# Construction of an Energy-Efficient Detour Non-Split Dominating Set in WSN 

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#### Abstract

Wireless sensor networks (WSNs) are one of the most important improvements due to their remarkable capacities and their continuous growth in various applications. However, the lifetime of WSNs is very confined because of the delimited energy limit of their sensor nodes. This is the reason why energy conservation is considered the main exploration worry for WSNs. For this energy-efficient routing is required to save energy and to subsequently drag out the lifetime of WSNs. In this report we use the Ant Colony Optimization (ACO) method and are evaluated using the Genetic Algorithm (GA), based on the Detour non-split dominant set (GA) In this research, we use the energy efficiency returnee non-split dominating set (DNSDS). A set $\mathrm{S} \subseteq$ $V$ is supposed to be a DNSDS of $G$ when the graph $G=(V, E)$ is expressed as both detours as well as a non-split dominating set of G. Let the detour non-split domination number be addressed as $\gamma \_$dns (G) and is the minimum order of its detour non-split dominating set. Any DNSDS of order $\gamma_{d n s}(G)$ is a $\gamma_{d n s}$-set of $G$. Here, the $\gamma_{\_}$dns (G) of various standard graphs is resolved and some of its general properties are contemplated. A connected graph usually has an order n with detour non-split domination number as n or $\mathrm{n}-1$ are characterized. Also connected graphs of order $n \geq 4$ and detour diameter $D \leq 4$ with detour non-split dominating number $n$ or $n-1$ or $n-2$ are additionally portrayed. While considering any pair of positive integers to be specific $a$ and $b$, there exists a connected graph $G$ which is normally indicated as $d n(G)=a, \gamma(G)=b$ and $\gamma_{d n s}(G)=\mathrm{a}+\mathrm{b}-2$, here $\gamma_{d n s}(G)$ indicates the detour domination number and $\operatorname{dn}(G)$ indicates the detour number of a graph. The time is taken for the construction and the size of DNSDS are considered for examining the performance of the proposed method. The simulation result confirms that the DNSDS nodes are energy efficient.


Keywords: Domination number; non-split domination number; detour number; detour non-split domination number

## 1 Introduction

In WSN, for its capability of detection and processing the substantial section of the sensor node is often set in massive sums. These sensor nodes collect information from the viewing zone and the base station. Communication is the primary role of the WSN. Each sensor node is energy-protected here [1]. The delay and the duration of transmission are to be reduced each time when the packet is exchanged. To do so, the nodes have to be energy-rich [2]. In WSN, communication is carried out with the backbones, only the set of nodes. It may be able to generate backbones through the use of DNSDS [3]. Treat the packet from one bundle to the next until DNSDS nodes reach the destination. The backbone usage reduces overhead communication [4], builds capacity for data transmission, and reduces packet latency.

In this article, we design a population-based search technique specifically for the ACO which is supported by ant behavior in the creation of EE-DNSDS to provide answers to the optimization problem [5]. It evaluates its performance against the DNSDS based on the GA. The GA approach is a search solution both for the population and for natural biological development [6]. The rest of the paper is as follows: The notion of developing a DNSDS is outlined in Section 2. The background of the work is presented in Section 3. The suggested work is presented in Section 4. Section 5 explains experimental evaluation together with settings of stimulation, performance measures, and evaluation results. Section 6 provides for the conclusion.

## 2 Related Work

We consider graph $G$ as finite, undirected, and connected lacking loops. Let the order of $G$ be denoted as p and its size be denoted as q respectively. For knowing more about the basic terminologies in graph theory, consider two edges, that are said to be adjacent when both the vertices (i.e.,) are in edge $G$. Incase when $u v \in E(G)$, then we can easily say that the edge $u$ is a neighbor of $v$ and it is represented using the notation $N(v)$ that is nothing but the neighbor set of edge $v$. The vertex degree $v \in V$ is $\operatorname{deg}(v)=|N(v)|$. A vertex $v$ is understood as a universal vertex when $\operatorname{deg}(v)=p-1$. The subgraph stimulated by set $S$ of vertices of $G$ is symbolized as $<S_{i}>$ with $\mathrm{V}\left(<S_{i}>\right)=\mathrm{S}$ and $\mathrm{E}\left(<S_{i}>\right)=$ $\{u v \in E(G): u, v \in S\}$. The path that exists between two vertices of a graph and the one which visits each vertex just one time is said to be the Hamiltonian path or Hamilton path. Incase if there is a Hamiltonian path with adjacent endpoints, the resultant graph cycle is described as a Hamiltonian cycle.

