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Emergency Decision-Making Based on *q*-Rung Orthopair Fuzzy Rough Aggregation Information

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Abstract: With the frequent occurrences of emergency events, emergency decision making (EDM) plays an increasingly significant role in coping with such situations and has become an important and challenging research area in recent times. It is essential for decision makers to make reliable and reasonable emergency decisions within a short span of time, since inappropriate decisions may result in enormous economic losses and social disorder. To handle emergency effectively and quickly, this paper proposes a new EDM method based on the novel concept of q-rung orthopair fuzzy rough (q-ROPR) set. A novel list of q-ROFR aggregation information, detailed description of the fundamental characteristics of the developed aggregation operators and the q-ROFR entropy measure that determine the unknown weight information of decision makers as well as the criteria weights are specified. Further an algorithm is given to tackle the uncertain scenario in emergency to give reliable and reasonable emergency decisions. By using proposed list of q-ROFR aggregation information all emergency alternatives are ranked to get the optimal one. Besides this, the q-ROFR entropy measure method is used to determine criteria and experts' weights objectively in the EDM process. Finally, through an illustrative example of COVID-19 analysis is compared with existing EDM methods. The results verify the effectiveness and practicability of the proposed methodology.

Keywords: q-rung orthopair fuzzy rough set; q-ROFR entropy measure; aggregation information; emergency decision making

1 Introduction

Catastrophic events such as earthquakes, hurricanes, flooding, and droughts, among others lead to mass destruction such as a large number of deaths, infrastructure damage, and adverse social instability and public security consequences [1]. For example, the 2005 Kashmir earthquake in Pakistan destroyed more than 780,000 buildings and killed 87,350 humans and over a billion animals. In 2019, the Super Typhoon Lekima brought catastrophic damages to mainland China and the direct economic losses amounted to approximately 52 billion yuan. The corona virus disease 2019 (COVID-19) spread to over 200 countries and about 370,000 people have died so far. While emergency response and immediate measures play a key role in addressing such situations, the implementation of emergency decision-making (EDM) with outdated procedures will ultimately lead to possible failures in emergency decisions. Therefore, the EDM process is a vital and essential part of the whole emergency response [2–4]. In fact, inappropriate information on decision-making and tight time pressures in the context of the unforeseen environment of decision-making make it hard for decision-makers to make an effective and reasonable choice [5]. Therefore, to detect the optimum solution for the EDM procedure in order to reduce the economic losses and casualties, it is very important to develop systematic and scientific EDM techniques [6]. Therefore, tackling EDM quickly and effectively has become an important research topic in recent years [7].

Nowadays, the information management and decision-making have become much more important because of increasing emergency situations. With the increasing complexity of the data, new and more accurate tools are necessary because they handle human inaccuracy or ambiguous knowledge more effectively when compared to the classical tools. Zadeh [8] introduced the concept of fuzzy sets (FSs) to deal with uncertain information in real-life situations. Atanassov [9] proposed in 1986 the notion of intuitionistic FSs (IFSs) by generalizing the well-known theory of FSs. Although IFSs are successful in a wide range of applications, they still have some limitations because of the restriction that the sum of membership grade and that of non-membership grade must not exceed 1. To handle this issue, Yager [10] further extended the theory of IFSs and proposed the notion of Pythagorean FSs (PFSs) for modeling the higher-level imprecise and vague information. After Yager's pioneering work, several researchers initiated the study in the field of PFS theory to show its applications in various disciplines. Khan et al. [11] introduced the Dombi norm based on PFSs and discussed their applications in decision-making problems (DMPs). Yager et al. [12] presented a link between Pythagorean fuzzy membership grades and complex numbers. Batool et al. [13] extended the PFSs to Pythagorean probabilistic hesitant FSs and elaborated their applications in DMPs. Peng et al. [14] established the division and subtraction operations under the Pythagorean fuzzy environment and studied their properties in detail. Ashraf et al. [15] proposed the novel approach using the sine function under Pythagorean fuzzy settings. Zhang [16] defined a similarity measure for DMPs under the Pythagorean fuzzy environment.

Let us assume that the value of membership grade (MG) is set to 0.8 and that of non-membership grade (NMG) is set to 0.9. From the available information, it is clear that $\mathrm{MG} + \mathrm{NMG} > 1$ and $\mathrm{MG}^2 + \mathrm{NMG}^2 > 1$, which does not satisfy the fundamental condition of IFSs as well as PFSs. To resolve this issue, Yager [17] proposed a more general concept, called q-rung orthopair FS (q-ROFS). The prominent feature of the q-ROFS is that the sum of qth power of the DM and DNM should be less than or equal to 1, which gives more flexibility to the decision-makers for providing MG and NMG more comprehensively. Note that the space of acceptable orthopair of membership grades increases as the value of q increases. A q-ROFS is reduced into the IFS and the PFS, respectively, when we take $q = 1$ and $q = 2$. Yager et al. [18] presented the approximate reasoning with q-ROFSs by defining the concepts of possibility and certainty. Khan et al. [19] proposed the q-ROFS-based knowledge measures and discussed their applications in decision-making. Hussain et al. [20] presented the aggregation operators under q-ROF soft sets and elaborated their applications to tackle the real-life DMPs. Liu et al. [21] defined some q-ROF weighted arithmetic/geometric aggregation operators and used them for real-world DMPs. Khan et al. [22] presented the novel ranking methodology under q-ROF environments. Joshi et al. [23]

established some novel aggregation methods for q-ROF information by considering the confidence levels of the experts. Verma [24] introduced the order-q-ROF divergence and entropy measures with their application in multi-attribute group decision-making (MAGDM).

Pawlak [25] initiated the important notion of rough set theory in 1982, which handles imprecise and ambiguous data more effectively. Investigation into the rough set, both theoretically and practically, in the recent era has made tremendous progress. The notion of rough sets has been enhanced in various ways by several scholars. Dubois et al. [26] established the structure of fuzzy rough sets (FRSs). Zhang et al. [27] established the decision-making methodology using FRSs to tackle the uncertain information in DMPs. Khan et al. [28] established the novel notion of probabilistic hesitant FRSs and discussed their applications in DMPs. Chinram et al. [29] proposed the evaluation based on distance from average solution methodology under intuitionistic FRSs to tackle the multi-attribute group decision-making. Zhou et al. [30] established the generalized approximation operators under intuitionistic FRSs.

In some real-life circumstances, decision-makers (DMs) have a strong point of view about the ranking or rating of plans, projects, or political statements of a government. For example, the construction of a cricket ground by a university to render its accomplishment and performance. The members of the university administration may rate their project highly by assigning a DM ($\mu = 0.9$), whereas others may rate the same project as a wastage of money and try to defame it by providing strong opposite points of view. So, they assign a DNM ($\nu = 0.7$). In this situation, $\mu + \nu > 1$ and $\mu^2 + \nu^2 > 1$, but $\mu^q + \nu^q < 1$ for $q > 3$ so that (μ, ν) is neither IFN nor PFN but it is q-ROFN. Thus, Yager's q-ROFNs are more efficient to deal with uncertainty in the data. q-rung orthopair fuzzy rough sets (q-ROFRSs), a hybrid intelligent structure of rough sets, and q-ROFS are advanced classification strategies that address ambiguous and incomplete data. We conclude from the analysis that in decision-making, aggregation operators have a significant role to play in aggregating the collective data from different sources to a single value. In accordance with the best available knowledge to date, the development of aggregation operators with the hybridization of q-ROFS with a rough set is not observed in the q-ROPF setting. Therefore, this motivates the current work of q-ROF rough study. Furthermore, we will investigate aggregation operators based on rough information that are q-rung orthopair fuzzy rough weighted averaging, order weighted averaging, hybrid weighted averaging, weighted geometric, and order weighted geometric and hybrid weighted geometric aggregation operators under t-norm and t-conorm.

