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Article

Colouring of COVID-19 Affected Region Based on Fuzzy Directed Graphs

Rupkumar Mahapatra[1], Sovan Samanta[2], Madhumangal Pal[1], Jeong-Gon Lee[3,*], Shah Khalid Khan[4], Usman Naseem[5] and Robin Singh Bhadoria[6]

[1]Department of Applied Mathematics with Oceanology and Computer Programming, Vidyasagar University, Midnapore, 721102, India
[2]Department of Mathematics, Tamralipta Mahavidyalaya, Tamluk, 721636, India
[3]Division of Applied Mathematics, Wonkwang University, Iksan-Si, Jeonbuk, 54538, Korea
[4]School of Engineering, RMIT University, Melbourne, 3001, Australia
[5]School of Computer Science, University of Sydney Sydney, 2006, Australia
[6]Department of Computer Science & Engineering, Birla Institute of Applied Sciences (BIAS), Bhimtal, Uttarakhand, 263136, India
[*]Corresponding Author: Jeong-Gon Lee. Email: jukolee@wku.ac.kr
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Abstract: Graph colouring is the system of assigning a colour to each vertex of a graph. It is done in such a way that adjacent vertices do not have equal colour. It is fundamental in graph theory. It is often used to solve real-world problems like traffic light signalling, map colouring, scheduling, etc. Nowadays, social networks are prevalent systems in our life. Here, the users are considered as vertices, and their connections/interactions are taken as edges. Some users follow other popular users' profiles in these networks, and some don't, but those non-followers are connected directly to the popular profiles. That means, along with traditional relationship (information flowing), there is another relation among them. It depends on the domination of the relationship between the nodes. This type of situation can be modelled as a directed fuzzy graph. In the colouring of fuzzy graph theory, edge membership plays a vital role. Edge membership is a representation of flowing information between end nodes of the edge. Apart from the communication relationship, there may be some other factors like domination in relation. This influence of power is captured here. In this article, the colouring of directed fuzzy graphs is defined based on the influence of relationship. Along with this, the chromatic number and strong chromatic number are provided, and related properties are investigated. An application regarding COVID-19 infection is presented using the colouring of directed fuzzy graphs.

Keywords: Graph colouring; chromatic index; directed fuzzy graphs

1 Introduction

The best way to represent the relations/links among objects is graphs. Edges in crisp graphs represent the linkage between two objects, but edges in fuzzy graphs measure the degree of connectedness by using some membership values varying from [0, 1]. Kaufmann [1] introduced the fuzzy graph in 1973, and Rosenfeld [2] developed it in 1975. Samanta et al. [3,4] proposed various types of fuzzy graphs and its applications. When representing some particular types of social networks, directed graphs are more relevant than undirected graphs. For example, on Twitter, some users are following some selected popular users. Here, users are considered as nodes of graphs. Here, if two users are just connected in the network, there exists an undirected edge. Again, if one user is following another, there exists a directed edge. So, these networks can be designed by a directed graph. Thus both types of edges occur in a graph simultaneously. The measure of influences is captured/calculated only by fuzzy membership values. Today, COVID-19 [5] is the most significant problem in the world. WHO declared COVID-19 outbreak as a world health emergency on 30th January 2020 [6–8]. They first detected this virus in Wuhan, China, in December 2019. After that, this virus spread globally. Within April 2020, almost all the countries were affected by this virus. Here, all the countries are considered as nodes of networks, and there exists a directed edge between two countries if one country is affected by the virus carried from another country. So, we get a directed graph [9]. The amount of influence can also be measured by some membership value. In this study, we are going to introduce a new class of fuzzy graph.

Graph colouring is one of the oldest problems. Samanta et al. [10] introduced fuzzy colouring in fuzzy graphs. Hansen et al. [11] and Sotskov et al. [12] introduced the mixed graph colouring. Mahapatra et al. [13–23] introduced the edge colouring of a fuzzy graph and radio k colouring in fuzzy graphs. In this article, the colouring of directed fuzzy graphs is also proposed.

2 Preliminaries

Definition 1 *A fuzzy graph [1]* $\zeta = (V, \sigma, \mu)$ *is a non-empty set* V *together with a pair of functions* $\sigma: V \to [0,1]$ *and* $\mu: V \times V \to [0,1]$ *such that for all* $x, y \in V$, $\mu(x,y) \le min\{\sigma(x), \sigma(y)\}$, *where* $\sigma(x)$ *and* $\mu(x,y)$ *represent the membership values of the vertex* x *and of the edge* (x,y) *in* ζ *respectively*.

The effectiveness of an edge (a,b) is defined by $I_{(a,b)} = \dfrac{\mu(a,b)}{\min\{\sigma(x), \sigma(y)\}}$.

Samanta et al. [10] defined fuzzy colours as the mixing of colours as follows

Definition 2 *Let's assume* $W = \{w_1, w_2, \ldots, w_\lambda\}, \lambda \ge 1$ *is the set of basic colours. Then the fuzzy set* (W, h)*where* $h: W \to (0,1]$ *may be called the set of fuzzy colours such that* $0 < h(w_i) \le 1$, *the membership value of the colour* w_i, *is the amount of* w_i *per unit of the mixture of* w_i *with white colour*.

Definition 3 $\xi = (V, \sigma, \mu, \vec{E})$ *is said to be a directed fuzzy graph, where* V *is a non-empty set, and* $\sigma: V \to [0,1]$ *is the membership function for vertices and* $\mu: \vec{E} \to [0,1]$ *is the membership function for edges,* $\vec{E}$ *is the set of all directed edges, such that* $\mu(a,b) \le \sigma(a) \wedge \sigma(b)$, *where* $\sigma(a)$ *denotes the vertex membership value of vertex* a *and* $\mu(a,b)$ *denotes the edge membership value of the edge* (a,b) *in* ξ.

2.1 Some Basic Notations

Some basic notations are shown in Tab. 1.