In a connected graph $G$ with two vertices namely u and v ; the distance denoted as $d(u, v)$ among two vertices is the length of a shortest $u-v$ path in $G$. Usually, the $u-v$ geodesic is indicated as the $u-v$ path which has the length $d(u, v)$. Let x be a vertex that is understood to lie on a $u-v$ geodesic $P, x$ is a vertex of $P$ together with the vertices namely $u$ and $v$. The closed interval $I[u, v]$ encloses the vertices $u$ and $v$ along with every vertex within the $u-v$ geodesic. Incase when $I[u, v]=\{u, v\}$ then $u$ and $v$ are said to be adjacent. For a set $S$ of vertices, let $I[S]=\cup_{u, v \in S} I[u, v]$. Then certainly $S \subseteq I[S]$. A set $S \subseteq V(G)$ is supposed to be a geodetic set of $G$ when $I[S]=V$. The geodetic number is usually denoted as $g(G)$ and a graph is expressed as the minimum order of its geodetic sets and any geodetic set of order $g(G)$ is a $g$-set of $G$. The $g(G)$ of graphs was studied in [7-14].

In any connected graph G , with two vertices $u$ and $v$, the detour distance $D(u, v)$ is defined as the length of the longest $u-v$ path in $G$. The $u-v$ detour is indicated as the $u-v$ path of length $D(u, v)$.). Let x be a vertex that is understood to lie on a $u-v$ detour $P, x$ is a vertex of $P$ together with the vertices namely $u$ and $v$. The utmost detour distance from $v$ to a vertex of $G$ is the detour eccentricity $e_{D}(v)$ of a vertex $v$ in $G$. The least $e_{D}(v)$ amid the vertices of $G$ is the detour radius, $\operatorname{rad}_{D}(G)$ of $G$ and the utmost
$e_{D}(v)$ is the detour diameter, $\operatorname{diam}_{D}(G)$ of G. We denote detour radius by R and detour diameter by $D$. The closed interval $I_{D}[u, v]$ for two vertices u and v , includes all the vertices that exist within a $u-v$ detour along with u and v. For a set $S$ of vertices, let $I_{D}[S]=\cup_{u, v \in D} I_{D}[u, v]$. Then certainly, $S \subseteq I_{D}[S]$. A set $S \subseteq V$ is termed as detour set if $I_{D}[S]=V$. The detour number $d_{n}(G)$ of $G$ is usually in the least order of its detour sets as well as any detour set of order $d_{n}(G)$ is called a $d_{n}-$ set of $G$ and is initiated and studied in [15-19]. A set $S \subseteq V$ is termed as dominating set of $G$ if for each $v \in V \backslash S$ is adjacent to some vertex in $S$. A dominating set $S$ is said to be minimal if no subset of $S$ is dominating set $G$. The domination number of $G$ is symbolized as $\gamma(G)$ and is the minimum cardinality of a minimal dominating set of $G$ and was studied in [20]. Dominating Sets and Domination Polynomial of Fan Related Graphs were studied in [21]. A dominating set $D$ is supposed to be a non-split dominating set of $G$ if $\langle V-D\rangle$ is connected. The minimum cardinality of a non-split dominating set of $G$ is called the non-split domination number of $G$ and is denoted by $\gamma_{n s}(G)$ is called $\gamma_{n s}$-set of $G$ and is deliberated in [22].

