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Article

A Novel Multiple Dependent State Sampling Plan Based on Time Truncated Life Tests Using Mean Lifetime

Pramote Charongrattanasakul[1], Wimonmas Bamrungsetthapong[2,*] and Poom Kumam[3]

[1]Division of Mathematics, Department of Mathematics and Computer Science, Faculty of Science and Technology, Rajamangala University of Technology Krungthep, Bangkok, 10120, Thailand
[2]Division of Applied Statistics, Department of Mathematics and Computer Science, Faculty of Science and Technology, Rajamangala University of Technology Thanyaburi, Pathum Thani, 12110, Thailand
[3]Center of Excellence in Theoretical and Computational Science (TaCS-CoE) & KMUTT Fixed Point Research Laboratory, Room SCL 802 Fixed Point Laboratory, Science Laboratory Building, Departments of Mathematics, Faculty of Science, King Mongkut's University of Technology Thonburi (KMUTT), Bangkok, 10140, Thailand
*Corresponding Author: Wimonmas Bamrungsetthapong. Email: wimonmas_b@rmutt.ac.th
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Abstract: The design of a new adaptive version of the multiple dependent state (AMDS) sampling plan is presented based on the time truncated life test under the Weibull distribution. We achieved the proposed sampling plan by applying the concept of the double sampling plan and existing multiple dependent state sampling plans. A warning sign for acceptance number was proposed to increase the probability of current lot acceptance. The optimal plan parameters were determined simultaneously with nonlinear optimization problems under the producer's risk and consumer's risk. A simulation study was presented to support the proposed sampling plan. A comparison between the proposed and existing sampling plans, namely multiple dependent state (MDS) sampling plans and a modified multiple dependent state (MMDS) sampling plan, was considered under the average sampling number and operating characteristic curve values. In addition, the use of two real datasets demonstrated the practicality and usefulness of the proposed sampling plan. The results indicated that the proposed plan is more flexible and efficient in terms of the average sample number compared to the existing MDS and MMDS sampling plans.

Keywords: Adaptive version of multiple dependent state sampling plan; time truncated life test; quality level; weibull distribution; mean lifetime

1 Introduction

An essential tool in product control techniques is the Acceptance Sampling Plan (ASP). This technique helps consumers decide to accept or reject a product by sampling items from the lot. At the same time, the manufacturer can select a minimum sample size from the ASP to provide the acceptance or rejection criteria for that lot. In the past, a single sampling plan (SSP) was the most commonly used for various industries, wherein the decision to accept or reject the lot sentencing depended on a single sampling. Although SSP is easy to use, it often results in a more average sampling number (ASN) than other sampling plans. In practice, if the producer and consumer want to make a clear and precise decision concerning acceptance or rejection of a lot, some situations cannot be decided with a single sample. They should use other sampling plans, such as the double sampling plan [1]. Additionally, manufacturers can use more effective sampling plans, such as MDS sampling plans, to reduce sample size and inspection costs as well as make decisions based on current and previous sample data. Baker et al. [2] presented an MDS sampling plan for a continuous manufacturing process and sent the lot to a serial inspection. The MDS reduces the sample size because accepting or rejecting the current lot depends on current and previous lots. Many researchers have presented the MDS sampling plans in various situations. Govindaraju et al. [3] proposed the development of MDS sampling plans to minimize the sum of producer's and consumer's risks under the specified acceptable quality level and limiting quality level. Balamurali et al. [4] studied the MDS based on a variable sampling plan under Normal distribution. Balamurali et al. [5] used the Bayesian methodology to investigate the MDS. Some studies [6–10] used the concept of MDS sampling plan in control chart design. Some researchers studied MDS sampling plans for coronavirus disease (COVID-19). Srinivasa et al. [11] applied the MDS sampling plan to COVID-19 outbreak data in China when the lifetime is Exponentiated Weibull Distribution. Aslam et al. [12] developed the MDS sampling plan based on a time truncated sampling plan for the COVID-19 data using gamma distribution under indeterminacy.

Nowadays, many products are highly reliable, such as electronic products. However, it may not be possible to test every item to ensure the mean lifetime of the product. Therefore, using an ASP can be very helpful in deciding lots before exporting to consumers. The item samples are selected and tested for the specified time in the life test. It can then be decided to accept or reject the lot using the number of failures and sample size entries allowed for the test. Many researchers have proposed various lifetime distributions for ASP designs to ensure product lifetime. For instance, Jun et al. [13] studied SSP and DSP plans by considering variable data under the Weibull distribution. They used sudden mortality testing to reduce inspection time and ASN. Rao [14] studied the group ASP to reduce the inspection time under the Weibull distribution and generalized exponential distribution. Lio et al. [15] suggested that the SSP provides a larger sample size to decide whether to accept or reject a lot. They found that larger sample sizes meant more time and expense spent testing life. Rao et al. [16] presented an MDS sampling plan wherein the life of products is truncated under exponentiated half logistic distribution. Balamurali et al. [17] studied the median life of products for the MDS sampling plan under generalized inverted exponential distribution. Nadi et al. [18] suggested a group MDS sampling plan following the Weibull distribution. They found that the actual mean lifetime was longer than the specified mean lifetime of products. Aslam et al. [19] studied the mean lifetime of products for a generalized MDS sampling plan under gamma distribution, Burr type XII distribution, and Birnbaum-Saunders distribution. MDS sampling plans generally reject the current lot immediately if the previous lot is of medium quality. However, the rest of the lot is of good quality. Aslam et al. [20] created an adapted version of the MDS Sampling Plan (MMDS), which they claimed was more flexible and efficient than the existing MDS plan as measured by sample size and inspection cost under a time truncated life. Al-Omari et al. [21] constructed the new ASP for Length-Biased Weighted Lomax distribution based on a truncated life test. Tripathi et al. [22] presented an attribute modified chain sampling inspection plan based on the time truncated life test under the Darna distribution. Abushal et al. [23] presented the acceptance sampling plan under the power inverted Topp–Leone distribution. The median life of products is used in the truncated life test.