This paper is organized as follows: In Section 2, we review some concepts related to q-ROFSs and rough sets. In Section 3, we proposed the novel notion of q-ROFRS and discussed its basic operations. In Section 4, we proposed the list of averaging/geometric aggregation operators for q-ROPFR information. In Section 5, we present the entropy measure and decision-making methodology. In Section 6, we demonstrate the numerical example of the public health emergency problem to show the applicability and effectiveness of the proposed methodology. Finally, Section 8 concludes the paper, illustrating achievements and setting future directions.

2 Preliminaries

In this section, we resolve the essential knowledge about q-ROFS and rough sets.

Definition 1 ([17]). Let M be a non-empty set. A q-ROFS Z of a set M is a set having the form

$$Z = \{(\delta, \mu_z(\delta), \nu_z(\delta)) : \delta \in M\},$$

where the values $\mu_z(\delta) \in [0, 1]$ and $\nu_z(\delta) \in [0, 1]$ are called the positive and negative membership grades of the element δ, subject to $(\mu_z(\delta))^q + (\nu_z(\delta))^q \leq 1$ with $q > 2 \forall \delta \in M$.

For simplicity, $Z = \{(\delta, \mu_z(\delta), \nu_z(\delta)) : \delta \in M\}$ is represented by $Z = (\mu_z, \nu_z)$, if there is no confusion and is called q-rung orthopair number (q-ROFN). The collections of all q-ROFNs in M will be represented by q-ROPFN(N).

Definition 2 ([25]). Let M be a non-empty set and $\mathbf{I} \in M \times M$ be any arbitrary relation on the set M. A mapping $\mathbf{I}^*: M \to P(M)$ is defined as $\mathbf{I}^*(\mathrm{b}) = \{\partial \in M: (\mathrm{b}, \partial) \in \mathbf{I}\}$, for $\mathrm{b} \in M$, where $\mathbf{I}^*(\mathrm{b})$ is the successor neighborhood of an object b w.r.t. $\mathbf{I}$. The pair $(M, \mathbf{I})$ is said to be crisp approximation space. For any $x \subseteq M$, the lower and upper approximation of x w.r.t. approximation space $(M, \mathbf{I})$ is denoted and defined by $\underline{\mathbf{I}}(x) = \{b \in M: \mathbf{I}^*(\mathrm{b}) \subseteq x\}$ and $\overline{\mathbf{I}}(x) = \{b \in M: \mathbf{I}^*(\mathrm{b}) \cap x \neq \varphi\}$, respectively. Therefore, $\left(\underline{\mathbf{I}}(x), \overline{\mathbf{I}}(x)\right)$ is known as a rough set and $\underline{\mathbf{I}}(x), \overline{\mathbf{I}}(x) : P(M) \to P(M)$ are upper and lower approximation operators.

3 q-Rung Orthopair Fuzzy Rough Aggregation Information

The aggregation information plays an important role in combining data into one format and addressing the DMP. In this section, we propose a list of novel aggregation information.

3.1 q-Rung Orthopair Fuzzy Rough Averaging Aggregation operators

Definition 3. Let us consider $(M, \mathbf{I})$ be the q-ROF approximation space. Suppose $\mathbf{I}(x_g) = \left(\underline{\mathbf{I}}(x_g), \overline{\mathbf{I}}(x_g)\right)$ q-ROFRS$(M) \in (g \in \mathrm{N})$. Then, the weighted averaging aggregation operator can be defined as

$$\mathrm{WA}\left(\mathbf{I}(x_1), \mathbf{I}(x_2), \ldots, \mathbf{I}(x_n)\right) = \left(\sum_{g=1}^{n} \beta_g \underline{\mathbf{I}}\left(x_g\right), \sum_{g=1}^{n} \beta_g \overline{\mathbf{I}}\left(x_g\right)\right),$$

where $(\beta_1, \beta_2, \ldots, \beta_n)^{\mathrm{T}}$ is the weight information of $(\mathbf{I}(x_1), \mathbf{I}(x_2), \ldots, \mathbf{I}(x_n))$, that is, $\beta_g \geq 0$ and $\sum_{g=1}^{n} \beta_g = 1$.

Theorem 1. Suppose $(M, \mathbf{I})$ be a q-ROF approximation space. Suppose $\mathbf{I}(x_g) = \left(\underline{\mathbf{I}}(x_g), \overline{\mathbf{I}}(x_g)\right) \in$ q-ROFRS(M) $(g \in \mathrm{N})$ and $(\beta_1, \beta_2, \ldots, \beta_n)^{\mathrm{T}}$ is weight information of $(\mathbf{I}(x_1), \mathbf{I}(x_2), \ldots, \mathbf{I}(x_n))$ such that $\beta_g \geq 0$ and $\sum_{g=1}^{n} \beta_g = 1$. Then, the WA aggregation operator is a mapping $D^n \to D$ such that

$$\mathrm{WA}\left(\mathbf{I}(x_1), \mathbf{I}(x_2), \ldots, \mathbf{I}(x_n)\right)$$
$$= \left(\sum_{g=1}^{n} \beta_g \underline{\mathbf{I}}\left(x_g\right), \sum_{g=1}^{n} \beta_g \overline{\mathbf{I}}\left(x_g\right)\right)$$
$$= \left\{\left(\sqrt[q]{s^{-1}\left(\sum_{g=1}^{n} \beta_g s\left(\underline{\mu_g}^q\right)\right)}, t^{-1}\left(\sum_{g=1}^{n} \beta_g t\left(\underline{\nu_g}\right)\right)\right), \left(\sqrt[q]{s^{-1}\left(\sum_{g=1}^{n} \beta_g s\left(\overline{\mu_g}^q\right)\right)}, t^{-1}\left(\sum_{g=1}^{n} \beta_g t\left(\overline{\nu_g}\right)\right)\right)\right\}.$$

In algebraic-strict Archimedean t-norm and t-conorm, if we assign values to generators t and s, then we obtain two algebraic operations for q-ROFRVs:

$$\mathrm{WA}^{(\mathrm{A})}\left(\mathbf{I}(x_1),\mathbf{I}(x_2),\ldots,\mathbf{I}(x_n)\right)$$
$$=\left(\sum_{g=1}^{n}\beta_g\underline{\mathbf{I}}\left(x_g\right),\sum_{g=1}^{n}\beta_g\bar{\mathbf{I}}\left(x_g\right)\right)$$
$$=\left\{\left(\sqrt[q]{1-\prod_{g=1}^{n}\left(1-\underline{\mu_g}^{q}\right)^{\beta_g}},\prod_{g=1}^{n}\left(\underline{\nu_g}\right)^{\beta_g}\right),\left(\sqrt[q]{1-\prod_{g=1}^{n}\left(1-\overline{\mu_g}^{q}\right)^{\beta_g}},\prod_{g=1}^{n}\left(\overline{\nu_g}\right)^{\beta_g}\right)\right\}.$$

Proof: The proof is straightforward by using mathematical induction.

Definition 4. Suppose $(M,\mathbf{I})$ be a q-ROF approximation space. Suppose $\mathbf{I}\left(x_g\right)=\left(\underline{\mathbf{I}}\left(x_g\right),\bar{\mathbf{I}}\left(x_g\right)\right)\in q\text{-ROFRS}\,(M)\,(g\in \mathrm{N})$. Then, the ordered weighted averaging aggregation operator is defined as

$$\mathrm{OWA}\left(\mathbf{I}(x_1),\mathbf{I}(x_2),\ldots,\mathbf{I}(x_n)\right)=\left(\sum_{g=1}^{n}\beta_g\underline{\mathbf{I}}\left(x_{\beta(g)}\right),\sum_{g=1}^{n}\beta_g\bar{\mathbf{I}}\left(x_{\beta(g)}\right)\right),$$

where $\beta(g)$ is denoted the order according to $(\beta(1),\beta(2),\beta(3),\ldots,\beta(n))$ and $(\beta_1,\beta_2,\ldots,\beta_n)^{\mathrm{T}}$ is the weight information of $(\mathbf{I}(x_1),\mathbf{I}(x_2),\ldots,\mathbf{I}(x_n))$, that is, $\beta_g\geq 0$ and $\sum_{g=1}^{n}\beta_g=1$.

