ding, L., Guo, G. (2015). Distributed event-triggered H-infinity consensus filtering in sensor networks. *Signal Processing*, 108(3), 365–375. DOI 10.1016/j.sigpro.2014.09.035.
21. Xia, N., Yang, F. W., Han, Q. L. (2017). Distributed event-triggered networked set-membership filtering with partial information transmission. *IET Control Theory and Applications*, 11(2), 155–163. DOI 10.1049/iet-cta.2016.0781.
22. Zhang, H., Wang, Z. P., Yan, H. C., Yang, F. W., Zhou, X. (2019). Adaptive event-triggered transmission scheme and H-infinity filtering co-design over a filtering network with switching topology. *IEEE Transactions on Cybernetics*, 49(12), 4296–4307. DOI 10.1109/TCYB.2018.2862828.
23. Ge, X. H., Han, Q. L., Wang, Z. D. (2019). A dynamic event-triggered transmission scheme for distributed set-membership estimation over wireless sensor networks. *IEEE Transactions on Cybernetics*, 49(1), 171–183. DOI 10.1109/TCYB.2017.2769722.
24. Zhang, H., Hong, Q. Q., Yan, H. C., Yang, F. W., Guo, G. (2017). Event-based distributed H-infinity filtering networks of 2-DOF quarter-car suspension systems. *IEEE Transactions on Industrial Informatics*, 13(1), 312–321. DOI 10.1109/TII.2016.2569566.
25. Velasco, M., Fuertes, J., Marti, P. (2003). The self-triggered task model for real-time control systems. *Work-in-Progress Session of the 24th IEEE Real-Time Systems Symposium (RTSS03)*, vol. 384, pp. 67–70.
26. Fan, Y., Liu, L., Feng, G., Wang, Y. (2015). Self-triggered consensus for multi-agent systems with Zeno-free triggers. *IEEE Transactions on Automatic Control*, 60(10), 2779–2784. DOI 10.1109/TAC.2015.2405294.

27. Heemels, W. P. M. H., Johansson, K. H., Tabuada, P. (2012). An introduction to event-triggered and self-triggered control. *51st IEEE Conference on Decision and Control (CDC)*, pp. 3270–3285. Maui, HI, USA. DOI 10.1109/CDC.2012.6425820
28. Dimarogonas, D. V., Frazzoli, E., Johansson, K. H. (2012). Distributed event-triggered control for multi-agent systems. *IEEE Transactions on Cybernetics*, *57(5)*, 1291–1297. DOI 10.1109/TAC.2011.2174666.
29. Ding, L., Han, Q. L., Ge, X., Zhang, X. M. (2018). An overview of recent advances in event-triggered consensus of multiagent systems. *IEEE Transactions on Cybernetics*, *48(4)*, 1110–1123. DOI 10.1109/TCYB.2017.2771560.
30. Liu, C. X., Li, H. P., Gao, J., Xu, D. (2018). Robust self-triggered min-max model predictive control for discrete-time nonlinear systems. *Automatica*, *89(6)*, 333–339. DOI 10.1016/j.automatica.2017.12.034.
31. Wan, H. Y., Luan, X. L., Karimi, H. R., Liu, F. (2021). Dynamic self-triggered controller code-sign for Markov jump systems. *IEEE Transactions on Automatic Control*, *66(3)*, 1353–1360. DOI 10.1109/TAC.2020.2992564.
32. Yang, H., Wang, Z. D., Shen, Y. X., Alsaadi, F. E. (2021). Self-triggered filter design for a class of nonlinear stochastic systems with Markovian jumping parameter. *Nonlinear Analysis-Hybrid Systems*, *40(1)*, 101022. DOI 10.1016/j.nahs.2021.101022.