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Article

Dynamic Multi-Attribute Decision-Making Method with Double Reference Points and Its Application

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Abstract: To better reflect the psychological behavior characteristics of loss aversion, this paper builds a double reference point decision making method for dynamic multi-attribute decision-making (DMADM) problem, taking bottom-line and target as reference pints. First, the gain/loss function is given, and the state is divided according to the relationship between the gain/loss value and the reference point. Second, the attitude function is constructed based on the results of state division to establish the utility function. Third, the comprehensive utility value is calculated as the basis for alternatives classification and ranking. Finally, the new method is used to evaluate the development level of smart cities. The results show that the new method can judge the degree to which the alternatives meet the requirements of the decision-maker. While the new method can effectively screen out the unsatisfactory alternatives, the ranking results of other alternatives are consistent with those of traditional methods.

Keywords: Double reference point; dynamic multi-attribute decision making; smart city evaluation; loss aversion

1 Introduction

Multi-attribute decision-making (MADM) is a type of decision-making problem in ranking and selection of finite alternatives with multiple attributes. It is an important part of modern decision theory and has a wide range of application backgrounds. As people face an increasingly complex environment, the MADM method that uses decision information of single period for static decision analysis can have difficulty meeting actual needs [1,2]. In the objective reality, economic investments, building maintenance [3], carbon emission permit allocation [4], semiconductor manufacturing [5], large-scale Web service component strategy [6], smart city evaluation, and other issues usually have to consider of decision-making information of multiple periods to improve the scientific of decision-making. This type of multi-attribute decision-making problem that takes the time dimension into account is called a dynamic multi-attribute decision-making (DMADM) problem.

Current methods for solving DMADM problems are mostly based on the Expected Utility theory [7–9] without considering the effects of loss aversion behavior on decision results. An increasing number of studies have proved that the psychological behavior characteristic of loss aversion is widespread in many fields such as politics, economy, and society, etc. [10–13]. That is, in the decision-making process, the decision-maker is bound to be rational and not seeking to maximize the expected utility but rather seeking to minimize the loss. Some scholars use the theory of Bounded Rationality as the basis and start from the perspective of loss aversion, combining Prospect theory [14], Cumulative Prospect theory [15], Regret theory [16] and other behavioral theories with decision-making methods for solving various MADM problems including DMADM problems. Prospect theory, Cumulative Prospect theory, and Regret theory do not use attribute values as the basis for decision-making and instead use the gap between attribute values and reference points as the basis for judgment [17], which makes decision results closer to reality than Expectancy theory. The reference point is the basis for decision-makers to make judgments and choices and has a decisive influence on decision results [14]. The reference points adopted by the Prospect theory, Cumulative Prospect theory, and Regret theory are static reference points, which cannot reflect the changes in the dynamic decision-making environment effectively. From the perspective of the selection and number of reference points, these theories establish a single reference point from the perspective of targets, which cannot reflect the bottom-line requirements of decision-makers effectively and have certain limitations [18]. As early as 1952, Roy proposed the first principle of safety for investment decision-making [19], and its essence is to place the bottom-line in the most important position [20]. March et al. [21] and Highhouse [22] pointed out that the bottom-line and targets have an important influence on the risk appetite of decision-makers. Wang et al. [23] demonstrated the necessity and rationality of setting the bottom-line and target as reference points. Huang et al. [24] solved the problem of complex fuzzy multi-attribute decision-making better by establishing two reference points of bottom-line and target.

According to previous literature, we can see that the trend has been to study the DMADM problem from the perspective of bounded rationality. However, related research results are based mainly on a single reference and static reference points. Meanwhile, the bottom-line and target have an important effect on the decision-making behavior and they can be used as the reference point to describe the psychological behavior characteristics of the decision-maker in more detail.

This paper assumes that decision-makers have loss aversion behaviors and proposes a double reference point decision method for DMADM problems. First, the bottom-line reference point and the target reference point are used to describe the decision-makers' psychological behavior preferences. Then, the dynamic double reference point is established in conjunction with the time dimension. Following the relationship between the two reference points and the attitude, the satisfaction function closer to the actual is constructed, and then the utility function is determined. Finally, the decision weights are assigned to different periods and attributes and the utility values are aggregated to realize the classification, ranking, and optimization of alternatives.

2 Problem Description and Reference Point Setting

For better explanation and use, in the DMADM problem, $A = \{a_1, a_2,\ldots,a_m\}$ denotes the set of alternatives containing m pieces of alternatives, $M = \{1, 2,\ldots, m\}$; $C = \{c_1, c_2,\ldots, c_n\}$ denotes the set of attributes containing n pieces oattributes, $N = \{1, 2,\ldots, n\}$. The set of benefit-type attribute subscripts is represented by N_b, and the set of cost-type attribute subscripts is represented by N_c, $N_c \cup N_b = N$, $N_c \cap N_b = \emptyset$. $T = \{t_1, t_2, \ldots, t_p\}$ suggests a set of periods containing p periods, $P = \{1, 2, \ldots, p\}$. $w(t_k) = w_1(t_k), w_2(t_k), \ldots, w_n(t_k)$ is the weight vector of attributes in the period t_k, where $w_j(t_k)$ represents the weight of the attribute c_j during the period t_k, and $w_j(t_k)$ is an unknown number, $0 \le w_j(t_k) \le 1$ and $\sum_{j=1}^{n} w_j(t_k) = 1$. The weight vector of $\eta = \{\eta(t_1), \eta(t_2), \ldots, \eta(t_p)\}$ is the weight vector of period, where $\eta(t_k)$ represents the weight of the period t_k, $\eta(t_k)$ is an unknown number, $0 \le \eta(t_k) \le 1$ and $\sum_{k=1}^{p} \eta(t_k) = 1$. The decision matrix of the t_k period is expressed as Eq. (1).

$$D(t_k) = \begin{bmatrix} x_{1,1}(t_k) & x_{1,2}(t_k) & \cdots & x_{1,n}(t_k) \\ x_{2,1}(t_k) & x_{2,2}(t_k) & \cdots & x_{2,n}(t_k) \\ \vdots & \vdots & \ddots & \vdots \\ x_{m,1}(t_k) & x_{m,2}(t_k) & \cdots & x_{m,n}(t_k) \end{bmatrix}. \tag{1}$$

where $x_{i,j}(t_k)$ indicates the measured value of alternative a_i in period t_k on attribute c_j.

