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New Decision-Making Technique Based on Hurwicz Criteria for Fuzzy Ranking

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Abstract: Efficient decision-making remains an open challenge in the research community, and many researchers are working to improve accuracy through the use of various computational techniques. In this case, the fuzzification and defuzzification processes can be very useful. Defuzzification is an effective process to get a single number from the output of a fuzzy set. Considering defuzzification as a center point of this research paper, to analyze and understand the effect of different types of vehicles according to their performance. In this paper, the multi-criteria decision-making (MCDM) process under uncertainty and defuzzification is discussed by using the center of the area (COA) or centroid method. Further, to find the best solution, Hurwicz criteria are used on the defuzzified data. A new decision-making technique is proposed using Hurwicz criteria for triangular and trapezoidal fuzzy numbers. The proposed technique considers all types of decision makers' perspectives such as optimistic, neutral, and pessimistic which is crucial in solving decision-making problems. A simple case study is used to demonstrate and discuss the Centroid Method and Hurwicz Criteria for measuring risk attitudes among

decision-makers. The significance of the proposed defuzzification method is demonstrated by comparing it to previous defuzzification procedures with its application.

Keywords: Defuzzification; decision-making; fuzzy numbers; Hurwicz; multi-criteria decision-making; ranking order

1 Introduction

Achieving fruitful decision-making is an important concern when choosing to apply machine learning techniques (or) aiming to impart artificial intelligence into machines that work upon the traditional logic theory. The idea behind this is to enable a machine to think like a human and in turn make humans ease the complexity in the decision-making process. The task of decision-making in everyday life is usually vague or uncertain. The ability to make decisions among humans differs significantly and depends on the knowledge and perception levels of an individual. Moreover, the decision-making process is more complex for blurred, vague, and qualitative data. It is thus a very big challenge under such uncertainty or vagueness to choose the best action from various available actions [1,2]. Recent research contributions insist that uncertainty, risk, and confidence should be taken into account with respect to multi-criteria decision-making (MCDM). The research addressed the uses of linguistic variables, triangular variables, and trapezoidal fuzzy numbers to represent the decision maker's confidence and risk attitudes so as to achieve a more complete MCDM Solution. With this regard, specifically, this work addresses a travel problem to illustrate the computation process and demonstrate the suitability of the results that are used previously to assess MCDM. Defuzzification is a procedure of conversion to a fuzzy set into a crisp set or values and the useful method was proposed by Norman Fenton and Wei Wang to deal with this kind of decision-making problem under uncertainty [3]. For triangular fuzzy numbers in multi-criteria decision making, Salamai et al. both have analyzed the confidence and risk-taking capacity of a decision-maker [4–6]. So far, various defuzzification methods [7–9] are proposed by different mathematicians. El-kenawy et al. [10] applied three methods which include i) discretization, ii) modified transformation function, and iii) for trapezoidal fuzzy numbers, performing the center of gravity (COG) defuzzification using the slope-based method. One of the most common and useful de-fuzzification techniques is the method of the center of gravity (COG). It is also called the centroid method [11]. If the data is precise and quantitative, the Hurwicz criterion is used for decision-making. It must be noted that the Hurwicz criterion is not suitable for imprecise data. El-kenawy et al. [12] introduced the concept of fuzzy sets to process imprecise linguistic information. Hurwicz Criterion has a defining feature, i.e., it allows the decision-makers to simultaneously consider both the best and the worst possible outcomes. For this, the decision-makers choose a "coefficient of pessimism", which is usually a decimal number between 0 and 1 and is denoted by alpha (α). The worst possible outcomes are emphasized by this number and the number $(1 - \alpha)$ determines the emphasis to be placed on the best outcomes. So, if the coefficient of pessimism is equal to 0.8 then, 0.2 is the emphasis on the best outcomes [13]. Given the aforementioned challenges, we present the following novel aspects of our study:

- To introduce a new ranking method using the centroid method or COA.
- To use Hurwicz criteria to defuzzify the triangular fuzzy numbers.
- To the extent the research to the trapezoidal fuzzy numbers which considers the risk attitude of the decision-maker.

The centroid method is used for defuzzifying the fuzzy numbers and a Hurwicz criterion is used to the defuzzified data. The Hurwicz criteria are used for triangular numbers and are extended for the trapezoidal fuzzy numbers as well which considers the risk attitude of the decision-maker.