From exploring the literature, we found that the existing MDS sampling plan and MMDS sampling plan are used to decide whether to accept or reject a current lot under a single sample. In some cases, decisions using a single sample can be a problem between producers and consumers regarding sample numbers or the acceptable number of defective products. Therefore, it is necessary to introduce an adaptive MDS sampling plan to eliminate the weakness of the MDS sampling plan and MMDS sampling plan. For this reason, we designed an adaptive version of the MDS (AMDS) sampling plan, intending to achieve a reduced sample size and increase the probability of current lot acceptance under the quality level. The concept of the DSP is applied together with the existing MDS and MMDS sampling plans. Suppose the quality of the product selected from the first sample is undecided. In that case, the second sample must be taken for inspection, after which the decision can be made to accept or reject the current lot based on the previous lots. The AMDS sampling plan is expected to be more flexible and efficient than the existing MDS and MMDS sampling plans. In this study, we endeavor to increase the probability of current lot acceptance and thus divide the acceptance quality into three levels: Excellent, good, or moderate. We focus on designing the AMDS sampling plan when the mean lifetime is based on the Weibull distribution under the time truncated life test. The comparison of the proposed sampling plan with existing sampling plans is given in terms of the ASN.

The rest of this paper is organized as follows: Section 2 shows a brief description of the Weibull distribution. Section 3 proposes the operating procedure and the design of the AMD sampling plan under a time truncated life test. The numerical results, comparison between the AMD sampling plan and the existing MDS sampling plan, and application of two real datasets are presented in Section 4. Finally, the concluding remarks and future work are discussed in Section 5.

2 Weibull Distribution

The Weibull distribution is the most widely used tool to model the lifetime of data because of its closed-form and flexibility. Many engineering works have studied the lifetime of products based on the Weibull distribution, such as ball bearings, automobile components, and electronic devices. Nowadays, the Weibull distribution is well-known for being used to design the ASP: Please see the example [24–28]. This research designs the AMDS sampling plan to approve the mean lifetime of the product under the Weibull distribution. Let t be the lifetime of a product under the Weibull distribution, and the cumulative distribution function is defined by

$$F(t; \lambda, \delta) = 1 - e^{-\left(\frac{t}{\lambda}\right)^{\delta}}, \ t \geq 0, \lambda > 0, \delta > 0, \tag{1}$$

where the shape parameter is δ, its value is known, while the scale parameter is λ, which is unknown. The mean lifetime of the product based on the Weibull distribution is given as follows:

$$\mu = \left(\frac{\lambda}{\delta}\right) \Gamma\left(\frac{1}{\delta}\right) \tag{2}$$

The failure probability of a product before time trial t_0 under the Weibull distribution is shown as the following equation:

$$p = 1 - e^{-\left(\frac{t_0}{\lambda}\right)^{\delta}} \tag{3}$$

The value of t_0 can be written in terms of the specified mean lifetime μ_0, e.g., $t_0 = a\mu_0$ for an experiment termination ratio (a). Reference [20] shows that from Eq. (3), the failure probability of product before the time trial can be rewritten as:

$$p = 1 - e^{-a^{\delta}\left(\frac{\mu_0}{\mu}\right)^{\delta}\left(\frac{1}{\delta}\Gamma\left(\frac{1}{\delta}\right)\right)^{\delta}}. \tag{4}$$

Eq. (4) show the failure probability in term of the experiment termination ratio, shape parameter, and mean lifetime based on the Weibull distribution.

3 Design of the Adaptive Version of a Multiple Dependent State Sampling Plan Based on a Time Truncated Life Test

Worth and Baker [2] constructed an MDS sampling plan as an attribute inspection process. This sampling plan decides to accept or reject the current lot and requires a continuous sample from the current and previous lots. This reason will result in a reduced sample size. Additionally, this sampling plan is usually used when production is ongoing with multiple lots, and each lot is submitted in sequence for inspection. However, the MDS sampling plan considers data from previous lots using a minimum sample size to eliminate a current lot of moderate quality. The MDS sampling plan will reject the current lot if the previous lot is of moderate quality, but all the remaining lots are of good quality. This reason will increase the risk for manufacturers. Regarding sample size and economic perspective, Nadi et al. [18] introduced the MMDS sampling plan to provide more flexibility and efficiency than the existing MDS sampling plan. As a result, both sampling plans provide more opportunities for producers when the current lot is of moderate quality. The difference is that the existing MDS sampling plan will only accept the current lot if the previous lot (m) is of good quality. Meanwhile, the MMDS sampling plan will accept the current lot if no more than one of the previous lots is of moderate quality; the remaining lots ($m - 1$) must satisfy the same conditions as MDS sampling plan. For the reasons mentioned earlier, the existing MDS sampling plan and MMDS sampling plan will accept or reject the current lot under a single sampling. In some cases, a single sampling may not be sufficient to accept or reject the current lot. Therefore, there is an opportunity to increase the producer's risk and reduce the consumer's risk.

This research aims to apply the concept of DSP together with the MDS and MMDS sampling plans. Suppose the product quality selected from the first sample is undecided. It is necessary to take the second sample for inspection and then decide to accept or reject the model. This sampling plan reduces the sample size over the MDS and MMDS sampling plans, recognizing that it will accept the current lot if they are of excellent, good, or moderate quality. This sampling plan will reduce the risk to the manufacturer. However, it will not affect consumers. Therefore, the proposed sampling plans are more flexible than the existing MDS and MMDS sampling plans concerning the reduced sample size.

Conditions and operating procedure

The AMDS sampling plan has the same conditions and operating procedures as the existing MDS and MMDS sampling plans:

(1) The product inspected consists of serial lots produced by continuous processes.
(2) In general, each lot examined should be of the same quality.
(3) The exact number of samples is chosen from each lot.
(4) The current lot will be the same quality as the previous lot, or it was immediately successful.
(5) Consumers have confidence in the honesty of the manufacturer.