Table 1: Some basic notations

Notation	Meaning
G	Crisp graph
ζ	Fuzzy graph
ξ	Directed fuzzy graph
V	Set of vertex
E	Set of edge
$\sigma(x)$	Membership value of the vertex x
$\mu(x,y)$	Membership value of the edge (x,y)
E_1	Set of all undirected edges of ξ
$\vec{E}$	Set of all directed edges of ξ
$\vec{\phi}(a,b)$	Measure of influence from the vertex a to b
$f(r_i)$	Depth of the colour r_i
$\mathrm{I}(a,b)$	Strength of an edge (a,b)
(K,N)	Chromatic number of the graph ξ
(K_s,N_s)	Strong chromatic number of the graph ξ

3 Measure of Influence of a Directed Fuzzy Graph

Sometimes two nodes of a network are connected by an edge, and their relationship may not be useful, but the amount of influence is significant. For example, the relationship between a student and a teacher may not always be good. Still, the student is influenced by the teacher. This concept is used here to measure the amount of influence. Now, we are introducing the term "the measure of influence" which can be used to measure the amount of influence between two vertices of a directed fuzzy graph. It can be formally defined In the following way:

Definition 4 *Let's assume V is a non-empty set. $\xi = (V,\sigma,\mu,\vec{E})$ is a fuzzy directed graph. Then the measure of influence between two vertices a, b is denoted by $\vec{\phi}(a,b)$ and is defined by $\vec{\phi}(a,b) \le |\sigma(a) - \sigma(b)|$.*

Example 1 *Fig. 1, shows a directed fuzzy graph and $V = \{a,b,c,d,e,f\}$ is the vertex set. Now, the vertices membership values are considered in Tab. 2, and the edge membership values are considered in Tab. 3. The amount of influence between two vertices is $\vec{\phi}(a,b) \le 0.6$, $\vec{\phi}(d,b) \le 0.6$, $\vec{\phi}(b,e) \le 0.3$, $\vec{\phi}(e,f) \le 0.2$, $\vec{\phi}(c,b) \le 0.1$, $\vec{\phi}(a,e) \le 0.3$, $\vec{\phi}(c,d) \le 0.5$, $\vec{\phi}(d,e) \le 0.3$.*

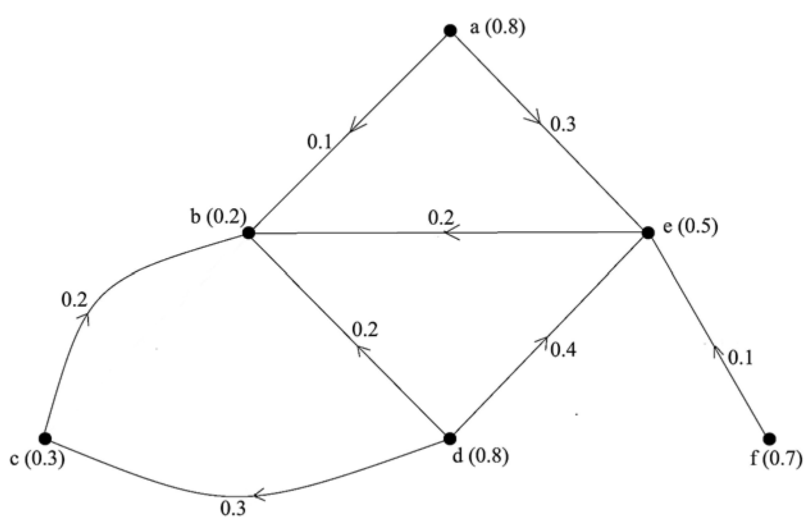

Figure 1: Fuzzy directed graph

Table 2: Vertex membership value of Fig. 1

Vertex	Membership value	Vertex	Membership value
a	0.8	b	0.2
c	0.3	d	0.8
e	0.5	f	0.7

Table 3: Edges membership value of Fig. 1

Edge	Membership value	Edge	Membership value
(a,b)	0.1	(a,e)	0.3
(b,c)	0.2	(b,e)	0.2
(b,d)	0.2	(c,d)	0.3
(d,e)	0.4	(e,f)	0.1

3.1 Complete Directed Fuzzy Graph

Let's assume $\xi = (V, \sigma, \mu, \vec{E})$ is a directed fuzzy graph. ξ can be called a complete directed fuzzy graph if the crisp underlying graph of ξ is a complete graph and $\mu(a,b) = \sigma(a) \wedge \sigma(b)$.

Fig. 2 shows a complete directed fuzzy graph.

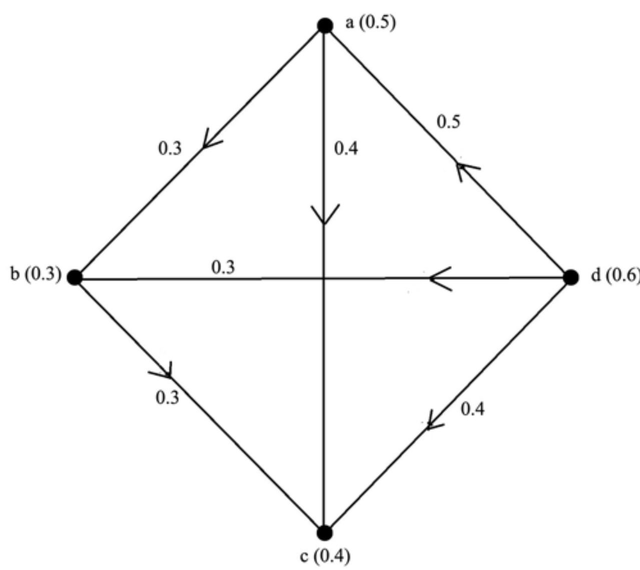

Figure 2: Complete directed fuzzy graph

Lemma 1 *Let's assume* $\xi = (V, \sigma, \mu, \vec{E})$ *is a directed fuzzy graph. Then* $0 \leq \vec{\phi}(a,b) < 1$, $\vec{\phi}(a,b)$ *is the measure of influence between the vertices* a *and* b.

Proof. ξ is a directed fuzzy graph. So, $\mu(a,b) \leq \sigma(a) \wedge \sigma(b)$, $\mu(a,b)$ is the edge membership value of the edge (a,b) and $\sigma(a)$ is the vertex membership value of the vertex a. The membership value of a vertex is $0 < \sigma(a) \le 1$. Also, $\vec{\phi}(a,b) \le |\sigma(a) - \sigma(b)|$ denotes the measure of influence between two nodes a and b. So, if $\sigma(a) = \sigma(b)$ then $\vec{\phi}(a,b)$ gets the minimum value, and the value is 0. And the maximum value of $\vec{\phi}(a,b)$ is $\vec{\phi}(a,b) < 1$ because of $0 < \sigma(a) \le 1$. So, $0 \le \vec{\phi}(a,b) < 1$ is true.