Theorem 2. Suppose $(M,\mathbf{I})$ be a q-ROF approximation space. Suppose $\mathbf{I}\left(x_g\right)=\left(\underline{\mathbf{I}}\left(x_g\right),\bar{\mathbf{I}}\left(x_g\right)\right)\in q\text{-ROFRS}\,(M)\,(g\in \mathrm{N})$ and $(\beta_1,\beta_2,\ldots,\beta_n)^{\mathrm{T}}$ is the weight information of $(\mathbf{I}(x_1),\mathbf{I}(x_2),\ldots,\mathbf{I}(x_n))$ such that $\beta_g\geq 0$ and $\sum_{g=1}^{n}\beta_g=1$. Then, OWA aggregation operator is a mapping $D^n\to D$ such that

$$\mathrm{OWA}\left(\mathbf{I}(x_1),\mathbf{I}(x_2),\ldots,\mathbf{I}(x_n)\right)$$
$$=\left(\sum_{g=1}^{n}\beta_g\underline{\mathbf{I}}\left(x_{\beta(g)}\right),\sum_{g=1}^{n}\beta_g\bar{\mathbf{I}}\left(x_{\beta(g)}\right)\right)$$
$$=\left\{\left(\sqrt[q]{s^{-1}\left(\sum_{g=1}^{n}\beta_g s\left(\underline{\mu_{\beta(g)}}^{q}\right)\right)},t^{-1}\left(\sum_{g=1}^{n}\beta_g t\left(\underline{\nu_{\beta(g)}}\right)\right)\right),\left(\sqrt[q]{s^{-1}\left(\sum_{g=1}^{n}\beta_g s\left(\overline{\mu_{\beta(g)}}^{q}\right)\right)},\right.\right.$$
$$\left.\left.t^{-1}\left(\sum_{g=1}^{n}\beta_g t\left(\overline{\nu_{\beta(g)}}\right)\right)\right)\right\}.$$

In algebraic-strict Archimedean t-norm and t-conorm, if we assign values to generators t and s, then we have algebraic operations for q-ROFRVs:

$$\mathrm{OWA}^{(\mathrm{A})}\left(\mathbf{I}(x_1), \mathbf{I}(x_2), \ldots, \mathbf{I}(x_n)\right)$$
$$= \left(\sum_{g=1}^{n} \beta_g \underline{\mathbf{I}}\left(x_{\beta(g)}\right), \sum_{g=1}^{n} \beta_g \overline{\mathbf{I}}\left(x_{\beta(g)}\right)\right)$$
$$= \left\{\left(\sqrt[q]{1-\prod_{g=1}^{n}\left(1-\underline{\mu_{\beta(g)}}^{q}\right)^{\beta_g}}, \prod_{g=1}^{n}\left(\underline{\nu_{\beta(g)}}\right)^{\beta_g}\right), \left(\sqrt[q]{1-\prod_{g=1}^{n}\left(1-\overline{\mu_{\beta(g)}}^{q}\right)^{\beta_g}}, \prod_{g=1}^{n}\left(\overline{\nu_{\beta(g)}}\right)^{\beta_g}\right)\right\}.$$

Proof: The proof is straightforward by using mathematical induction.

Definition 5. Suppose $(M, \mathbf{I})$ be a q-ROF approximation space and suppose $\mathbf{I}(x_g) = \left(\underline{\mathbf{I}}(x_g), \overline{\mathbf{I}}(x_g)\right) \in q\text{-ROFRS}(M)$ $(g \in \mathrm{N})$. Then, the hybrid weighted averaging aggregation operator is defined as

$$\mathrm{HWA}\left(\mathbf{I}(x_1), \mathbf{I}(x_2), \ldots, \mathbf{I}(x_n)\right) = \left(\sum_{g=1}^{n} \beta_g \underline{\mathbf{I}}\left(x'_{\beta(g)}\right), \sum_{g=1}^{n} \beta_g \overline{\mathbf{I}}\left(x'_{\beta(g)}\right)\right),$$

where $\beta(g)$ is the order according to $(\beta(1), \beta(2), \beta(3), \ldots, \beta(n))$ such that $\underline{\mathbf{I}}\left(x'_{\beta(g)}\right)\left(\underline{\mathbf{I}}\left(x'_{\beta(g)}\right) = n\beta_g \underline{\mathbf{I}}\left(x_{\beta(g)}\right) : g \in \mathrm{N}\right)$, $\overline{\mathbf{I}}\left(x'_{\beta(g)}\right)\left(\overline{\mathbf{I}}\left(x'_{\beta(g)}\right) = n\beta_g \overline{\mathbf{I}}\left(x_{\beta(g)}\right) : g \in \mathrm{N}\right)$, and $(\beta_1, \beta_2, \ldots, \beta_n)^{\mathrm{T}}$ is the weight information of $(\mathbf{I}(x_1), \mathbf{I}(x_2), \ldots, \mathbf{I}(x_n))$, that is, $\beta_g \geq 0$ and $\sum_{g=1}^{n} \beta_g = 1$. Also, $(\eta_1, \eta_2, \ldots, \eta_n)^{\mathrm{T}}$ is the associated weight information of $(\mathbf{I}(x_1), \mathbf{I}(x_2), \ldots, \mathbf{I}(x_n))$, that is, $\eta_g \geq 0$ and $\sum_{g=1}^{n} \eta_g = 1$.

Theorem 3. Suppose $(M, \mathbf{I})$ be a q-ROF approximation space. Suppose $\mathbf{I}(x_g) = \left(\underline{\mathbf{I}}(x_g), \overline{\mathbf{I}}(x_g)\right) \in q\text{-ROFRS}(M)$ $(g \in \mathrm{N})$ and $(\beta_1, \beta_2, \ldots, \beta_n)^{\mathrm{T}}$ be the weight information of $(\mathbf{I}(x_1), \mathbf{I}(x_2), \ldots, \mathbf{I}(x_n))$ such that $\beta_g \geq 0$ and $\sum_{g=1}^{n} \beta_g = 1$. Then, HWA aggregation operator is a mapping $D^n \to D$ with associated weight information $(\eta_1, \eta_2, \ldots, \eta_n)^{\mathrm{T}}$, that is, $\eta_g \geq 0$ and $\sum_{g=1}^{n} \eta_g = 1$, such that

$$\mathrm{HWA}\left(\mathbf{I}(x_1), \mathbf{I}(x_2), \ldots, \mathbf{I}(x_n)\right)$$
$$= \left(\sum_{g=1}^{n} \beta_g \underline{\mathbf{I}}\left(x'_{\beta(g)}\right), \sum_{g=1}^{n} \beta_g \overline{\mathbf{I}}\left(x'_{\beta(g)}\right)\right)$$
$$= \left\{\left(\sqrt[q]{s^{-1}\left(\sum_{g=1}^{n} \beta_g s\left(\underline{\mu'_{\beta(g)}}^{q}\right)\right)}, t^{-1}\left(\sum_{g=1}^{n} \beta_g t\left(\underline{\nu'_{\beta(g)}}\right)\right)\right), \left(\sqrt[q]{s^{-1}\left(\sum_{g=1}^{n} \beta_g s\left(\overline{\mu'_{\beta(g)}}^{q}\right)\right)},\right.\right.$$
$$\left.\left. t^{-1}\left(\sum_{g=1}^{n} \beta_g t\left(\overline{\nu'_{\beta(g)}}\right)\right)\right)\right\}.$$