The AMDS sampling plan is based on time truncated life tests under the Weibull distribution. Let the life test terminate at time trial t_0. There are six parameters, namely portraying the AMDS sampling plan, n_1, n_2, c_{a1}, c_w, c_{a2}, and m. The operating procedure for the AMDS sampling plan includes the following steps:

Step 1. Choose the first random sample size n_1 from the current lot. Put sample items on a truncated life test and count the number of nonconforming items before the time trial t_0, denoted as d_1.

Step 2. The current lot is accepted if $d_1 \leq c_{a1}$, called Type I acceptance, and the current lot is rejected if $d_1 > c_{a2}$. Otherwise, go to Step 3.

Step 3. The current lot is accepted if $c_{a1} < d_1 \leq c_w$, in which previous lots were accepted with $d_1 \leq c_{a1}$, called Type II acceptance. Otherwise, go to Step 4.

Step 4. If $c_w < d_1 \leq c_{a2}$, choose the second sample size n_2, Put sample items on a truncated life test and count the number of nonconforming items before the time trial t_0; denote it as d_2. Accept the current lot if $d_1 + d_2 \leq c_{a2}$ and no more than one previous lot with good quality and the remaining $m-1$ previous lots are of excellent quality, called Type III acceptance. Otherwise, reject the current lot.

We can summarize the above steps in a flow chart, as presented in Fig. 1.

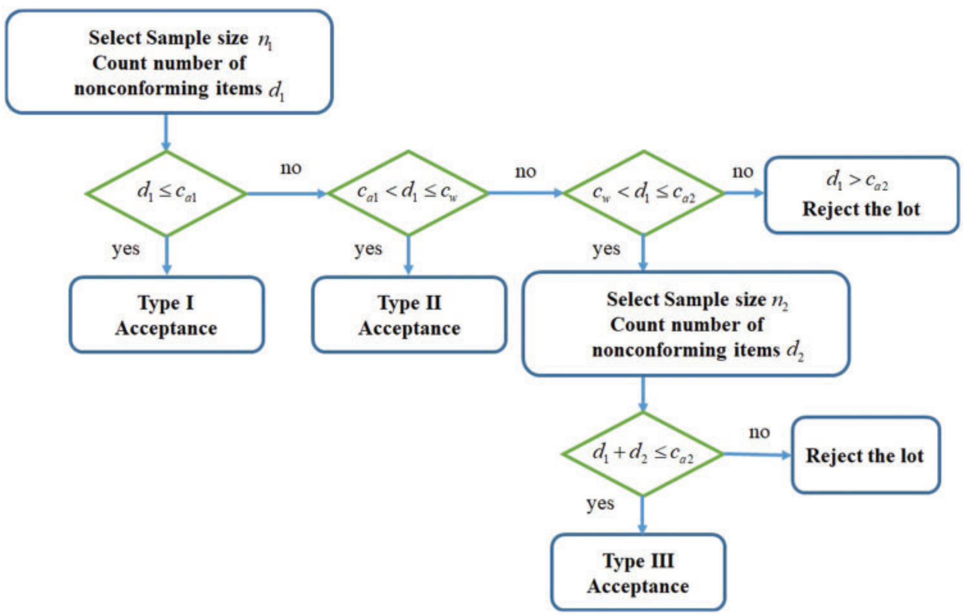

Figure 1: Operating procedure for the AMDS sampling plan

where n_1 is the sample size for the first inspection,

n_2 is the sample size for the second inspection,

c_{a1} is the maximum acceptable number of nonconforming items for unconditional acceptance $c_{a1} \geq 0$,

c_w is the acceptable warning number of additional nonconforming items for conditional acceptance $c_{a1} < c_w < c_{a2}$,

c_{a2} is the maximum acceptable number of additional nonconforming items for conditional acceptance $c_{a2} > c_w$, and

m is the number of previous lots.

This research aims to develop an existing MDS sampling plan to accept the current lot better based on the time truncated life test. Therefore, an acceptable warning number (c_w) is proposed to increase the probability of current lot acceptance.

Type I acceptance ($P_I(p)$) is the probability of current lot acceptance when its quality is excellent without considering the quality of m previous lots, given as follows:

$$P_I(p) = P(d_1 \le c_{a1}) \tag{5}$$

Type II acceptance ($P_{II}(p)$) is the probability of current lot acceptance when its quality is good provided previous lots are of excellent quality ($d_1 \le c_{a1}$) and obtained as follows:

$$P_{II}(p) = P(c_{a1} < d_1 \le c_w)\, P(d_1 \le c_{a1})^m \tag{6}$$

Type III acceptance ($P_{III}(p)$) is the probability of current lot acceptance when the quality is moderate, then the second sampling occurs. Provided that no more than one of the previous lot is of good quality whereas the remaining $m-1$ previous lots are under excellent quality and obtained as follows:

$$P_{III}(p) = P(c_w < d_1 \le c_{a2})\, P(d_1 + d_2 \le c_{a2}) \times \left[P(d_1 \le c_{a1})^m + mP(c_{a1} < d_1 \le c_w)\, P(d_1 \le c_{a1})^{m-1}\right]. \tag{7}$$

Thus, the performance of the AMDS sampling plan is interpreted by the operating characteristic (OC) function, which is defined by

$$\begin{aligned} P_a(p) &= P_I(p) + P_{II}(p) + P_{III}(p) \\ &= P(d_1 \le c_{a1}) + P(c_{a1} < d_1 \le c_w)\, P(d_1 \le c_{a1})^m \\ &\quad + P(c_w < d_1 \le c_{a2})\, P(d_1 + d_2 \le c_{a2}) \\ &\quad \times \left[P(d_1 \le c_{a1})^m + mP(c_{a1} < d_1 \le c_w)\, P(d_1 \le c_{a1})^{m-1}\right]. \end{aligned} \tag{8}$$