2 Related Works

In linguistic multi-criteria decision-making problems, the ranking of fuzzy numbers plays a very significant role. Numerous researchers have used several fuzzy ranking methods to solve decision-making problems using triangular and trapezoidal fuzzy numbers. Ye et al. [14] proposed a direct and simplified approach for computing triangular fuzzy Banzhaf values of cooperative games for a special class of cooperative games with coalitions' values represented by triangular fuzzy numbers. They have suggested that interval-valued cooperative games are associated with the triangular fuzzy cooperative games and have proved that the Banzhaf values of the associated cooperative games together with the interval-valued cooperative games are non-decreasing functions and monotonic under same size monotonicity-like conditions. Furthermore, they have proved some properties of the triangular fuzzy Banzhaf values and illustrate the validity and applicability of the developed approach. Jing-Jing An et al. develop a new method for solving bi-matrix games with payoffs of intuitionistic fuzzy numbers (IFNs). In this method, a weighted mean-area ranking method based on a non-linear programming approach of intuitionistic fuzzy numbers was proposed [15]. Deng-Feng et al. [16,17] proposed some ranking methods which are based on the concept of value and ambiguity, a ratio of the value index to the ambiguity index, and applied it to solve multi-attribute decision-making problems where the ratings of alternatives are expressed using triangular intuitionistic fuzzy numbers. Savitha et al. [18] proposed some new methods for ranking Trapezoidal Fuzzy numbers based on the extent fuzzy analytic hierarchy processes which are used in developing fuzzy multi-criteria decision-making models. For compiling experts' judgment arithmetic mean operations of fuzzy numbers have been used. Yadong et al. in [19] developed a long-term forecasting model based on static and dynamic fuzzy information granules for interval-valued time series (ITS). Hassib et al. [20] addresses the green supplier selection problem in and suggest the solution through the MCDM technique based on the novel Hamming distance function. Traditional ranking algorithms, on the other hand, are unsuitable for many real-time problems involving data with inherent ambiguity or impreciseness. Nguyen [21] proposed a method for ranking fuzzy numbers based on the Unified Index that combines the weighted mean known as the centroid value and the weighted area known as the attitude-incorporated left and right area. According to the authors, the unified index has the consistency property for ranking fuzzy numbers and their images, as well as for symmetric fuzzy numbers with an identical altitude, which is a critical property for accurate matching and/or retrieval of information in the field of computer vision and image pattern recognition. Many such popular examples are used for a wide range of fuzzy number comparative studies, and ranking performance is compared between the unified index and some recent representative indices. Wang [22] proposed a method for ranking triangular and trapezoidal fuzzy numbers based on the relative preference relation, in which a fuzzy preference relation with a membership function representing preference degree is used to compare two fuzzy numbers, and a relative preference relation is built on top of the fuzzy preference relation to rank a set of fuzzy numbers. According to Eid et al. [23], the difference function is used from the fuzzy preference relation between alternatives and extreme solutions to obtain weakness and strength matrices, which are then multiplied by the weight matrix to produce weighted weakness and strength indices, which are then converted into positive and negative indices. These two indices are combined to produce a total performance index, and then alternatives are sorted based on their related performance indices.

3 Preliminaries

3.1 Fuzzy Number

A fuzzy number is a fuzzy set Ã defined on the set of real numbers R if its membership functions Ã: R → [0, 1] has the following properties.

(i) If it is a convex fuzzy set.
   i.e., $\mu_{\tilde{A}} (\lambda x_1 + (1 - \lambda) x_2) \geq \min (\mu_{\tilde{A}} (x_1), \mu_{\tilde{A}}(x_2)), \forall x \in [0, 1], \lambda \in [0, 1]$

(ii) If it is normal i.e., there exists an x ∈R such that if max $\mu_{\tilde{A}}(x) = 1$.
(iii) If it is continuous piecewise.

3.2 Triangular Fuzzy Number

A fuzzy number $\tilde{A} = (a, b, c)$ is called a triangular fuzzy number if its membership function $\mu_{\tilde{A}}(X)$ is given in Eq.(1) [24,25]. The graphical representation of a triangular fuzzy number is shown in Fig. 1.

$$\mu_{\tilde{A}}(x) = \begin{cases} 0 & , x \leq a \& x \geq c \\ \dfrac{x}{b-a} - \dfrac{a}{b-a} & , a < x \leq b \\ 1 & , x = b \\ \dfrac{c}{c-b} - \dfrac{x}{c-b} & , b < x \leq c \end{cases} \tag{1}$$
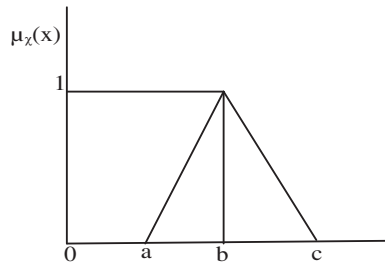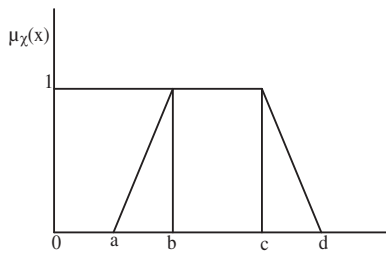