In algebraic-strict Archimedean t-norm and t-conorm, if we assign values to generators t and s, then we have two algebraic operations for q-ROFRVs:

$$\mathrm{HWA}^{(\mathrm{A})}\left(\mathbf{I}(x_1), \mathbf{I}(x_2), \ldots, \mathbf{I}(x_n)\right)$$

$$= \left(\sum_{g=1}^{n} \beta_g \underline{\mathbf{I}}\left(x_{\beta(g)}\right), \sum_{g=1}^{n} \beta_g \overline{\mathbf{I}}\left(x_{\beta(g)}\right)\right)$$

$$= \left\{\left(\sqrt[q]{1-\prod_{g=1}^{n}\left(1-\underline{\mu'_{\beta(g)}}^{q}\right)^{\beta_g}}, \prod_{g=1}^{n}\left(\underline{\nu'_{\beta(g)}}\right)^{\beta_g}\right), \left(\sqrt[q]{1-\prod_{g=1}^{n}\left(1-\overline{\mu'_{\beta(g)}}^{q}\right)^{\beta_g}}, \prod_{g=1}^{n}\left(\overline{\nu'_{\beta(g)}}\right)^{\beta_g}\right)\right\}.$$

Proof: The proof is straightforward by using mathematical induction.

4 Development of *q*-ROFR Entropy Measure

To calculate the differences between two q-ROFRVs, this segment developed the generalized and weighted generalized distance measures of q-ROFR information. To measure the fuzziness of q-ROFRVs, we propose entropy measures for q-ROFRS based on the developed distance operators.

Definition 6. Suppose $(M, \mathbf{I})$ be a q-ROF approximation space. Suppose $\mathbf{I}\left(x_g\right) = \left(\underline{\mathbf{I}}\left(x_g\right), \overline{\mathbf{I}}\left(x_g\right)\right), K\left(x_g\right) = \left(\underline{K}\left(x_g\right), \overline{K}\left(x_g\right)\right) \in q\text{-ROFRS}(M)\ (g \in \mathrm{N})$. Then, the generalized distance measure (GDM) is described for any $\wp > 0\ (\in \mathrm{R})$ as

$$d^{g}(\mathbf{I}, K) = \left(\frac{1}{2n}\sum_{g=1}^{n}\left(\left|\left(\underline{\mu_g^{\mathbf{I}}}\right)^2 - \left(\underline{\mu_g^{K}}\right)^2\right|^{\wp} + \left|\left(\underline{\nu_g^{\mathbf{I}}}\right)^2 - \left(\underline{\nu_g^{K}}\right)^2\right|^{\wp}\right) + \frac{1}{2n}\sum_{g=1}^{n}\left(\left|\left(\overline{\mu_g^{\mathbf{I}}}\right)^2 - \left(\overline{\mu_g^{K}}\right)^2\right|^{\wp} + \left|\left(\overline{\nu_g^{\mathbf{I}}}\right)^2 - \left(\overline{\nu_g^{K}}\right)^2\right|^{\wp}\right)\right)^{1/\wp}.$$

Definition 7. Suppose $(M, \mathbf{I})$ be a q-ROF approximation space. Suppose $\mathbf{I}\left(x_g\right) = \left(\underline{\mathbf{I}}\left(x_g\right), \overline{\mathbf{I}}\left(x_g\right)\right), K\left(x_g\right) = \left(\underline{K}\left(x_g\right), \overline{K}\left(x_g\right)\right) \in q\text{-ROFRS}(M)\ (g \in \mathrm{N})$. Then, the weighted generalized distance measure (WGDM) is described for any $\wp > 0\ (\in \mathrm{R})$ as

$$d^{\mathrm{WG}}(\mathbf{I}, K) = \left(\frac{1}{2n}\sum_{g=1}^{n}\beta_g\left(\left|\left(\underline{\mu_g^{\mathbf{I}}}\right)^2 - \left(\underline{\mu_g^{K}}\right)^2\right|^{\wp} + \left|\left(\underline{\nu_g^{\mathbf{I}}}\right)^2 - \left(\underline{\nu_g^{K}}\right)^2\right|^{\wp}\right) + \frac{1}{2n}\sum_{g=1}^{n}\beta_g\left(\left|\left(\overline{\mu_g^{\mathbf{I}}}\right)^2 - \left(\overline{\mu_g^{K}}\right)^2\right|^{\wp} + \left|\left(\overline{\nu_g^{\mathbf{I}}}\right)^2 - \left(\overline{\nu_g^{K}}\right)^2\right|^{\wp}\right)\right)^{1/\wp},$$

where $\beta_g\ (g \in N)$ are the weight information such that $\beta_g \geq 0$ and $\sum_{g=1}^{n} \beta_g = 1$.

Definition 8. Suppose $(M, \mathbf{I})$ be q-ROF approximation space. Suppose $\mathbf{I}(x_g) = (\underline{\mathbf{I}}(x_g), \overline{\mathbf{I}}(x_g)) \in$ q-ROFRS(M) $(g = \{1, 2\} \in \mathrm{N})$. Then, the GDM is reduced to

$$d^{g}(\mathbf{I}(x_1), \mathbf{I}(x_2)) = \left(\frac{1}{2}\left(\left|\left(\underline{\mu_1^{\mathbf{I}}}\right)^2 - \left(\underline{\mu_2^{\mathbf{I}}}\right)^2\right|^{\wp} + \left|\left(\underline{\nu_1^{\mathbf{I}}}\right)^2 - \left(\underline{\nu_2^{\mathbf{I}}}\right)^2\right|^{\wp}\right)\right.$$
$$\left. + \frac{1}{2}\left(\left|\left(\overline{\mu_1^{\mathbf{I}}}\right)^2 - \left(\overline{\mu_2^{\mathbf{I}}}\right)^2\right|^{\wp} + \left|\left(\overline{\nu_1^{\mathbf{I}}}\right)^2 - \left(\overline{\nu_2^{\mathbf{I}}}\right)^2\right|^{\wp}\right)\right)^{1/\wp}.$$

Novel entropy measure for q-ROFRVs is developed in this segment.

Definition 9. Suppose $(M, \mathbf{I})$ be a q-ROF approximation space and $\mathbf{I}(x_g) = (\underline{\mathbf{I}}(x_g), \overline{\mathbf{I}}(x_g)) \in$ q-ROFRS(M) $(g \in \mathrm{N})$. Then, the q-ROFR entropy measure is described as

$$E(\mathbf{I}(x_g)) = \frac{1}{n}\sum_{g=1}^{n}\left[\left\{1 - d\left(\mathbf{I}(x_g), (\mathbf{I}(x_g))^c\right)\right\}\frac{1 + \left(\upsilon_{\mathbf{I}(x_g)}\right)^q}{2}\right],$$

where $\upsilon_{\mathbf{I}(x_g)}$ is the indeterminacy of $\mathbf{I}(x_g)$.

For q-ROF approximation space $(M, \mathbf{I})$, suppose $\mathbf{I}(x_g) = (\underline{\mathbf{I}}(x_g), \overline{\mathbf{I}}(x_g)), K(x_g) = (\underline{K}(x_g), \overline{K}(x_g)) \in q$-ROFRS$(M)$ $(g \in \mathrm{N})$. Then, the q-ROFR entropy measure satisfies the following properties:

(1) $E(\mathbf{I}(x_g)) = 0$ iff $\mathbf{I}(x_g)$ is the crisp set,
(2) $E(\mathbf{I}(x_g)) \leq E(\mathbf{I}(x_g)^c)$, and
(3) $E(\mathbf{I}(x_g)) \leq E(K(x_g))$ if $\mathbf{I}(x_g) \leq K(x_g)$, that is, $\underline{\mathbf{I}}(x_g) \leq \underline{K}(x_g)$ and $\overline{\mathbf{I}}(x_g) \leq \overline{K}(x_g)$.