In this study, from Eq. (8), the OC function can be determined by the binomial distribution, which is shown as follows.

$$\begin{aligned} P_a(p) &= \sum_{d_1=0}^{c_{a1}} \binom{n_1}{d_1} p^{d_1}(1-p)^{n_1-d_1} + \left(\sum_{d_1=c_{a1}+1}^{c_w} \binom{n_1}{d_1} p^{d_1}(1-p)^{n_1-d_1}\right) \cdot \left(\sum_{d_1=0}^{c_{a1}} \binom{n_1}{d_1} p^{d_1}(1-p)^{n_1-d_1}\right)^m \\ &+ \left(\sum_{d_1=c_w+1}^{c_{a2}} \binom{n_1}{d_1} p^{d_1}(1-p)^{n_1-d_1}\right) \cdot \left(\sum_{d_2=0}^{c_{a2}-d_1} \binom{n_2}{d_2} p^{d_2}(1-p)^{n_2-d_2}\right) \\ &\times \left[\left(\sum_{d_1=0}^{c_{a1}} \binom{n_1}{d_1} p^{d_1}(1-p)^{n_1-d_1}\right)^m + m\left(\sum_{d_1=c_{a1}+1}^{c_w} \binom{n_1}{d_1} p^{d_1}(1-p)^{n_1-d_1}\right) \cdot \left(\sum_{d_1=0}^{c_{a1}} \binom{n_1}{d_1} p^{d_1}(1-p)^{n_1-d_1}\right)^{m-1}\right]. \end{aligned} \tag{9}$$

Another essential point to note is that we can switch the AMDS sampling plan to SSP and DSP by considering the following:

(1) Considering that $m \to \infty$, the AMDS sampling plan is reduced to SSP with an acceptance number c_{a1}.

(2) Considering that $m \to 0$ and $c_{a1} = c_w$ or $c_{a2} = c_w$, then the AMDS sampling plan is reduced to DSP with acceptance numbers c_{a1} and c_{a2}. The ASN function of the AMDS sampling plan(ASN_{AMDS}) is derived by

$$\begin{aligned} ASN_{AMDS} &= n_1 P_I + (n_1 + n_2)(1 - P_I) \quad ; P_I = P(d_1 \le c_w) + P(d_1 > c_{a2}) \\ &= n_1 + n_2 (1 - P_I) \\ &= n_1 + n_2 \left[1 - \sum_{d_1 = c_w + 1}^{c_{a2}} \binom{n_1}{d_1} p^{d_1} (1-p)^{n_1 - d_1} \right] \end{aligned} \tag{10}$$

4 Results, Comparative Study, and Application of Real Datasets

In order to provide good quality electronic products, time and costs are necessary for the inspection process. The ASP that can reduce time and costs for the inspection processes is the economic sampling plan, which minimizes ASN to reduce time and costs. The AMDS sampling plan aims to obtain an ASN lower than the existing MDS and MMDS sampling plans. This sampling plan ensures that the mean lifetime and the ratio between the true mean lifetime and the specified mean lifetime $\left(\mu / \mu_0\right)$ are essential to the product. We consider that the failure probability of a product is related to the mean ratio, which affects the quality level of the product. The requirements for the producer's risks (α) and consumer's risks (β) are considered with the acceptable quality level (AQL or p_1) and limiting quality level (LQL or p_2). The AMDS sampling plan is practical with two points (AQL , $1 - \alpha$) and (LQL, β), and will be considered for changes in the OC curve. A producer expects that the probability of current lot acceptance should be greater than $1-\alpha$ at p_1. On the other hand, a customer also expects that the probability of current lot acceptance should be less than β at p_2. The nonlinear optimization technique is used to determine the optimal parameters, which results in the reduced size of the ASN at p_1. The optimization problem is as follows.

Objective function:

Minimize ASN_{AMDS} (11)

Subject to: $P_a(p_1) \ge 1 - \alpha,\ P_a(p_2) \le \beta,$

$n_1, n_2 > 1,\ m \ge 1,\ c_{a2} > c_w > c_{a1} \ge 0.$

Let the mean ratio of the producer's risk be $\mu / \mu_0 = 2, 4, 6, 8, 10$. In contrast, consumers expect to get good products, and we assume the mean ratio $\mu / \mu_0 = 1$ as the consumer's risk. Under the Weibull distribution, the values of p_1 and p_2 are calculated using Eq. (4) for different values μ / μ_0. The optimal parameters $(n_1, n_2, c_{a1}, c_w, c_{a2}, m)$ of the AMDS sampling plan under the Weibull distribution are determined and selected to simultaneously satisfy the producer's and consumer's risks with the minimum sample size. In this section, the producer's risk is set at $\alpha = 0.05$ with different consumer's risk values at $\beta = 0.25$, 0.10, 0.05, and 0.01. Also, two cases of the experiment termination ratio are considered as $a = 0.5$ and 1.0. The values for shape parameters under the Weibull distribution are $\delta = 2$, 2.5, and 3.0.

4.1 Numerical Results

Tabs. 1–3 present the optimal parameters for the AMDS sampling plan under the Weibull distribution. The results show that, for the fixed values of a and μ/μ_0, if β decreases or δ increases, the ASN_{AMDS} increases. Additionally, the ASN_{AMDS} tends to decrease if μ/μ_0 increases. For the fixed values of β and δ, the ASN decreases if a or μ/μ_0 increases. We consider that, for fixed β, a, and δ, the ASN_{AMDS} tends to be closer or equal to n_1 if μ/μ_0 increases. It means that the higher the μ/μ_0, the lower the ASN_{AMDS} used in the inspection lot.

