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Article

Novel Computing for the Delay Differential Two-Prey and One-Predator System

Prem Junsawang[1], Zulqurnain Sabir[2], Muhammad Asif Zahoor Raja[3], Soheil Salahshour[4], Thongchai Botmart[5,*] and Wajaree Weera[3]

[1]Department of Statistics, Faculty of Science, Khon Kaen University, Khon Kaen, 40002, Thailand
[2]Department of Mathematics and Statistics, Hazara University, Mansehra, Pakistan
[3]Future Technology Research Center, National Yunlin University of Science and Technology, Douliou, 64002, Taiwan
[4]Faculty of Engineering and Natural Sciences, Bahcesehir University, Istanbul, Turkey
[5]Department of Mathematics, Faculty of Science, Khon Kaen University, Khon Kaen, 40002, Thailand
*Corresponding Author: Thongchai Botmart. Email: thongbo@kku.ac.th
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Abstract: The aim of these investigations is to find the numerical performances of the delay differential two-prey and one-predator system. The delay differential models are very significant and always difficult to solve the dynamical kind of ecological nonlinear two-prey and one-predator system. Therefore, a stochastic numerical paradigm based artificial neural network (ANN) along with the Levenberg-Marquardt backpropagation (L-MB) neural networks (NNs), i.e., L-MBNNs is proposed to solve the dynamical two-prey and one-predator model. Three different cases based on the dynamical two-prey and one-predator system have been discussed to check the correctness of the L-MBNNs. The statistic measures of these outcomes of the dynamical two-prey and one-predator model are chosen as 13% for testing, 12% for authorization and 75% for training. The exactness of the proposed results of L-MBNNs approach for solving the dynamical two-prey and one-predator model is observed with the comparison of the Runge-Kutta method with absolute error ranges between 10^{-05} to 10^{-07}. To check the validation, constancy, validity, exactness, competence of the L-MBNNs, the obtained state transitions (STs), regression actions, correlation presentations, MSE and error histograms (EHs) are also provided.

Keywords: Delay differential model; dynamical system; prey-predator; Levenberg-Marquardt backpropagation; MSE; neural networks

1 Introduction

In the population ecology, a predator-prey dynamical system is considered one of the significant factors. It provides the different species distributions in the ecological model and in a few variations, it predicts the extinction or abundance of various classes. Based on the environmental effects, the prey and the predators share different relationships amongst themselves. Mutualism and competition are two significant communications among the various species. Krebs performed the competition, when two classes share similar harm or resources to each other for finding possessions [1]. The mutualism is a longstanding, close connotation between two classes that can benefit both the partners. The fundamental basic predator-prey model is the Lotka-Volterra, which was anticipated to clarify the oscillating levels of confident fish in the Adriatic Ocean during the World War-1 [2]. A few predator-prey models is examined by the researchers with the traditional way by Volterra [3] and Lotka [4], particularly the system of predator-prey describe the communication among numerous species, because of more multifaceted associations among classes. Meng et al. [5] discussed the mathematical formulation of the one-predator and two-prey models. Mukhopadhyay et al. [6] examined the effects of interference and harvesting of the predator for a system that consist of a single prey and two modest predators. Upadhyay et al. [7] investigated the disaster in the ecological model to examine the chaotic subtleties. Beside the three species dynamics of predator-prey system, which has been investigated by numerous researchers [8–12].

The delay differential kind of systems have a long history based on the modeling of predator-prey to consider and represent the essential feeding time, growth period, reaction time [13–19]. Ignoring the time-delays in various species of dynamical system mean disregarding the authenticity [20]. To introduce the time-delays in dynamical systems have more complex than ordinary systems, as it can undermine the equilibrium points and increase the limit stable cycle [21]. Kundu et al. expressed a predator-prey three species system with the cooperation of prey to consider the multi-delay system [22]. They investigated the time-delay impacts of the system and proved the appropriate conditions to exist the Hopf bifurcation by selecting the time-delays. Rihan et al. [23] discussed and examined one-predator and two-prey system with two-discrete of delays. They investigated the qualitative conduct of system, where the evolution of the populations of both prey is exposed to Allee impact.

Functional response is one of the significant components in the dynamics of the predator-prey. Holling [24] discussed the three forms of functional comebacks, like as the form of Holling 1, 2 and 3, known as the functional responses of prey. The types of Hassell-Varley, Beddington-DeAngelis and ratio dependent are known as the functional responses of the predator [25]. Mishra et al. [26] examined the predator-prey model with the involvement of one-predator and two-prey using the types of Holling II and Monod-Haldane. They supposed the 1st prey is perilous, and the 2nd prey is innocent for the predator. Moreover, prey-predator networks have been extended to food chain systems by using various responses of the function [27–29].

In the natural creation, each species endures in the wild, few live flocks, alone, packs, schools, hives, and herds. Few kinds of animals show the best system to active is to live close to the animals. The best teamwork cooperation is achieved, when the requirements of distinct member are satisfied. Furthermore, cooperating and forming a team is an individual tool with a member of the group always achieved good results and justifies their requirements effortlessly. The two major advantages have been noticed to design a team for the animals, reduction of predation risk and the food finding as a team than doing together. Consequently, it is stimulating to examine the system of predator-prey with the postulation of predation that the prey's members help one another. Elettreby [30] discussed a one-predator and two-prey model in which the team of prey help one another and investigated the local as well as global constancy of the addressed network. Tripathi et al. [31] described two-predators and one-predator system in which prey team help each other in the predator's presence, whereas participate in the nonappearance of predator. The authors performed two-prey and one-predator competitive system with the functional response of Beddington-DeAngelis and they examined durability as well as a Hopf bifurcation of the network.