5 Algorithm for DMPs

Here, we have developed a framework for addressing uncertainty in DM under q-rung orthopair fuzzy rough information. Consider a DM problem with $\{\lambda_1, \lambda_2, \ldots, \lambda_g\}$ as a set of m alternatives and a set of attributes $\{\tau_1, \tau_2, \ldots, \tau_h\}$ with $(\beta_1, \beta_2, \ldots, \beta_h)^{\mathrm{T}}$ being the weights, that is, $\beta_t \in [0, 1]$ and $\sum_{t=1}^{h}\beta_t = 1$. To test the reliability of kth alternative λ_k under the tth attribute τ_t, let a set of DMs $\{D_1, D_2, \ldots, D_{\hat{j}}\}$ and $\left(\eta_1, \eta_2, \ldots, \eta_{\hat{j}}\right)^{\mathrm{T}}$ be DM weights such that $\eta_s \in [0, 1]$ and $\sum_{s=1}^{\hat{j}}\eta_s = 1$.

Step 1. The expert's evaluation matrices are constructed. $E^{j} = \left[\mathbf{I}\left(x_{i\mathbf{I}}^{\hat{j}}\right)\right]_{g\times h} = \left(\underline{\mathbf{I}}\left(x_{i\mathbf{I}}^{\hat{j}}\right), \overline{\mathbf{I}}\left(x_{i\mathbf{I}}^{\hat{j}}\right)\right)$, where $\underline{\mathbf{I}}(x_{i\mathbf{I}}) = \left\{\left(b, \mu_{\underline{\mathbf{I}}(x_{i\mathbf{I}})}(b), \nu_{\underline{\mathbf{I}}(x_{i\mathbf{I}})}(b)\right) : b \in M\right\}$ and $\overline{\mathbf{I}}(x_{i\mathbf{I}}) = \left\{\left(b, \mu_{\overline{\mathbf{I}}(x_{i\mathbf{I}})}(b), \nu_{\overline{\mathbf{I}}(x_{i\mathbf{I}})}(b)\right) : b \in M\right\}$ such that $0 \leq \left(\mu_{\underline{\mathbf{I}}(x_{i\mathbf{I}})}(b)\right)^q + \left(\nu_{\underline{\mathbf{I}}(x_{i\mathbf{I}})}(b)\right)^q \leq 1$ and $0 \leq \left(\mu_{\overline{\mathbf{I}}(x_{i\mathbf{I}})}(b)\right)^q + \left(\nu_{\overline{\mathbf{I}}(x_{i\mathbf{I}})}(b)\right)^q \leq 1$ are the q-ROF rough values.

Step 2(a). The expert ideal matrix (EIM) is calculated using a q-ROFRWA aggregation operator, which is closer to each expert information. $\text{EIM} = \begin{pmatrix} \text{EI}_{11} & \text{EI}_{12} & \ldots & \text{EI}_{1\text{h}} \\ \text{EI}_{21} & \text{EI}_{22} & \ldots & \text{EI}_{2\text{h}} \\ M & M & O & M \\ \text{EI}_{g1} & \text{EI}_{g1} & \ldots & \text{EI}_{g\text{h}} \end{pmatrix}$, where

$$\text{EI}_{i\mathbf{I}} = \sum_{k=1}^{\hat{j}} \frac{1}{\hat{j}} N_{i\mathbf{I}}^{(k)} = \left\{ \left(\sqrt{1 - \prod_{k=1}^{\hat{j}} \left(1 - \left(\underline{\mu_{i\mathbf{I}}}^{(k)}\right)^2\right)^{1/\hat{j}}}, \prod_{k=1}^{\hat{j}} \left(\underline{\nu_{i\mathbf{I}}}^{(k)}\right)^{1/\hat{j}} \right), \left(\sqrt{1 - \prod_{k=1}^{\hat{j}} \left(1 - \left(\overline{\mu_{i\mathbf{I}}}^{(k)}\right)^2\right)^{1/\hat{j}}}, \prod_{k=1}^{\hat{j}} \left(\overline{\nu_{i\mathbf{I}}}^{(k)}\right)^{1/\hat{j}} \right) \right\}.$$

Step 2(b). Compute the expert right ideal matrix (ERIM) and expert left ideal matrix (ELIM) as follows:

$$\text{ERIM} = \begin{pmatrix} \text{RIM}_{11} & \text{RIM}_{12} & \ldots & \text{RIM}_{1\text{h}} \\ \text{RIM}_{21} & \text{RIM}_{22} & \ldots & \text{RIM}_{2\text{h}} \\ M & M & O & M \\ \text{RIM}_{g1} & \text{RIM}_{g2} & \ldots & \text{RIM}_{gh} \end{pmatrix}, \quad \text{where } \text{RIM}_{i\mathbf{I}} = \left\{ \left(N_{i\mathbf{I}}^{(k)}\right) : \max_{k \in [1,\hat{j}]} \left[\text{sc}\left(N_{i\mathbf{I}}^{(k)}\right)\right] \right\}.$$

$$\text{ELIM} = \begin{pmatrix} \text{LIM}_{11} & \text{LIM}_{12} & \ldots & \text{LIM}_{1\text{h}} \\ \text{LIM}_{21} & \text{LIM}_{22} & \ldots & \text{LIM}_{2\text{h}} \\ M & M & O & M \\ \text{LIM}_{g1} & \text{LIM}_{g1} & \ldots & \text{LIM}_{gh} \end{pmatrix}, \quad \text{where } \text{LIM}_{i\mathbf{I}} = \left\{ \left(N_{i\mathbf{I}}^{(k)}\right) : \min_{k \in [1,\hat{j}]} \left[\text{sc}\left(N_{i\mathbf{I}}^{(k)}\right)\right] \right\}.$$

Step 2(c). The distance of $N_{i\mathbf{I}}^{(k)}$ with EIM, ERIM, and ELIM is evaluated as DEIM, DERIM, and DELIM, respectively, as follows:

$$\text{DEIM}_i^{(k)} = \left(\frac{1}{2n} \sum_{g=1}^{n} \left(\left| \left(\underline{\mu_{N_{i\mathbf{I}}^{(k)}}}\right)^2 - \left(\underline{\mu_{\text{EI}_{i\mathbf{I}}}}\right)^2 \right|^{\wp} + \left| \left(\underline{\nu_{N_{i\mathbf{I}}^{(k)}}}\right)^2 - \left(\underline{\nu_{\text{EI}_{i\mathbf{I}}}}\right)^2 \right|^{\wp} \right) + \frac{1}{2n} \sum_{g=1}^{n} \left(\left| \left(\overline{\mu_{N_{i\mathbf{I}}^{(k)}}}\right)^2 - \left(\overline{\mu_{\text{EI}_{i\mathbf{I}}}}\right)^2 \right|^{\wp} + \left| \left(\overline{\nu_{N_{i\mathbf{I}}^{(k)}}}\right)^2 - \left(\overline{\nu_{\text{EI}_{i\mathbf{I}}}}\right)^2 \right|^{\wp} \right) \right)^{1/\wp},$$

$$\text{DERIM}_i^{(k)} = \left(\frac{1}{2n} \sum_{g=1}^{n} \left(\left| \left(\underline{\mu_{N_{i\mathbf{I}}^{(k)}}}\right)^2 - \left(\underline{\mu_{\text{RID}_{i\mathbf{I}}}}\right)^2 \right|^{\wp} + \left| \left(\underline{\nu_{N_{i\mathbf{I}}^{(k)}}}\right)^2 - \left(\underline{\nu_{\text{RID}_{i\mathbf{I}}}}\right)^2 \right|^{\wp} \right) \right.$$