From the perspective of psychological behavior characteristics of loss aversion, the bottom-line and targets have an important effect on decision-making behavior [21–24]. Therefore, this paper chooses the bottom-line and the target as two reference points for decision-making. The bottom-line reference point represents the minimum requirements adhered to by the decision-maker, which cannot be easily broken, while the target reference point is the ideal target that the decision-maker expects to achieve. Taking smart city evaluation as an example, policymakers may have a bottom-line requirement and an ideal target for the development level of smart cities. When the development level of smart cities is poor and cannot meet the bottom-line requirements of decision-makers, the development level of smart cities will not be recognized by decision-makers. Meanwhile, when the development level of the smart city reaches or even surpasses the ideal target of the decision-maker, the development level of the smart city will be recognized by the decision-makers. When the development level of smart cities is between the bottom-line requirements and ideal targets, decision-makers exhibit hesitation and contradiction between approval and disapproval. At present, few studies have focused on the method of setting reference points in MADM problems and the static reference points are mainly used. In a DMADM environment, the reference point often changes with time [14]. Some scholars have also clearly pointed out that the dynamic reference points exist objectively in the fields of portfolio optimization [25], multi-agent path selection [26], emergency decision-making [27], and passenger behavior under flight delay [28]. Hence, the setting of dynamic reference points is very necessary. In summary, in this paper, the dynamic double reference points $(B(t_k), G(t_k))(k \in P)$ are set, where $B(t_k)$ is the bottom-line reference point at the t_k period, $B(t_k) = (B_1(t_k), B_2(t_k), \ldots, B_n(t_k))$, and $B_j(t_k)$ represents the bottom-line value of attribute c_j at the t_k period. $G(t_k)$ is the target reference point at the t_k period and $G(t_k) = (G_1(t_k), G_2(t_k), \ldots, G_n(t_k))$, and $G_j(t_k)$ represents the target value of attribute c_j at the t_k period.

3 Decision Model

3.1 Calculation of Gain/Loss Values

Losses and gains are relative to the reference point. When the attribute value is better than the reference point, it appears as a gain. Meanwhile, when the attribute value is less than the reference point, it appears as a loss. Taking the benefit-type attribute as an example (the cost-type attribute is similar), the judgment results of the bottom-line reference point B and target reference point G on losses and gains are shown in Fig. 1.

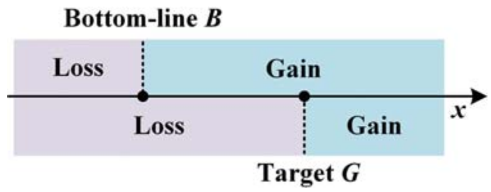

Figure 1: Relationship between reference points and losses and gains

According to the psychological characteristics of loss aversion and Equity Theory, decision-makers are often concerned not with the absolute value of gain or loss but with the relative value. When multiple reference points are observed, the calculation of the gain/loss value is suitable for adopting the mode of processing each reference point separately [29] and then the results are combined. Following this idea, the gain/loss value of attribute $x_{ij}(t_k)$ relative to the bottom-line reference point $B_j(t_k)$ can be expressed as Eq. (2), and its gain/loss value relative to the target reference point $G_j(t_k)$ can be expressed as Eq. (3).

$$y_{ij}^{b}(t_k) = \begin{cases} \dfrac{x_{ij}(t_k) - B_j(t_k)}{B_j(t_k)}, & j \in N_b, \\ \dfrac{B_j(t_k) - x_{ij}(t_k)}{B_j(t_k)}, & j \in N_c, \end{cases} \quad i \in M, k \in P. \tag{2}$$

$$y_{ij}^{g}(t_k) = \begin{cases} \dfrac{x_{ij}(t_k) - G_j(t_k)}{G_j(t_k)}, & j \in N_b, \\ \dfrac{G_j(t_k) - x_{ij}(t_k)}{G_j(t_k)}, & j \in N_c, \end{cases} \quad i \in M, k \in P. \tag{3}$$

y is a gain when $y > 0$, and y is a loss when $y < 0$. Based on the separate calculation of the gain/loss value of the two reference points, the gain/loss value based on the two reference points is obtained through the combination as shown in Eq. (4).

$$y_{ij}(t_k) = r y_{ij}^{b}(t_k) + (1 - r)\, y_{ij}^{g}(t_k), \tag{4}$$

where r is the coefficient of the decision mechanism, indicating the relative importance of the decision-makers on the two reference points, $1 > r > 0$. When $r > 0.5$, the decision-maker pays more attention to the bottom-line reference point, while when $r < 0.5$, the decision-maker pays more attention to the target reference point. When $r = 0.5$, the decision-maker attaches equal importance to the two reference points while when $r = 1$, the decision-maker only pays attention to the bottom-line reference point and not the target reference point. When $r = 0$, it means that the decision-maker focuses only on the target reference point and not the bottom-line reference point. $\alpha_j(t_k)$ was used to represent the gain/loss value corresponding to the target value $G_j(t_k)$, as shown in Eq. (5). $\beta_j(t_k)$ was used to represent the gain/loss value corresponding to the bottom-line value $B_j(t_k)$, as shown in Eq. (6).