## 3 Background

Ant Colony Optimization (ACO) is appropriate to track optimum paths depending on the behavior of the ants used to look through the food. When a food source is found, it goes back to the province by leaving 'marks' (predominantly called pheromones) which signals how much food is available. If others approach the marks, they have a certain probability and they will follow the path. In this event, it is not an easy for others to replenish the food with their own markings. The pathway is located by other ants and is further grounded till some ants flood the province from diverse food sources. As they release pheromones when transporting the food, a shorter path is bound to be more grounded, improving the "solution." Meanwhile, few ants continue to search for food sources closer to home. When the food resource is depleted, the path is no longer established with pheromones and eventually decays. Because the ant-colony moves in a fairly dynamic manner, and the ACO performs better in graphs with changing topologies. Examples of such frameworks include computer networks and worker artificial intelligence simulations.

### 3.1 The Detour Non-Split Domination Number of a Graph

A set $\mathrm{S} \subseteq \mathrm{V}$ is a Detour Non-Split Dominating Set (DNSDS) of G when the graph $\mathrm{G}=(\mathrm{V}, \mathrm{E})$ is expressed as both detours as well as a non-split dominating set of G. Let the detour non-split domination number be addressed as $\gamma_{\_}$dns $(G)$ and is the least order of its DNSDS. Any DNSDS of order $\gamma_{d n s}(G)$ is a $\gamma_{d n s}$-set of $G$.

Example 3.2.2: Assume a graph $G$ in Fig. 1, with no two-element subset of $G$ is a DNSDS of $G$ and so $\gamma_{d n s}(G) \geq 3$. Let $S=\left\{v_{1}, v_{4}, v_{9}\right\}$. Then $S$ is a DNSDS of $G$ consequently $\gamma_{d n s}(G)=3$.

## Observation:

(i) All end vertex of a connected graph, G belongs to every DNSDS of G.
(ii) Let order of $G$ be $n \geq 3$ with $v$ as its cut vertex, then every DNSDS of $G$ carries a minimum of a single element from each component of $G-v$.
(iii) For the star $G=K_{1, n-1}(n \geq 3), \gamma_{d n s}(G)=n$.


Figure 1: Graph $G$ is a DNSDS of $G$

- Theorem 1: For the path $G=P_{n}(n \geq 4), \gamma_{\text {dns }}(G)=n-2$.
- Proof: Let $P_{n}$ be $v_{1}, v_{2}, \ldots, v_{n}$. Then $S=V-\left\{v_{2}, v_{3}\right\}$ is a DNSDS of $G$ accordingly $\gamma_{\text {dns }}(G) \leq$ $n-2$. We prove that $\gamma_{d n s}(G)=n-2$. In contrast, suppose $\gamma_{d n s}(G) \leq n-3$. In that case, $S^{\prime}$ of $G$ is available then $\left|S^{\prime}\right| \leq n-3$. Now $<\mathrm{V}-S^{\prime}>$ is a path $P$ such that $|P| \geq 3$. Let $v_{i}^{\prime}$ be an internal vertex of P. Then $v_{i}^{\prime}$ is not dominated by any vertex of $S^{\prime}$. Hence $S^{\prime}$ is not a DNSDS of $G$, there is a negation. As a result $\gamma_{d n s}(G)=n-2$.
- Theorem 2: For the cycle $G=C_{n}(n \geq 4), \gamma_{d n s}(G)=-2$.
- Proof: This is alike the attestation of Theorem 3.2.4.
- Theorem 3: For the complete bipartite graph $G=K_{m, n}(2 \leq m \leq n), \gamma_{d n s}(G)=2$.
- Proof: Assume $L$ and $W$ as bipartite sets of $G$ and $x y \in E(G)$. Then $S=\{x, y\}$ is a DNSDS of $G$ thus $\gamma_{d u s}(G)=2$.
- Theorem 4: For wheel $G=W_{n}=K_{1}+C_{n-1}(n \geq 4), \gamma_{d n s}(G)=2$.

Proof: Let $V\left(K_{1}\right)=x$ and $y \in V\left(C_{n-1}\right)$. Then $S=\{x, y\}$ is a DNSDS of $G$ with the intention that $\gamma_{d n s}(G)=2$.