Theorem 1 *Let's assume* $\xi = (V, \sigma, \mu, \vec{E})$ *is a complete directed fuzzy graph. Then* $\vec{\phi}(a,b) + \mu(a,b) \ge \sigma(a) \vee \sigma(b)$.

Proof. ξ is a complete directed fuzzy graph. So, $\mu(a,b) = \sigma(a) \wedge \sigma(b)$, $\mu(a,b)$ is the edge membership value of the edge (a,b) and $\sigma(a)$ is the vertex membership value of the vertex a in ξ. Also, $\vec{\phi}(a,b) \le |\sigma(a) - \sigma(b)|$ denotes the measure of influence between two nodes a and b. When, $\sigma(a) > \sigma(b)$, then the value of $\vec{\phi}(a,b) \le \sigma(a) - \sigma(b)$. Then, $\vec{\phi}(a,b) + \mu(a,b) \ge \sigma(a) - \sigma(b) + \sigma(b)$. So, $\vec{\phi}(a,b) + \mu(a,b) \ge \sigma(a)$. So, $\vec{\phi}(a,b) + \mu(a,b) \ge \sigma(a) \vee \sigma(b)$. Similarly, we can easily show that if $\sigma(a) < \sigma(b)$, then $\vec{\phi}(a,b) + \mu(a,b) \ge \sigma(a) \vee \sigma(b)$.

3.2 Colouring of Directed Fuzzy Graph

In crisp graph colouring, two adjacent vertices have to have two different colours. Same colours in two adjacent vertices will create problems. Here, we use fuzzy colour to mark the directed fuzzy graph. Suppose, red, blue, green etc. are considered the basic colours, and $(red, 0.6)$ is the fuzzy colour, where red is the basic colour, and 0.6 is the depth of the colour. Here, we use fuzzy colour to mark the directed fuzzy graph as follows-

Let's assume $\xi = (V, \sigma, \mu, \vec{E})$ is a directed fuzzy graph and $R = (r_1, r_2, \ldots, r_k)$ is a set of basic colours. Now, two vertices will be taken different basic colour if they are connected by a directed edge whose effectiveness is ≥ 0.5 or amount of influence is ≥ 0.5, i.e., two vertices "a" and "b" take different basic colour if the directed edge connects them with $I_{(a,b)} \ge 0.5$ or $\vec{\phi}(a,b) \ge 0.5$, otherwise the basic colour may be the same. Suppose, a vertex "a" gets a colour $(r_i, f(r_i))$, where r_i are the basic colour and the $f(r_i)$ is the depth of the basic colour. For simplicity, let's consider, $f(r_i) = \sigma(a)$, where $\sigma(a)$ is the vertex membership value of the vertex a in ξ. If a is connected to another vertex b, with an edge either effectiveness greater than 0.5 or influence greater than 0.5, b will get different basic colour $(r_j, f(r_j))$ with $r_i \ne r_j$ *and* $f\left(r_j\right) = \sigma(b)$.

3.3 Chromatic Index of Directed Fuzzy Graph

The minimum number of basic colours needed to colour a directed fuzzy graph is called the chromatic index of a directed fuzzy graph.

Suppose, such minimum number of basic colours is K. Now, this crisp chromatic index is not sufficient to mention the vertex's nature or weight. Hence, the chromatic index is associated with some weight. So, the chromatic index of a directed fuzzy graph is denoted by (K, N), where K is the number of the basic colours used to colour the directed fuzzy graph, and N is the weight. The weight N is defined by

$$N = \sum_{i=1}^{K} \{max_j f_{v_j}(r_i)\}$$

3.4 Algorithm to Colour the Directed Fuzzy Graph

Input: A directed fuzzy graph $\xi = (V, \sigma, \mu, \vec{\mathrm{E}})$, $|V| = n$.

Output: Complete coloured, directed fuzzy graph.

Step 1: All the vertices are labelled as $1, 2, \ldots, n$.

Step 2: Vertex "1" is assigned a colour $(r_i, f(r_i))$, where r_i are the basic colour and the $f(r_i)$ is the depth of the basic colour. Then, $f(r_i) = \sigma(1)$, where $\sigma(1)$ is the vertex membership value of the vertex "1" in ξ.

Step 3: All the directly connected vertices of "1" are labelled as 11, 12, ..., 1m, where m number of vertices are directly connected with the vertex "1".

Step 4: The formula does the calculation of the strength of all directly connected edges with "1" $I_{(1,1i)} = \frac{\mu(1,1i)}{\min\{\sigma(1),\sigma(1i)\}}$, where $i = 1, 2, \ldots, m$. Also, the measure of influence $\vec{\phi}(1, 1i)$ of all directly connected vertices with "1" is considered by the formula $\vec{\phi}(1, 1i) \leq |\sigma(1) - \sigma(1i)|$, where $i = 1, 2, \ldots, m$.

Step 5: Basic colours, different from what is given to "1", are alloted to all the adjacent vertices of "1" and whose $I_{(1,1i)} \geq 0.5$ or $\vec{\phi}(1, 1i) \geq 0.5$. The depth of the colour given is the same as their vertex membership value. The rest of the adjacent vertices of "1", whose $I_{(1,1i)} < 0.5$ or $\vec{\phi}(1, 1i) < 0.5$, are assigned the equal basic colour as alloted to vertex 1. The target is to use a minimum number of basic colours to mark this directed fuzzy graph.

Step 6: Steps 3–5 are repeated until all of this directed graph's vertices have been coloured.

Step 7: Calculation is done for the chromatic index of this directed fuzzy graph (K, N) where K is the number of the basic colours used to mark the directed fuzzy graph, and N is the weight. N is calculated by the formula $N = \sum_{i=1}^{K} \{max_j f_{v_j}(r_i)\}$.