$$\alpha_j(t_k) = r \left| \frac{G_j(t_k) - B_j(t_k)}{B_j(t_k)} \right|, \quad k \in P \tag{5}$$

$$\beta_j(t_k) = -(1-r)\left|\frac{B_j(t_k) - G_j(t_k)}{G_j(t_k)}\right|, \quad k \in P \tag{6}$$

Fig. 1 and Eq. (4) suggest the following: (1) When the measured value of attribute $x_{i,j}(t_k)$ is better than the target reference point $G_j(t_k)$, $y_{ij}^b(t_k) > 0$ and $y_{ij}^g(t_k) > 0$, $y_{ij}^b(t_k)$ and $y_{ij}^g(t_k)$ both represent gain, $y_{ij}(t_k) > \alpha_j(t_k)$. This state is called a double-gain state. (2) When the measured value of attribute $x_{ij}(t_k)$ is worse than the bottom-line reference point $B_j(t_k)$, $y_{ij}^b(t_k) < 0$ and $y_{ij}^g(t_k) < 0$, $y_{ij}^b(t_k)$and $y_{ij}^g(t_k)$ represent loss, $y_{ij}(t_k) < \beta_j(t_k)$. This state is called a double-loss state. (3) When the measured value of attribute $x_{i,j}(t_k)$ is between the bottom-line reference point $B_j(t_k)$ and target reference point $G_j(t_k)$, $y_{ij}^b(t_k) > 0$ and $y_{ij}^g(t_k) < 0$, $y_{ij}^b(t_k)$is the gain, $y_{ij}^g(t_k)$ is the loss, and $\beta_j(t_k) \le y_{ij}(t_k) \le \alpha_j(t_k)$. This state is called the coexistence state of gain and loss. Using $\delta_j(t_k)$ present the x value when $y_{ij}(t_k) = 0$. $\delta_j(t_k) = \frac{G_j(t_k)B_j(t_k)}{G_j(t_k)r + B_j(t_k)(1-r)}$ can be derived. Under the coexistence state of gain and loss, when $x_{ij}(t_k)$ is between the bottom-line reference point $B_j(t_k)$ and $\delta_j(t_k)$, the loss is greater than the gain, $\beta_{ij}(t_k) \le y_{ij}(t_k) \le 0$. This state is called an incomplete loss state. When $x_{ij}(t_k)$ is between the target reference point $G_j(t_k)$ and $\delta_j(t_k)$, the gain value is greater than the loss value, $0 \le y_{ij}(t_k) \le \alpha_j(t_k)$, this state is called an incomplete gain state. In summary, the division of the whole domain of attribute measured value x by double reference, and the comparison results of the division on the whole domain of gain/loss values y are shown in Fig. 2.

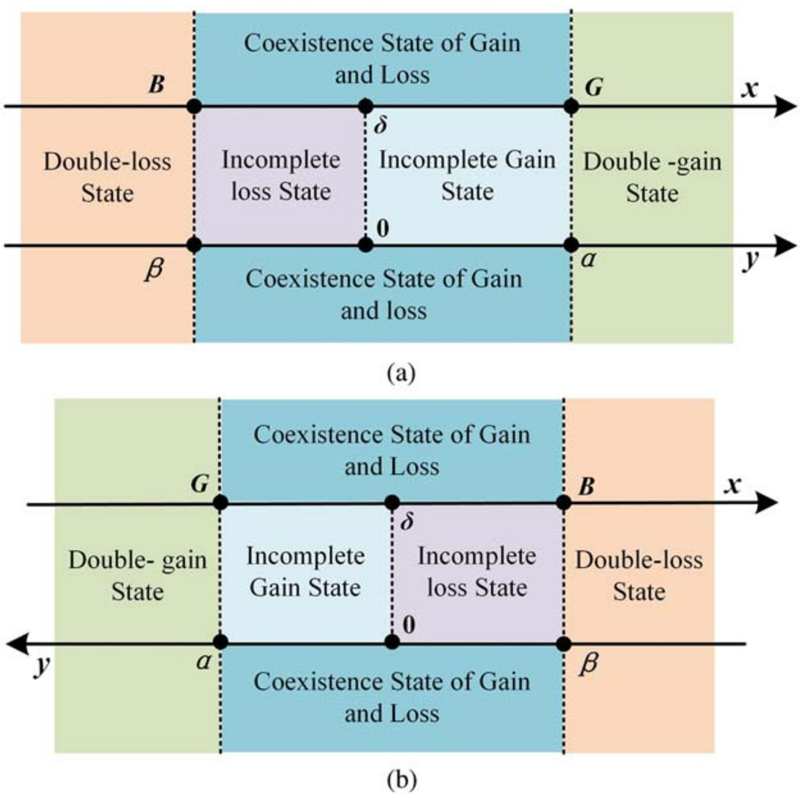

Figure 2: Double reference points and area division. (a) Attribute value is benefit type, (b) attribute value is cost type

3.2 Construction of Attitude Function

Attitude is the essential reflection of decision-makers on the gain/loss [24]. Attitude value is a quantitative description of attitude characteristics. When the attribute value is in the double-gain state, the decision-makers are satisfied. Meanwhile, when the attribute value is in the double-loss state, the decision-maker is dissatisfied. When the attribute value is under the coexistence state of gain and loss, the decision-makers feel hesitant and contradictory. The relationship between gain and loss status and the attitude of decision-makers is shown in Fig. 3.