The current study is to find the numerical performances of the delay differential two-prey and one-predator system. The delay differential models are considered very significant and difficult to solve these the dynamical ecological types of nonlinear two-prey and one-predator system. Therefore, a stochastic paradigm based artificial neural network (ANN) along with the Levenberg-Marquardt backpropagation (L-MB) neural networks (NNs), i.e., L-MBNNs is presented to solve the dynamical two-prey and one-predator system.

The remaining sections are provided as: The presentation of the mathematical model is described in Section 2. The stochastic solvers along with the novel features are shown in Section 3. The proposed structure is presented in Section 4. The numerical results are provided in Section 5. The conclusions are reported in the last Section.

2 Mathematical Form of the Delay Differential Two-Prey and One-Predator System Model

In this section, the classification of the mathematical form of the delay differential dynamical kind of ecological two-prey and one-predator model is presented. The mathematical representation of the ecological model is given as:

$$\begin{cases} \dfrac{du(y)}{dy} = \alpha_1\left(1-\dfrac{u(y)}{k_1}\right)u(y) - \eta_1 u(y)\,v(y) - \dfrac{\sigma_1 u(y)\,p(y-\tau_1)}{\gamma_1+\xi u^2(y)} + \rho_1 u(y)\,v(y)\,p(y)\; u(0)=i_1, \\ \dfrac{dv(y)}{dy} = \alpha_2\left(1-\dfrac{v(y)}{k_2}\right)v(y) - \eta_2 u(y)\,v(y) - \dfrac{\sigma_2 v(y)\,p(y-\tau_2)}{\gamma_1+v(y)} + \rho_2 u(y)\,v(y)\,p(y)\; v(0)=i_2, \\ \dfrac{dp(y)}{dy} = \dfrac{\mu\sigma_1 u(y)\,p(y-\tau_1)}{\gamma_1+\xi u^2(y)} + \dfrac{\mu\sigma_2 v(y)\,p(y-\tau_2)}{\gamma_2+v(y)} - \delta_1 p(y) - \delta_2 p^2(y)\,p(0)=i_3. \end{cases} \tag{1}$$

The prey-predator model has been solved by using the stochastic procedures [32–33]. But the delay differential form of the ecological two-prey and one-predator system has never been solved by using the applications of stochastic computing numerical schemes. The detailed parameters used in the above system are provided in the Tab. 1.

Table 1: Parameter details of the ecological two-prey and one-predator system

Parameters	Details
τ_1, τ_2	Discrete time delays
α_1, α_2	Fundamental growth rate of the preys
η_1, η_2	Competition coefficient
μ	Same rate of transformation of predator to preys
γ_1, γ_2	Intra precise components of $u(y)$ and $v(y)$
ρ_1, ρ_2	Cooperation rate of preys
σ_1, σ_2	Predation rate
ξ	Inverse ration of inhibitory impacts
k_1, k_2	Carrying size for $u(y)$ and $v(y)$
y	Time
δ_1	Predator's death rate
$i_1,\ i_2, i_3$	ICs
δ_2	Intra-species competition rate in the predator

3 Novel Features and Stochastic Applications

In this section, the stochastic L-MBNNs is presented to solve the dynamical two-prey and one-predator form. The local and global form of the stochastic solvers has been provided to present the numerical performances of the singular, stiff, nonlinear, and complicated and dynamical system [34–36]. Some recent submissions of the stochastic computational solvers are 4th order singular models [37,38], periodic systems [39,40], UAV-based traffic monitoring [41] food-chain systems [42], HIV nonlinear systems [43] and differential form of the smoke models [44].

The solution of the dynamical two-prey and one-predator system has been presented by using the stochastic L-MBNNs procedures. The novel features of the proposed study are described as:

- A numerical computing stochastic L-MBNNs technique is proposed to solve the dynamical two-prey and one-predator form.
- The stochastic computing techniques implemented effectively to solve the dynamical two-prey and one-predator form.
- Three different cases of the dynamical two-prey and one-predator system have been discussed to check the correctness of the L-MBNNs.
- The brilliance and perfection of the proposed stochastic L-MBNNs technique is checked with the comparison of the reference (Runge–Kutta) solutions.
- The performance and accuracy of stochastic L-MBNNs technique is checked based on the absolute error (AE) for the dynamical two-prey and one-predator system.
- The performances based STs, MSE, regression, EHs and correlation signify the dependability of the L-MBNNs technique for the dynamical two-prey and one-predator system.

4 Proposed L-MBNNs Structures

This section of the study shows the structure of the L-MBNNs technique for the dynamical two-prey and one-predator system. The methodology based on the stochastic schemes is described as:

- The significant operator performances-based L-MBNNs technique is provided.
- The execution performances of the L-MBNNs technique is provided to solve the two-prey and one-predator system.

Fig. 1 represents the optimization procedures based on the multi-layer performances of the L-MBNNs technique. The L-MBNNs technique is provided by the data selection as 13% for testing, 12% for authorization and 75% for training.