Table 1: Optimal plan parameters for the AMDS sampling plan with $\delta = 2$

β	μ/μ_0	$a = 0.5$								$a = 1.0$							
		n_1	n_2	c_{a1}	c_w	c_{a2}	m	ASN_{AMDS}	$P_a(p_1)$	n_1	n_2	c_{a1}	c_w	c_{a2}	m	ASN_{AMDS}	$P_a(p_1)$
0.25	2	18	18	1	3	5	1	18.1664	0.9543	*	*	*	*	*	*	*	*
	4	16	16	1	3	5	2	16.0200	0.9995	9	6	1	2	3	1	9.0413	0.9938
	6	14	14	0	2	4	3	14.0008	0.9850	8	8	1	2	3	1	8.0956	0.9998
	8	11	11	0	3	5	4	11.0004	0.9958	7	5	0	2	3	1	7.0003	0.9932
	10	10	10	0	3	5	2	10.0001	0.9993	5	5	0	1	2	1	5.0003	0.9985
0.10	2	24	24	2	4	7	1	24.1195	0.9887	6	6	2	3	6	1	6.0669	0.9942
	4	22	22	1	4	6	2	22.0001	0.9983	10	3	1	2	3	3	10.0281	0.9821
	6	19	19	0	2	4	3	19.0028	0.9737	9	1	0	1	2	1	9.0144	0.9696
	8	16	15	0	3	5	2	16.0018	0.9955	8	8	0	1	4	2	8.0317	0.9840
	10	13	13	0	3	5	2	13.0002	0.9987	7	3	0	3	4	3	7.0000	0.9919
0.05	2	28	28	2	4	6	1	28.2677	0.9733	8	8	2	4	6	2	8.0496	0.9531
	4	27	27	1	3	5	1	27.0084	0.9982	11	4	1	3	4	3	11.0049	0.9756
	6	27	24	1	4	6	2	27.0014	0.9998	10	7	1	3	4	2	10.0003	0.9993
	8	26	25	1	4	6	3	26.0004	1.0000	9	9	0	1	4	2	9.0455	0.9802
	10	22	22	0	4	6	1	22.0001	0.9982	9	5	0	1	3	4	9.0106	0.9836
0.01	2	32	32	2	5	7	1	32.1160	0.9602	11	9	3	5	8	3	11.0586	0.9640
	4	32	32	1	3	6	2	32.0194	0.9935	12	10	1	2	3	2	12.1555	0.9724
	6	30	29	0	2	5	1	30.0170	0.9973	12	2	1	2	3	3	12.0036	0.9979
	8	30	26	0	2	5	1	30.0029	0.9923	12	3	0	3	4	3	12.0011	0.9511
	10	29	29	0	2	5	1	29.0008	0.9969	10	4	0	3	4	4	10.0008	0.9796

Note: * There is no optimal no optimal plan

Table 2: Optimal plan parameters of the AMDS sampling plan with $\delta = 2.5$

β	μ/μ_0	$a = 0.5$								$a = 1.0$							
		n_1	n_2	c_{a1}	c_w	c_{a2}	m	ASN_{AMDS}	$P_a(p_1)$	n_1	n_2	c_{a1}	c_w	c_{a2}	m	ASN_{AMDS}	$P_a(p_1)$
0.25	2	24	24	1	3	5	2	24.0483	0.9793	10	10	2	4	7	1	10.0412	0.9867
	4	22	22	0	2	4	4	22.0022	0.9739	11	9	0	1	4	1	11.2264	0.9543
	6	16	15	0	4	6	1	16	0.9994	9	1	0	3	4	4	9	0.9810
	8	13	12	0	4	6	3	13	0.9997	8	2	0	3	4	4	8	0.9960
	10	12	10	0	4	6	3	12	0.9999	8	8	0	2	4	3	8	0.9990
0.10	2	34	32	1	3	5	1	34.2326	0.9653	11	11	2	4	7	1	11.0748	0.9789
	4	32	31	1	4	6	3	32.0010	0.9998	13	4	1	3	4	3	13.0006	0.9965
	6	28	23	0	3	5	1	28.0004	0.9983	11	11	0	1	5	3	11.0403	0.9794

(Continued)

Table 2: Continued

β	μ/μ_0	$a = 0.5$								$a = 1.0$							
		n_1	n_2	c_{a1}	c_w	c_{a2}	m	ASN_{AMDS}	$P_a(p_1)$	n_1	n_2	c_{a1}	c_w	c_{a2}	m	ASN_{AMDS}	$P_a(p_1)$
	8	26	23	0	3	5	2	26	0.9993	10	2	0	3	4	4	10.0004	0.9939
	10	25	21	0	3	5	2	25	0.9998	9	4	0	1	3	3	9.0008	0.9987
0.05	2	40	39	2	4	7	2	40.0829	0.9923	13	13	3	5	8	1	13.0354	0.9952
	4	37	36	0	2	5	1	37.0172	0.9803	13	9	1	2	3	2	13.0245	0.9971
	6	36	36	2	5	1	2	36.0008	0.9973	13	1	0	3	4	3	13.0110	0.9710
	8	34	33	0	2	5	1	34.0001	0.9994	12	12	0	1	4	4	12.0129	0.9916
	10	30	29	0	2	5	1	30	0.9998	11	6	0	3	4	2	11	0.9987
0.01	2	60	55	2	4	6	1	60.6434	0.9704	12	10	2	4	7	1	12.1048	0.9693
	4	52	51	1	3	5	2	52.0033	0.9866	13	13	1	2	4	2	13.0375	0.9977
	6	43	41	0	3	5	1	43.0004	0.9962	13	13	0	2	4	4	13.0020	0.9634
	8	40	39	0	2	4	1	40.0001	0.9992	13	3	0	3	4	4	13.0025	0.9901
	10	38	35	0	2	4	2	38	0.9995	12	11	0	1	3	4	12.0039	0.9971