Figure 3: Gain and loss status and attitude

To describe the attitude characteristics quantitatively, numbers greater than 1 are used to express satisfaction; the larger the value, the higher the satisfaction. Meanwhile, numbers less than −1 are used to express dissatisfaction and the smaller the value, the higher the dissatisfaction. The numbers in [−1, 1] indicate contradictory and hesitant attitudes, the closer the value is to 1, the closer to satisfaction, and the closer the value is to −1, the closer to dissatisfaction. If the decision-maker exhibits a risk-neutral attitude to the gain/loss value, then the correspondence between the attitude value v and the gain/loss value y can be simply expressed as Eq. (7) using a linear function.

$$v_{ij}(t_k) = \begin{cases} \dfrac{y_{ij}(t_k)}{\alpha_{ij}(t_k)}, & y_{ij}(t_k) \geq 0 \\ \dfrac{y_{ij}(t_k)}{\beta_{ij}(t_k)}, & y_{ij}(t_k) < 0 \end{cases}, \tag{7}$$

In reality, decision-makers often respond to gains with a risk-aversion attitude and deal with losses with a risk-seeking attitude [30,31]. Based on this idea, when the gain/loss value $y \geq 0$, the attitude function behaves as a monotonically increasing convex function. Meanwhile, when the gain/loss value $y < 0$, the attitude function is a monotonically increasing concave function. In other words, the attitude function should be an S type function whose inflection point is at the position of $y = 0$. The correspondence between attitude value v and the gain/loss value y can be expressed by the power function as Eq. (8).

$$v_{ij}(t_k) = \begin{cases} \left(\dfrac{y_{ij}(t_k)}{\alpha_j(t_k)}\right)^{\varphi}, & y_{ij}(t_k) \geq 0 \\ -\left(\left|\dfrac{y_{ij}(t_k)}{\beta_j(t_k)}\right|\right)^{\varphi}, & y_{ij}(t_k) < 0 \end{cases}, \tag{8}$$

where φ is the preference coefficient, $0 < \varphi < 1$.

By comparing Eqs. (7) and (8), we can see that if the prefer ence coefficient φ discards value constraints, Eq. (8) becomes Eq. (7) when taking the value 1. That is, when $\varphi = 1$, decision-makers exhibit risk-neutral attitudes towards losses and gains. When the value range of φ is expanded from (0, 1) to [0, 1], Eq. (7) will be unified into Eq. (8). The function curve when the constant φ takes different values is shown in Fig. 4.

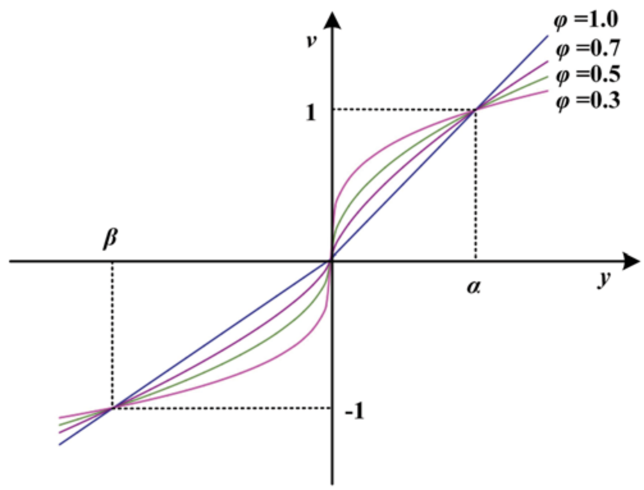

Figure 4: Attitude function

3.3 Construction of Utility Function

In the coexistence state of gain and loss, the more efficient the attitude value, the greater the utility. The utility value increases with the increase of attitude value and decreases with a decrease in attitude value. The utility function $u(\cdot)$ at this time can be expressed as Eq. (9).

$$u\left(v_{ij}\left(t_k\right)\right) = v_{ij}\left(t_k\right), \quad -1 \le v_{ij}\left(t_k\right) \le 1, \tag{9}$$

Han [18] and Wang et al. [32] posited that decision-makers will become very sensitive near the bottom-line reference point and a small drop in the attribute value across the bottom-line reference point will cause a "catastrophic" decline in the utility of the decision-makers. Decision-makers extremely circumvent such risks. In other words, when the coexistence state of gain and loss becomes the double-loss state, a qualitative change occurs and the utility value will drop significantly. Supposing $0 < \varepsilon \ll |B - G|$, the relationship between gain and loss status and attitude in Fig. 3 shows that attitude value $v = -1 - \varepsilon$ corresponds to the double-loss state and attitude value $v = -1 + \varepsilon$ corresponds to the coexistence state of gain and loss. Therefore, $u\left(-1-\varepsilon\right) \ll u\left(-1+\varepsilon\right)$. Eq. (9) shows that $\min u\left(-1+\varepsilon\right) \approx -1$, and thus, $u\left(-1-\varepsilon\right) \ll -1$. In addition, because the reference point effect has the characteristic of decreasing sensitivity [33], the utility value under the double-loss state has the characteristic of diminishing margin. According to the above analysis, assuming that δ represents the "catastrophic" reduction in utility value, the utility function $u(\cdot)$ in the double-loss state can be expressed as Eq. (10).

$$u\left(v_{i,j}\left(t_k\right)\right) = -\ln\left(\left|v_{i,j}\left(t_k\right)\right|\right) - \delta, \quad v_{i,j}\left(t_k\right) < -1, \tag{10}$$

where $\delta \gg 1$, $i \in M$, $j \in N$, and $k \in p$.