$$+\frac{1}{2n}\sum_{g=1}^{n}\left(\left|\left(\overline{\mu_{N_{i\mathbf{I}}^{(k)}}}\right)^2-\left(\overline{\mu_{\mathrm{RID}_{i\mathbf{I}}}}\right)^2\right|^{\wp}+\left|\left(\overline{\nu_{N_{i\mathbf{I}}^{(k)}}}\right)^2-\left(\overline{\nu_{\mathrm{RID}_{i\mathbf{I}}}}\right)^2\right|^{\wp}\right)\Bigg)^{1/\wp},$$

$$\mathrm{DELIM}_i^{(k)}=\Bigg(\frac{1}{2n}\sum_{g=1}^{n}\left(\left|\left(\underline{\mu_{N_{i\mathbf{I}}^{(k)}}}\right)^2-\left(\underline{\mu_{\mathrm{LID}_{i\mathbf{I}}}}\right)^2\right|^{\wp}+\left|\left(\underline{\nu_{N_{i\mathbf{I}}^{(k)}}}\right)^2-\left(\underline{\nu_{\mathrm{LID}_{i\mathbf{I}}}}\right)^2\right|^{\wp}\right)$$

$$+\frac{1}{2n}\sum_{g=1}^{n}\left(\left|\left(\overline{\mu_{N_{i\mathbf{I}}^{(k)}}}\right)^2-\left(\overline{\mu_{\mathrm{LID}_{i\mathbf{I}}}}\right)^2\right|^{\wp}+\left|\left(\overline{\nu_{N_{i\mathbf{I}}^{(k)}}}\right)^2-\left(\overline{\nu_{\mathrm{LID}_{i\mathbf{I}}}}\right)^2\right|^{\wp}\right)\Bigg)^{1/\wp},$$

for $i=1,2,\ldots,m$ and $k=1,2,\ldots,\hat{j}$.

Step 2(d). Evaluate closeness indices (CIs) as follows:

$$\mathrm{CI}^{(k)}=\frac{\sum_{i=1}^{m}\mathrm{DERIM}_i^{(k)}+\sum_{i=1}^{m}\mathrm{DELIM}_i^{(k)}}{\sum_{i=1}^{m}\mathrm{DEIM}_i^{(k)}+\sum_{i=1}^{m}\mathrm{DERIM}_i^{(k)}+\sum_{i=1}^{m}\mathrm{DELIM}_i^{(k)}},\quad \text{for } k=1,2,\ldots,\hat{j}.$$

Step 2(e). Expert weight information is evaluated as

$$\Upsilon^{(k)}=\frac{\mathrm{CI}^{(k)}}{\sum_{k=1}^{\hat{j}}\mathrm{CI}^{(k)}}.$$

Step 3(a). Evaluate revised expert ideal matrix (R_vEIM) based on the developed entropy measure as

$$R_v\,\mathrm{EIM}_{i\mathbf{I}}=\sum_{k=1}^{\hat{j}}\Upsilon^{(k)}N_{i\mathbf{I}}^{(k)}=\left\{\left(\sqrt{1-\prod_{k=1}^{\hat{j}}\left(1-\left(\underline{\mu_{i\mathbf{I}}}^{(k)}\right)^2\right)^{\Upsilon^{(k)}}},\prod_{k=1}^{\hat{j}}\left(\underline{\nu_{i\mathbf{I}}}^{(k)}\right)^{\Upsilon^{(k)}}\right),\right.$$

$$\left.\left(\sqrt{1-\prod_{k=1}^{\hat{j}}\left(1-\left(\overline{\mu_{i\mathbf{I}}}^{(k)}\right)^2\right)^{\Upsilon^{(k)}}},\prod_{k=1}^{\hat{j}}\left(\overline{\nu_{i\mathbf{I}}}^{(k)}\right)^{\Upsilon^{(k)}}\right)\right\}.$$

Step 3(b). The entropy measure corresponding to each attribute is computed as

$$\mathrm{EA}_{\mathbf{I}}=\left(E_{\mathbf{I}}\left(\underline{\mathbf{H}}\right),E_{\mathbf{I}}\left(\overline{\mathbf{I}}\right)\right)=E\left(R_v\,\mathrm{EIM}_{1\mathbf{I}},R_v\,\mathrm{EIM}_{2\mathbf{I}},\ldots,R_v\,\mathrm{EIM}_{i\mathbf{I}}\right),\quad \mathbf{I}=1,2,\ldots,n.$$

Step 3(c). The attribute weight information is calculated as follows:

$$\beta_{\mathbf{I}}=\frac{1-\left(\frac{E_{\mathbf{I}}(\underline{\mathbf{H}})+E_{\mathbf{I}}(\overline{\mathbf{I}})}{2}\right)}{n-\prod_{\mathbf{I}=1}^{n}\left(\frac{E_{\mathbf{I}}(\underline{\mathbf{H}})+E_{\mathbf{I}}(\overline{\mathbf{I}})}{2}\right)},\quad \mathbf{I}=1,2,\ldots,n$$

Step 4. Aggregate the revised expert ideal matrix based on the proposed aggregation operators to construct the aggregated matrix using attribute weights $\beta_{\mathbf{I}}$.

Step 5. Compute the score (according to Definition 7) of overall values $\mathbf{F}_t$ $(t = 1, 2, \ldots, g)$ for the alternatives λ_k.

Step 6. According to Definition 8, the alternatives λ_k $(k = 1, 2, \ldots, g)$ are ranked and the optimal one that has the higher value is chosen.

6 Numerical Application of the Proposed Algorithm

In this section, a practical EDM problem concerning a public health emergency is considered to validate the applicability and practicality of the developed methodology.

6.1 Real-Life Case Study

Wuhan Province of China has reported many unexplained cases in December 2019. The cause of pneumonia was identified as the new coronavirus, later labeled corona virus disease 2019 (COVID-19). Since 1–14 days of the incubation period is required, infected persons without symptoms can quickly pass on the virus through drops and intimate contact with others. A total of 81,000 people had been diagnosed in China by 22 March 2020, of whom more than 3000 died. Wuhan was the center of the epidemic with approximately 50,000 people, representing 81.31% of all patients, and the mortality rate stood at approximately 5.02%. This acute, rapidly spreading disease has led to enormous economic disorders for the catering, entertainment, retail, and tourism industries. Controlling virus sources and virus transmission are generally the essential solutions for the prevention and control of such infectious diseases. Quarantine measures must, therefore, be taken on time and the movements of the population must be monitored. Four alternative emergency responses are recommended to Wuhan citizens on the basis of the above-mentioned analysis:

(1) The infected individuals are quarantined and closely monitored (λ_1).
(2) Suspected individuals with infections and those who have recently been in close contact with infected individuals are also quarantined. Moreover, uninfected people are advised to work for themselves, for example, wearing masks (λ_2).
(3) Participation in public meetings is strictly prohibited. If people go out, they must take protective measures such as wearing masks, measuring temperature if they enter public places, and so on (λ_3).
(4) All classes and work are suspended, all people must stay at home, and their travel freedom is restricted (λ_4).

In addition, four emergency response alternatives are assessed using four criteria: (1) life satisfaction (τ_1); (2) the rate of epidemic transmission (τ_2); (3) economic losses (τ_3); (4) the consumption of medical supplies (τ_4).

The invited DMs are divided into three expert panels: Expert Information $= \left\{(E)^1, (E)^2, (E)^3\right\}$, where each expert panel is required to provide unified evaluation results in the form of q-rung orthopair fuzzy rough values with unknown expert and criteria weight information.

Step 1(a). Tabs. 1a–1c presents the expert evaluation information in the form of q-rung orthopair fuzzy rough.