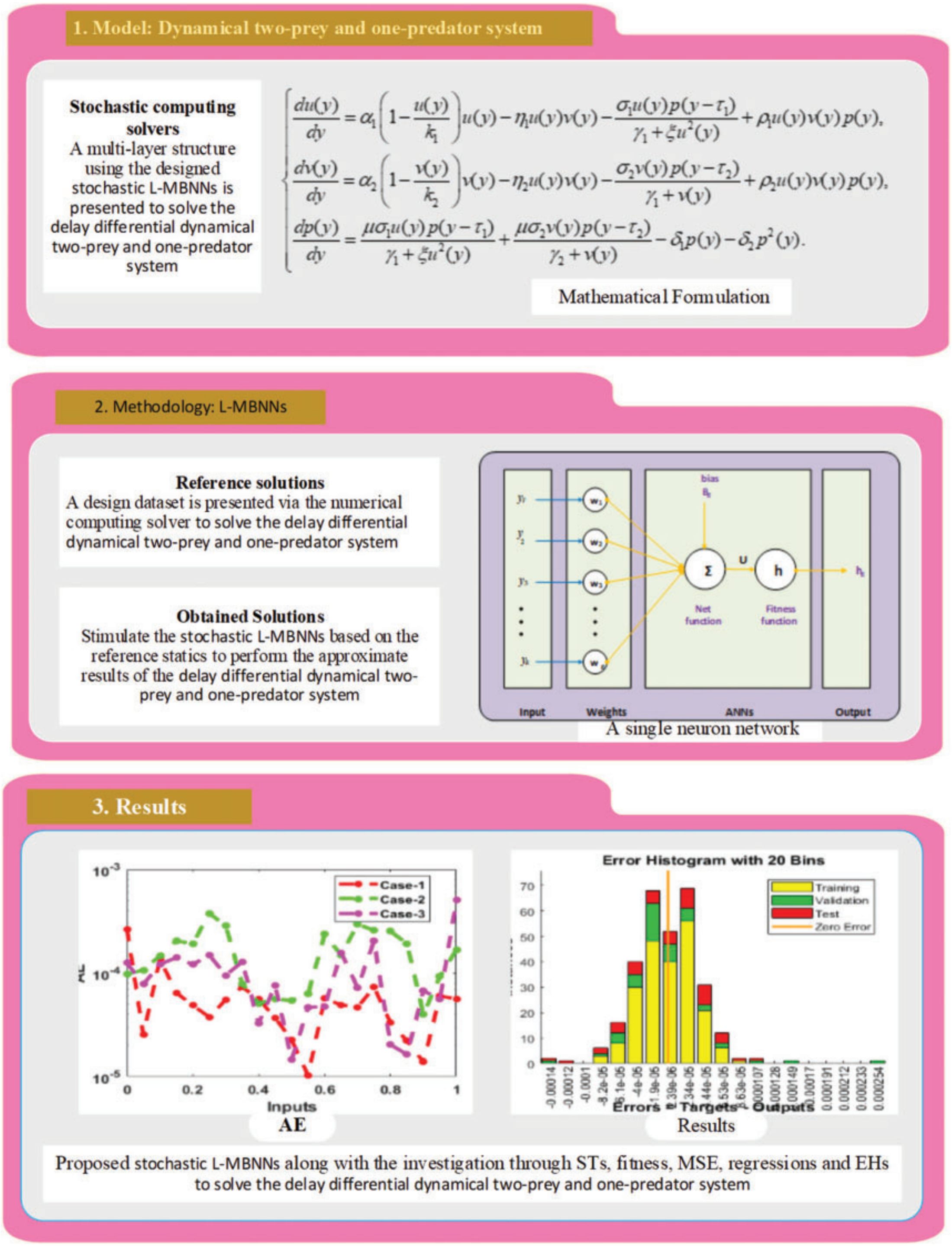


Figure 1: Structure of L-MBNNs technique for the dynamical form of two-prey and one-predator

5 Results and Discussions

This section shows the three different cases based on the dynamical form of two-prey and one-predator using the L-MBNNs. The mathematical representation of each variation is presented as:

Case 1: Consider a dynamical two-prey and one-predator system is discussed by using $\alpha_1 = 0.05$, $k_1 = 1$, $\eta_1 = 0.1$, $\sigma_1 = 0.15$, $\tau_1 = 1$, $\sigma_2 = 0.25$, $\gamma_1 = 0.2$, $\alpha_2 = 0.1$, $k_2 = 2$, $\eta_2 = 0.2$, $\tau_2 = 1$, $\gamma_2 = 0.3$, $\xi = 0.35$, $\mu = 0.45$, $\sigma_1 = 0.12$, $\delta_2 = 0.24$, $i_1 = 0.1$, $i_2 = 0.2$ and $i_3 = 0.3$ are shown as:

$$\begin{cases} \dfrac{du(y)}{dy} = 0.05(1-u(y))u(y) - 0.1u(y)v(y) - \dfrac{\sigma_1 u(y)p(y-1)}{0.2+0.35u^2(y)} + 0.2u(y)v(y)p(y)\, u(0) = 0.1, \\ \dfrac{dv(y)}{dy} = 0.1\left(1-\dfrac{v(y)}{2}\right)v(y) - 0.2u(y)v(y) - \dfrac{\sigma_2 v(y)p(y-1)}{0.3+v(y)} + 0.3u(y)v(y)p(y)\, v(0) = 0.2, \\ \dfrac{dp(y)}{dy} = \dfrac{0.054_1 u(y)p(y-1)}{0.2+0.35u^2(y)} + \dfrac{0.108v(y)p(y-\tau_2)}{0.3+v(y)} - 0.12p(y) - 0.24p^2(y)\, p(0) = 0.3. \end{cases} \tag{2}$$