From the coexistence state of gain and loss to the double-gain state, it reflects more a change of quantity than a sudden change of quality. Therefore, when the attitude value changes from $v = 1 - \varepsilon$ to $v = 1 + \varepsilon$, the utility value will change smoothly without a great difference. Similar to the double-loss state, because the reference point effect has the characteristic of decreasing sensitivity [33], the utility value under the double-loss state also has the characteristic of diminishing margin. The utility function $u(\cdot)$ at this time can be expressed as Eq. (11).

$$u\left(v_{ij}\left(t_k\right)\right) = \ln\left(v_{ij}\left(t_k\right)\right) + 1, v_{ij}\left(t_k\right), \quad v_{ij}\left(t_k\right) > 1, \tag{11}$$

where $i \in M$, $j \in N$, and $k \in p$.

In summary, the utility function $u(\cdot)$ can be obtained by combining Eqs. (9)–(11) as shown in Eq. (12). The utility curve obtained from Formula (12) is shown in Fig. 5.

$$u\left(v_{ij}\left(t_k\right)\right) = \begin{cases} \ln\left(v_{ij}\left(t_k\right)\right) + 1, & v_{ij}\left(t_k\right) > 1 \\ v_{ij}\left(t_k\right), & -1 \leq v_{i,j}\left(t_k\right) \leq 1 \\ -\ln\left(\left|v_{ij}\left(t_k\right)\right|\right) - \delta, & v_{ij}\left(t_k\right) < -1 \end{cases}, \tag{12}$$

where $\delta \gg 1$, $i \in M$, $j \in N$, and $k \in p$.

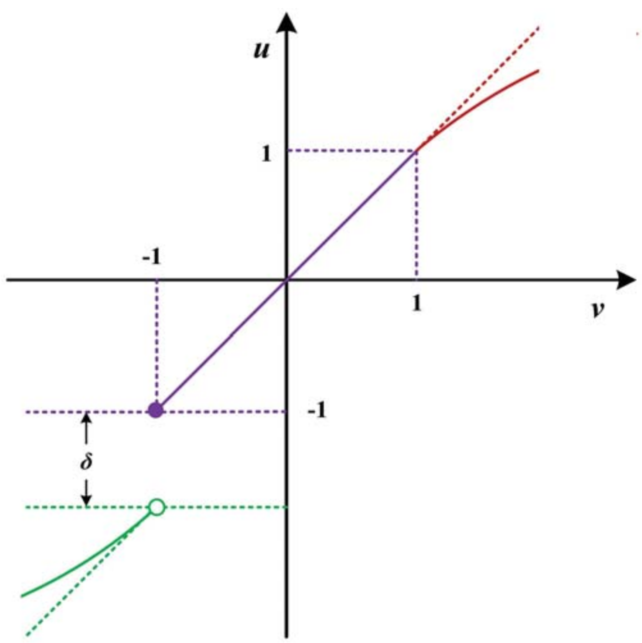

Figure 5: Utility curve

3.4 Calculation of Weights

3.4.1 Time Weight Calculation

Determining the time weight reasonably is a key issue in the DMADM model. Generally speaking, the value of information will decay over time. At present, most methods for determining the weight of time are based on the principle of "preference of the new to the old." That is, new information is given greater weight, and old information is given a lower weight. Assuming that the attenuation rate of information is $\lambda(0 \leq \lambda \leq 1)$ and the amount of information in the current period (t_p) is 1, the time decay model of information can be expressed as Eq. (13).

$$z\left(t_k\right) = (1-\lambda)^{t_p - t_k}, \quad k \in P, \tag{13}$$

where $t_p - t_k$ is the interval of period t_k and the current period t_p, $t_p - t_k = p - k$. When the attenuation rate λ takes different values, the attenuation function curve can be expressed as shown in Fig. 6.

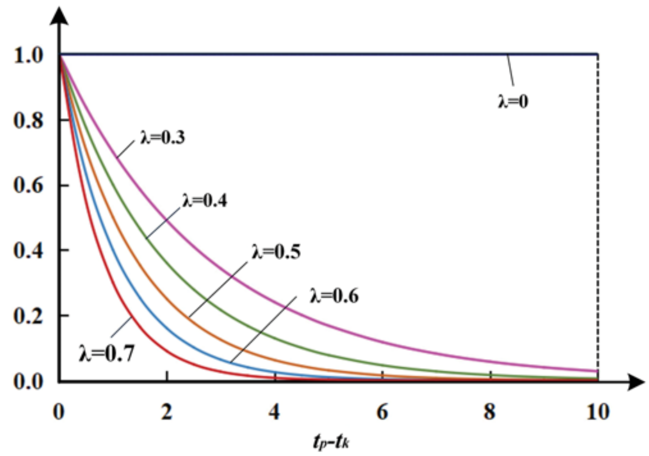

Figure 6: Time attenuation function curve

The time weight can be obtained by normalizing $z(t_k)$, as shown in Eq. (14). In particular, when $\lambda = 0$, $z(t_k) \equiv 1$, $\eta = \left(\frac{1}{p}, \frac{1}{p}, \ldots, \frac{1}{p}\right)$; when $\lambda = 1$, $z(t_p) = 1$, $z(t_k) = 0$ $(k < p)$, $\eta = (1, 0, \cdots, 0)$.

$$\eta\left(t_k\right) = z\left(t_k\right) \Big/ \sum_{k=1}^{p} z\left(t_k\right), \quad k \in P. \tag{14}$$