Example 2 *In Fig. 1 a directed fuzzy graph has been considered, and the vertices membership values are considered in Tab. 2, and the edges membership values are considered in Tab. 3. Also, the amount of influences is considered in Tab. 4. Also, the strength of all edges of this graph is shown in Tab. 5. Now, the coloured directed fuzzy graph of Fig. 1 has been shown in Fig. 3. Here, three basic colours are used to colour this directed fuzzy graph. Also, the strength of all edges is greater than or equal to 0.5 except the edge* (e,f) *and the influence between the vertex* e *and* f *is* 0.1. *So, the vertex* e *and* f *can take the same basic colour. Three basic colours red, green and blue are used for colouring this directed fuzzy graph, i.e.,* $K = 3$. *The depth of colour is the vertex membership value of the corresponding vertex. Here, the red colour is given in the vertices* b *with membership values* 0.2. *The green colour is assigned to the vertices* c, e, f *with membership value* $0.3, 0.5, 0.7$ *respectively. The blue colour is assigned to the vertices* a, d *with membership value* $0.8, 0.8$ *respectively. So, weight* $N = 0.2 + 0.8 + 0.7 = 1.7$. *Thus, the chromatic index of this directed fuzzy graph is* $(3, 1.7)$. *The coloured directed fuzzy graph of Fig. 1 has been shown in Fig. 3.*

Table 4: Amount of influence between two vertices of Fig. 1

Influence between two vertices	Amount of influence	Influence between two vertices	Amount of influence
$\vec{\phi}(a,b)$	0.5	$\vec{\phi}(a,e)$	0.3
$\vec{\phi}(b,c)$	0.1	$\vec{\phi}(b,e)$	0.2
$\vec{\phi}(b,d)$	0.55	$\vec{\phi}(c,d)$	0.2
$\vec{\phi}(d,e)$	0.2	$\vec{\phi}(e,f)$	0.1

Table 5: Strength of all edges of Fig. 1

Edge	Strength of all edges	Edge	Strength of all edges
(a, b)	0.5	(a, e)	0.6
(b, c)	1.0	(b, e)	1.0
(b, d)	1.0	(c, d)	1.0
(d, e)	0.8	(e, f)	0.2

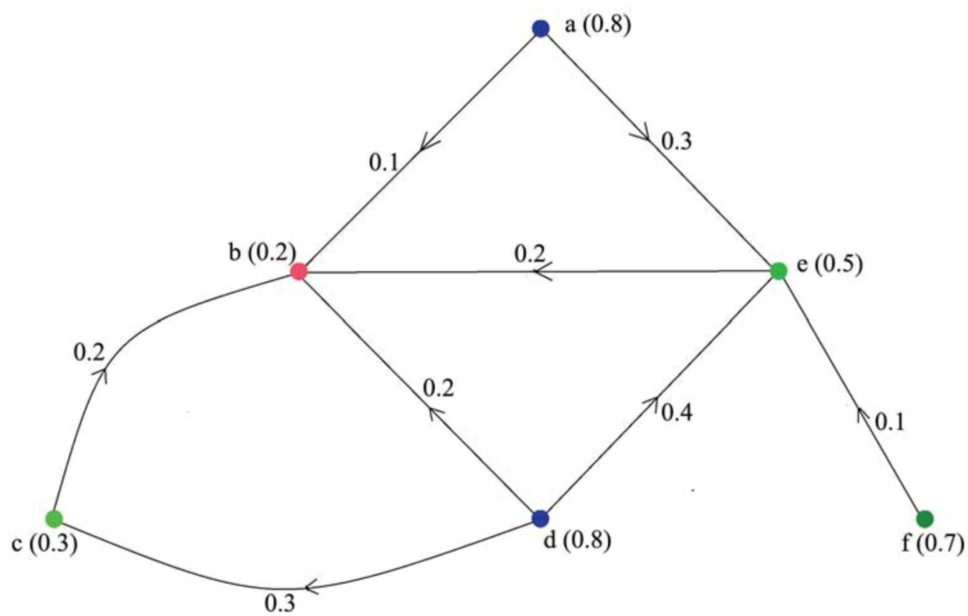

Figure 3: Colouring of directed fuzzy graph

Lemma 3 *If the chromatic index of a directed fuzzy graph is* (K, N), *then* $0 < N \leq K$.

Proof. Let's assume $\xi = (V, \sigma, \mu, \vec{E})$ is a directed fuzzy graph with chromatic index (K, N). It is obvious that the value of weight is always positive, i.e., the value of N is always positive, So, $N > 0$. Also, the membership value of each basic colour is less than or equal to 1. Now, N is the sum of the maximum membership values of each K basic colours. Hence, $N \leq K$, i.e., $0 < N \leq K$.

3.5 Strong Chromatic Index of Directed Fuzzy Graph

Let's assume $\xi = (V, \sigma, \mu, \vec{E})$ is a directed fuzzy graph, and strong chromatic index is denoted by (K_s, N_s), where K_s is the number of basic colours used for colouring ξ with each of depth greater or equal to 0.5 and N_s is the sum of maximum depth of each basic colour whose depth is greater than or equal to 0.5.

Example 3 *A colouring in the directed fuzzy graph has been shown in Fig. 3. Here, the red colour is given in the vertices* b *with membership values* 0.2. *The green colour is assigned to the vertices* c, e, f *with membership value* $0.3, 0.5, 0.7$ *respectively. The blue colour is given to the vertices* a, d *with membership* *value* $0.8, 0.8$ *respectively. So, the value of* K_s *is* 2 *and the weight* $Ns = 0.8 + 0.7 = 1.5$. *Thus, the strong chromatic index of this directed fuzzy graph is* $(2, 1.5)$.

Theorem 3 *Let's assume* ξ *is a directed fuzzy graph and* (K, N), (K_s, N_s) *are the chromatic index and strong chromatic index of this directed fuzzy graph respectively. Then* $K \geq K_s$ *and* $N \geq N_s$.

Proof. Let's assume $\xi = (V, \sigma, \mu, \vec{E})$ be a directed fuzzy graph. If all basic colours are strong, then chromatic index and strong chromatic index of this directed fuzzy graph are the same, i.e., $K = K_s$ and $N = N_s$.

