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Article

Quantum Remote State Preparation Based on Quantum Network Coding

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Abstract: As an innovative theory and technology, quantum network coding has become the research hotspot in quantum network communications. In this paper, a quantum remote state preparation scheme based on quantum network coding is proposed. Comparing with the general quantum remote state preparation schemes, our proposed scheme brings an arbitrary unknown quantum state finally prepared remotely through the quantum network, by designing the appropriate encoding and decoding steps for quantum network coding. What is worth mentioning, from the network model, this scheme is built on the quantum k-pair network which is the expansion of the typical bottleneck network—butterfly network. Accordingly, it can be treated as an efficient quantum network preparation scheme due to the characteristics of network coding, and it also makes the proposed scheme more applicable to the large-scale quantum networks. In addition, the fact of an arbitrary unknown quantum state remotely prepared means that the senders do not need to know the desired quantum state. Thus, the security of the proposed scheme is higher. Moreover, this scheme can always achieve the success probability of 1 and 1-max flow of value k. Thus, the communication efficiency of the proposed scheme is higher. Therefore, the proposed scheme turns out to be practicable, secure and efficient, which helps to effectively enrich the theory of quantum remote state preparation.

Keywords: Quantum remote state preparation; quantum network coding; quantum k-pair network

1 Introduction

In 2000, quantum remote state preparation (RSP) was first proposed by Lo [1]. RSP enables the senders to prepare known states for remote receivers with the assistance of entangled resources [2,3] and classical communications [4,5]. With so many years of development, RSP has already made significant advances in theories [6–8] and applications [9–11]. In these researches, respect to the different target state, such as Bell state [12], GHZ state [13], W state [14], cluster state [15] and so on, various kinds of RSP schemes have been proposed. In addition, respect to the different implementation techniques, such as deterministic RSP [16], joint RSP [17] and controlled RSP [18], several schemes have also been proposed.

However, with the development of quantum communication and the requirement for multi-user's quantum communication, point-to-point communication mode will necessarily toward to the direction of network communication. Thus, the research of RSP over the quantum network have become a meaningful task. In quantum network communications, as a breakthrough technology, quantum network coding (QNC) can effectively improve the quantum communication efficiency and quantum network throughput as compared to the traditional technology of routing. Since the first QNC scheme proposed by Hayashi et al. [19] in 2006, a series of QNC schemes have been put forward in theories [20–23] and applications [24–26] over the past decade. By QNC, the intermediate nodes are allowed to encode the received information and ultimately the destination nodes can recover the original unknown quantum state by decoding. Therefore, it is reasonable to believe that if combining RSP and QNC, the problem of quantum state preparation over quantum networks can be solved effectively.

In this paper, a novel RSP scheme based on QNC is proposed. This scheme is assisted by the resource of prior entanglement and classical communication, and the specified encoding or decoding operations are performed at the corresponding nodes. In terms of the quantum state, an arbitrary unknown quantum state possessed by the source nodes are finally prepared at the sink nodes; in terms of the success probability, this scheme can always achieve the success probability of 1; in terms of the network model, the quantum k-pair network as an expansion of the typical bottleneck network—butterfly network [27–29] in QNC is constructed to propose the scheme. Thus, the proposed scheme can effectively improve the communication efficiency and is more applicable to the scenarios of large-scale quantum networks, which can enrich the theory of quantum remote state preparation over quantum network.

The remainder of this paper is organized as follows. In Section 2, preliminaries including notations, quantum k-pair network, quantum operators and quantum measurements are presented. In Section 3, the specific RSP scheme based on QNC is proposed. In Section 4, the performance analyses including correctness, security, practicability and achievable rate region are discussed. At last, some concluding remarks are given in Section 5.

2 Preliminaries

2.1 Notations

The following notations will be used throughout this paper:

$\oplus$: The addition module d; $\otimes$: The tensor product of vector space.

$\mathbb{C}^d$: The d-dimension complex field; $\mathbb{Z}_d$: The integer ring with respect to addition $\oplus$ module d.

σ_x: The Pauli operator over 2-dimension space; X/R: The Pauli operator over d-dimension space.

2.2 Quantum K-Pair Network

Let us consider a directed acyclic graph (DAG) $G = (V,E)$, where V is the set of nodes and E is the set of edges that connect pairs of nodes in V. K pairs of nodes $(s_1, t_1), (s_2, t_2), \ldots, (s_k, t_k)$ constitute a subset of V. Next, a directed acyclic network (DAN) $\mathcal{N}$ is deemed a quantum network if it comprises a DAG G and the edge quantum capacity function $c : E \to \mathbb{Z}^+$. The quantum k-pair network $\mathcal{N}$ explored herein is shown in Fig. 1.

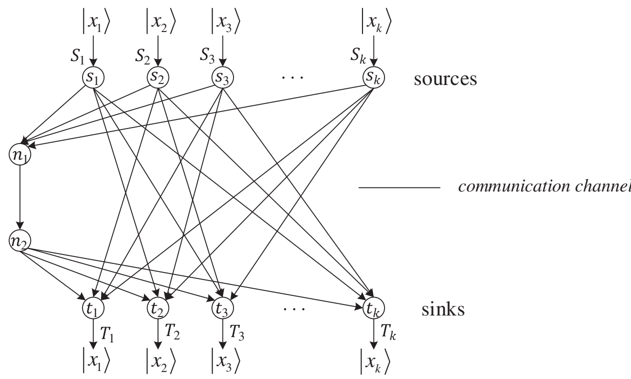

Figure 1: Quantum k-pair network $\mathcal{N}$

In this figure, the solid lines represent the quantum physical channels and the arrows indicate the transmission direction of the quantum information. Let $\mathcal{H} = \mathbb{C}^d$ denote a Hilbert space. According to the task of the quantum RSP and QNC, one needs to make a quantum state $|\Psi\rangle_{S_1,\dots,S_k} \in \mathcal{H}^{\otimes k}$ supported on the source nodes $s_1, \dots, s_k$ (in this order) remotely prepared at the sink nodes $t_1, \dots, t_k$ (in this order) through $\mathcal{N}$, under the condition that $c(e) \equiv 1, e \in E$, i.e., each edge of $\mathcal{N}$ can transmit no more than one qudit state over $\mathcal{H}$. For $i \in \{1, 2, \dots, k\}$, each quantum register S_i is possessed by the source node s_i, while the quantum register T_i is possessed by the sink node t_i. A RSP scheme based on QNC over the quantum k-pair network refers to the corresponding protocol, which contains certain quantum operations for all nodes in V that enable the above preparation to be accomplished successfully.