Figure 1: Illustration of triangular fuzzy number

3.3 Trapezoidal Fuzzy Number

A fuzzy number $\tilde{A} = (a, b, c, d)$ is said to be a trapezoidal fuzzy number whose membership function $\mu_{\tilde{A}}(X)$ is given in Eq. (2) [24,25]. The graphical representation of a trapezoidal fuzzy number is shown in Fig. 2.

$$\mu_{\tilde{A}}(x) = \begin{cases} 0 & , x \leq a \& x \geq d \\ \dfrac{x}{b-a} - \dfrac{a}{b-a} & , a < x \leq b \\ 1 & , b \leq x \leq c \\ \dfrac{c}{c-b} - \dfrac{x}{c-b} & , c < x \leq d \end{cases} \tag{2}$$

Figure 2: Illustration of trapezoidal fuzzy number

3.4 Hurwicz Criteria

Hurwicz's rule (Criteria) [26] is a procedure that is applied within the decision-making process under uncertainty. This Hurwicz criterion strikes a balance between an optimistic and a pessimistic decision. First, take a value between 0 and 1, for the coefficient of realism "α". A decision-maker is optimistic about the future when "α" is close to 1, and the decision-maker is pessimistic about the future when "α" is close to 0. The weighted outcome is given in Eq. (3).

Weighted Outcome $= (1 - \alpha)$ *(minimum payoff)* $+ \alpha$ *(maximum payoff)* (3)

3.5 General Fuzzy MCDM Approach

A general fuzzy multi-criteria decision making problem can be expressed as a decision matrix "$\tilde{D}$" where $\tilde{D} = [x_{ij}]$, where $i = 1, 2, \ldots, m$ and $j = 1, 2, \ldots, n$ and the entry x_{ij} represents the rating of m-alternatives A_i with respect to the n-criteria C_j. and $\tilde{W} = (w_j)$. In general, these criteria can be classified as a Benefit criterion in which higher the value of x_{ij}, better for the decision-maker, or Cost criteria in which lower the value of x_{ij}, better for the decision-maker [27].

3.6 Normalization

This normalization process deals with the criteria having different scales. To particularly obtain a performance matrix $\tilde{P} = [\tilde{P}_{ij}]$, the fuzzy numbers in the decision matrix $\tilde{D}$ are normalized. Where, $\tilde{P}ij$ is given by in Eq. (4).

$$\tilde{P}_{ij} = \begin{cases} \left(\dfrac{xij1}{M}, \dfrac{xij2}{M}, \dfrac{xij3}{M}\right) \\ \left(\dfrac{N - Xij1}{N}, \dfrac{N - Xij2}{N}, \dfrac{N - Xij3}{N}\right) \end{cases}, \quad (4)$$

Here $M = \max(x_{ij3})$, for benefit criteria, $N = \max(x_{ij3})$, for cost criteria.

It should be noted that this normalization process conserves the ranges of normalized triangular fuzzy numbers between 0 and 1 inclusive.

3.7 Weighting the Criteria

To obtain a weighted performance matrix, a weighted vector is multiplied with the fuzzy numbers in the decision matrix as $\tilde{P}^w = [\tilde{P}^w_{ij}]$, Where, $\tilde{P}^w_{ij1} = w_{j1}p_{ij1}$, $\tilde{P}^w_{ij2} = w_{j2}p_{ij2}$, $\tilde{P}^w_{ij3} = w_{j3}p_{ij3}$, $i = 1, 2, 3\ldots, m$ and $j = 1, 2, 3\ldots, n$.