Again, if some of the basic colours are strong, then $K > K_s$. Then N is the sum of the maximum membership value of each basic colour and N_s is the sum of all the maximum membership values of all strong basic colours. So, in this case, $N > N_s$.

Lastly, none of the basic colours is strong. So, it is obvious that $K_s = 0$ and $K > K_s$. Also, $N_s = 0$ and $N > N_s$. So from these three cases, it is concluded that $K \geq K_s$ and $N \geq N_s$.

Lemma 5 *If* ξ *is a directed fuzzy graph and* (K_s, N_s) *is a strong chromatic index of* ξ, *then* $2N_s - K_s \geq 0$.

Proof. Let's assume $\xi = (V, \sigma, \mu, \vec{E})$ is a directed fuzzy graph and (K_s, N_s) is the strong chromatic index. So ξ is coloured by K_s the number of strong basic colours and each strong basic colour's membership value is greater than or equal to 0.5. N_s is the sum of the maximum membership value of each basic colour. So, $N_s \geq 0.5 \times K_s$. Hence, $2N_s - K_s \geq 0$.

Theorem 4 *Let's assume* ξ *is a directed fuzzy graph and* (K_s, N_s) *is the strong chromatic index. Then* $\frac{K_s}{2} \leq N_s \leq K_s$ *is true.*

Proof. Let's assume $\xi = (V, \sigma, \mu, \vec{E})$ is a directed fuzzy graph and (K_s, N_s) is the strong chromatic index of this directed fuzzy graph ξ. So, the directed fuzzy graph ξ is colored by K_s number of strong basic colours. So, the membership value of all the strong colours is greater than and equal to 0.5. So, minimum value of each basic colour is 0.5, then $N_s \geq \{0.5 + 0.5 + \ldots K_s \ times\} = \frac{K_s}{2}$.

So, the minimum value of strong weight is $\frac{K_s}{2}$, i.e., $\frac{K_s}{2} \leq N_s$. Also, the maximum depth of each basic colour is 1. So, $N_s \leq \{1 + 1 + 1 + \ldots K_s \ times\} = K_s$. Thus, $N_s \leq K_s$. So, $\frac{K_s}{2} \leq N_s \leq N_s$ is true.

Lemma 6 *If* ξ *is a directed fuzzy graph and* (K, N), (K_s, N_s) *are the chromatic index and strong chromatic index of this directed fuzzy graph. If* $K - K_s \neq 0$ *then* $\frac{N - N_s}{K - K_s} \leq 0.5$ *is true.*

Proof. Let's assume $\xi = (V, \sigma, \mu, \vec{E})$ is a directed fuzzy graph and $K - K_s \neq 0$. $(K - K_s)$ is the number of basic colours whose membership value is less than 0.5. And $(N - N_s)$ is the weight of this $(K - K_s)$ number of colours. Thus, $N - N_s$ = sum of maximum membership values of each basic colour. So, $N - N_s \leq 0.5 \times (K - K_s)$.

Hence, $\frac{N - N_s}{K - K_s} \leq \frac{1}{2}$.

4 Application of Colouring of Directed Fuzzy Graph

Graph colouring and fuzzy graphs have many real-life applications. Today, COVID-19 is a major threat to human lives. So people are conscious of tracking the places where the majority of the COVID-19 infected patients have been found. Many websites are providing live updates about COVID-19. These websites use the tool of the colouring of maps. But they are not maintaining graph colouring techniques. In this section, to capture uncertainty and direction of links of COVID19, the colouring of the directed fuzzy graphs is used to colour such infected regions.

This virus was first detected in Wuhan, China, in December 2019. Since then, it has been spreading globally. Within April 2020, almost all countries were affected by this virus. The transmission is happening due to the inter-links among countries. Here, we have considered a graph of top 10 COVID-19 affected countries. Data are taken from the website (https://www.worldometers.info/coronavirus/) dated 4th April. These countries are assumed as vertices of a directed fuzzy graph. Also, there exists an edge if any two (vertices) countries have been affected by the COVID-19.

In Tab. 6 and Fig. 4, the top ten affected countries and the number of affected patients by COVID-19 have been shown. Here, all countries are considered nodes of the concerned network. There is a directed edge between two countries (vertex) if there is any travel history between them because they are affected by another. So, we get a directed graph and Fig. 5 has shown this directed graph.

Table 6: Top ten affected counties by COVID-19 and no. of cases

SI No.	Country name	Total cases	Normalized score	Vertex Me tubers hip value
1	USA	258,409	1.00	1.00
2	Italy	119,827	0.46	0.46
3	Spain	117,710	0.46	0.46
4	Germany	89,451	0.35	0.35
5	China	81,620	0.32	0.32
6	Franco	59,105	0.23	0.23
7	Iran	53,183	0.21	0.21
8	UK	38,168	0.15	0.15
9	Switzerland	19,303	0.07	0.07
10	Turkey	18,135	0.07	0.07