Case 2: Consider a dynamical two-prey and one-predator system is discussed by using $\alpha_1 = 0.05$, $k_1 = 1$, $\eta_1 = 0.1$, $\sigma_1 = 0.15$, $\tau_1 = 1$, $\sigma_2 = 0.25$, $\gamma_1 = 0.2$, $\alpha_2 = 0.1$, $k_2 = 2$, $\eta_2 = 0.2$, $\tau_2 = 1$, $\gamma_2 = 0.3$, $\xi = 0.35$, $\mu = 0.45$, $\sigma_1 = 0.12$, $\delta_2 = 0.24$, $i_1 = 0.15$, $i_2 = 0.25$ and $i_3 = 0.35$ are shown as:

$$\begin{cases} \dfrac{du(y)}{dy} = 0.05(1-u(y))u(y) - 0.1u(y)v(y) - \dfrac{\sigma_1 u(y)p(y-1)}{0.2+0.35u^2(y)} + 0.2u(y)v(y)p(y)\, u(0) = 0.15, \\ \dfrac{dv(y)}{dy} = 0.1\left(1-\dfrac{v(y)}{2}\right)v(y) - 0.2u(y)v(y) - \dfrac{\sigma_2 v(y)p(y-1)}{0.3+v(y)} + 0.3u(y)v(y)p(y)\, v(0) = 0.25, \\ \dfrac{dp(y)}{dy} = \dfrac{0.054_1 u(y)p(y-1)}{0.2+0.35u^2(y)} + \dfrac{0.108v(y)p(y-\tau_2)}{0.3+v(y)} - 0.12p(y) - 0.24p^2(y)\, p(0) = 0.35. \end{cases} \tag{3}$$

Case 3: Consider a dynamical two-prey and one-predator system is discussed by using $\alpha_1 = 0.05$, $k_1 = 1$, $\eta_1 = 0.1$, $\sigma_1 = 0.15$, $\tau_1 = 1$, $\sigma_2 = 0.25$, $\gamma_1 = 0.2$, $\alpha_2 = 0.1$, $k_2 = 2$, $\eta_2 = 0.2$, $\tau_2 = 1$, $\gamma_2 = 0.3$, $\xi = 0.35$, $\mu = 0.45$, $\sigma_1 = 0.12$, $\delta_2 = 0.24$, $i_1 = 0.2$, $i_2 = 0.3$ and $i_3 = 0.4$ are shown as:

$$\begin{cases} \dfrac{du(y)}{dy} = 0.05(1-u(y))u(y) - 0.1u(y)v(y) - \dfrac{\sigma_1 u(y)p(y-1)}{0.2+0.35u^2(y)} + 0.2u(y)v(y)p(y)\, u(0) = 0.2, \\ \dfrac{dv(y)}{dy} = 0.1\left(1-\dfrac{v(y)}{2}\right)v(y) - 0.2u(y)v(y) - \dfrac{\sigma_2 v(y)p(y-1)}{0.3+v(y)} + 0.3u(y)v(y)p(y)\, v(0) = 0.3, \\ \dfrac{dp(y)}{dy} = \dfrac{0.054_1 u(y)p(y-1)}{0.2+0.35u^2(y)} + \dfrac{0.108v(y)p(y-\tau_2)}{0.3+v(y)} - 0.12p(y) - 0.24p^2(y)\, p(0) = 0.4. \end{cases} \tag{4}$$

The numerical representations using the performances of the dynamical two-prey and one-predator system is presented by using the L-MBNNs technique. 16 numbers of neurons have been used to solve the dynamical form of two-prey and one-predator along with the selection of data as 13% for testing, 12% for authorization and 75% for training. The hidden, output and input layer construction is exemplified in Fig. 2.

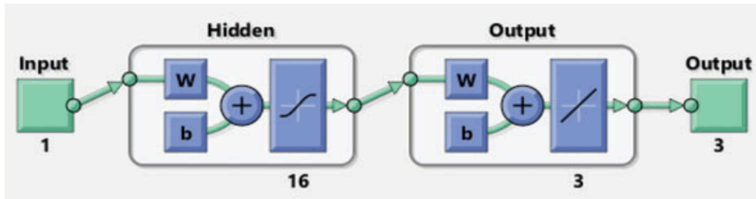


Figure 2: Proposed L-MBNNs technique to solve the dynamical form of two-prey and one-predator

The numerical results have been plotted in Figs. 3 to 5 to solve the dynamical form of two-prey and one-predator by using the proposed L-MBNNs. The STs and best results have been performed in Figs. 3 and 4. The values based on the STs and MSE for verification, best curves and training are provided in Fig. 3 to solve the system. The best achieved performances of the dynamical two-prey and one-predator system have been measured at iterations 14, 10 and 12 and calculated at 3.8521×10^{-09}, 3.7202×10^{-08} and 8.1111×10^{-09}, respectively. The gradient performances have been plotted in Fig. 3 for the delay differential based dynamical form of two-prey and one-predator. These gradient measures have been performed as 2.3067×10^{-09}, 2.3637×10^{-08} and 1.8131×10^{-08} for each case of the delay differential system. These graphical representations indicate the convergence of designed L-MBNNs technique to solve the dynamical two-prey and one-predator system.