Table 3: Optimal plan parameters for the AMDS sampling plan with $\delta = 3$

β	μ/μ_0	$a = 0.5$								$a = 1.0$							
		n_1	n_2	c_{a1}	c_w	c_{a2}	m	ASN_{AMDS}	$P_a(p_1)$	n_1	n_2	c_{a1}	c_w	c_{a2}	m	ASN_{AMDS}	$P_a(p_1)$
0.25	2	34	26	1	5	7	2	34	0.9942	12	8	2	5	8	3	12.0018	0.9840
	4	22	13	0	4	7	4	22	0.9965	12	2	0	1	3	3	12.0150	0.9608
	6	20	13	0	4	6	2	20	0.9999	11	9	0	2	4	4	11.0001	0.9952
	8	20	20	0	3	5	4	20	1.0000	9	7	0	2	3	4	9	0.9994
	10	17	16	0	4	6	3	17	1.0000	9	4	0	2	3	4	9	0.9998
0.10	2	51	46	1	3	5	1	51.1129	0.9877	*	*	*	*	*	*	*	*
	4	37	34	0	3	5	1	37	0.9975	13	13	0	1	4	2	13.1145	0.9681
	6	33	24	0	4	6	2	33	0.9996	13	1	0	3	4	4	13	0.9934
	8	33	32	0	3	5	3	33	0.9999	12	9	0	1	3	4	12.0011	0.9989
	10	31	29	0	3	5	2	31	1.0000	11	8	0	1	3	4	11.0002	0.9998
0.05	2	59	56	1	3	5	1	59.2326	0.9801	*	*	*	*	*	*	*	*
	4	47	43	0	3	5	1	47	0.9960	13	13	0	1	5	2	13.1145	0.9681
	6	47	44	0	3	5	1	47	0.9996	13	3	0	3	4	4	13	0.9934
	8	44	41	0	3	5	2	44	0.9999	13	13	0	2	3	4	13	0.9987
	10	42	42	0	3	5	2	42	1.0000	12	11	0	1	3	4	12.0004	0.9997
0.01	2	65	64	1	3	5	1	65.3744	0.9727	*	*	*	*	*	*	*	*
	4	63	61	0	3	5	4	63.0001	0.9752	13	13	0	1	4	3	13.1145	0.9551
	6	62	61	0	3	5	1	62	0.9994	13	5	0	1	3	4	13.0041	0.9935
	8	59	59	0	2	4	2	59	0.9998	13	12	0	3	5	4	13	0.9987
	10	56	50	0	2	4	2	56	1.0000	13	11	0	2	3	3	13	0.9997

Note: * There is no optimal no optimal plan

The OC curves present the effect of m values based on the probability of current lot acceptance with different m values, but n_1, n_2, c_{a1}, c_w, and c_{a2} are equal. The optimal plan parameters are considered for fixed $a = 0.5$, $\delta = 2$, $\mu/\mu_0 = 8$, $\alpha = 0.05$ and $\beta = 0.10$. As a result, the optimal plan parameters are $(n_1, n_2, c_{a1}, c_w, c_{a2}, m) = (37, 37, 1, 3, 6, 2)$, and the OC curve for the AMDS sampling plan with $m = 1, 2, 3$, and 4 are shown in Fig. 2. It can be seen from Fig. 2 that $m = 2, 3$, and 4 give the probability of current lot acceptance lower than $m = 1$. As a result, there is a high probability of current lot acceptance. It depends only on the acceptance of the previous lot. In addition, if the proportion of nonconformity is increased, the value of m does not significantly affect the probability of current lot acceptance.

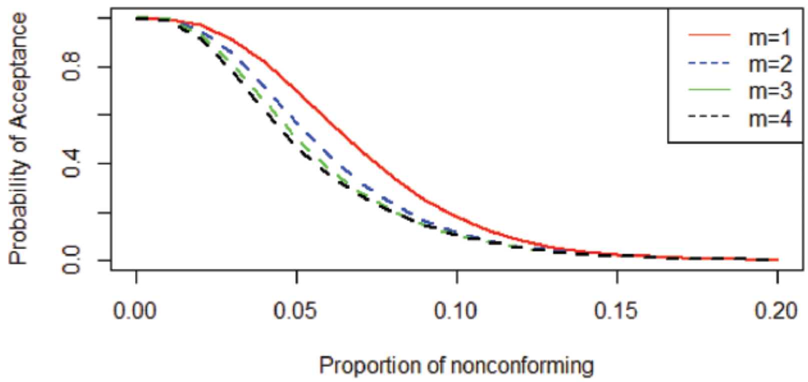

Figure 2: OC curves for the AMDS sampling plan with different m values

Illustrative example: To consider applying the AMDS sampling plan from Tabs. 1–3, we suppose that a manufacturer wants to test the mean lifetime of items where the lifetime is based on the Weibull distribution with shape parameter 2. Let the specified lifetime of the product (μ_0) be 500 hrs, and the time trial (t_0) be 250 hrs; then, the experiment termination ratio of this test (a) is 0.5. In addition, it is assumed that $\alpha = 0.05, \beta = 0.25$, and $\mu/\mu_0 = 2$. Tab. 1 gives the optimal plan parameters for the AMDS sampling plan as $(n_1, n_2, c_{a1}, c_w, c_{a2}, m) = (18, 18, 1, 3, 5, 1)$, wherein the probability of current lot acceptance is 0.9543, and ASN_{AMDS} is 18.1664. The inspection procedure is as follows:

Step 1: Choose a first random sample of 18 items from the current lot and put it on the life test. Count the number of nonconforming items (d_1) before $t_0 = 250$ hrs.

Step 2: If $d_1 \leq 1$, the current lot will be accepted regardless of the quality of the previous lot, which is called **excellent quality**. If $d_1 > 5$, the current lot will be rejected. Otherwise, go to step 3.

Step 3: If $1 < d_1 \leq 3$, the current lot is accepted provided that only one previous lot ($m = 1$) is of excellent quality, which means the current lot is given **good quality**. Otherwise, go to step 4.

Step 4: If $d_1 + d_2 \leq 5$, the current lot is accepted if no more than one of the previous lots is of good quality, which means the current lot is given **moderate quality**. Otherwise, reject the current lot.