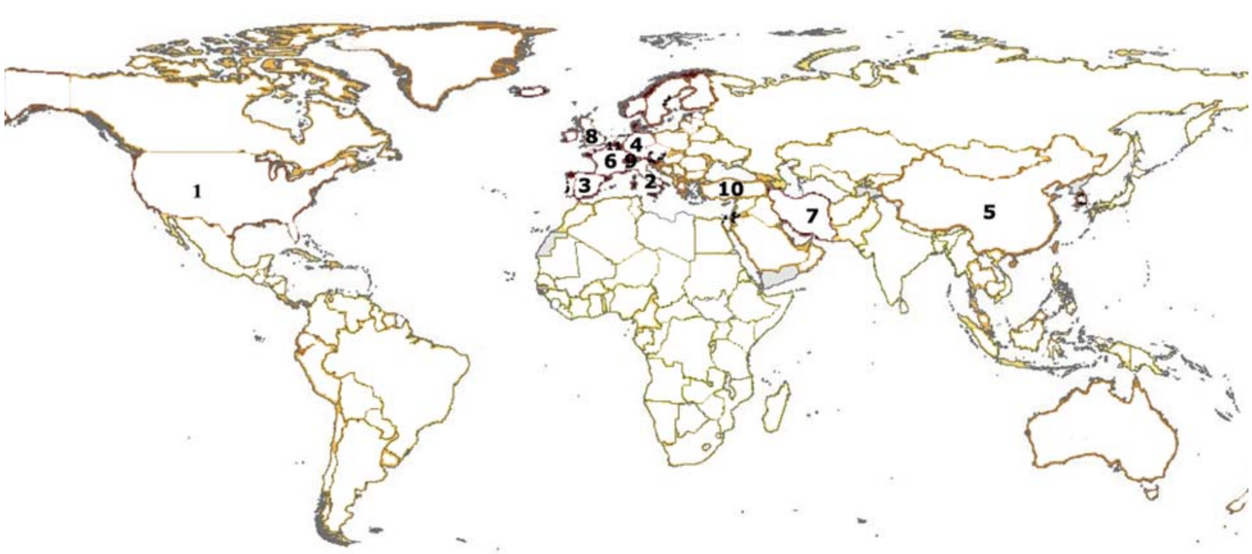

Figure 4: Top ten affected counties in a world map

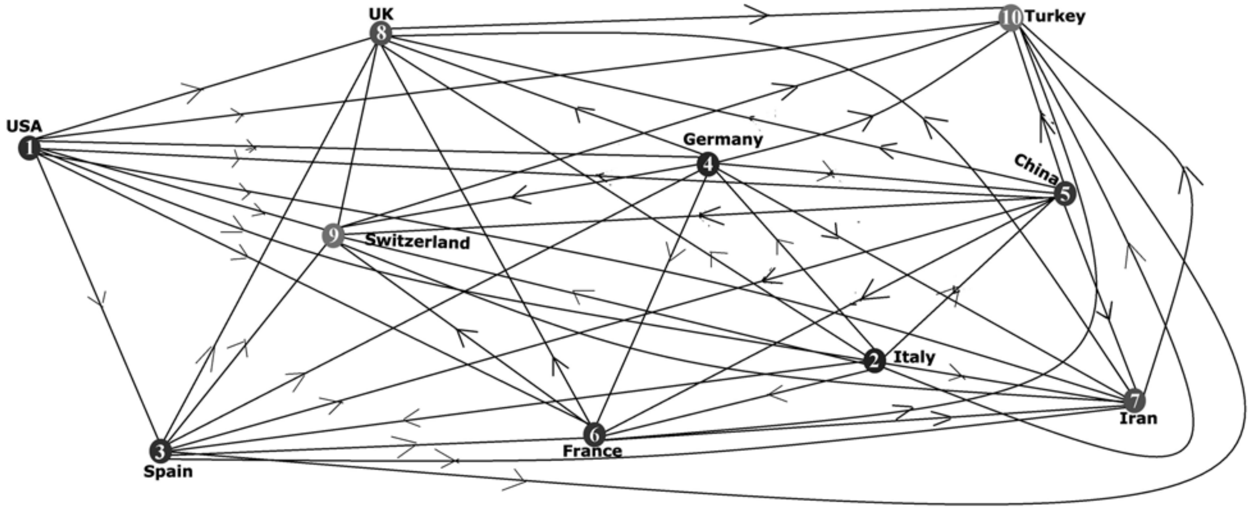

Figure 5: Directed graph of top ten affected counties by COVID-19

The normalized score is considered as a vertex membership value, and Tab. 6 has shown it. Tab. 7 has shown the travel history between the two countries last year. Here, the edge membership value of an edge = "normalized score of travel history between two counties × minimum vertex membership value of the end vertices". Tab. 9 has shown the edge membership value of all edges. Fig. 6 has revealed this directed fuzzy graph. Now, the measure of influence between two vertices is considered a difference between two vertices membership value for this particular case. So, the influence between vertices is calculated by this formula $\vec{\phi}(a,b) = \sigma(a) - \sigma(b)$ and this calculation are shown in Tab. 8, and the strength of all edges is shown in Tab. 10.

Table 7: Travel history between the two countries last year

	USA	Italy	Spain	Germany	China	France	Iran	UK	Switzerland	Turkey
USA	∓	5.656,740	1,151,956	2,062,462	2,991 813	1,767,461	350,000	4,930,013	5987000	5,7110.740
Italy		–	2.175,267	12,184,502	3,200,847	4,737,464	555,223	3,781,882	2,925,32 1	4,025,695
Spain			–	1,362,431	374,755	11,258,540	295,656	I7,675,367	1.703,481	297,625
Germany					I.362,4319	1,725.854	256,565	2,551,061	3,115,456	5,027,472
China					–	1,661,000	356,664	2,656,566	1,142,4111	397465
France						–	456,856	1,564,566	6,200,000	875,957
Iran							–		514502,1	2,1012,890
UK								–	1,667,437	2,562,064
Switzerland										1452323
Turkey										

Now, two vertices are assigned different basic colours if they are connected by a directed edge whose effectiveness is ≥ 0.5, or the amount of influence is ≥ 0.5. So, four basic colours are used to colour this directed graph. The coloured directed fuzzy graph is shown in Fig. 7, with the same depth as of vertex membership values. The colouring of the world map of affected countries is shown in Fig. 8, with the same depth in relation to their number of affected people.