- Theorem 5: For the complete graph $G=K_{n}(n \geq 3), \gamma_{d n s}(G)=2$.
- Proof: Let $x y \in E(G)$. Then $S=\{x, y\}$ is a DNSDS of $G$ thus $\gamma_{d n s}(G)=2$.
- Theorem 6: For double star $G$ of order $(n \geq 4), \gamma_{\text {dns }}(G)=2$.
- Proof: Let the set, $S$ carries $n-2$ end vertices of $G$. By Observation 3.2.3 (i), $S$ is a subset of each DNSDS of $G$ and as a result $\gamma_{d n s}(G) \geq n-2$. Since S is a DNSDS of $G$, it goes with $\gamma_{d n s}(G)$ $=n-2$.
- Theorem 7: For helm graph $G=H_{r}, \gamma_{d n s}(G)=r+1$.
- Proof: Assume $x$ as central vertex and $Z$ as the set of $r$ end vertices of $G$. By Observation 3.2.3 (i), $Z$ is a subset of each DNSDS of $G$. Since $x$ is not subjugated by any vertex of $Z, Z$ is not a DNSDS of $G$ thus $\gamma_{d n s}(G) \geq r+1$. Let $Z^{\prime}=Z \cup\{x\}$. Then $I_{D}\left[Z^{\prime}\right]=V$ and $\left\langle V-Z^{\prime}\right\rangle$ doesn't have isolated vertices. As a result $Z^{\prime}$ is a DNSDS of $G$ and $\gamma_{\text {dns }}(G)=r+1$.
- Theorem 8: For banana tree graph $G=B_{r, s}, \gamma_{d n s}(G)=r+1$.
- Proof: Assume $x$ as central vertex and $Z$ as the set of $r$ end vertices of $G$. By Observation 3.2.3 (i), $Z$ is a subset of each DNSDS of $G$. Since $x$ is not subjugated by any vertex of $Z, Z$ is not a DNSDS of $G$ thus $\gamma_{\text {dns }}(G) \geq r+1$. Let $Z^{\prime}=Z \cup\{x\}$. Then $I_{D}\left[Z^{\prime}\right]=V$ and $<\mathrm{V}-Z^{\prime}>$ doesn't have isolated vertices. As a result $Z^{\prime}$ is a DNSDS of G and $\gamma_{\text {dns }}(G)=r+1$.
- Theorem 9: Assume Regard $G$ as a connected graph with order $n \geq 3$ with $D \geq 2$. Then $\gamma_{d n s}(G)$ $\leq n-1$.
- Proof: Assume $P: v_{0}, v_{1}, v_{2}, \ldots, v_{D}$ as a detour diametral path in $G$. As $D \geq 2, P$ contains at least one internal vertex. Let $S=V-\left\{v_{1}\right\}$. The S is a DNSDS of $G$ with $\gamma_{d n s}(G) \leq n-1$.
- Remark 10: The bound in Theorem 3.2.12 is spiky. For path $G=P_{3}, \gamma_{d n s}(G)=2=n-1$.
- Theorem 11: Assume Regard $G$ as a connected graph with order $n \geq 2$. Also $\gamma_{d n s}(G)=n$ as long as $G$ is $K_{2}$.