Step 2(a). The EIM is calculated in Tab. 2.

Step 2(b). The ERIM and ELIM are calculated in Tabs. 3 and 4.

Table 1: (a) Expert information $(E)^1$ (b) Expert information $(E)^2$ (c) Expert information $(E)^3$

	τ_1	τ_2	τ_3	τ_4
(a)				
λ_1	((0.6, 0.7), (0.2, 0.5))	((0.8, 0.4), (0.4, 0.6))	((0.9, 0.2), (0.4, 0.3))	((0.6, 0.5), (0.5, 0.2))
λ_2	((0.9, 0.4), (0.6, 0.3))	((0.7, 0.6), (0.4, 0.2))	((0.5, 0.7), (0.5, 0.1))	((0.8, 0.4), (0.5, 0.2))
λ_3	((0.7, 0.5), (0.4, 0.2))	((0.8, 0.3), (0.5, 0.2))	((0.6, 0.5), (0.5, 0.3))	((0.4, 0.9), (0.4, 0.3))
λ_4	((0.9, 0.3), (0.5, 0.3))	((0.6, 0.5), (0.3, 0.5))	((0.8, 0.4), (0.5, 0.2))	((0.7, 0.6), (0.5, 0.2))
(b)				
λ_1	((0.8, 0.5), (0.3, 0.6))	((0.3, 0.9), (0.2, 0.7))	((0.8, 0.3), (0.5, 0.4))	((0.7, 0.5), (0.5, 0.3))
λ_2	((0.7, 0.6), (0.5, 0.1))	((0.8, 0.4), (0.5, 0.3))	((0.4, 0.7), (0.6, 0.2))	((0.9, 0.4), (0.4, 0.3))
λ_3	((0.9, 0.4), (0.5, 0.3))	((0.7, 0.6), (0.6, 0.2))	((0.7, 0.6), (0.6, 0.1))	((0.3, 0.9), (0.5, 0.4))
λ_4	((0.6, 0.8), (0.5, 0.4))	((0.6, 0.5), (0.2, 0.6))	((0.9, 0.4), (0.5, 0.1))	((0.8, 0.5), (0.6, 0.3))
(c)				
λ_1	((0.7, 0.4), (0.2, 0.7))	((0.5, 0.7), (0.5, 0.4))	((0.8, 0.5), (0.4, 0.5))	((0.9, 0.2), (0.2, 0.6))
λ_2	((0.8, 0.3), (0.5, 0.2))	((0.6, 0.5), (0.6, 0.4))	((0.6, 0.7), (0.5, 0.3))	((0.7, 0.4), (0.4, 0.1))
λ_3	((0.6, 0.5), (0.6, 0.3))	((0.9, 0.4), (0.5, 0.2))	((0.7, 0.4), (0.5, 0.3))	((0.6, 0.8), (0.5, 0.4))
λ_4	((0.3, 0.9), (0.4, 0.5))	((0.8, 0.3), (0.5, 0.4))	((0.9, 0.3), (0.6, 0.2))	((0.8, 0.5), (0.5, 0.2))

Table 2: Expert ideal matrix

	τ_1	τ_2	τ_3	τ_4
λ_1	(0.7171, 0.5192), (0.2431, 0.5943))	((0.6331, 0.6316), (0.4059, 0.5517))	((0.8429, 0.3107), (0.4393, 0.3914))	((0.7836, 0.3684), (0.4441, 0.3301))
λ_2	((0.8228, 0.4160), (0.5388, 0.1817))	((0.7171, 0.4932), (0.5158, 0.2884))	((0.5158, 0.7000), (0.5388, 0.1817))	((0.8228, 0.4000), (0.4393, 0.1817))
λ_3	((0.7836, 0.4641), (0.5158, 0.2620))	((0.8228, 0.4160), (0.5388, 0.2000))	((0.6717, 0.4932), (0.5388, 0.2080))	((0.4735, 0.8653), (0.4719, 0.3634))
λ_4	((0.7421, 0.6000), (0.4719, 0.3914))	((0.6914, 0.4217), (0.3797, 0.4932))	((0.8751, 0.3634), (0.5388, 0.1587))	((0.7725, 0.5313), (0.5388, 0.2289))

Table 3: Expert right ideal matrix

	τ_1	τ_2	τ_3	τ_4
λ_1	((0.8, 0.5), (0.3, 0.6))	((0.8, 0.4), (0.4, 0.6))	((0.9, 0.2), (0.4, 0.3))	((0.9, 0.2), (0.2, 0.6))
λ_2	((0.9, 0.4), (0.6, 0.3))	((0.8, 0.4), (0.5, 0.3))	((0.5, 0.7), (0.5, 0.1))	((0.8, 0.4), (0.5, 0.2))
λ_3	((0.9, 0.4), (0.5, 0.3))	((0.9, 0.4), (0.5, 0.2))	((0.7, 0.6), (0.6, 0.1))	((0.6, 0.8), (0.5, 0.4))
λ_4	((0.9, 0.3), (0.5, 0.3))	((0.8, 0.3), (0.5, 0.4))	((0.9, 0.3), (0.6, 0.2))	((0.8, 0.5), (0.6, 0.3))

Step 2(c). The distance of $N_{i\mathbf{I}}^{(k)}$ with EIM, ERIM, and ELIM is calculated using Definition 6 and the information is given in Tab. 5 (DEIM), Tab. 6 (DERIM), and Tab. 7 (DELIM), respectively.

Table 4: Expert left ideal matrix

	τ_1	τ_2	τ_3	τ_4
λ_1	((0.6, 0.7), (0.2, 0.5))	((0.3, 0.9), (0.2, 0.7))	((0.8, 0.5), (0.4, 0.5))	((0.6, 0.5), (0.5, 0.2))
λ_2	((0.7, 0.6), (0.5, 0.1))	((0.7, 0.6), (0.4, 0.2))	((0.4, 0.7), (0.6, 0.2))	((0.7, 0.4), (0.4, 0.1))
λ_3	((0.7, 0.5), (0.4, 0.2))	((0.7, 0.6), (0.6, 0.2))	((0.6, 0.5), (0.5, 0.3))	((0.3, 0.9), (0.5, 0.4))
λ_4	((0.3, 0.9), (0.4, 0.5))	((0.6, 0.5), (0.2, 0.6))	((0.8, 0.4), (0.5, 0.2))	((0.7, 0.6), (0.5, 0.2))

Table 5: *DEIM*

	λ_1	λ_2	λ_3	λ_4
Expert-1 $(E)^1$	0.38995	0.17374	0.17928	0.31988
Expert-2 $(E)^2$	0.43168	0.25816	0.28402	0.29407
Expert-3 $(E)^3$	0.36411	0.22224	0.26319	0.49680

Table 6: *DERIM*

	λ_1	λ_2	λ_3	λ_4
Expert-1 $(E)^1$	0.52139	0.19118	0.37702	0.34117
Expert-2 $(E)^2$	0.72945	0.33279	0.35798	0.59828
Expert-3 $(E)^3$	0.47838	0.31772	0.37496	0.73657

Table 7: *DELIM*

	λ_1	λ_2	λ_3	λ_4
Expert-1 $(E)^1$	0.65379	0.32672	0.25039	0.73657
Expert-2 $(E)^2$	0.32542	0.30157	0.29034	0.31384
Expert-3 $(E)^3$	0.65479	0.34227	0.41273	0.36646

Step 2(d). The CIs are calculated as follows:

$CI^{(1)}$	$CI^{(2)}$	$CI^{(3)}$
0.761748868	0.719333678	0.73234948

Step 2(e). Expert weight information is calculated as follows:

$\Upsilon^{(1)}$	$\Upsilon^{(2)}$	$\Upsilon^{(3)}$
0.344148	0.324985	0.330866

Step 3(a). The revised expert ideal matrix is given in Tab. 8.