4 Proposed Technique

This section describes a novel decision-making technique for ranking fuzzy numbers that combine the centroid method and the Hurwicz criteria. In order to aid in the decision-making process, the area of region method is used for ranking fuzzy numbers. The proposed decision-making technique is unique in that it combines the centroid method and the Hurwicz criteria for ranking fuzzy numbers which are shown in Fig. 3.
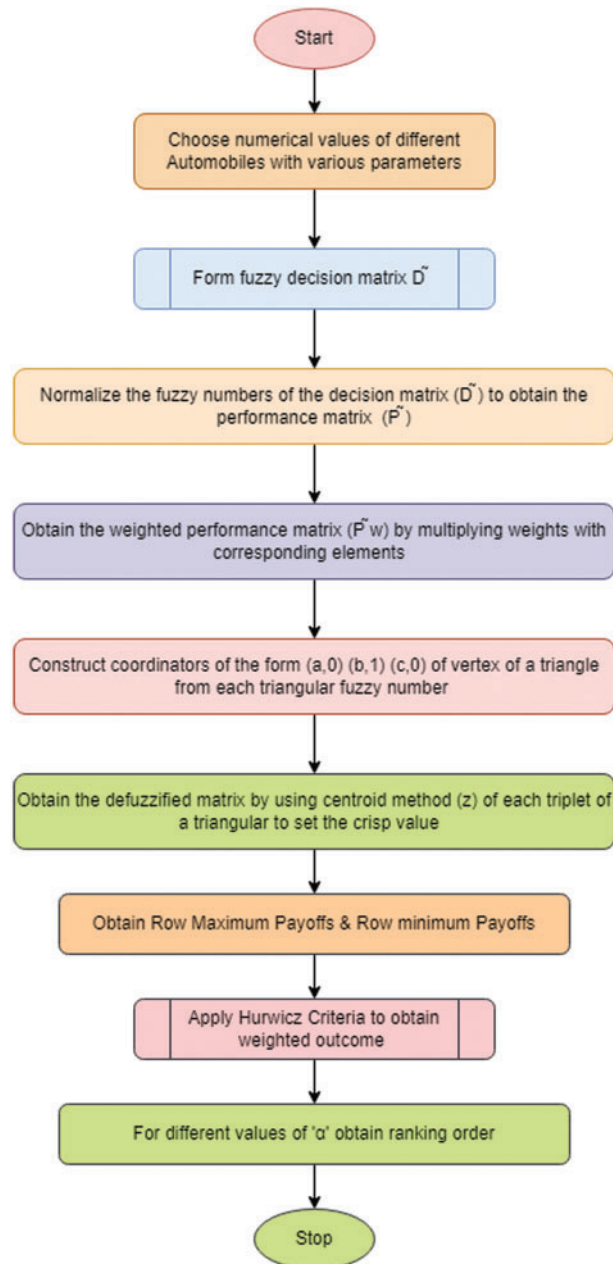
Figure 3: The architecture of the proposed flow diagram

Unlike conventional ranking methods, in this paper, we have extended our research with trapezoidal fuzzy numbers. The process flow diagram is shown in Fig. 3 and step by step process of the proposed technique is mentioned below:

1. Arrange fuzzy numbers of the problem in the form of a fuzzy decision matrix.
2. Obtain the performance matrix by normalizing the element of the decision matrix.
3. Find the weighted performance matrix by multiplying the weight with the corresponding elements of the performance matrix.
4. The centroid method is used after considering each triangular fuzzy number as the vertex of a triangle in the form $(a, 0)$, $(b, 1)$, $(c, 0)$. (Same procedure can be done for the trapezoidal fuzzy numbers as well).
5. To find the weighted outcome for each row applies Hurwicz criteria.
6. The maximum value of the entire weighted outcome will be the best action.

4.1 Case Studies and Analysis of Results

In this section, the proposed method presented in the previous section is applied to a travel problem specification by considering different types of automobiles which include two, three, and four-wheeler vehicles. The decision matrix and its corresponding weight for different types of automobiles are shown in Tab. 1. In this example, the price criterion is measured in dollars and the journey time is measured in minutes. The comfort and satisfaction criterion is the value criterion measured on a scale of one to ten. Suppose that, the ratings are expressed in the decision matrix as triangular fuzzy numbers (for example, a two-wheeler vehicle journey such as a bike journey to the college costs 20 dollars, however, it can be as high as 25 dollars and as low as 15 dollars) [6]. This problem is illustrated with two different assumptions one is triangular fuzzy numbers and the second one is the trapezoidal fuzzy numbers. The numerical values in the fuzzy decision matrix are chosen arbitrary depending upon the type of the automobile as well as the distance e.g., in the case of two-wheeler the expenditure for instance the fuel cost is less (varies from 6 to 12 dollars) as compared to four-wheeler (varies from 15 to 25 dollars), but the four-wheeler is more comfort (its value varies from 7 to 10) as compared to the two-wheeler (its value varies from 2 to 4). The journey time for a two-wheeler (varies from 40 to 110 min) is more as compared to the four-wheeler (varies from 35 to 100 min) so on and so forth [26].

Table 1: Decision matrix

Types of vehicles	Expenditure (in Dollars) $W = 0.4$	Journey time (in min) $W = 0.5$	Satisfaction (1–10) $W = 0.3$	Comfort (1–10) $W = 0.6$
Two Wheeler	(6, 10, 12)	(40, 80, 110)	(2, 4, 6)	(2, 3, 4)
Three Wheeler	(10, 14, 20)	(60, 100, 130)	(4, 5, 7)	(3, 5, 6)
Four Wheeler	(15, 19, 25)	(35, 60, 100)	(5, 7 , 9)	(7, 8, 10)

Step 1. Arrange fuzzy numbers of the problem in the form of a fuzzy decision matrix

Tab. 1 shows ratings in the decision matrix as triangular fuzzy numbers.