3.4.2 Calculation of Attribute Weights

The weight of attributes in a dynamic decision model may change with time, in contrast to the static decision model. The objective weighting method is used for weighting to utilize fully the loss information in different periods and improve the scientific of decision-making. Common objective weighting methods include the variation coefficient method, entropy weight method, and mean-variance method. The attribute weight is obtained based on the gain/loss value to reflect the difference in profit and loss information. The value range of the gain/loss value is not suitable for weighting using the variation coefficient method and the entropy weight method. Hence, to reflect the degree of difference between gain/loss values, the mean-variance method can be used for weighting. Because the gain/loss values are related closely to the reference point, the mean square error should be calculated separately according to different reference points. The mean square deviation of the gain/loss value based on the bottom-line reference point is shown in Eq. (15). The mean square deviation of the gain/loss value based on the target reference point is shown in Eq. (16).

$$\sigma_j^b\left(t_k\right) = \sqrt{\frac{\sum_{i=1}^{m}\left(y_{ij}^b\left(t_k\right) - \overline{y_j^b\left(t_k\right)}\right)^2}{m}}, \quad k \in P, \tag{15}$$

where $\overline{y_j^b(t_k)} = \frac{y_{1j}^b(t_k)+y_{2j}^b(t_k)+\cdots+y_{mj}^b(t_k)}{m}$.

$$\sigma_j^g(t_k) = \sqrt{\frac{\sum_{i=1}^{m}\left(y_{ij}^g(t_k) - \overline{y_j^g(t_k)}\right)^2}{m}}, \quad k \in P, \tag{16}$$

where, $\overline{y_j^g(t_k)} = \frac{y_{1j}^g(t_k)+y_{2j}^g(t_k)+\cdots+y_{mj}^g(t_k)}{m}$.

Finally, the two mean square errors are combined and normalized to obtain attribute weights, as shown in Eq. (17).

$$w_j(t_k) = \frac{\sigma_j^b(t_k) + \sigma_j^g(t_k)}{\sum_{j=1}^{n}\left(\sigma_j^b(t_k) + \sigma_j^g(t_k)\right)}, \quad k \in P. \tag{17}$$

3.5 Calculation and Ranking of Comprehensive Utility Value

Following previous calculation results, the comprehensive utility value of the alternative can be expressed as shown in Eq. (18).

$$u_i = \sum_{k=1}^{p}\eta(t_k)\sum_{j=1}^{n}u\left(v_{ij}(t_k)\right), \quad i \in M. \tag{18}$$

The larger the comprehensive utility value u_i, the better the alternative a_i. Decision-makers will become very sensitive near the bottom-line reference point and a small drop across the bottom-line reference point will cause a huge decline in the utility [18,32]. If the virtual alternative with the attribute value equal to the bottom-line value is called the bottom-line alternative a_b, then following the calculation of the utility value and the aggregation method, the comprehensive utility value of the bottom-line alternative is $u_b = -1$. Similarly, if the virtual alternative with the attribute value equal to the target value is called the target alternative a_g, then following the calculation of the utility value and the aggregation method, the comprehensive utility value of the target alternative is $u_g = 1$. Regarding the division of decision-makers' attitudes in Fig. 3, the alternative with comprehensive utility value $u > 1$ is called the satisfaction alternative, the alternative with comprehensive utility value $u < -1$ is called the dissatisfaction alternative, and the alternative with integrated utility value $u \in [-1, 1]$ is called hesitation alternative. The relationship between the different types of alternatives and utility values is shown in Fig. 7. The satisfaction alternative is better than the hesitation alternative and the hesitation alternative is better than the dissatisfaction alternative.

Figure 7: Alternative classification

3.6 Decision Steps

The steps of the MADM method based on double reference points in a dynamic environment are as follows:

Step 1: Start making decisions.

Step 2: Obtain the decision matrix and double reference points in different periods through a survey.

Step 3: Calculate the gain/loss values according to Eqs. (2)–(4) and obtain the gain/loss matrix for different periods.

Step 4: Calculate the attitude values corresponding to different gain/loss values according to Eq. (8) and obtain the attitude matrix for different periods.

Step 5: Calculate the utility value corresponding to different attitude values according to Eq. (12).

Step 6: Calculate the weight of the period according to Eqs. (13), (14); and according to Eqs. (15)–(17), calculate the attribute weight vector under each period using the mean-variance method.

Step 7: Calculate the comprehensive utility value of alternatives according to Eq. (18), and then classify, rank, and select alternatives.

Step 8: End.

4 Application of Decision-Making Methods in Smart City Evaluation

4.1 Description of Smart City Evaluation Issues

With the rapid development of RFID technology [34,35], internet of things technology [36], network technology [37,38], big data [39] and other technologies, the construction and development of smart cities have been given considerable attention by many governments and organizations [40–42]. The analysis of the development level of smart cities has also attracted the attention of many scholars. For example, Shen et al. [43] used the entropy weight method and Technique for Order of Preference by Similarity to Ideal Solution (TOPSIS) method to evaluate the development level of smart cities in 44 cities of China. Ren et al. [44] built an evaluation index system from five aspects that include smart infrastructure, smart government, and smart people's livelihood to evaluate the development level of smart cities. Zhang et al. [45] established an evaluation index system based on the needs of residents and used a fuzzy analytic hierarchy process to evaluate the development level of smart cities. In general, most existing studies have used static methods for analysis and evaluation and did not consider the dynamic perspective. The construction of smart cities is a long-term and gradual process. The static evaluation method has obvious shortcomings in describing its intellectualization process and development stage. Moreover, the development stage of urban intelligence should also be measured from the perspective of dynamic evaluation [46].