4.2 Comparative Study

ASN perspective: To demonstrate the efficacy of the AMDS sampling plan in reducing ASN better than the existing MDS and MMDS sampling plans. The ASN of three sampling plans are shown in Tab. 4, in which the concepts and results of ASN under the MMDS and the existing MDS sampling plans are referenced in [20] and [27], respectively. This study compared the ASNs of the three sampling plans using the same parameters. Also, the ASN of the AMDS sampling plan is calculated by substituting $c_w = c_{a1} = c_1$ in the OC function. Tab. 4 shows that the AMDS sampling plan has a smaller ASN than the existing MDS and MMDS sampling plans. For example, suppose that $\alpha = 0.05, \beta = 0.01$, $a = 0.5$, $\delta = 2$, and $\mu/\mu_0 = 4$, then the ASN of the AMDS sampling plan is 22.2689, while the ASN of the MDS and MMDS sampling plans are 24 and 35, respectively. We found that the ASN of the AMDS sampling plan decreases as the mean ratio increases, whereas the ASNs of the existing MDS and MMDS sampling plans remain constant. Moreover, the ASN of the AMDS plan is the lowest as the consumer's risk decreases, while the OC function is the greatest. Following the Weibull distribution, the AMDS sampling plan can ensure a better mean lifetime for the product than the existing MDS and MMDS sampling plans based on the minimum sample size.

Table 4: ASN of the AMDS sampling plan, MMDS sampling plan, and MDS sampling plan for ensuring mean lifetime under the Weibull distribution with $\delta = 2$ and $a = 0.5$

β	μ/μ_0	AMDS		MMDS [20]		MDS [27]	
		ASN	$P_a(p_1)$	*ASN*	$P_a(p_1)$	*ASN*	$P_a(p_1)$
0.25	4	6.0688	0.9877	8	0.9930	8	0.9801
	6	6.0318	0.9974	8	0.9989	8	0.9957
	8	5.0303	0.9994	8	0.9997	8	0.9986
	10	5.0195	0.9997	8	0.9999	8	0.9994
0.10	4	7.0793	0.9837	12	0.9818	12	0.9583
	6	6.0318	0.9974	12	0.9971	12	0.9906
	8	6.0181	0.9992	12	0.9992	12	0.9969
	10	6.0117	0.9996	12	0.9997	12	0.9987
0.05	4	10.2184	0.9788	16	0.9747	16	0.9551
	6	10.0518	0.9957	16	0.9956	16	0.9900
	8	9.0269	0.9989	16	0.9987	16	0.9967
	10	9.0174	0.9995	16	0.9995	16	0.9986
0.01	**4**	**22.2689**	**0.9939**	**24**	**0.9794**	**35**	**0.9871**
	6	15.0756	0.9907	24	0.9892	24	0.9783
	8	15.0440	0.9969	24	0.9969	24	0.9927
	10	13.0249	0.9990	24	0.9989	24	0.9969

OC curve perspective: This section compares the probability of current lot acceptance between the proposed sampling plans, MMDS and MDS, based on the OC curves of those plans given the same parameters. Suppose $n_1 = 8$, $n_2 = 8$, $c_{a1} = 0$, $c_w = 1$, $c_{a2} = 4$ and $m = 2$ for AMDS sampling plan and $n = 8$, $c_1 = 0$, $c_2 = 4$ and $m = 2$ for MDS and MMDS sampling plans, respectively. All three sampling plans are considered based on the time truncated life test for the Weibull distribution when $\delta = 2$, $a = 1$, $\mu/\mu_0 = 6$, $\alpha = 0.05$, and $\beta = 0.25$. From Fig. 3, it can be seen that the AMDS sampling plan provides a higher probability of current lot acceptance compared to the existing MDS and MMDS sampling plans for small values of the proportion of nonconformity. As the proportion of nonconformity increased, we found that the OC curve of the proposed sampling plan aligned with the OC curves of the other two sampling plans. Based on this comparison, it can be concluded that the proposed sampling plan had better usability than the existing MDS and MMDS sampling plans.

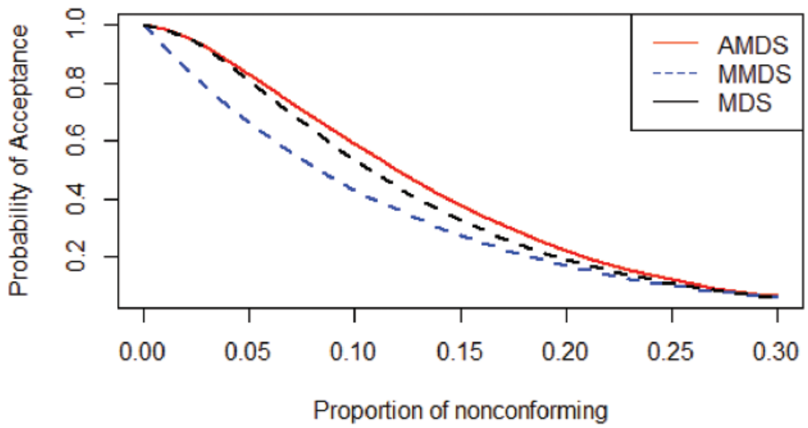

Figure 3: OC curves for AMDS, MMDS, and MDS sampling plan

4.3 Application of Real Datasets

Two real datasets were used to investigate the efficiency of the AMDS sampling plan under the Weibull distribution. First, we needed to check the Weibull distribution fits for both datasets. The unknown parameters were estimated using the maximum likelihood method, and the goodness of the fitness value was judged using the Kolmogorov-Smirnov (K-S) test. The model-fitting results for two real datasets are shown in Tab. 5.