- Proof: Let $\gamma_{d n s}(G)=n$. In contrast when $G \neq K_{2}$. By Theorem 3.2.12, $\gamma_{d n s}(G) \leq n-1$, which is a contradiction. As a result, $D=1$. Hence $G=K_{2}$. The reverse is apparent.
- Theorem 12: Regard $G$ as a connected graph with order $n \geq 4$ which is not a star. Then $\gamma_{d n s}(G)$ $\leq n-2$.
- Proof: Assume $G$ as a tree. Since $G \neq K_{1, n-1}$, $G$ holds two adjacent vertices, say $x$ and $y$. Then $S=V(G)-\{x, y\}$ is a DNSDS of $G$ so that $\gamma_{d n s}(G) \leq \mathrm{n}-2$. After that imagine that $G$ is not a tree. Then $G$ includes a cycle says $C$. Let $C: v_{1}, v_{2}, \ldots, v_{r}(r \geq 3)$ be the longest cycle in $G$. Suppose that all the vertices of $C$ are cut-vertices of $G$. Then $S=V(G)-V(C)$ is a DNSDS of $G$ and so $\gamma_{d n s}(G) \leq n-|V(G)| \leq n-3$, therefore is a negation. Suppose that $G$ holds as a minimum one cut-vertex, say $v_{1}$. Then $S=V(G)-\left\{v_{1}, v_{2}\right\}$ is a DNSDS of $G$ as a result $\gamma_{d n s}(G)$ $\leq n-2$, which is a contradiction. If no vertex of $G$ is a cut vertex of $G$, by the similar argument, it can show that $\gamma_{\text {dns }}(G) \leq n-2$, which is a contradiction.
- Remark 13: The bound in Theorem 2.15 is spiky. For cycle $G=C_{4}, \gamma_{d n s}(G)=2=n-2$.
- Theorem 14: Assume Regard $G$ as a connected graph with order $n \geq 3$. Also $\gamma_{d n s}(G)=n-1$ as long as $G=K_{1, n-1}$ or $K_{3}$.
- Proof: Let $\gamma_{\text {dns }}(G)=n-1$. If $n=3$, then $G=K_{1.2}$ or $K_{3}$, which satisfies the requirements of this theorem. So we have done. Let $n \geq 4$. But when $G \neq K_{1, n-1}$, then according to the Theorem 3.2.15, $\gamma_{d n s}(G) \leq n-2$, therefore is a negation. For that reason $G=K_{1, n-1}$. The converse is clear. Now we distinguish connected graph with order $n \geq 4$ and detour diameter $D \leq 4$ with $\gamma_{d n s}(G)$ $=n-2$. For this purpose, we introduce family $\mathfrak{I}$ of graph
- Theorem 15: Assume $G$ as a connected graph with $n \geq 4$ and $D \leq 4$. Then $\gamma_{d n s}(G)=n-2$ as long as $G$ is either $C_{4}$ or $K_{4}$ or $K_{4}-\{\mathrm{e}\}$ or a double star of the graph $G$ specified in Fig. 2 of the family $\mathfrak{I}$.