The researchers are ready to evaluate the development level of smart cities of six cities a_1, a_2, a_3, a_4, a_5, and a_6. Considering that the development of smart cities is a dynamic process, the evaluation information covers three periods, that is, t_1, t_2, and t_3. Drawing on [44], five aspects, including smart infrastructure (c_1), smart government (c_2), smart livelihood (c_3), smart production (c_4), and innovation drive (c_5) are taken as starting points and the evaluation values of each city in each attribute in different periods are obtained using the expert scoring method, as shown in Tab. 1. According to the development background and requirements of the different periods and following the principle of gradually increasing requirements, the bottom-line reference point and target reference point are determined as shown in Tab. 2. Now it is required to evaluate and analyze the development level of smart cities of the six cities according to the above information.

Table 1: Evaluation information

Cities	Period t_1					Period t_2					Period t_3				
	c_1	c_2	c_3	c_4	c_5	c_1	c_2	c_3	c_4	c_5	c_1	c_2	c_3	c_4	c_5
a_1	67	63	70	68	60	76	72	78	74	70	81	76	82	78	76
a_2	74	70	71	73	68	80	73	82	81	74	85	81	86	86	79
a_3	82	78	80	82	65	88	80	86	89	71	92	89	92	92	82
a_4	68	64	70	68	62	81	74	80	81	74	92	95	93	95	92
a_5	84	78	82	81	58	86	80	83	81	65	87	81	85	80	65
a_6	86	82	84	86	81	92	83	93	92	85	96	93	96	98	92

Table 2: Double reference points

Reference points	Period t_1					Period t_2					Period t_3				
	c_1	c_2	c_3	c_4	c_5	c_1	c_2	c_3	c_4	c_5	c_1	c_2	c_3	c_4	c_5
$B_j(t_k)$	65	60	65	62	55	70	65	70	68	62	75	72	75	75	70
$G_j(t_k)$	85	80	85	85	80	90	82	90	92	84	95	90	95	96	90

4.2 Evaluation of Smart City Development Level

The decision-making method mentioned above is used to evaluate the development level of smart cities. First, following Eqs. (2) and (3), the gain/loss matrix relative to bottom-line reference point B and target reference point G can be obtained. Assuming that the decision-maker pay more attention to the bottom-line reference point than to the target reference point, the decision mechanism coefficient r is taken as 0.6, and the comprehensive gain/loss matrix can be obtained according to Eq. (4), as shown in Tabs. 3–5.

Table 3: Gain/loss matrix for period t_1

Cities	c_1	c_2	c_3	c_4	c_5
a_1	−0.066	−0.055	−0.024	−0.022	−0.045
a_2	0.031	0.050	−0.010	0.050	0.082
a_3	0.143	0.170	0.115	0.179	0.034
a_4	−0.052	−0.040	−0.024	−0.022	−0.014
a_5	0.171	0.170	0.143	0.165	−0.077
a_6	−0.052	−0.025	−0.066	−0.065	−0.173

Table 4: Gain/loss matrix for period t_2

Cities	c_1	c_2	c_3	c_4	c_5
a_1	−0.011	0.016	0.015	−0.025	0.011
a_2	0.041	0.030	0.067	0.067	0.069
a_3	0.145	0.129	0.119	0.172	0.025
a_4	0.054	0.044	0.041	0.067	0.069
a_5	0.119	0.129	0.080	0.067	−0.061
a_6	−0.063	−0.041	−0.050	−0.052	−0.047

Table 5: Gain/loss matrix for period t_3

Cities	c_1	c_2	c_3	c_4	c_5
a_1	−0.011	−0.029	0.001	−0.051	−0.011
a_2	0.038	0.035	0.050	0.046	0.028
a_3	0.123	0.137	0.123	0.119	0.067
a_4	0.123	0.214	0.136	0.156	0.197
a_5	0.062	0.035	0.038	−0.027	−0.154
a_6	−0.048	−0.042	−0.048	−0.027	−0.050

Then, based on the gain/loss matrix, the preference coefficient φ is 0.5 according to experience and the attitude matrix can be obtained using Eq. (8). The utility value corresponding to different attitude values are determined according to Eq. (12), where δ is 10 according to the preference of the evaluator. The results are shown in Tabs. 6–8.

Table 6: Utility matrix for period t_1

Cities	c_1	c_2	c_3	c_4	c_5
a_1	−0.839	−0.742	−0.510	−0.450	−0.603
a_2	0.412	0.500	−0.334	0.474	0.548
a_3	0.880	0.922	0.789	0.898	0.354
a_4	−0.745	−0.632	−0.510	−0.450	−0.330
a_5	0.962	0.922	0.880	0.861	−0.786
a_6	1.036	1.070	0.962	1.031	1.028

Then, the attenuation rate λ takes 0.5 and the weight vector $\eta = \{0.143, 0.286, 0.571\}$ for each period is calculated according to Eqs. (13) and (14). The weight vectors of the attributes in each period are obtained according to Eqs. (15)–(17) as $w(t_1) = \{0.204, 0.211, 0.160, 0.195, 0.231\}$, $w(t_2) = \{0.196, 0.165, 0.177, 0.217, 0.245\}$, and $w(t_3) = \{0.147, 0.210, 0.143, 0.214, 0.288\}$.

Finally, the comprehensive utility value is obtained by using Eq. (18), and the alternatives are classified and ranked accordingly. The results are shown in Tab. 9. It shows that the smart city development level of city a_6 satisfied the evaluators, city a_5 dissatisfied the evaluator, and other cities are between satisfaction and dissatisfaction. The specific ranking from good to bad is $a_6 > a_3 > a_4 > a_2 > a_1 > a_5$.