2.3 Quantum Operators

Firstly, let us introduce the controlled-X operation on a d-dimension quantum system $\mathcal{H} = \mathbb{C}^d$, which is written as follows:

$$\Lambda X_{A\to B} := \sum_{r\in\mathbb{Z}_d} |r\rangle\langle r|_A \otimes X_B^r, \tag{1}$$

where $X|i\rangle = |i \oplus 1 \bmod d\rangle$ is an analogue of the unitary Pauli operator σ_x on qubits [30,31]. Based on this, for $\forall t \in \mathbb{Z}_d$, the quantum operation constructed by performing t times the controlled-X operation in d-dimension quantum system $\mathcal{H} = \mathbb{C}^d$ is also presented here, defined as

$$\Lambda X^t_{A\to B} := \sum_{r\in\mathbb{Z}_d} |r\rangle\langle r|_A \otimes X_B^{rt}. \tag{2}$$

Then, another quantum operation named controlled-R operation on d-dimension quantum system $\mathcal{H} = \mathbb{C}^d$ is also presented here, defined as

$$\Lambda R_{A\to B} := \sum_{i=0}^{d-1} |i\rangle\langle i|_A \otimes R_B^i, \tag{3}$$

where $R|i\rangle = |i - 1 \bmod d\rangle$ is the reverse transformation of X on qudits.

2.4 Quantum Measurements

In the quantum system $\mathcal{H} = \mathbb{C}^d$, quantum Fourier transform $\mathcal{F}$ is a unitary transformation that transforms the computing basis states $\{|k\rangle\}_{k\in\mathbb{Z}_d}$ to the Fourier basis as

$$|w_k\rangle = \mathcal{F}|k\rangle = \frac{1}{\sqrt{d}}\sum_{l=0}^{d-1} e^{2\pi\iota kl/d}|l\rangle, \tag{4}$$

where $\iota^2 = -1$, $\mathcal{F} = \frac{1}{\sqrt{d}}\sum_{l,k=0}^{d-1} e^{2\pi\iota kl/d}|l\rangle\langle k|$. Thus, the basis states $\{|w_k\rangle\}_{k\in\mathbb{Z}_d}$ are called quantum Fourier basis and the quantum measurement in Fourier basis is called quantum Fourier measurement [21,22]. Quantum Fourier measurement is usually used to measure a single-particle state over $\mathcal{H} = \mathbb{C}^d$.

In the quantum system $\mathcal{H} = \mathbb{C}^d$, the Bell states (EPR pair) are represented as follows:

$$|\phi(M_1, M_2)\rangle = \frac{1}{\sqrt{d}}\sum_{j=0}^{d-1} e^{2\pi\iota jM_1/d}|j, j \oplus M_2\rangle, M_1, M_2 \in \mathbb{Z}_d. \tag{5}$$

Hence, the basis states $\{|\phi(M_1, M_2)\rangle\}_{M_1,M_2\in\mathbb{Z}_d}$ are called the Bell basis, and the quantum measurement in the Bell basis is called the Bell measurement. In general, Bell measurement is typically used to jointly measure a two-particle state over $\mathcal{H} = \mathbb{C}^d$.

3 A Quantum Remote Preparation Scheme Based on Quantum Network Coding

3.1 Protocol Specification

Suppose that the arbitrary k-qudit quantum state originally possessed at the source nodes $s_1, \ldots, s_k$ or desired to be prepared at the sink nodes $t_1, \ldots, t_k$ is written as

$$|\Psi\rangle_S = \sum_{x_1,\ldots,x_k\in\mathbb{Z}_d} \alpha_{x_1,\ldots,x_k} e^{\iota\theta_{x_1,\ldots,x_k}} |x_1\rangle_{S_1} \otimes \cdots \otimes |x_k\rangle_{S_k}, \tag{6}$$

where all the amplitude coefficient $\alpha_{x_1,\ldots,x_k}$ and the phase coefficient $\theta_{x_1,\ldots,x_k}$ are real numbers such that $\sum_{x_1,\ldots,x_k\in\mathbb{Z}_d} |\alpha_{x_1,\ldots,x_k}|^2 = 1$, the $S_i(i \in \{1, 2, \ldots k\})$ represent the registers possessed by the source node s_i. And then suppose that the source nodes $s_1, \ldots, s_k$ and the sink nodes $t_1, \ldots, t_k$ initially share a $2k$-qudit entangled GHZ state as

$$|\phi\rangle_{ST} = \frac{1}{\sqrt{d}}\sum_{h\in\mathbb{Z}_d} |h, h, \ldots, h\rangle_{S_1{}',\ldots,S_k{}',T_1{}',\ldots,T_k{}'}, \tag{7}$$

where for $i \in \{1, 2, \ldots, k\}$, each quantum register S_i' is possessed by the source node s_i, while quantum register T_i' is possessed by the sink node t_i.

Thus, the whole quantum system now becomes as

$$|\Psi\rangle_0 = \frac{1}{\sqrt{d}}\sum_{x_1,\ldots,x_k,h\in\mathbb{Z}_d} \alpha_{x_1,\ldots,x_k} e^{\iota\theta_{x_1,\ldots,x_k}} |x_1, x_2, \ldots, x_k\rangle_{S_1,\ldots,S_k} \otimes |h, h, \ldots, h\rangle_{S_1{}',\ldots,S_k{}',T_1{}',\ldots,T_k{}'}. \tag{8}$$

According to the task of RSP over the quantum k-pair network $\mathcal{N}$, the coefficients of $|\Psi\rangle_S^{(k)}$ is known only for the source nodes $s_1, \ldots, s_k$ and ultimately this desired k-qudit quantum should be prepared at the sink nodes $t_1, \ldots, t_k$. Now, the specific steps of the proposed RSP scheme based on QNC are described below in detail.

Step 1: At each source node $s_i(i \in \{1, 2, \ldots, k\})$, ancillary registers R_{ii} and $R_{i,i+1}$ are introduced, in which each quantum state is initialized to $|0_{\mathcal{H}}\rangle$. Then, the quantum operators $\Lambda X_{S_i \to R_{ii}}$ and $\Lambda X_{S_i' \to R_{ii}}$ are applied to the registers S_i , S_i' and R_{ii}, the operator $\Lambda R_{S_i \to R_{i,i+1}}$ is applied to the registers S_i and $R_{i,i+1}$. Thus, the quantum state becomes

$$|\Psi\rangle_1 = \frac{1}{\sqrt{d}} \sum_{x_1,\ldots,x_k,h \in \mathbb{Z}_d} \alpha_{x_1,\ldots,x_k} e^{\iota\theta x_1,\ldots,x_k} \bigotimes_{i=1}^{k} |x_i\rangle_{S_i} |h\rangle_{S_i'} |h\rangle_{T_i'} |x_i \oplus h\rangle_{R_{ii}} |-x_i\rangle_{R_{i,i+1}}. \tag{9}$$

Then, for $i \in \{1, 2, \ldots, k\}$, the quantum registers R_{ii} are sent from each node s_i to node n_1; for $i \in \{1, 2, \ldots, k-1\}$, registers $R_{i,i+1}$ are sent to the sink nodes t_{i+1}; for $i = k$, register $R_{i,i+1}$ is sent to the sink nodes t_1. And the registers S_i and S_i' are maintained at the node s_i.