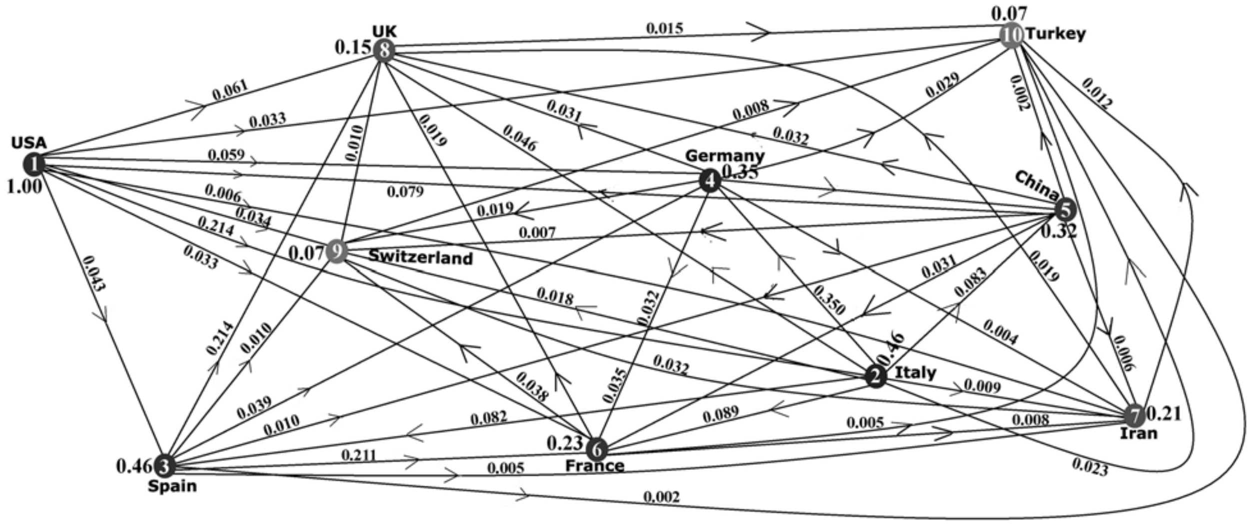

Figure 6: Directed fuzzy graph

Table 8: Measure of influence between the two vertices

Vertices	Influence	Vertices	Influence
USA-Italy	0.54	Spain-Switzerland	0.39
USA-Spain	0.54	Spain-Turkey	0.39
USA-Germany	0.65	Germany-China	0.03
USA-China	0.68	Germany-France	0.12
USA-France	0.77	Germany-Iran	0.14
USA-Iran	0.79	Germany-UK	0.2
USA-UK	0.85	Germany-Switzerland	0.28
USA-Switzerland	0.93	Germany-Turkey	0.28
USA-Turkey	0.93	China-France	0.09
Italy-Spain	0	China-Iran	0.11
Italy-Germany	0.11	China-UK	0.17
Italy-China	0.14	China-Switzerland	0.25
Italy-France	0.23	China-Turkey	0.25
Italy-Iran	0.25	France-Iran	0.02
Italy-UK	0.31	France-UK	0.08
Italy-Switzerland	0.39	France-Switzerland	0.16
Italy-Turkey	0.39	France-Turkey	0.16
Spain-Germany	0.11	Iran-UK	0.06
Spain-China	0.14	Iran-Switzerland	0.14
Spain-France	0.23	Iran-Turkey	0.14
Spain-Iran	0.25	UK-Switzerland	0.08
Spain-UK	0.31	UK-Turkey	0.08
		Switzerland-Turkey	0

4.1 Analysis of the Result

Fig. 8 shows that four basic colours have been used. In this colouring technique, two nodes are given different colours, if there exists a directed edge between them whose effectiveness is ≥ 0.5, or the amount of influence is ≥ 0.5. The depths of colours among the countries are given based on its number of affected cases.

Table 9: Edge membership value of the directed fuzzy graph of Fig. 6

Edge	Number of tourists visiting	Normalized score(S)	Edge membership value (S X minimum vertex membership value nf the end vertices)
US A-Italy	5,656,740	0.46	0.214
USA-Spain	1,151,956	0.09	0.043
USA-Germany	2,062,462	0.17	0.059
USA-China	2,991.813	0.25	0.079
USA-Frame	1,767.461	0.15	0.033
USA-Iran	350,000	0.03	0.006
USA-UK	4,930,013	0.40	0.061
USA-Switzerland	5987000	0.49	0.034
USA-Turkey	5,780,740	0.47	0.033
Italy-Spain	2,175,267	0.18	0.082
Italy-Germany	12.184,502	1.00	0.350
Italy-China	3,200,847	0.26	0.083
Italy-France	4,737,464	0.39	0.089
Italy-Iran	555,223	0.05	0.009
Italy-UK	3,781,882	0.31	0.046
Italy-Switzerland	2,925,321	0.24	0.018
Italy-Turkey	4,025,695	0.33	0.023
Spain-Germany	1,362,431	0.11	0.039
Spain-China	374,755	0.03	0.010
Spain-Franee	11.258,540	0.92	0.211
Spain-Iran	295,656	0.02	0.005
Spain-UK	17,675,367	1.45	0.214
Spain-Switzerland	1.703,481	0.14	0.010
Spain-Turkey	297,625	0.02	0.002
Germany-China	1,363,979	0.11	0.035
Germany-France	1,725,854	0,14	0.032
Germany-Iran	256,565	0.02	0.004
Germany-UK	2,551,061	0.21	0.031
Germany-Switzerland	3,115,456	0,26	0.019
German y-Turkey	5,027,472	0,41	0.029
China-France	1,661,000	0.14	0.031
China-] ran	356,664	0.03	0.006
China-UK	2,656,566	0.22	0.032
China-Switzerland	1,142.438	0.09	0.007
China-Turkcy	397465	0.03	0.002
France-Iran	456,856	0.04	0.008
France-UK	1,564,566	0.13	0.019
France-Switzerland	6,200,000	0.51	0.038
France-Turkey	875,957	0.07	0.005
Iran-UK	1.584.665	0.13	0.019
Iran-Switzerland	5145023	0.42	0.032
Iran-Turkey	2,102,890	0.17	0.012
UK-Switzerland	1,667,437	0.14	0.010
UK-Turkey	2,562,064	0.21	0.015
Switzerland-Turkey	1452323	0.12	0.008