Fig. 4 represents the performances of the fitting cure plots to solve the dynamical form of two-prey and one-predator. These graphical measures show the result comparisons of each case of the delay differential dynamical model. The error plots using the substantiation, testing, and training have been presented to solve the delay differential dynamical model based on the designed L-MBNNs technique. The EHs illustrations along with the regression are plotted in Fig. 4 for the dynamical form of two-prey and one-predator using the designed L-MBNNs technique. The EHs have been calculated as 2.39×10^{-06}, 1.70×10^{-05} and 1.75×10^{-06} for each case of the dynamical two-prey and one-predator system using the designed L-MBNNs technique.

Fig. 5 shows the correlation to authenticate the performance of regression. One can notice that the values of the correlation are calculated as 1 for each case of the dynamical two-prey and one-predator system. The training, substantiation and testing values designate the precision and accuracy of the L-MBNNs technique to solve the delay differential dynamical model. The convergence based MSE based on the training, testing, verification, complexity, iterations, and backpropagation is provided in Tab. 1 to solve the delay differential dynamical model using the L-MBNNs technique.

Figs. 6 and 7 represents the comparison of the results and AE for the dynamical two-prey and one-predator system using the designed L-MBNNs procedure. These numerical values have been used to perform the correctness of the designed numerical L-MBNNs procedure for the dynamical form of two-prey and one-predator. The comparison of the achieved performances and the reference solutions are provided in Fig. 6 and the overlapping of the results is performed. This comparison authenticates the exactness of the designed L-MBNNs procedure for the delay differential model. The AE performances for the delay differential model using the stochastic L-MBNNs procedure is plotted in Fig. 7. The AE for $u(y)$ calculated as 10^{-05} to 10^{-06}, 10^{-03} to 10^{-05} and 10^{-04} to 10^{-05} for case 1, 2 and 3 of the nonlinear delayed differential dynamical model. The AE for $v(y)$ calculated as 10^{-05} to 10^{-06}, 10^{-03} to 10^{-05} and 10^{-04} to 10^{-06} for case 1, 2 and 3 of the nonlinear delayed differential dynamical model. Likewise, the AE for $p(y)$ calculated as 10^{-04} to 10^{-06} for each case of the nonlinear delayed differential dynamical model.