Step 2. Obtain the performance matrix by normalizing the element of the decision matrix

Tab. 2 shows a normalized fuzzy decision matrix which is obtained by the normalization process.

Table 2: Performance matrix

Types of vehicles	Expenditure (in Dollars) $W = 0.4$	Journey time (in min) $W = 0.5$	Satisfaction (1–10) $W = 0.3$	Comfort (1–10) $W = 0.6$
Two-Wheeler	(0.52, 0.60, 0.76)	(0.15, 0.38, 0.69)	(0.22, 0.44, 0.66)	(0.20, 0.3, 0.4)
Three-Wheeler	(0.20, 0.44, 0.60)	(0.00, 0.23, 0.54)	(0.44, 0.55, 0.77)	(0.30, 0.50, 0.60)
Four-Wheeler	(0.00, 0.24, 0.40)	(0.23, 0.54, 0.73)	(0.55, 0.77 , 1.00)	(0.70, 0.80, 1.00)

Step 3. Find the weighted performance matrix by multiplying the weight with the corresponding elements of the performance matrix.

Given below is the weighted performance matrix "*A*" which is obtained by multiplying the weight with its corresponding performance matrix.

$$A = \begin{pmatrix} (0.21, 0.24, 0.30) & (0.07, 0.19, 0.34) & (0.07, 0.13, 0.19) & (0.12, 0.18, 0.24) \\ (0.08, 0.18, 0.24) & (0.00, 0.12, 0.27) & (0.13, 0.16, 0.23) & (0.18, 0.30, 0.36) \\ (0.00, 0.09, 0.16) & (0.12, 0.27, 0.36) & (0.16, 0.23, 0.30) & (0.42, 0.48, 0.60) \end{pmatrix}.$$

Here it should be noted that each triangular fuzzy number can be considered as the vertex of a triangle in the form $(a, 0)$, $(b, 1)$, $(c, 0)$. Given below is the triangular fuzzy decision matrix "A".

$A =$

$$\begin{pmatrix} [(0.21, 0) (0.24, 1) (0.30, 0)] & [(0.07, 0) (0.19, 1) (0.34.0)] & [(0.07, 0) (0.13, 1) (0.19, 0)] & [(0.12, 0) (0.18, 1) (0.24, 0)] \\ [(0.08, 0) (0.18, 1) (0.24.0)] & [(0.00, 0) (0.12, 1) (0.27, 0)] & [(0.13, 0) (0.16, 1) (0.23, 0)] & [(0.18, 0) (0.30, 1) (0.36, 0)] \\ [(0.00, 0) (0.09, 1) (0.16, 0)] & [(0.12, 0) (0.27, 1) (0.36, 0)] & [(0.16, 0) (0.23, 1) (0.30, 0)] & [(0.42, 0) (0.48, 1) (0.60, 0)] \end{pmatrix}$$

Step 4. The centroid method is used after considering each triangular fuzzy number as the vertex of a triangle in the form (a, 0), (b, 1), (c, 0). (Same procedure can be done for the trapezoidal fuzzy numbers as well).

The defuzzified matrix in Tab. 3 can be obtained by using the centroid method where each triplet represents a triangle, where the Centroid method is given as $z^* = \dfrac{\int \mu \tilde{A} \ (z) \,.z \ dz}{\int \mu \tilde{A} \ (z) \ dz}$

Table 3: Defuzzified matrix

Types of vehicles	Expenditure $W = 0.4$	Journey time (in min) $W = 0.5$	Satisfaction (1–10) $W = 0.3$	Comfort (1–10) $W = 0.6$
Two-Wheeler	0.2578	0.3333	0.1566	0.1815
Three-Wheeler	0.1654	0.1007	0.1701	0.1669
Four-Wheeler	0.08335	0.1604	0.2288	0.5011

For the first element of matrix *A,* the crisp is: $z^* = \dfrac{\int_{0.21}^{0.24} 33 \, (z - 0.21) \, z \ dz + \int_{0.24}^{0.3} (16z + 0.76) \ z \ dz}{\int_{0.21}^{0.24} 33 \, (z - 0.21) \ dz + \int_{0.24}^{0.3} (16z + 0.76) \ dz}$

$= 0.2578$.

Similarly, all the triplets can be obtained which are mentioned in Tab. 3.

The matrix in Tab. 4 represents the maximum and minimum payoffs.