Table 7: Utility matrix for period t_2

Cities	c_1	c_2	c_3	c_4	c_5
a_1	−0.348	0.318	0.298	−0.493	0.225
a_2	0.491	0.437	0.627	0.562	0.567
a_3	0.921	0.906	0.834	0.902	0.344
a_4	0.563	0.530	0.491	0.562	0.567
a_5	0.834	0.906	0.684	0.562	−0.766
a_6	1.071	1.043	1.103	1.000	1.033

Table 8: Utility matrix for period t_3

Cities	c_1	c_2	c_3	c_4	c_5
a_1	−0.361	−0.601	0.089	−0.763	−0.348
a_2	0.487	0.483	0.560	0.525	0.406
a_3	0.878	0.956	0.878	0.843	0.627
a_4	0.878	1.177	0.921	0.963	1.071
a_5	0.624	0.483	0.487	−0.552	−10.275
a_6	1.037	1.114	1.037	1.068	1.071

Table 9: Utility value and ranking

Cities	Period t_1		Period t_2		Period t_3		Comprehensive		
	Utility value	Ranking	Utility value	Ranking	Utility value	Ranking	Utility value	Ranking	Type
a_1	−0.636	6	−0.015	6	−0.429	5	−0.340	5	Hesitation
a_2	0.3544	4	0.540	4	0.481	4	0.480	4	Hesitation
a_3	0.7563	2	0.757	2	0.815	3	0.790	2	Hesitation
a_4	−0.531	5	0.546	3	1.021	2	0.663	3	Hesitation
a_5	0.5175	3	0.368	5	−2.82	6	−1.431	6	Dissatisfaction
a_6	1.0286	1	1.047	1	1.069	1	1.057	1	Satisfaction

Notes: "Hesitation" refers to the satisfaction of the evaluator on the development level of the smart city being between satisfaction and dissatisfaction.

4.3 Comparative of Methods

We use the expected utility method, weighted TOPSIS method, and Regret theory to evaluate the development level of smart cities and compare the results with the results of the new decision-making method to further illustrate the difference between the decision-making method proposed in this study and the traditional method. The traditional method is required to provide the time and attribute weights in advance. The weights are calculated using the new decision method to make the results comparable. When using the expected utility method, the weighted arithmetic average operator is used twice to obtain the evaluation result. Meanwhile, when using the weighted TPOSIS method, the closeness of each city in each period is first calculated and the weighted arithmetic average operator is used to fuse the closeness of different periods. When using the Regret theory, the average of bottom-line reference point B and target reference point G is taken as the reference point and then the perception utility of each city in each period is calculated. Then, the weighted arithmetic average operator is used to aggregate the perception utility of different periods. The evaluation results of the development level of smart cities through different methods are shown in Tab. 10.

Table 10: Comparison of different methods

Methods	Sort results:	Type division	Empowerment
Expected utility method	$a_6 > a_3 > a_4 > a_2 > a_5 > a_1$	No	No
Weighted TPOSIS method	$a_6 > a_4 \approx a_3 > a_2 > a_5 > a_1$	No	No
Regret theory	$a_6 > a_3 > a_4 > a_2 > a_5 > a_1$	No	No
New decision method	$(a_6) > (a_3 > a_4 > a_2 > a_1) > (a_5)$	Yes	Yes

Notes: The cities in "( )" indicate that their satisfaction is the same type.

The above comparison shows the following: (1) The results of traditional methods for evaluating the development level of smart cities are basically consistent. (2) The new method can classify the development level of smart cities and determine that the smart city development level of city a_6 is in a satisfaction state of decision-makers, city a_5 is in a state of dissatisfaction, and other cities are somewhere between satisfaction and dissatisfaction. (3) The ranking result in a new decision-making method for cities with satisfactory and intermediate states (a_1, a_2, a_3, a_4, a_6) is basically consistent with that of the traditional method. (4) The new method can effectively weigh periods and attributes.

5 Conclusions and Future Work

As people face an increasingly complex environment, the use of multi-period decision information for dynamic decision analysis has become a growing trend. The bottom-line and target have important influence on decision-making behavior and they can be used as reference points to describe in more detail the psychological behavior characteristics of the decision-maker. Hence, this paper proposes a DMADM method based on two reference points, namely, bottom-line and target. First, the bottom-line reference point and target reference point are set to reflect the psychological and behavioral characteristics of decision-makers. The two reference points are used to divide the entire range of attribute values into three state intervals of "double gain," "double loss," and "coexistence of gain and loss" The state interval "coexistence of gain and loss" can be divided into "incomplete income" and "incomplete loss". Second, gain/loss function, attitude function, and utility function are established according to the psychological behavior characteristics of decision-makers. The weight of the period is determined using the principle of information attenuation, while the attribute weight was calculated based on the mean square error. Finally, the methods of alternatives classification and ranking are given based on the comprehensive utility value.

The new decision-making method is compared with the expected utility method, weighted TOPSIS method, and Regret theory through the application of examples. The new decision-making method has the following advantages: (1) The new decision-making method can divide the alternatives into three types, namely, satisfaction, hesitation, and dissatisfaction, and it can effectively judge the degree to which the alternatives meet the requirements of the decision-makers. (2) The new decision-making method can effectively solve the weighting problem of periods and attributes. (3) While the new decision-making method screens out unsatisfactory alternatives effectively, the ranking results of other alternatives are consistent with traditional methods.

This study can provide a reference for research on multi-reference MADM and DMADM problems, and further enrich the research connotation of MADM theory and methods. However, this paper only studies the DMADM problem with double reference points and decision information as crisp number. The decision mechanism coefficient r, attenuation rate λ, and other related coefficients in this paper require further discussion. The DMADM problem with more than two reference points and the double reference points DMADM problem with fuzzy number or linguistic variable will be discussed in the future work.

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