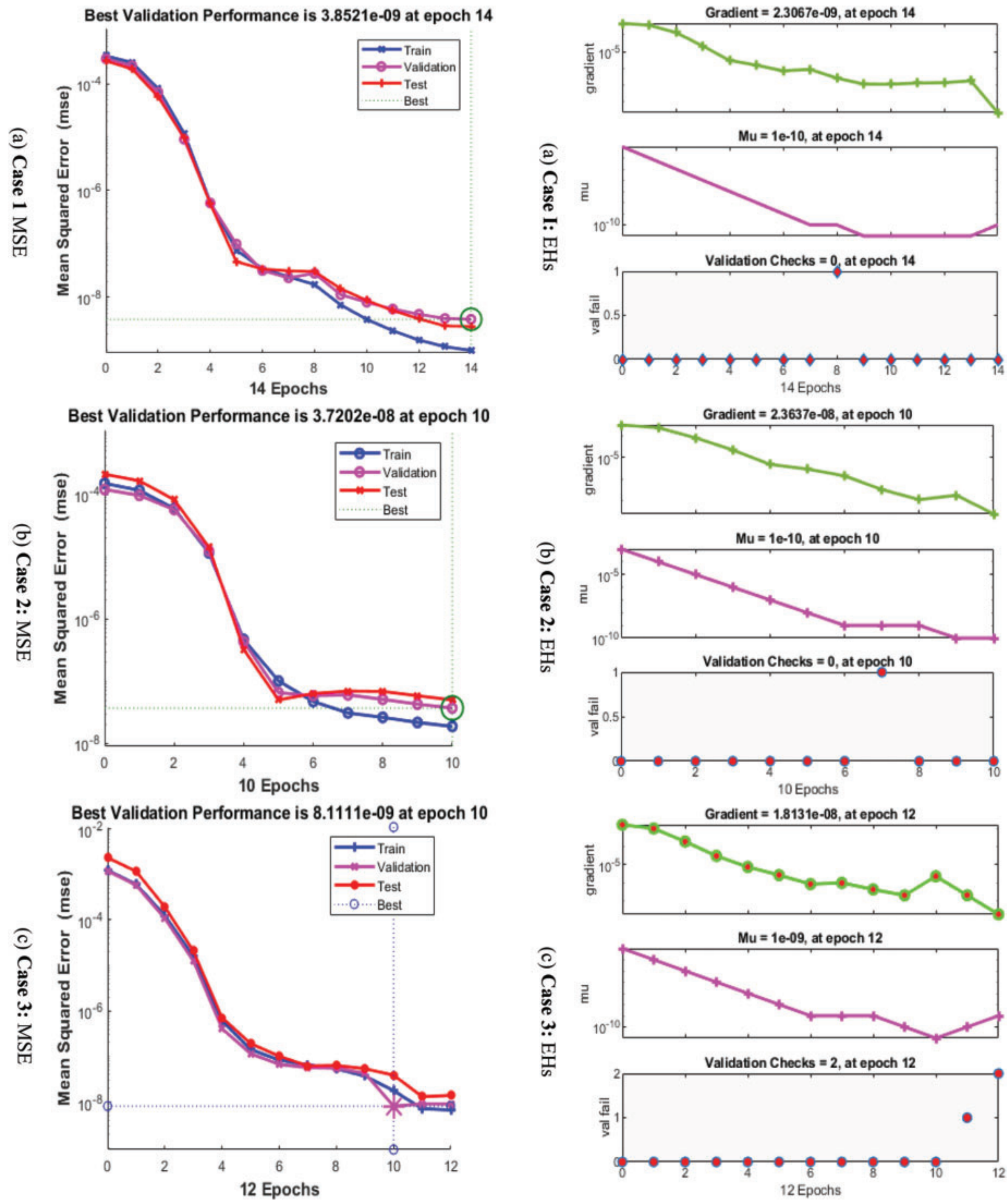


Figure 3: STs and MSE for the dynamical form of two-prey and one-predator using the designed L-MBNNs procedure

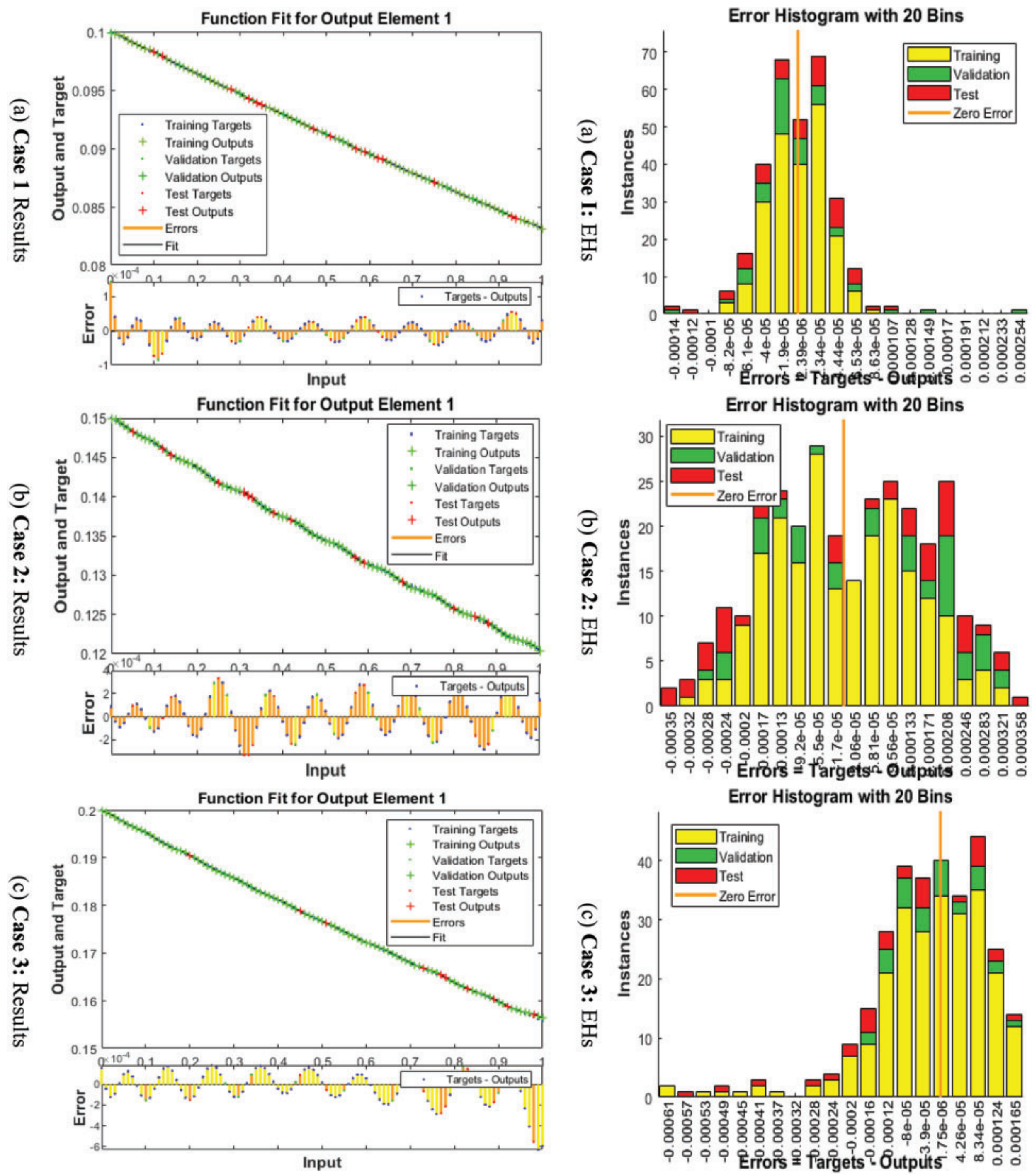
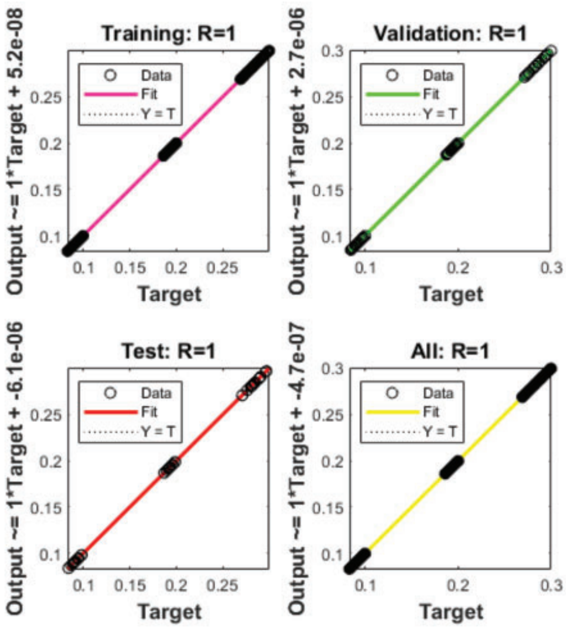
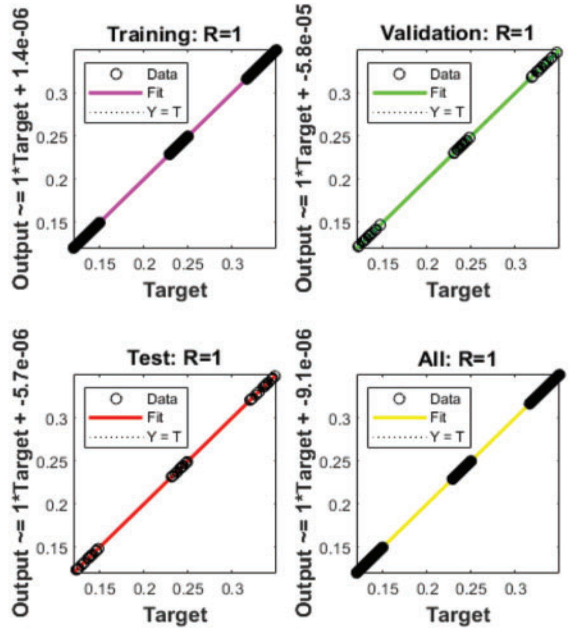