Table 4: Row maximum and row minimum payoffs

Types of vehicles	Expenditure $W=0.4$	Journey time (in min) $W=0.5$	Satisfaction (1–10) $W=0.3$	Comfort (1–10) $W=0.6$	Row maximum	Row minimum
Two Wheeler	0.2578	0.3333	0.1566	0.1815	0.3333	0.1566
Three Wheeler	0.1654	0.1007	0.1701	0.1669	0.1701	0.1007
Four Wheeler	0.08335	0.1604	0.2288	0.5011	0.5011	0.0833

Step 5. To find the weighted outcome for each row apply Hurwicz criteria

We now use Hurwicz criteria to obtain the weighted outcome which is represented in Tab. 5.

Table 5: Weighted outcome, for different values of α

Types of vehicles	$\alpha = 0$	$\alpha = 0.1$	$\alpha = 0.2$	$\alpha = 0.3$	$\alpha = 0.4$	$\alpha = 0.5$	$\alpha = 0.6$	$\alpha = 0.7$	$\alpha = 0.8$	$\alpha = 0.9$	$\alpha = 1$
Two Wheeler	0.1566	0.1442	0.1919	0.2095	0.2272	0.2449	0.1565	0.2802	0.2979	0.3155	0.3333
Three Wheeler	0.1007	0.1076	0.1145	0.1214	0.1284	0.1353	0.1422	0.1492	0.1561	0.1630	0.1701
Four Wheeler	0.0833	0.1260	0.1668	0.2086	0.2504	0.2921	0.3339	0.3757	0.4175	0.4592	0.5011

Where, *Weighted outcome = (1 − α) (minimum payoff) + α (maximum payoff).*

Tab. 6 below shows the ranking order obtained for different values of α.

Table 6: Ranking order, for different values of α

Types of vehicles	$\alpha = 0$	$\alpha = 0.1$	$\alpha = 0.2$	$\alpha = 0.3$	$\alpha = 0.4$	$\alpha = 0.5$	$\alpha = 0.6$	$\alpha = 0.7$	$\alpha = 0.8$	$\alpha = 0.9$	$\alpha = 1$
Two Wheeler	1	1	1	1	2	2	2	2	2	2	2
Three Wheeler	2	3	3	3	3	3	3	3	3	3	3
Four Wheeler	3	2	2	2	1	1	1	1	1	1	1

The ranking order from Tab. 6 shows that with a fair (or) extreme optimistic approach the decision-maker will choose the four-wheeler vehicle as to the first alternative and the two-wheeler vehicle as to the second alternative.

4.2 For Trapezoidal Fuzzy Numbers

Tab. 7 represents the ratings in the decision matrix as trapezoidal fuzzy numbers. Three types of vehicles are considered. They are i) Two-wheeler, ii) Three-wheeler and iii) Four-wheeler. The parameters or attributes considered are expenditure, journey time, satisfaction, and comfort.

Tab. 8 represents the normalized fuzzy decision matrix.

Table 7: Decision matrix

Types of vehicles	Expenditure (in Dollars) $W = 0.4$	Journey time (in min) $W = 0.5$	Satisfaction (1–10) $W = 0.3$	Comfort (1–10) $W = 0.6$
Two Wheeler	(6, 8, 10, 12)	(40, 60, 80, 110)	(2, 3, 4, 6)	(2, 3, 3, 4)
Three Wheeler	(10, 14, 16, 20)	(60, 80, 100, 130)	(4, 5, 6, 7)	(3, 4, 5, 6)
Four Wheeler	(15, 19, 23, 25)	(35, 60, 80, 100)	(5, 7, 8, 9)	(7, 8, 9, 10)

Table 8: Performance matrix

Types of vehicles	Expenditure (in Dollars) $W = 0.4$	Journey time (in min) $W = 0.5$	Satisfaction (1–10) $W = 0.3$	Comfort (1–10) $W = 0.6$
Two-Wheeler	(0.52, 0.60,.68, 0.76)	(0.15, 0.38, 0.53, 0.69)	(0.22, 0.33, 0.44, 0.66)	(0.20, 0.3, 0.3, 0.4)
Three-Wheeler	(0.20, 0.36, 0.44, 0.6)	(0.0, 0.23, 0.38, 0.54)	(0.44, 0.55, 0.66, 0.77)	(0.30, 0.4, 0.5, 0.6)
Four-Wheeler	(0.0, 0.08, 0.24, 0.40)	(0.23, 0.38, 0.54, 0.73)	(0.55, 0.77, 0.88, 1.0)	(0.7, 0.8, 0.9, 1.0)

Given below is the weighted performance matrix "B" which is obtained by multiplying the weight with its corresponding performance matrix.