Table 8: Revised expert ideal matrix (R_vEIM)

	τ_1	τ_2	τ_3	τ_4
λ_1	((0.7153, 0.5214), (0.2422, 0.5929))	((0.6375, 0.6265), (0.4065, 0.5516))	((0.8441, 0.3089), (0.4384, 0.3900))	((0.7825, 0.3692), (0.4446, 0.3281))
λ_2	((0.8247, 0.4149), (0.5399, 0.1835))	((0.7163, 0.4951), (0.5147, 0.2869))	((0.5161, 0.7), (0.5379, 0.1801))	((0.8219, 0.4), (0.4404, 0.1814))
λ_3	((0.7816, 0.4650), (0.5147, 0.2609))	((0.8230, 0.4133), (0.5379, 0.200))	((0.6707, 0.4927), (0.5379, 0.2099))	((0.4734, 0.8656), (0.4709, 0.3622))
λ_4	((0.7463, 0.5934), (0.4721, 0.3900))	((0.6909, 0.4222), (0.3795, 0.4927))	((0.8742, 0.3636), (0.5385, 0.1596))	((0.7716, 0.5323), (0.5379, 0.2281))

Step 3(b). The entropy measure corresponding to each attribute is computed as follows:

$\left(E_1(\underline{\mathbf{I}}), E_1(\overline{\mathbf{I}})\right)$	$\left(E_2(\underline{\mathbf{I}}), E_2(\overline{\mathbf{I}})\right)$	$\left(E_3(\underline{\mathbf{I}}), E_3(\overline{\mathbf{I}})\right)$	$\left(E_4(\underline{\mathbf{I}}), E_4(\overline{\mathbf{I}})\right)$
((0.470118) (0.725892))	((0.546861) (0.755896))	((0.428504) (0.735745))	((0.373381) (0.798391))

Step 3(c). The attribute weight information is given as

β_1	β_2	β_3	β_4
0.254008	0.220283	0.264043	0.261666

Step 4. The collective preference values of each alternative in the revised expert ideal matrix are calculated using the proposed list of aggregation operators as follows:

Case 1. Using $\text{WA}^{(A)}$ **Aggregation Operator.** The collective preference values of each alternative using the $\text{WA}^{(A)}$ aggregation operator are given in Tab. 9.

Case 2. Using $\text{OWA}^{(A)}$ **Aggregation Operator**. The collective preference values of each alternative using the $\text{OWA}^{(A)}$ aggregation operator are given in Tab. 10.

Table 9: Overall preference value ($WA^{(A)}$)

λ_1	((0.7655, 0.4319), (0.4002, 0.4475))
λ_2	((0.7544, 0.4905), (0.5116, 0.2008))
λ_3	((0.7205, 0.5412), (0.5162, 0.2531))
λ_4	((0.7893, 0.4701), (0.4955, 0.2818))

Table 10: Overall preference value $OWA^{(A)}$

λ_1	((0.75927, 0.44407), (0.39722, 0.45907))
λ_2	((0.74972, 0.49447), (0.51420, 0.20495))
λ_3	((0.72280, 0.53913), (0.51707, 0.25089))
λ_4	((0.78529, 0.46799), (0.48912, 0.29359))

Case 3. Using HWA$^{(A)}$ **Aggregation Operator.** The collective preference values of each alternative using the HWA$^{(A)}$ aggregation operator are given in Tab. 11.

Case 4. Using WG$^{(A)}$ **Aggregation Operator.** The collective preference values of each alternative using the WG$^{(A)}$ aggregation operator are given in Tab. 12.

Table 11: Overall preference value $HWA^{(A)}$

λ_1	((0.76077, 0.43217), (0.41949, 0.42111))
λ_2	((0.77924, 0.45428), (0.49685, 0.20847))
λ_3	((0.74781, 0.49174), (0.52286, 0.23949))
λ_4	((0.78684, 0.49790), (0.50808, 0.26942))

Table 12: Overall preference value ($WG^{(A)}$)

λ_1	((0.74586, 0.48659), (0.37220, 0.49028))
λ_2	((0.70579, 0.54401), (0.50601, 0.21474))
λ_3	((0.66587, 0.66609), (0.51371, 0.27712))
λ_4	((0.77172, 0.49886), (0.48201, 0.36003))

Case 5. Using OWG$^{(A)}$ **Aggregation Operator.** The collective preference values of each alternative using the OWG$^{(A)}$ aggregation operator are given in Tab. 13.

Case 6. Using HWG$^{(A)}$ **Aggregation Operator.** The collective preference values of each alternative using the HWG$^{(A)}$ aggregation operator are given in Tab. 14.

Table 13: Overall preference value ($OWG^{(A)}$)

λ_1	((0.73814, 0.49995), (0.36907, 0.49952))
λ_2	((0.70233, 0.54582), (0.50924, 0.22021))
λ_3	((0.66703, 0.66550), (0.51447, 0.27570))
λ_4	((0.76698, 0.49689), (0.47448, 0.37266))

Table 14: Overall preference value ($HWG^{(A)}$)

λ_1	((0.74039, 0.49174), (0.40616, 0.46379))
λ_2	((0.75333, 0.48620), (0.49038, 0.22456))
λ_3	((0.71738, 0.56578), (0.52151, 0.25352))
λ_4	((0.77463, 0.52196), (0.49963, 0.33022))

Step 5. The score of collective overall preference values of each alternative is given in Tab. 15.

Step 6. The ranking of the alternatives λ_k $(k = 1, 2, \ldots, 4)$ is given in Tab. 15.

Table 15: Score values and ranking of the alternatives

Operators	$Sc(\lambda_1)$	$Sc(\lambda_2)$	$Sc(\lambda_3)$	$Sc(\lambda_4)$	Ranking
$WA^{(A)}$	0.571567	0.643664	0.610595	0.633194	$\lambda_2 > \lambda_4 > \lambda_3 > \lambda_1$
$OWA^{(A)}$	0.563334	0.641125	0.612461	0.628209	$\lambda_2 > \lambda_4 > \lambda_3 > \lambda_1$
$HWA^{(A)}$	0.581746	0.653334	0.634858	0.631902	$\lambda_2 > \lambda_4 > \lambda_3 > \lambda_1$
$WG^{(A)}$	0.535299	0.613263	0.559093	0.598711	$\lambda_2 > \lambda_4 > \lambda_3 > \lambda_1$
$OWG^{(A)}$	0.526935	0.611384	0.560077	0.592977	$\lambda_2 > \lambda_4 > \lambda_3 > \lambda_1$
$HWG^{(A)}$	0.547754	0.633236	0.604897	0.605519	$\lambda_2 > \lambda_4 > \lambda_3 > \lambda_1$

From the above computational process, we concluded that alternative λ_2 is the best among others, and, therefore, it is highly recommended.

7 Conclusion

In this study, we proposed a novel method to deal with EDM problems based on the novel notion of q-ROFRS and the list of aggregation operators. First, the q-ROFRSs provide a flexible and natural way for DMs with different backgrounds to express uncertain assessment information on emergency alternatives. Then, the novel methodology based on the aggregation operators is modified to rank emergency alternatives to help DMs to determine the best one. The expert and the criteria weights are calculated by the entropy measure method, which are derived from initial evaluation information directly avoiding human intervention and secondary information collection. Eventually, to demonstrate the effectiveness and practicability of our proposed method, it is applied to a real EDM example of COVID-19 and compared against those of the existing EDM method.

Our established methodology can be extended to cover heterogeneous information because different types of information are closer to the actual situation and suitable for various criteria. We can use Hamacher, Yager, and Dombi norms to develop generalized aggregation operators to address the uncertain information more accurately in EDM problems. These will be used in future research directions.

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