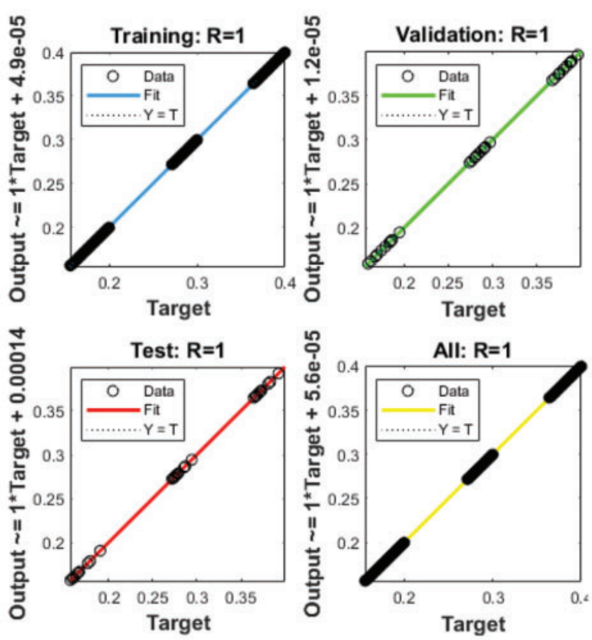
Figure 4: Results and EHs for the dynamical form of two-prey and one-predator using the designed L-MBNNs procedure