$$B = \begin{pmatrix} (0.21, 0.24, 0.27, 0.30) & (0.07, 0.19, 0.26, 0.34) & (0.07, 0.09, 0.13, 0.19) & (0.12, 0.18, 0.18, 0.24) \\ (0.08, 0.14, 0.18, 0.24) & (0.00, 0.12, 0.19, 0.27) & (0.13, 0.16, 0.19, 0.23) & (0.18, 0.24, 0.30, 0.36) \\ (0.00, 0.03, 0.09, 0.16) & (0.12, 0.19, 0.27, 0.36) & (0.16, 0.23, 0.26, 0.30) & (0.42, 0.48, 0.54, 0.60) \end{pmatrix}.$$

Here, it should be noted that each triangular fuzzy number can be considered as the vertex of a triangle in the form $(a, 0), (b, 1), (c, 1), (d, 0)$. Given below is the trapezoidal fuzzy decision matrix "B".

$B =$

$$\begin{pmatrix} [(0.21,0)\,(0.24,1)\,(0.27,1)\,(0.30,0)] & [(0.07,0)\,(0.19,1)\,(0.26,1)\,(0.34,0)] & [(0.07,0)\,(0.09,1)\,(0.13,1)\,(0.19,0)] & [(0.12,0)\,(0.18,1)\,(0.18,1)\,(0.24,0)] \\ [(0.08,0)\,(0.14,1)\,(0.18,1)\,(0.24,0)] & [(0.00,0)\,(0.12,1)\,(0.19,1)\,(0.27,0)] & [(0.13,0)\,(0.16,1)\,(0.19,1)\,(0.23,0)] & [(0.18,0)\,(0.24,1)\,(0.30,1)\,(0.36,0)] \\ [(0.00,0)\,(0.03,1)\,(0.09,1)\,(0.16,0)] & [(0.12,0)\,(0.19,1)\,(0.27,1)\,(0.36,0)] & [(0.16,0)\,(0.23,1)\,(0.26,1)\,(0.30,0)] & [(0.42,0)\,(0.48,1)\,(0.54,1)\,(0.60,0)] \end{pmatrix}$$

The defuzzified matrix in Tab. 9 can be obtained by using the centroid method where each of the four tuples represents vertices of a quadrilateral.

Table 9: Defuzzified matrix

Types of vehicles	Expenditure $W = 0.4$	Journey time (in min) $W = 0.5$	Satisfaction (1–10) $W = 0.3$	Comfort (1–10) $W = 0.6$
Two-Wheeler	0.193	0.082	0.452	0.181
Three-Wheeler	0.095	0.152	0.183	0.261
Four-Wheeler	0.076	0.223	0.405	0.521

The matrix in Tab. 10 represents the maximum and minimum payoffs.

Table 10: Row maximum and row minimum payoffs

Types of vehicles	Expenditure $W = 0.4$	Journey time (in min) $W = 0.5$	Satisfaction (1–10) $W = 0.3$	Comfort (1–10) $W = 0.6$	Row Maximum	Row Minimum
Two-Wheeler	0.193	0.082	0.452	0.181	0.452	0.082
Three-Wheeler	0.095	0.152	0.183	0.261	0.261	0.095
Four-Wheeler	0.076	0.223	0.405	0.521	0.521	0.076

We now use Hurwicz criteria to obtain the weighted outcome which is represented in Tab. 11. The expression for the weighted outcome is considered as *Weighted outcome = (1 − α) (minimum payoff) + α (maximum payoff)*.

Table 11: Weighted outcome, for different values of α

Types of vehicles	$\alpha = 0$	$\alpha = 0.1$	$\alpha = 0.2$	$\alpha = 0.3$	$\alpha = 0.4$	$\alpha = 0.5$	$\alpha = 0.6$	$\alpha = 0.7$	$\alpha = 0.8$	$\alpha = 0.9$	$\alpha = 1$
Two-Wheeler	0.082	0.119	0.156	0.192	0.229	0.267	0.303	0.340	0.377	0.414	0.452
Three-Wheeler	0.095	0.111	0.128	0.144	0.161	0.177	0.194	0.210	0.227	0.234	0.261
Four-Wheeler	0.076	0.120	0.165	0.209	0.256	0.298	0.342	0.386	0.431	0.475	0.521

Step 6 Maximum value of the entire weighted outcome will be the best action. Finally, Tab. 12 below shows the ranking order obtained for different values of α.