Step 2: At the intermediate node n_1, the ancillary register R_n initialized to $|0_{\mathcal{H}}\rangle$ is introduced. Afterwards, the quantum operators $\Lambda X_{R_{ii} \to R_n}$ are applied on the registers $R_{ii}(i = 1, 2, \ldots k)$ and R_n, we have the quantum state:

$$|\Psi\rangle_2 = \frac{1}{\sqrt{d}} \sum_{x_1,\ldots,x_k,h \in \mathbb{Z}_d} \alpha_{x_1,\ldots,x_k} e^{\iota\theta x_1,\ldots,x_k} |\bigoplus_{i=1}^{k} x_i \oplus kh\rangle_{R_n} \bigotimes_{i=1}^{k} |x_i\rangle_{S_i} |h\rangle_{S_i'} |h\rangle_{T_i'} |x_i \oplus h\rangle_{R_{ii}} |-x_i\rangle_{R_{i,i+1}}. \tag{10}$$

Then, the quantum register R_n is sent from the node n_1 to the node n_2 and the registers $R_{ii}(i = 1, 2, \ldots k)$ are kept at the node n_1.

Step 3: At the intermediate node n_2, quantum registers $r_i(i = 1, 2, \ldots, k)$ each initialized to $|0_{\mathcal{H}}\rangle$ are introduced and then the quantum operator $\Lambda X_{R_n \to r_i}$ is applied to the registers R_n and r_i. Thus, the quantum state becomes

$$|\Psi\rangle_3 = \frac{1}{\sqrt{d}} \sum_{x_1,\ldots,x_k,h \in \mathbb{Z}_d} \alpha_{x_1,\ldots,x_k} e^{\iota\theta x_1,\ldots,x_k} |\bigoplus_{i=1}^{k} x_i \oplus kh\rangle_{R_n} \bigotimes_{i=1}^{k} |\bigoplus_{i=1}^{k} x_i \oplus kh\rangle_{r_i} |x_i\rangle_{S_i} |h\rangle_{S_i'} |h\rangle_{T_i'} |x_i \oplus h\rangle_{R_{ii}} |-x_i\rangle_{R_{i,i+1}}. \tag{11}$$

Then, k quantum registers $r_i(i = 1, 2, \ldots, k)$ are transmitted from the node n_2 to the sink nodes t_i respectively, and the register R_n are maintained at the node n_2.

Step 4: For each sink node $t_i(i \in \{1, 2, \ldots, k\})$, the quantum register T_i initialized to $|0_{\mathcal{H}}\rangle$ is introduced. Remembering that node $t_i(i \in \{2, \ldots, k\})$ has received the register $R_{i-1,i}$ and the node t_1 has received the registers $R_{k,1}$ in Step 1, and register r_i in Step 3, now the quantum operator $\Lambda X_{r_i \to T_i}$ is applied to r_i and T_i, $\Lambda X_{R_{i-1,i} \to T_i}(i \in \{2, \ldots, k\})$ is applied to $R_{i-1,i}$ and T_i($\Lambda X_{R_{k,1} \to T_1}$ is applied to $R_{k,1}$ and T_1). And then the quantum operator $\Lambda X^{-k}_{T_i' \to T_i}$ is applied to T_i' and T_i, hence the resulting state becomes

$$|\Psi\rangle_4 = \frac{1}{\sqrt{d}} \sum_{x_1,\ldots,x_k,h \in \mathbb{Z}_d} \alpha_{x_1,\ldots,x_k} e^{\iota\theta x_1,\ldots,x_k} |\bigoplus_{i=1}^{k} x_i \oplus kh\rangle_{R_n} \bigotimes_{i=1}^{k} |\bigoplus_{i=1}^{k} x_i \oplus kh\rangle_{r_i} |x_i\rangle_{S_i} |h\rangle_{S_i'} |x_i\rangle_{T_i} |h\rangle_{T_i'} |x_i \oplus h\rangle_{R_{ii}} |-x_i\rangle_{R_{i,i+1}}. \tag{12}$$

Step 5: At the intermediate node n_1, quantum Fourier measurement is performed on $R_{ii}(i = 1, 2, \ldots k)$, giving measurement results m_i. Because of

$$|l\rangle = \frac{1}{\sqrt{d}} \sum_{k=0}^{d-1} e^{-2\pi\iota kl/d} |k\rangle, \tag{13}$$

we have the quantum state:

$$|\Psi\rangle_5 = \frac{1}{\sqrt{d}} \sum_{x_1,\ldots,x_k,h\in\mathbb{Z}_d} \alpha_{x_1,\ldots,x_k} e^{\iota\theta x_1,\ldots,x_k} \prod_{i=1}^{k} e^{-2\pi\iota(x_i\oplus h)m_i/d} |\bigoplus_{i=1}^{k} x_i \oplus kh\rangle_{R_n} \bigotimes_{i=1}^{k} |\bigoplus_{i=1}^{k} x_i \oplus kh\rangle_{r_i} |x_i\rangle_{S_i} |h\rangle_{S_i'} |x_i\rangle_{T_i} |h\rangle_{T_i'} | - x_i\rangle_{R_{i,i+1}}. \tag{14}$$

Then, $k \lceil \log d \rceil$ bits classical information m_i is sent from node n_1 to the node s_i.

Step 6: At each source node $s_i(i \in \{1, 2, \ldots, k\})$, once receiving the information m_i, the quantum unitary operator mapping of the state $|x_i\rangle$ to $e^{2\pi\iota m_i x_i/d}|x_i\rangle$ for each $x_i \in \mathbb{Z}_d$ is applied on the register S_i, and the quantum unitary operator mapping of the state $|h\rangle$ to $e^{2\pi\iota m_i h/d}|h\rangle$ is applied on the register S_i'. Thus the phase caused by Bell measurement in step 5 is corrected. Afterward, considering the owned registers S_i and S_i', the quantum operation $\Lambda X_{S_i' \to S_i}$ and then the Bell measurement are performed on this two qudits, providing the measurement result $u_{i1}u_{i2}$. Because of

$$|j, j \oplus u_2\rangle = \frac{1}{\sqrt{d}} \sum_{u_1=0}^{d-1} e^{-2\pi\iota j u_1/d} |\phi(u_1, u_2)\rangle, \tag{15}$$

we obtain the quantum state:

$$|\Psi\rangle_6 = \frac{1}{\sqrt{d}} \sum_{x_1,\ldots,x_k,h\in\mathbb{Z}_d} \alpha_{x_1,\ldots,x_k} e^{\iota\theta x_1,\ldots,x_k} \prod_{i=1}^{k} e^{-2\pi\iota(\bigoplus_{i=1}^{k} x_i \oplus kh)u_{i1}/d} |\bigoplus_{i=1}^{k} x_i \oplus kh\rangle_{R_n} \bigotimes_{i=1}^{k} |\bigoplus_{i=1}^{k} x_i \oplus kh\rangle_{r_i} |x_i\rangle_{T_i} |h\rangle_{T_i'} | - x_i\rangle_{R_{i,i+1}}. \tag{16}$$

Then, $2k \lceil \log d \rceil$ bits classical information $u_{i1}u_{i2}$ are sent from the nodes s_i to the sink node t_{i+1}. Here, the security of classical channels can be guaranteed by message authentication mechanism such as HMAC [32], UMAC [33], etc., preventing outside attackers from tampering the transmitted classical information. It is noted that the classical information transmission involved in the following steps is all the same and there will be no repeated mention.