Figure 2: Graph $G$ specified in the family $\mathfrak{I}$

- Proof: Let $\gamma_{d n s}(G)=n-2$. So we enclose the two subsequent cases.
- Case (i): If $G$ is a tree. According to Theorem 3.2.17, $G \neq K_{1, n-1}$. Suppose $G$ is a double star, then $G$ satisfies the requirements of this theorem. So, we have done. Let us assume that $G$ is neither a star nor a double star. Then $G$ contains a path $P: x, y, z$. Let $S=V-\{x, y, z\}$. Then $S$ is a DNSDS of a graph as a consequence $\gamma_{d n s}(G) \leq n-3$, therefore is a negation.
- Case (ii): If $G$ is not a tree. Then it holds as a minimum of one cycle $C$. Let $C$ be a girth in $G$ and $C(G)$ be its length. Since $D \leq 4$. We have that $C(G) \leq 4$. Let $C$ be $v_{1}, v_{2}, v_{3}, v_{4}, v_{1}$. If $G=K_{4}-\{e\}$, then we are done. If $G=K_{4}$, then we are done. Suppose that $G$ is neither $C_{4}$ nor $K_{4}-\{e\}$ nor $K_{4}$. Then there exists one vertex $x$ to such a degree which is adjacent to $v_{1}$, say. Then $S=V-\left\{v_{1}, v_{2}, v_{3}\right\}$ is a detour non-split domination number of a graph as a result, $\gamma_{\text {dns }}(G) \leq$ $n-3$, therefore is a negation. Let $C(G)=3$. Let $C$ be $v_{1}, v_{2}, v_{3}, v_{1}$. Since $D \leq 4$, there exists a minimum of one vertex $(x)$ thereby $x v_{1} \in E(G)$. If $d\left(v_{2}\right)=d\left(v_{3}\right)=2$ and the edges incident with $v_{1}$ are end edges, then the graph $G$ is given in family $\mathfrak{I}$ of Fig. 2 a. This satisfies the requirements of this theorem. If at least one edge incident with $x$ is not an end edge, subsequently $\gamma_{d n s}(G) \leq$ $n-3$, which is a contradiction. If $\operatorname{deg}\left(v_{1}\right)=2, \operatorname{deg}\left(v_{2}\right) \geq 3$ and $\operatorname{deg}\left(v_{3}\right) \geq 3$, then since $D \leq 4$, the edges incident at $v_{2}$ and $v_{3}$ are end edges. Then graph $G$ is given in the family $\mathfrak{I}$ of Fig. 2 b. Since $D \leq 4, \operatorname{deg}\left(v_{i}\right) \geq 3$ for all $\mathrm{i}(1 \leq \mathrm{i} \leq 3)$ is not possible. The reverse is apparent.
- Theorem 16: While considering whichever pair of positive integers to be specific a and b , there exists a connected graph $G$ thereby $d n(G)=a, \gamma(G)=b$ and $\gamma_{d n s}(G)=a+b-2$.
Proof: Let $P_{2(b-2)+1}: x, v_{1}, v_{2}, \ldots, v_{2(b-2)+2}, y$ be a path on $2(\mathrm{~b}-2)+2$ vertices. $H$ as a graph attained from $P_{2(b-2)+2}$ by accumulating the new vertices $x_{1}, x_{2}, \ldots, x_{a-1}$ and introduced as edge $x x_{i}(1 \leq i \leq a-1)$. Assume graph $G$ gained from $H$ by summing up new vertices $u_{1}, u_{2}, \ldots, u_{b-2}$ with initiating the edges $u_{i} v_{i}(1 \leq i \leq 2(b-2)-1)$ and $u_{i} v_{i+1}(2 \leq i \leq 2(b-2)$ is revealed in Fig. 3. Since $I_{D}[X]=V, X$ is a detour set of $G$ therefore, $d n(G)=a$. Subsequently, we illustrate that $\gamma(G)=b$. We view that all $\gamma$-set of $G$ contains $u_{i}(1 \leq i \leq b-2)$ and the vertices $x$ and $y$ and $\gamma(G)=b-2+2=b$. Let $S=\left\{x, y, u_{1}, u_{2}, \ldots, u_{b-2}\right\}$. Then $S$ is a dominating set of $G$ so that $\gamma(G)=b$. After that, we show that $\gamma_{d n s}(G)=a+b-2$. The end vertices of $G$ be $X=\left\{x, x_{1}, x_{2}, \ldots, x_{a-1}, y\right\}$. By Observing 3.2.3 (i), X is a subset of every DNSDS of $G$ and so $\gamma_{d n s}(G) \geq a$. It is handily seen that each DNSDS of $G$ holds each $u_{i}(1 \leq i \leq b-2)$ and so $\gamma_{d n s}(G)$ $\geq a+b-2$. Let $S^{\prime}=\mathrm{X} \cup\left\{u_{1}, u_{2}, \ldots, u_{b-2}\right\}$. Then $S^{\prime}$ is DNSDS of $G$ so that $\gamma_{d n s}(G)=a+b-2$.


Figure 3: Graph $G$ gained from $H$ by summing up new vertices

### 3.2 The Detour Non-Split Domination Number of Join of Graph

Assume $H$ and as we $K$ as two graphs. The combination of two graphs namely $G$ and $H$ is symbolized as $G+H$ and defined as the graph with $V(G+H)=V(G) \cup V(H)$ and $E(G+H)=$ $E(G) \cup E(H) \cup\{u v: u \in V(G), v \in V(H)\}$.