(a) Regression: **Case 1**

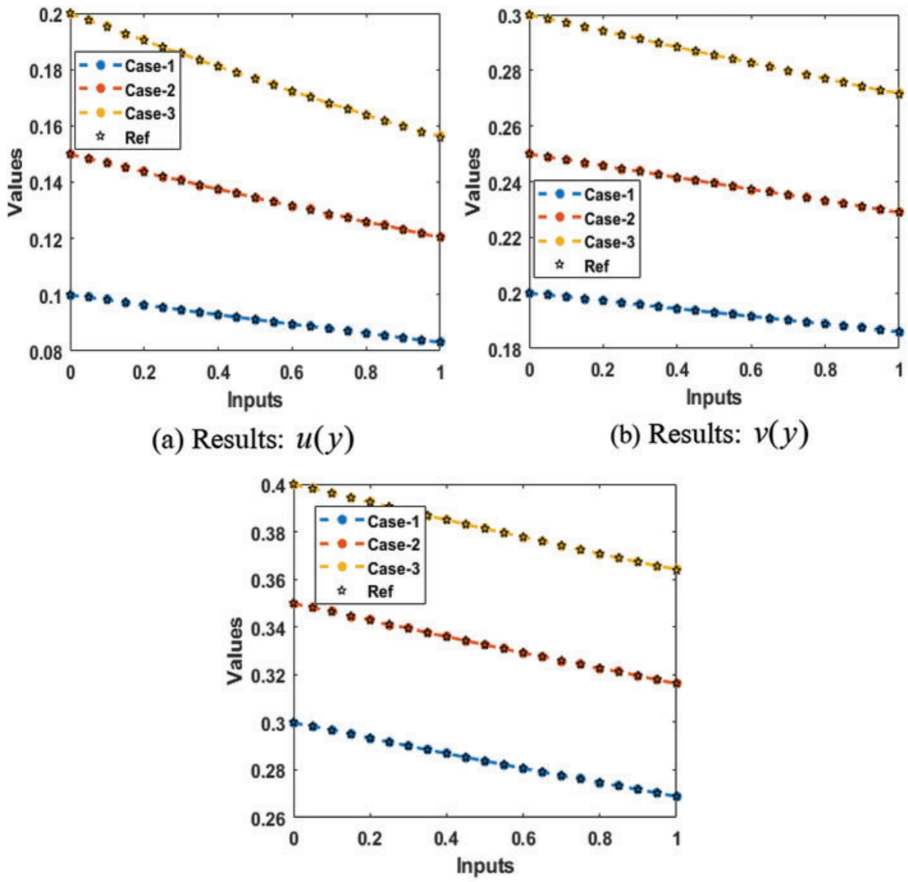


(b) Regression: **Case 2**



(c) Regression: **Case 3**

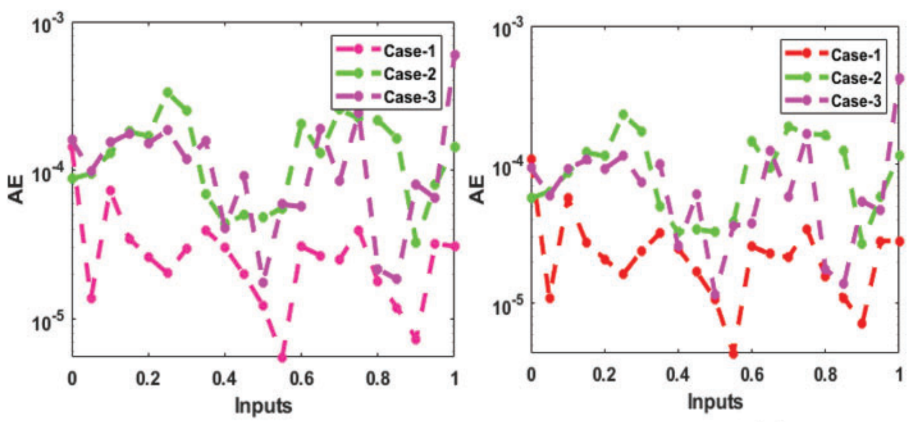
Figure 5: Regression measures for the dynamical form of two-prey and one-predator using the designed L-MBNNs procedure. (a) Regression: Case 1. (b) Regression: Case 2. (c) Regression: Case 3



(a) Results: $u(y)$ (b) Results: $v(y)$

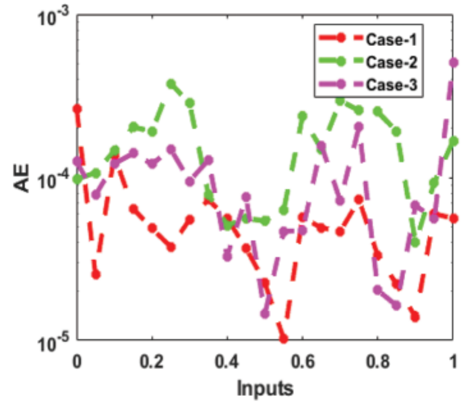
(c) Results: $p(y)$

Figure 6: Results of the dynamical two-prey and one-predator system using the designed L-MBNNs procedure. (a) Results: $u(y)$. (b) Results: $v(y)$. (c) Results: $p(y)$



(a) AE: $u(y)$ (b) AE: $v(y)$

Figure 7: (Continued)



(c) AE: $p(y)$

Figure 7: AE for the dynamical two-prey and one-predator system using the designed L-MBNNs procedure. (a) AE: $u(y)$. (b) AE: $v(y)$. (c) AE: $p(y)$

6 Conclusion

These investigations represent to perform the numerical performances of the delay differential two-prey and one-predator system. It is always found to be difficult to solve the dynamical kind of ecological nonlinear two-prey and one-predator system. Therefore, a stochastic numerical paradigm based artificial neural network along with the Levenberg–Marquardt backpropagation neural networks is proposed to solve the delay differential dynamical two-prey and one-predator system. The numerical solutions of the delay differential dynamical system have never been presented before nor solved by applying the stochastic L-MBNNs. Three different cases based on the dynamical form of two-prey and one-predator have been discussed to check the correctness of the stochastic L-MBNNs. Sixteen numbers of neurons have been used to solve the dynamical form of two-prey and one-predator along with the selection of data as 13% for testing, 12% for authorization and 75% for training. The correctness of the scheme is observed by comparing the proposed and Runge–Kutta results. To reduce the performance of MSE, the achieved results using the stochastic L-MBNNs is proposed. The capability and consistency of stochastic L-MBNNs is observed using the correlation, STs, MSE, regression and EHs. The designed scheme performance is traditional using the reliability and consistency of the stochastic L-MBNNs.

In future, the stochastic L-MBNNs can be applied to solve the numerical representations of nonlinear systems of utmost significance [45–50].

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