Step 7: At the sink node $t_i(i \in \{1, 2, \ldots, k\})$, upon receiving the information $u_{i-1,1}u_{i-1,2}$, the quantum unitary operator mapping of the state $|x\rangle$ to $e^{2\pi\iota u_{i-1,1}x/d}|x\rangle$ for each $x \in \mathbb{Z}_d$ is applied on its register r_i. Thus the phase caused by Bell measurement in step 5 is corrected. And then quantum Fourier measurement is performed on r_i, giving measurement results l_i. The quantum state becomes

$$|\Psi\rangle_7 = \frac{1}{\sqrt{d}} \sum_{x_1,\ldots,x_k,h\in\mathbb{Z}_d} \alpha_{x_1,\ldots,x_k} e^{\iota\theta x_1,\ldots,x_k} \prod_{i=1}^{k} e^{-2\pi\iota(\bigoplus_{i=1}^{k} x_i \oplus kh)l_i/d} |\bigoplus_{i=1}^{k} x_i \oplus kh\rangle_{R_n} \bigotimes_{i=1}^{k} |x_i\rangle_{T_i} |h\rangle_{T_i'} | - x_i\rangle_{R_{i,i+1}}. \tag{17}$$

Afterwards, $k \lceil \log d \rceil$ bits classical information l_i is sent from node t_i to the intermediate node n_2.

Step 8: At the intermediate node n_2, once receiving the information l_i, the quantum unitary operator mapping of the state $|x\rangle$ to $e^{2\pi\iota l_i x/d}|x\rangle$ for each $x_i \in \mathbb{Z}_d$ is applied on the register R_n to correct the phase. Following this, quantum Fourier measurement is performed on R_n, giving measurement results p. Thus, we have the quantum state

$$|\Psi\rangle_8 = \frac{1}{\sqrt{d}} \sum_{x_1,\dots,x_k,h\in\mathbb{Z}_d} \alpha_{x_1,\dots,x_k} e^{\iota\theta_{x_1,\dots,x_k}} e^{-2\pi\iota(\overset{k}{\underset{i=1}{\oplus}} x_i \oplus kh)p/d} \bigotimes_{i=1}^{k} |x_i\rangle_{T_i} |h\rangle_{T_i'} |-x_i\rangle_{R_{i,i+1}}. \tag{18}$$

Then, 1 $\lceil \log d \rceil$ bits classical information p are sent from the nodes n_2 to the sink node t_i.

Step 9: At the sink node $t_i (i \in \{1, 2, \dots, k\})$, upon receiving the information p, the quantum unitary operator mapping of the state $|h\rangle$ to $e^{2\pi\iota(\overset{k}{\underset{i=1}{\oplus}} x_i \oplus kh)p}|h\rangle$ for each $x_i \in \mathbb{Z}_d$ is applied on the register T_i' to correct the phase. Then, the Bell measurement is performed on the registers $R_{i-1,i}$ and T_i', giving measurement results $q_1 q_2$. The quantum state becomes

$$|\Psi\rangle_9 = \frac{1}{\sqrt{d}} \sum_{x_1,\dots,x_k,h\in\mathbb{Z}_d} \alpha_{x_1,\dots,x_k} e^{\iota\theta_{x_1,\dots,x_k}} e^{2\pi\iota x_i q_1/d} \bigotimes_{i=1}^{k} |x_i\rangle_{T_i}. \tag{19}$$

Next, the phase is corrected by performing the quantum unitary operator mapping of the state $|x\rangle$ to $e^{-2\pi\iota q_1}|x\rangle$ for each $x_i \in \mathbb{Z}_d$ on the register T_i. Thus, the final quantum state becomes the desired state, as follows:

$$|\Psi\rangle_T = \sum_{x_1,\dots,x_k\in\mathbb{Z}_d} \alpha_{x_1,\dots,x_k} e^{\iota\theta_{x_1,\dots,x_k}} |x_1\rangle_{T_1} \otimes \cdots \otimes |x_k\rangle_{T_k}, \tag{20}$$

That is, the arbitrary k-qudit quantum state originally possessed by the source nodes is successfully prepared remotely at the sink nodes.

3.2 Example: The Butterfly Network Over $\mathcal{H} = \mathbb{C}^2$

For the quantum k-pair network $\mathcal{N}$ illustrated in Fig. 1, when $k = 2$, the network becomes the typical bottleneck network—butterfly network as shown as Fig. 2. This subsection describes the techniques developed in the previous descriptions using the example of the butterfly network over Hilbert space $\mathcal{H} = \mathbb{C}^2$.

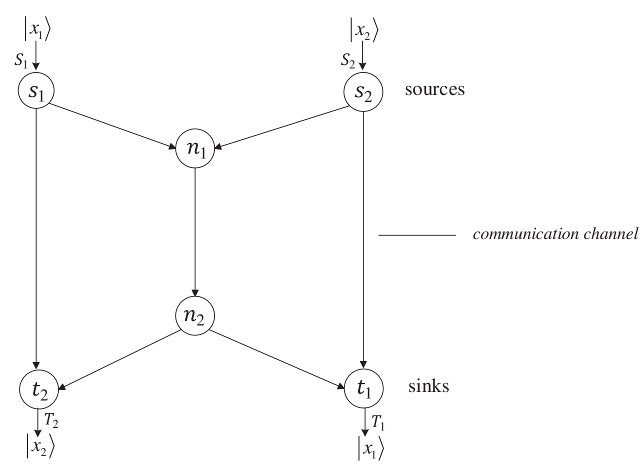

Figure 2: Quantum butterfly network

Here, the arbitrary 2-qubit quantum state originally possessed at the source nodes s_1, s_2 and desired to be prepared at the sink nodes t_1, t_2 is written as

$$|\Psi\rangle_S^{(2)} = (\alpha_{00}e^{\iota\theta_{00}}|00\rangle + \alpha_{01}e^{\iota\theta_{01}}|01\rangle + \alpha_{10}e^{\iota\theta_{10}}|10\rangle + \alpha_{11}e^{\iota\theta_{11}}|11\rangle)_{S_1S_2}. \tag{21}$$

And the source nodes s_1, s_2 and the sink nodes t_1, t_2 initially share a 4-qubit entangled GHZ state as

$$|\phi\rangle_{ST}^{(2)} = \frac{1}{\sqrt{2}}(|0000\rangle + |1111\rangle)_{S_1'S_2'T_1'T_2'}. \tag{22}$$

Thus, the initial quantum state of the whole system is

$$|\Psi\rangle_0^{(2)} = \frac{1}{\sqrt{2}}(\alpha_{00}e^{\iota\theta_{00}}|00\rangle + \alpha_{01}e^{\iota\theta_{01}}|01\rangle + \alpha_{10}e^{\iota\theta_{10}}|10\rangle + \alpha_{11}e^{\iota\theta_{11}}|11\rangle)_{S_1S_2} \otimes (|0000\rangle + |1111\rangle)_{S_1'S_2'T_1'T_2'}. \tag{23}$$

Hereafter, we proceed to the specific steps of the RSP scheme based on QNC. For step 1-step 4, it can be considered as the encoding process and the object is to make the particles in the quantum registers mutually entangled. The corresponding schematic diagram of quantum circuit can be shown in Fig. 3.