Table 12: Ranking order, for different values of α

Types of vehicles	$\alpha = 0$	$\alpha = 0.1$	$\alpha = 0.2$	$\alpha = 0.3$	$\alpha = 0.4$	$\alpha = 0.5$	$\alpha = 0.6$	$\alpha = 0.7$	$\alpha = 0.8$	$\alpha = 0.9$	$\alpha = 1$
Two-Wheeler	2	2	2	2	2	2	2	2	2	2	2
Three-Wheeler	1	3	3	3	3	3	3	3	3	3	3
Four-Wheeler	3	1	1	1	1	1	1	1	1	1	1

The ranking order from Tab. 12 shows that with a fair (or) extreme optimistic approach the decision-maker may choose the four-wheeler vehicle as to the first alternative and the two-wheeler vehicle as to the second alternative. It should be noted that the ranking order for triangular fuzzy numbers as well as for trapezoidal fuzzy numbers is the same for this example.

4.3 Comparison with State-of-the-art Methods

This section compares our ranking method to those proposed by Lee-Kwang et al. [26], Baldwin et al. [27], Ibrahim et al. [28], Ghoneim et al. [29], Eid et al. [30], and Peneva et al. [31]. It's worth noting that the pessimistic, neutral, and optimistic targets correspond to Lee-Kwang et al. method [26]. The fuzzy sets considered for comparison are shown in Fig. 4, and the comparative results are shown in Tab. 13.
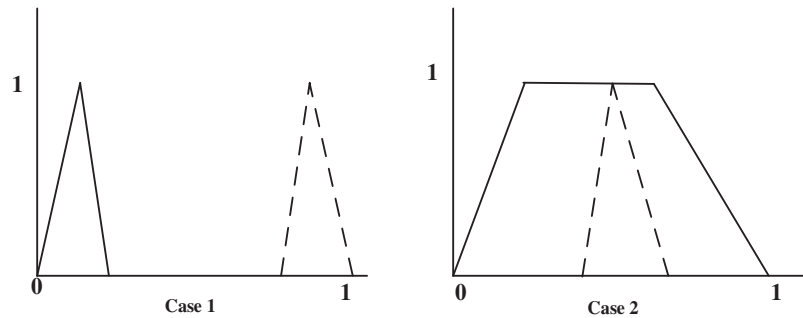
Figure 4: Fuzzy sets considered for comparison

Table 13: Comparison of ranking methods with other methods

Methods	Target	Case 1		Case 2	
		A	B	A	B
Lee-Kwang et al. [26]	Pessimistic	0.34	0.99	0.74	0.77
	Neutral	0.10	0.90	0.40	0.40
	Optimistic	0.01	0.66	0.16	0.09
Baldwin et al. [27]	Pessimistic	0.19	0.99	0.61	0.64
	Neutral	0.10	0.90	0.41	0.40
	Optimistic	0.01	0.81	0.22	0.16
Ibrahim et al. [28]	$K = 0.5$	0.11	0.69	0.33	0.46
	$K = 1$	0.00	0.82	0.43	0.42
	$K = 2$	0.00	0.82	0.47	0.22
Ghoneim et al. [29]	$K = 0.5$	0.40	0.95	0.78	0.66
	$K = 1$	0.18	0.90	0.76	0.45
	$K = 2$	0.03	0.84	0.54	0.23
Eid et al. [30]	$K = 0.5$	0.05	0.85	0.05	0.35
	$K = 1$	0.10	0.90	0.40	0.40
	$K = 2$	0.15	0.95	0..75	0.45
Peneva et al. [31]	$K = 0.5$	0.09	0.82	0.09	0.33
	$K = 1$	0.14	0.86	0..38	0.39
	$K = 2$	0.18	0.91	0.67	0.45
Proposed method	$K = 0.5$	0.15	0.90	0.30	0.40
	$K = 1$	0.15	0.90	0.46	0.40
	$K = 2$	0.15	0.90	0.63	0.40

From the outcomes, it should be underscored here that though all considered ranking methods permit an adjustment of the assessment methodology, it is challenging to see plainly how the difference in parameters in the last five strategies reflects the decision-makers' attitude of evaluation. In Lee-Kwang et al. [26] and our techniques, fuzzy targets have an unmistakable semantics related to

a well-deciphered assessment system, and henceforth, the adjustment of target reflects obviously and straightforwardly the comparing change in the demeanor of the decision-maker.

5 Conclusions

In this paper, to support the decision-making process, a new hybrid approach is used for ranking and defuzzify the fuzzy numbers into crisp numbers. The proposed method takes into consideration all types of decision makers' perspectives which is crucial in solving decision-making problems. Hurwicz criteria are used under different risk attitudes of the decision-maker to find the best action. The defuzzification process is done by using the center of gravity (COG) or centroid method which is one of the best methods among several defuzzification methods that converts the triangular fuzzy quantity as well as the trapezoidal fuzzy quantity into a crisp quantity. Through a case study, it is shown that the proposed method provides strong discrimination ability which supports the decision-making process.

Conflicts of Interest: The authors declare that they have no conflicts of interest to report regarding the present study.

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