Table 5: Model fitting results for two real datasets

Dataset	Parameter estimate	L-L	AIC	BIC	K-S	*p*-value
1	$\hat{\delta} = 2.0065$ $\hat{\lambda} = 9.8989$	−142.5215	289.043	292.8266	0.0879	0.8430
2	$\hat{\delta} = 1.1569$ $\hat{\lambda} = 60.3748$	−251.3237	506.6475	510.4715	0.1086	0.5967

The first dataset: Dataset of the stress-rupture life of Kevlar 49/epoxy strands under constant pressure at 70% stress level until everything fails (in 1, 000 h) is considered, as shown in Cooray et al. [29]. A total of 49 observations are discussed below:

1.051 1.137 1.389 1.921 1.942 2.322 3.629 4.006 4.012 4.063 4.921

5.445 5.620 5.917 5.905 5.956 6.068 6.121 6.473 7.501 7.886 8.108

8.546 8.666 8.831 9.106 9.711 9.806 10.205 10.396 10.861 11.026 11.214

11.362 11.604 11.608 11.745 11.762 11.895 12.044 13.520 13.670 14.110 14.496

15.395 16.179 17.092 17.568 17.568

From Tab. 5, the K-S test is 0.087937 with a *p*-value of 0.843. Therefore, first dataset is fit for the Weibull distribution. The maximum likelihood estimate of shape parameter $\hat{\delta} = 2.0064806 \approx \mathbf{2}$

resulted in the value $\hat{\mu} = 8.77$. Given that μ_0 is 8.77 and t_0 is 4.385, then a is 0.5. From Tab. 1, consider $\alpha = 0.05$, $\beta = 0.25$, and $\mu/\mu_0 = 4$, the optimal plan parameter for the AMDS sampling plan is $(n_1, n_2, c_{a1}, c_w, c_{a2}, m)$ = (16, 16, 1, 3, 5, 2), the probability of current lot acceptance is 0.9995, and ASN_{AMDS} is 16.02. For illustration purposes, the first random sample is chosen 16 items, then put on the life test for the sample items in the time trial from the current lot as follows:

15.395 **3.629** 14.496 17.568 6.068 4.921 8.546 11.214

5.917 5.905 5.956 **4.063** 6.121 **1.137** 7.501 7.886

From 16 items, three failures ($d_1 = 3$) are recorded before the time trial 4.385; the results show $1 < d_1 \leq 3$ ($c_{a1} < d_1 \leq c_w$). Therefore, the second sample is chosen 16 items, put on the life test, and counted for the nonconforming item during the time trial 4.385 as follows:

4.006 8.831 5.620 8.108 10.396 11.604 13.670 14.110

8.666 5.445 11.745 9.711 9.806 2.322 10.861 11.026

Under the second inspection, only one failure ($d_2 = 1$) is observed from the current lot; the results show $d_1 + d_2 < 5$ ($d_1 + d_2 \leq c_{a2}$). The current lot is accepted if the two previous lots ($m = 2$) are of excellent quality.

The second dataset: The following dataset represents the failure times for a sample of 50 electronic devices, as shown in Kenett et al. [30]. A total of 50 observations are discussed below:

26.3 78.5 29.8 22.6 113.1 157.4 2.4 51.9 29.3 40.3 216.6 30.5 31.6

57.5 38.1 113.7 1.0 96.8 63.3 72.1 107.4 39.6 29.0 11.0 105.2 36.7

7.1 85.5 24.6 28.0 23.6 14.7 24.3 46.9 56.9 293.4 33.0 47.0 51.9

20.0 20.3 158.9 54.0 14.8 81.2 46.0 42.8 8.9 35.7 32.3

From Tab. 5, The K-S test is 0.0980 with a p-value of 0.5967. Therefore, second dataset is fit for the Weibull distribution. The maximum likelihood estimate of shape parameter $\hat{\delta} = 1.1569$ resulted in the value $\hat{\mu} = 57.3601$. Given that μ_0 is 57.3601 and t_0 is 28.7, then a is 0.5. From the optimization technique, consider $\alpha = 0.05$, $\beta = 0.10$ and $\mu/\mu_0 = 4$, the optimal plan parameter of the AMDS sampling plan is $(n_1, n_2, c_{a1}, c_w, c_{a2}, m)$ = (11, 8, 1, 3, 5, 1), the probability of current lot acceptance is 0.9511, and ASN_{AMDS} is 11.07. For illustration purposes, we chose a first random sample of 11 items and put the sample items on the life test for the time trial from the current lot as follows:

30.5 31.6 57.5 38.1 113.7 **1.0** 96.8 63.3 29.0 **11.0** 105.2

From 11 items, we note that two failures ($d_1 = 2$) are recorded before the time trial 28.7; the results show $1 < d_1 < 3$ ($c_{a1} < d_1 \leq c_w$). Therefore, the second sample is chosen 8 items, put on the life test, and counted for nonconforming items during the time trial 28.7 as follows:

33.0 47.0 51.9 **20.0** **20.3** 158.9 54.0 **14.8**

Under the second inspection, three failures ($d_2 = 3$) are observed from the current lot; the results show that $d_1 + d_2 \leq 5$ ($d_1 + d_2 \leq c_{a2}$). Then, the current lot is accepted if only one previous lot ($m = 1$) is of excellent quality.

5 Conclusions

This paper proposes a new adaptive version of the multiple dependent state (AMDS) sampling plan for a truncated life test based on the Weibull distribution. The concept of DSP is used to apply together with the existing MDS and MMDS sampling plans. The proposed sampling plan will accept the current lot if the quality of the product is of excellent, good, or moderate quality. The nonlinear optimization technique is used to determine the optimal plan parameters to satisfy the consumer's and producer's risks simultaneously. Tables for optimal plan parameters are presented under different shape parameter values, manufacturer's risk, consumer's risk, and test termination ratio. A comparison between the proposed plan and the existing plan is considered under the ASN and OC curve values. The illustrative example shows the operational process for the proposed plan. In addition, the application of two real datasets demonstrates the usability and usefulness of the proposed plan. Finally, we conclude that the AMDS sampling plan under the Weibull distribution is more flexible and efficient in terms of the average sample number than the existing MDS and MMDS sampling plans. The AMDS sampling plan will be considered under accelerated testing techniques and cost determination in future work.

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