Table 10: Strength of all edges of the directed graph of Fig. 5

Edge	Edge membership value	Strength of edges	Edge	Edge membership value	Strength of edges
USA-Italy	0.147	0.32	Spain-Switzerland	0.007	0.10
USA-Spain	0.030	0.07	Spain-Turkey	0.001	0.02
USA-Germany	0.041	0.12	Germany-China	0.024	0.08
USA-China	0.054	0.17	Germany-France	0.022	0.10
USA-France	0.023	0.10	Germany-Iran	0.003	0.01
USA-Iran	0.004	0.02	Germany-UK	0.021	0.14
USA-UK	0.042	0.28	Germany-Switzerland	0.013	0.19
USA-Switzerland	0.024	0.34	Germany-Turkey	0.020	0.29
USA-Turkey	0.023	0.33	China-France	0.021	0.09
Italy-Spain	0.057	0.12	China-Iran	0.004	0.02
Italy-Germany	0.241	0.69	China-UK	0.022	0.15
Italy-China	0.057	0.18	China-Switzerland	0.005	0.07
Italy-France	0.061	0.27	China-Turkey	0.002	0.02
Italy-Iran	0.006	0.03	France-Iran	0.005	0.03
Italy-UK	0.032	0.21	France-UK	0.013	0.09
Italy-Switzerland	0.012	0.18	France-Switzerland	0.026	0.37
Italy-Turkey	0.016	0.23	France-Turkey	0.003	0.05
Spain-Germany	0.027	0.08	Iran-UK	0.013	0.09
Spain-China	0.007	0.02	Iran-Switzerland	0.022	0.31
Spain-France	0.146	0.63	Iran-Turkey	0.008	0.12
Spain-Iran	0.003	0.02	UK-Switzerland	0.007	0.10
Spain-UK	0.148	0.98	UK-Turkey	0.010	0.15
			Switzerland-Turkey	0.006	0.08

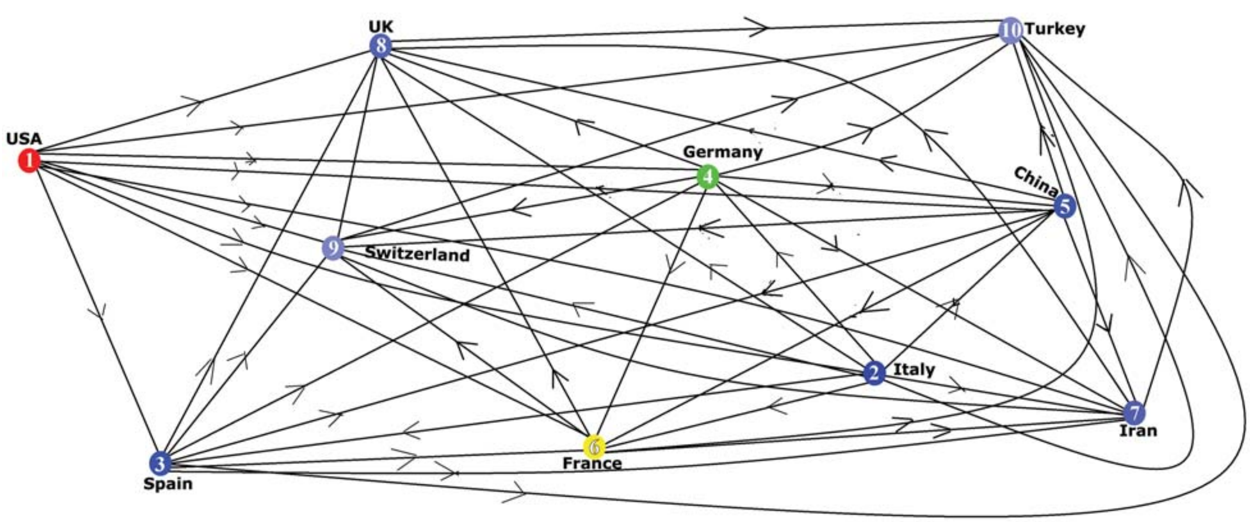

Figure 7: Colouring of the directed fuzzy graph

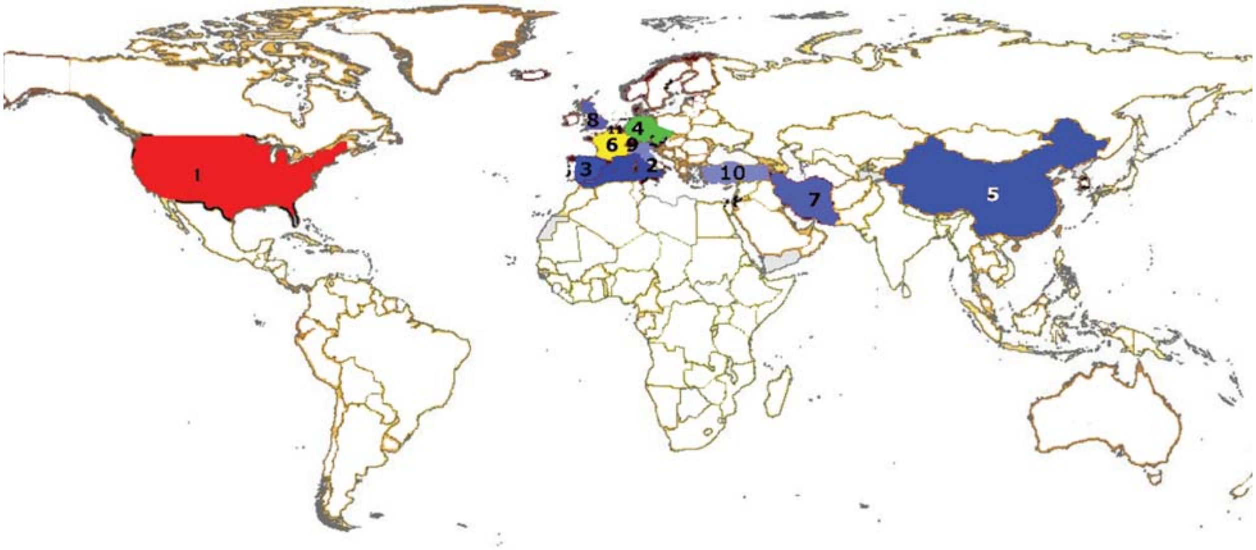

Figure 8: World map colouring using the fuzzy directed fuzzy graph colouring

5 Conclusion

In this paper, a new technic of a fuzzy graph colouring based on edges' influence value has been introduced. Moreover, the term, chromatic index and strong chromatic index are defined differently. By this directed graph colouring we have represented this COVID-19 outbreak all over the world. There is one limitation of this study too. The data relating to the travellers who have been wandering across the borders has not been fully detailed.This can be made possible if the data is collected on a larger basis. Due to the limitation of the fund, the collection is not feasible for this study, but capturing the exact figure will be done in the future. Besides, the literature on directed fuzzy graphs will be explored on the basics of this article.

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Conflicts of Interest: The authors unanimously declare that they have no interest in reporting the present study.

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