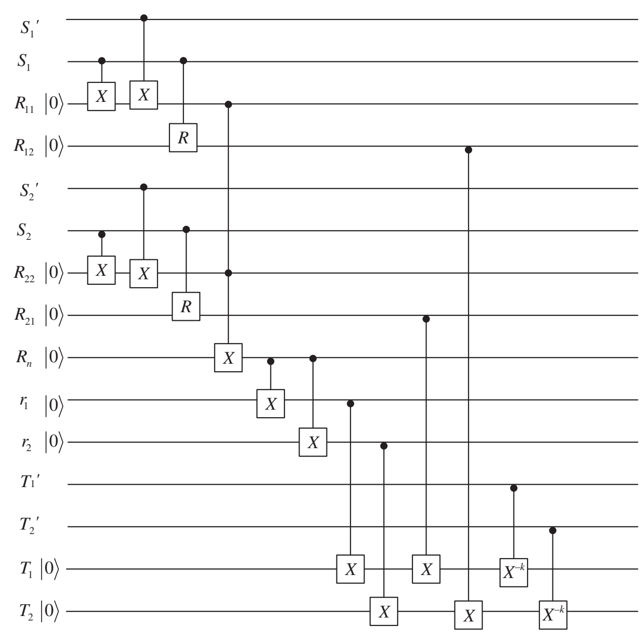

Figure 3: Schematic diagram of quantum circuit for steps 1–4

According to the specific calculations, after Step 1–4, the quantum states of the of the whole system becomes

$$
\begin{aligned}
|\Psi\rangle_4^{(2)} = \frac{1}{\sqrt{2}}[&\alpha_{00}e^{\iota\theta_{00}}(|000\rangle_{R_nr_1r_2}|000\rangle_{S_1R_{11}R_{12}}|000\rangle_{S_2R_{22}R_{21}}|0000\rangle_{S_1'S_2'T_1'T_2'}|00\rangle_{T_1T_2} + |000\rangle_{R_nr_1r_2}|010\rangle_{S_1R_{11}R_{12}}\\
&|010\rangle_{S_2R_{22}R_{21}}|1111\rangle_{S_1'S_2'T_1'T_2'}|00\rangle_{T_1T_2})\\
+&\alpha_{01}e^{\iota\theta_{01}}(|111\rangle_{R_nr_1r_2}|000\rangle_{S_1R_{11}R_{12}}|111\rangle_{S_2R_{22}R_{21}}|0000\rangle_{S_1'S_2'T_1'T_2'}|01\rangle_{T_1T_2} + |111\rangle_{R_nr_1r_2}|010\rangle_{S_1R_{11}R_{12}}|101\rangle_{S_2R_{22}R_{21}}\\
&|1111\rangle_{S_1'S_2'T_1'T_2'}|01\rangle_{T_1T_2})\\
+&\alpha_{10}e^{\iota\theta_{10}}(|111\rangle_{R_nr_1r_2}|111\rangle_{S_1R_{11}R_{12}}|000\rangle_{S_2R_{22}R_{21}}\ |0000\rangle_{S_1'S_2'T_1'T_2'}|10\rangle_{T_1T_2} + |111\rangle_{R_nr_1r_2}|101\rangle_{S_1R_{11}R_{12}}|010\rangle_{S_2R_{22}R_{21}}\\
&|1111\rangle_{S_1'S_2'T_1'T_2'}|10\rangle_{T_1T_2})\\
+&\alpha_{11}e^{\iota\theta_{11}}(|000\rangle_{R_nr_1r_2}|111\rangle_{S_1R_{11}R_{12}}|111\rangle_{S_2R_{22}R_{21}}\ |0000\rangle_{S_1'S_2'T_1'T_2'}|11\rangle_{T_1T_2} + |000\rangle_{R_nr_1r_2}|101\rangle_{S_1R_{11}R_{12}}|101\rangle_{S_2R_{22}R_{21}}\\
&|1111\rangle_{S_1'S_2'T_1'T_2'}|11\rangle_{T_1T_2}),
\end{aligned}
\tag{24}
$$

Then, For Step 5-Step 9, it can be considered as the decoding process and the object is to remove the particles in the unnecessary quantum registers. The corresponding schematic diagram of quantum circuit can be shown in Fig. 4.

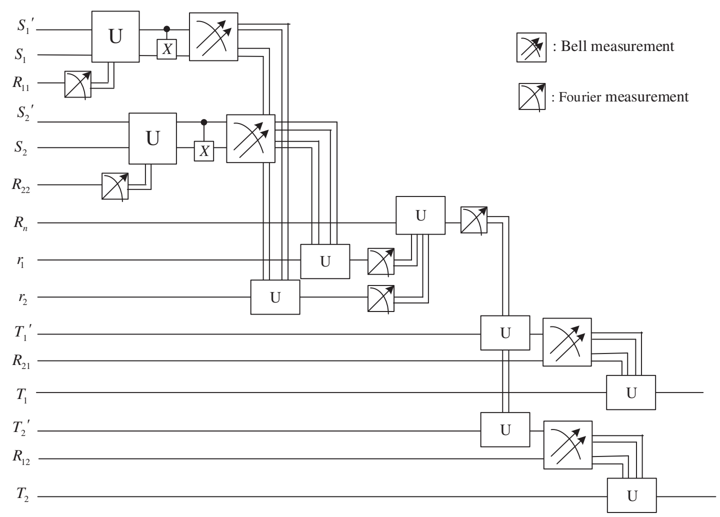

Figure 4: Schematic diagram of quantum circuit for steps 5–9

According to the specific calculations, the quantum states of Step 5–9 are detailed as follows:

$$
\begin{aligned}
|\Psi\rangle_5^{(2)} = \frac{1}{\sqrt{2}}[&\alpha_{00}e^{\iota\theta_{00}}(|000\rangle_{R_n r_1 r_2}|00\rangle_{S_1R_{12}}|00\rangle_{S_2R_{21}}|0000\rangle_{S_1'S_2'T_1'T_2'}|00\rangle_{T_1T_2} + |000\rangle_{R_n r_1 r_2}|00\rangle_{S_1R_{12}}|00\rangle_{S_2R_{21}}\\
&|1111\rangle_{S_1'S_2'T_1'T_2'}|00\rangle_{T_1T_2})\\
+&\alpha_{01}e^{\iota\theta_{01}}(|111\rangle_{R_n r_1 r_2}|00\rangle_{S_1R_{12}}|11\rangle_{S_2R_{21}}|0000\rangle_{S_1'S_2'T_1'T_2'}|01\rangle_{T_1T_2} + |111\rangle_{R_n r_1 r_2}|00\rangle_{S_1R_{12}}|11\rangle_{S_2R_{21}}|1111\rangle_{S_1'S_2'T_1'T_2'}\\
&|01\rangle_{T_1T_2})\\
+&\alpha_{10}e^{\iota\theta_{10}}(|111\rangle_{R_n r_1 r_2}|11\rangle_{S_1R_{12}}|00\rangle_{S_2R_{21}}|0000\rangle_{S_1'S_2'T_1'T_2'}|10\rangle_{T_1T_2} + |111\rangle_{R_n r_1 r_2}|11\rangle_{S_1R_{12}}|00\rangle_{S_2R_{21}}|1111\rangle_{S_1'S_2'T_1'T_2'}\\
&|10\rangle_{T_1T_2})\\
+&\alpha_{11}e^{\iota\theta_{11}}(|000\rangle_{R_n r_1 r_2}|11\rangle_{S_1R_{12}}|11\rangle_{S_2R_{21}}|0000\rangle_{S_1'S_2'T_1'T_2'}|11\rangle_{T_1T_2} + |000\rangle_{R_n r_1 r_2}|11\rangle_{S_1R_{12}}|11\rangle_{S_2R_{21}}|1111\rangle_{S_1'S_2'T_1'T_2'}\\
&|11\rangle_{T_1T_2}),
\end{aligned} \tag{25}
$$

$$
\begin{aligned}
|\Psi\rangle_6^{(2)} = \frac{1}{\sqrt{2}}[&\alpha_{00}e^{\iota\theta_{00}}(|000\rangle_{R_n r_1 r_2}|0\rangle_{R_{12}}|0\rangle_{R_{21}}|00\rangle_{T_1'T_2'}|00\rangle_{T_1T_2} + |000\rangle_{R_n r_1 r_2}|0\rangle_{R_{12}}|0\rangle_{R_{21}}|11\rangle_{T_1'T_2'}|00\rangle_{T_1T_2})\\
+&\alpha_{01}e^{\iota\theta_{01}}(|111\rangle_{R_n r_1 r_2}|0\rangle_{R_{12}}|1\rangle_{R_{21}}|00\rangle_{T_1'T_2'}|01\rangle_{T_1T_2} + |111\rangle_{R_n r_1 r_2}|0\rangle_{R_{12}}|1\rangle_{R_{21}}|11\rangle_{T_1'T_2'}|01\rangle_{T_1T_2})\\
+&\alpha_{10}e^{\iota\theta_{10}}(|111\rangle_{R_n r_1 r_2}|1\rangle_{R_{12}}|0\rangle_{R_{21}}|00\rangle_{T_1'T_2'}|10\rangle_{T_1T_2} + |111\rangle_{R_n r_1 r_2}|1\rangle_{R_{12}}|0\rangle_{R_{21}}|11\rangle_{T_1'T_2'}|10\rangle_{T_1T_2})\\
+&\alpha_{11}e^{\iota\theta_{11}}(|000\rangle_{R_n r_1 r_2}|1\rangle_{R_{12}}|1\rangle_{R_{21}}|00\rangle_{T_1'T_2'}|11\rangle_{T_1T_2} + |000\rangle_{R_n r_1 r_2}|1\rangle_{R_{12}}|1\rangle_{R_{21}}|11\rangle_{T_1'T_2'}|11\rangle_{T_1T_2}),
\end{aligned} \tag{26}
$$

$$
\begin{aligned}
|\Psi\rangle_7^{(2)} = \frac{1}{\sqrt{2}}[&\alpha_{00}e^{\iota\theta_{00}}(|0\rangle_{R_n}|0\rangle_{R_{12}}|0\rangle_{R_{21}}|00\rangle_{T_1'T_2'}|00\rangle_{T_1T_2} + |0\rangle_{R_n}|0\rangle_{R_{12}}|0\rangle_{R_{21}}|11\rangle_{T_1'T_2'}|00\rangle_{T_1T_2})\\
+&\alpha_{01}e^{\iota\theta_{01}}(|1\rangle_{R_n}|0\rangle_{R_{12}}|1\rangle_{R_{21}}|00\rangle_{T_1'T_2'}|01\rangle_{T_1T_2} + |1\rangle_{R_n}|0\rangle_{R_{12}}|1\rangle_{R_{21}}|11\rangle_{T_1'T_2'}|01\rangle_{T_1T_2})\\
+&\alpha_{10}e^{\iota\theta_{10}}(|1\rangle_{R_n}|1\rangle_{R_{12}}|0\rangle_{R_{21}}|00\rangle_{T_1'T_2'}|10\rangle_{T_1T_2} + |1\rangle_{R_n}|1\rangle_{R_{12}}|0\rangle_{R_{21}}|11\rangle_{T_1'T_2'}|10\rangle_{T_1T_2})\\
+&\alpha_{11}e^{\iota\theta_{11}}(|0\rangle_{R_n}|1\rangle_{R_{12}}|1\rangle_{R_{21}}|00\rangle_{T_1'T_2'}|11\rangle_{T_1T_2} + |0\rangle_{R_n}|1\rangle_{R_{12}}|1\rangle_{R_{21}}|11\rangle_{T_1'T_2'}|11\rangle_{T_1T_2}),
\end{aligned} \tag{27}
$$

$$
\begin{aligned}
|\Psi\rangle_8^{(2)} = \frac{1}{\sqrt{2}}[&\alpha_{00}e^{\iota\theta_{00}}(|0\rangle_{R_{12}}|0\rangle_{R_{21}}|00\rangle_{T_1'T_2'}|00\rangle_{T_1T_2} + |0\rangle_{R_{12}}|0\rangle_{R_{21}}|11\rangle_{T_1'T_2'}|00\rangle_{T_1T_2})\\
+&\alpha_{01}e^{\iota\theta_{01}}(|0\rangle_{R_{12}}|1\rangle_{R_{21}}|00\rangle_{T_1'T_2'}|01\rangle_{T_1T_2} + |0\rangle_{R_{12}}|1\rangle_{R_{21}}|11\rangle_{T_1'T_2'}|01\rangle_{T_1T_2})\\
+&\alpha_{10}e^{\iota\theta_{10}}(|1\rangle_{R_{12}}|0\rangle_{R_{21}}|00\rangle_{T_1'T_2'}|10\rangle_{T_1T_2} + |1\rangle_{R_{12}}|0\rangle_{R_{21}}|11\rangle_{T_1'T_2'}|10\rangle_{T_1T_2})\\
+&\alpha_{11}e^{\iota\theta_{11}}(|1\rangle_{R_{12}}|1\rangle_{R_{21}}|00\rangle_{T_1'T_2'}|11\rangle_{T_1T_2} + |1\rangle_{R_{12}}|1\rangle_{R_{21}}|11\rangle_{T_1'T_2'}|11\rangle_{T_1T_2}),
\end{aligned} \tag{28}
$$

$$
|\Psi\rangle_9^{(2)} = \frac{1}{\sqrt{2}}(\alpha_{00}e^{\iota\theta_{00}}|00\rangle_{T_1T_2} + \alpha_{01}e^{\iota\theta_{01}}|01\rangle_{T_1T_2} + \alpha_{10}e^{\iota\theta_{10}}|10\rangle_{T_1T_2} + \alpha_{11}e^{\iota\theta_{11}}|11\rangle_{T_1T_2}). \tag{29}
$$

Here, the global phases 1/2 can be ignored during computation, so the final quantum state at the sink nodes t_1, t_2 becomes the desired state, i.e., $|\Psi\rangle_9^{(2)} = |\Psi\rangle_S^{(2)} = \alpha_{00}e^{\iota\theta_{00}}|00\rangle + \alpha_{01}e^{\iota\theta_{01}}|01\rangle + \alpha_{10}e^{\iota\theta_{10}}|10\rangle + \alpha_{11}e^{\iota\theta_{11}}|11\rangle$.

That is, the arbitrary 2-qubit quantum state originally possessed by the source nodes is successfully prepared remotely at the sink nodes.

4 Scheme Analysis

4.1 Correctness

The correctness of the proposed RSP scheme based on QNC can be verified by the specific steps. From Section 2, in the Step 1–4, the particles at every network node are entangled to the whole quantum system by applying relevant quantum encoding operations. The resulting quantum state after the entanglement of each time is presented specifically. In the Step 5–9, by applying relevant quantum measurements called decoding operations, all the unnecessary particles are disentangled from the whole quantum system and leave alone the certain particles on the sink nodes. The resulting quantum state after the disentanglement of each time is also presented specifically. Thus, after all the encoding and decoding steps, the final quantum state at the sink nodes formed $|\Psi\rangle_T = \sum_{x_1,\dots,x_k \in \mathbb{Z}_d} \alpha_{x_1,\dots,x_k} e^{\iota\theta_{x_1,\dots,x_k}} |x_1\rangle_{T_1} \otimes \cdots \otimes |x_k\rangle_{T_k}$, is exactly equal to the initial source state $|\Psi\rangle_S = \sum_{x_1,\dots,x_k \in \mathbb{Z}_d} \alpha_{x_1,\dots,x_k} e^{\iota\theta_{x_1,\dots,x_k}} |x_1\rangle_{S_1} \otimes \cdots \otimes |x_k\rangle_{S_k}$, at the source nodes. Therefore, according to all the calculating procedure and numerical results, the correctness of the proposed RSP scheme is verified. What is worth mentioning, in this proposed scheme, the probability of successful preparation can always reach 1.

4.2 Security

In the existing RSP schemes [12–18], the quantum state desired to be prepared remotely is known for the senders, i.e., the amplitude coefficient and the phase coefficient is known. However, in many actual scenarios, if the senders are not honest, then the final quantum state prepared remotely is likely to be mistaken. In this paper, the proposed RSP scheme is based on QNC, which means that the senders do not need to know the desired quantum state and finally it can be prepared remotely. Therefore, the security of the proposed RSP scheme here is higher.

4.3 Practicability

In the existing RSP schemes [12–18], the point-to-point communication mode is used. With the development of quantum communication, quantum network communication containing more users has become a necessity. In this paper, the proposed RSP scheme based on QNC is established over the quantum k-pair network which is the typical bottleneck network model. Using this scheme, the quantum network congestion can be solved effectively and the communication efficiency is higher. Thus, the proposed scheme can be more applicable to the scenarios of large-scale quantum networks.

4.4 Achievable Rate Region

As is known, the communication rate [34] between s_i and t_i in n network uses is defined as $r_i^{(n)} = \frac{1}{n} log|\mathcal{H}_i|$, where $\mathcal{H}_i$ denotes the Hilbert space of the transmitted quantum state owned by s_i, and $|\cdot|$ denotes the dimension of the Hilbert space. Meanwhile, an edge capacity constraint [35], i.e., $log|\mathcal{H}_{(u,v)}| \le n \cdot c((u, v))$, exists when the quantum state is transmitted with the fidelity of one over the edge $(u, v) \in E$ in n uses.

Accordingly, in the presented RSP scheme based on QNC above, the desired arbitrary unknown quantum state is finally prepared successfully over the quantum k-pair network in single use of the network. Thus, the 1-flow [34] value reaches

$$\sum_{i=1}^{k} r_i^{(1)} = \sum_{i=1}^{k} log|\mathcal{H}_i| \le \sum_{i=1}^{k} c((u, v)) = \sum_{i=1}^{k} 1 = k, \tag{30}$$

under the condition that the capacity $c((u, v))$ of each edge (u, v) always remains equal to 1 according to the quantum k-pair network $\mathcal{N}$. In fact, the 1-max flow is the supremum of 1-flow over all achievable rate. Hence, 1-max flow of value k is achieved, and the achievable rate region [34,36] can be written as $\{(r_1, \dots, r_k) | \sum_{i=1}^{k} r_i \le k\}$.

5 Conclusions

In this paper, we propose a novel RSP scheme based on QNC over the quantum k-pair network. In terms of the quantum state, an arbitrary unknown quantum state is finally prepared at the sink nodes; in terms of the success probability, this scheme can always achieve the success probability of 1; in terms of the network model, the quantum k-pair network is the expansion of the typical bottleneck network—butterfly network. Thus, the proposed scheme can effectively improve the security and communication efficiency, and is more applicable to the large-scale quantum networks, which can enrich the theory of quantum remote state preparation.

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Conflicts of Interest: The authors declare that they have no conflicts of interest to report regarding the present study.

References

[1] H. K. Lo, "Classical communication cost in distributed quantum information processing-A generalization of quantum communication complexity," *Physical Review A*, vol. 62, no. 1, pp. 012313, 2000.
[2] Z. G. Qu, Y. M. Huang and M. Zheng, "A novel coherence-based quantum steganalysis protocol," *Quantum Information Processing*, vol. 19, no. 362, pp. 1–19, 2020.
[3] X. B. Chen, Y. R. Sun, G. Xu and Y. X. Yang, "Quantum homomorphic encryption scheme with flexible number of evaluator based on (k, n)-threshold quantum state sharing," *Information Sciences*, vol. 501, pp. 172–181, 2019.
[4] Z. G. Qu, H. R. Sun and M. Zheng, "An efficient quantum image steganography protocol based on improved EMD algorithm," *Quantum Information Processing*, vol. 20, no. 53, pp. 1–29, 2021.
[5] L. Sun, Y. L. Wang, Z. G. Qu and N. N. Xiong, "BeatClass: A sustainable ECG classification system in IoT-based eHealth," *IEEE Internet of Things Journal*, 2021. DOI 10.1109/JIOT.2021.3108792.
[6] C. H. Bennett, D. P. DiVincenzo, P. W. Shor, J. A. Smolin, B. M. Terhal *et al.,* "Remote state preparation," *Physical Review Letters*, vol. 87, no. 7, pp. 077902, 2001.
[7] A. K. Pati, "Minimum classical bit for remote preparation and measurement of a qubit," *Physical Review A*, vol. 63, no. 1, pp. 014302, 2000.
[8] I. Devetak and T. Berger, "Low-entanglement remote state preparation," *Physical Review Letters*, vol. 87, no. 19, pp. 197901, 2001.
[9] D. W. Berry and B. C. Sanders, "Optimal remote state preparation," *Physical Review Letters*, vol. 90, no. 5, pp. 057901, 2003.
[10] N. A. Peters, J. T. Barreiro, M. E. Goggin, T. C. Wei and P. G. Kwiat, "Remote state preparation: Arbitrary remote control of photon polarization," *Physical Review Letters*, vol. 94, no. 15, pp. 150502, 2005.

[11] X. Peng, X. Zhu, X. Fang, M. Feng, M. Liu *et al.,* "Experimental implementation of remote state preparation by nuclear magnetic resonance," *Physics Letters A*, vol. 306, no. 5-6, pp. 271–276, 2003.
[12] D. W. Leung and P. W. Shor, "Oblivious remote state preparation," *Physical Review Letters*, vol. 90, no. 12, pp. 127905, 2003.
[13] Y. Xia, J. Song and H. S. Song, "Remote preparation of the N-particle GHZ state using quantum statistics," *Optics Communications*, vol. 277, no. 1, pp. 219–222, 2007.
[14] N. N. Wu and M. Jiang, "A highly efficient scheme for joint remote preparation of multi-qubit W state with minimum quantum resource," *Quantum Information Processing*, vol. 17, no. 12, pp. 1–17, 2018.
[15] K. Hou, "Joint remote preparation of four-qubit cluster-type states with multiparty," *Quantum Information processing*," vol. 12, no. 12, pp. 3821–3833, 2013.
[16] S. Y. Ma, C. Gao, P. Zhang and Z. G. Qu, "Deterministic remote preparation via the brown state," *Quantum Information Processing*, vol. 16, no. 4, pp. 1–22, 2017.
[17] N. B. An, "Joint remote state preparation via W and W-type states," *Optics Communications*, vol. 283, no. 20, pp. 4113–4117, 2010.
[18] X. B. Chen, S. Y. Ma, Y. Su, R. Zhang and Y. X. Yang, "Controlled remote state preparation of arbitrary two and three qubit states via the brown state," *Quantum Information Processing*, vol. 11, no. 6, pp. 1653–1667, 2012.
[19] M. Hayashi, K. Iwama and H. Nishimura, "Quantum network coding," in *Proc. Symp. Theoretical Aspects of Computer Science, Lecture Notes in Computer Science*, Aachen, Germany, vol. 4393, pp. 610–621, 2006.
[20] M. Hayashi, "Prior entanglement between senders enables perfect quantum network coding with modification," *Physical Review A*, vol. 76, no. 4, pp. 040301(R), 2007.
[21] H. Kobayashi, F. Le Gall, H. Nishimura and M. Roetteler, "General scheme for perfect quantum network coding with free classical communication," in *Proc. of the 36th Int. Colloquium on Automata, Languages and Programming, Lecture Note in Computer Science*, Rhodes, Greece, pp. 622–633, 2009.
[22] H. Kobayashi, F. Le Gall, H. Nishimura and M. Roetteler, "Constructing quantum network coding schemes from classical nonlinear protocols," in *Proc. of the 2011 IEEE Int. Symp. Information Theory (ISIT)*, St. Petersburg, Russia, pp. 109–113, 2011.
[23] S. Chen, G. Xu, X. Chen, H. Ahmad and Y. Chen, "Measurement-based quantum repeater network coding," *Intelligent Automation & Soft Computing*, vol. 30, no.1, pp. 273–284, 2021.
[24] P. Pathumsoot, T. Matsuo, T. Satoh, M. Hajdušek, S. Suwanna *et al.,* "Modeling of measurement-based quantum network coding on a superconducting quantum processor," *Physical Review A*, vol. 101, no. 5, pp. 052301, 2020.
[25] Z. Li, Z. Li, X. Chen, Z. Qu, X. Wang *et al.,* "A practical quantum network coding protocol based on non-maximally entangled state," *Computers, Materials & Continua*, vol. 68, no. 2, pp. 2651–2663, 2021.
[26] X. Pan, X. Chen, G. Xu, H. Ahmad, T. Shang *et al.,* "Controlled quantum network coding without loss of information," *Computers, Materials & Continua*, vol. 69, no. 3, pp. 3967–3979, 2021.
[27] T. Satoh, F. Le Gall and H. Imai, "Quantum network coding for quantum repeaters," *Physical Review A*, vol. 86, no. 3, pp. 032331, 2012.
[28] T. Satoh, K. Ishizaki, S. Nagayamaand and R. Van Meter, "Analysis of quantum network coding for realistic repeater networks," *Physical Review A*, vol. 93, no. 3, 032302, 2016.
[29] Z. X. Zhang and Z. G. Qu, "Anti-noise quantum network coding protocol based on bell states and butterfly network model," *Journal of Quantum Computing*, vol. 1, no. 2, pp. 89–109, 2019.
[30] M. A. Nielsen and I. L. Chuang, "*Quantum Computation and Quantum Information*," Cambridge University Press, Cambridge, 2000.
[31] G. Xu, Y. Cao, S. Xu, K. Xiao, X. Liu *et al.,* "A novel post-quantum blind signature for log system in blockchain," *Computer Systems Science and Engineering*, vol. 41, no. 3, pp. 945–958, 2022.
[32] M. Bellare, R. Canetti and H. Krawczyk, "Message authentication using hash functions: The HMAC construction," *RSA Laboratories' CryptoBytes*, vol. 2, no. 1, pp. 12–15, 1996.
[33] J. Black, S. Halevi, H. Krawczyk, T. Krovetz and P. Rogaway, "UMAC: Fast and secure message authentication," in *Proc. of Annual Int. Cryptology Conf.*, California, USA, pp. 216–233, 1999.

[34] S. Akibue and M. Murao, “Network coding for distributed quantum computation over cluster and butterfly networks,” *IEEE Transactions on Information Theory*, vol. 62, no. 11, pp. 6620–6637, 2016.
[35] A. Jain, M. Franceschetti and D. A. Meyer, “On quantum network coding,” *Journal of Mathematical Physics*, vol. 52, no. 3, pp. 032201, 2011.
[36] H. Nishimura, “Quantum network coding–how can network coding be applied to quantum information?,” in *Proceeding of the 2013 IEEE Int. Symposium on Network Coding*, Calgary, AB, Canada, pp. 1–5, 2013.