- Theorem 1: If $K$ and $H$ are two connected graphs that contain either a Hamiltonian path or a Hamiltonian cycle. Then $\gamma_{\text {dns }}(K+H)=2$.
- Proof: Let $P_{1}: u_{0}, u_{1}, u_{2}, \ldots, u_{l}$ be a Hamiltonian path in K , similarly $P_{2}: v_{0}, v_{1}, v_{2}, \ldots, v_{k}$ be a Hamiltonian path in $G$, where $l+k=n$. Then $P_{1} \cup P_{2}$ is a Hamiltonian path in $K+H$. Let $S=\left\{u_{0}, v_{0}\right\}$. Then $S$ is a DNSDS of $K+H$ consequently $\gamma_{\text {dns }}(\mathrm{K}+\mathrm{H})=2$.
- Corollary 2:
(i) Let $K=P_{n}(n \geq 4)$ and $H=P_{m}(m \geq 4)$. Then $\gamma_{d n s}(\mathrm{~K}+\mathrm{H})=2$.
(ii) Let $K=P_{n}(n \geq 4)$ and $H=C_{m}(m \geq 4)$. Then $\gamma_{\text {dns }}(K+H)=2$.
(iii) Let $K=C_{n}(n \geq 4)$ and $H=C_{m}(m \geq 4)$. Then $\gamma_{d n s}(K+H)=2$.
(iv) Let $K=K_{n}(n \geq 3)$ and $H=K_{m}(m \geq 4)$. Then $\gamma_{\text {dns }}(K+H)=2$.
(v) Let $K=K_{n}(n \geq 4)$ and $H=P_{m}(m \geq 4)$. Then $\gamma_{d n s}(K+H)=2$.
(vi) Let $K=K_{n}(n \geq 4)$ and $H=C_{m}(m \geq 4)$. Then $\gamma_{d n s}(\mathrm{~K}+\mathrm{H})=2$.


### 3.3 The Detour Non-Split Domination Number of Corona Product of Graph

The corona product $K \circ H$ is described as the graph gained from $K$ and $H$ by attaining one copy of $K$ and $|V(K)|$ copies of $H$ and then by joining an edge of, all the vertices from the $\mathrm{i}^{\text {th }}$-copy of $H$ to the $\mathrm{i}^{\text {ith }}$-vertex of $K$, where $i=1,2, \ldots,|V(H)|$.

- Theorem 1: Assume two connected graphs notably, $K$ as well as $H$ with orders $n_{1}$ and $n_{2}$ respectively. If H contains either a Hamiltonian path or a Hamiltonian cycle, then $\gamma_{d n s}(K \circ H)=$ $n_{1}$.
- Proof: If $n_{2}=1$, then the result is obvious. Let $H_{1}=\left(V_{1}, E_{1}\right), H_{2}=\left(V_{2}, E_{2}\right), \ldots, H_{n_{1}}=\left(V_{n_{1}}, E_{n_{1}}\right)$ be the $n_{1}$ copies of $H$ in $K \circ H$. Let $Q_{i}: v_{i 1}, v_{i 2}, \ldots, v_{i n_{2}},\left(1 \leq i \leq n_{1}\right)$ be a Hamiltonian path in $H_{i}\left(1 \leq i \leq n_{1}\right)$. Assume $V$ as the vertex of $K$ and $V=\left\{v_{1}, v_{2}, \ldots, v_{n_{1}}\right\}$. Then set of cut vertices in $K \circ H$ is $V$. By Observing 3.2.3 (ii), every DNSDS of $K \circ H$ holds minimum vertex from every $Q_{i}\left(1 \leq i \leq n_{1}\right)$ consequently $\gamma_{d n s}(G) \geq n_{1}$. Let $\mathrm{S}=\left\{v_{11}, v_{21}, \ldots, v_{n_{1}}\right\}$. Then $S$ is a DNSDS of $K \circ H$ so that $\gamma_{d n s}(G)=n_{1}$.
- Corollary 2 :
(i) Let $K=P_{n}(n \geq 4)$ and $H=P_{m}(m \geq 4)$. Then $\gamma_{d n s}(K \circ H)=n$.
(ii) Let $K=P_{n}(n \geq 4)$ and $H=C_{m}(m \geq 4)$. Then $\gamma_{d n s}(K \circ H)=n$.
(iii) Let $K=C_{n}(n \geq 4)$ and $H=C_{m}(m \geq 4)$. Then $\gamma_{\text {dns }}(K \circ H)=\mathrm{n}$.
(iv) Let $K=K_{n}(n \geq 3)$ and $H=K_{m}(m \geq 4)$. Then $\gamma_{d n s}(K \circ H)=\mathrm{n}$.
(v) Let $K=K_{n}(n \geq 4)$ and $H=P_{m}(m \geq 4)$. Then $\gamma_{d n s}(K \circ H)=n$.
(vi) Let $K=K_{n}(n \geq 4)$ and $H=C_{m}(m \geq 4)$. Then $\gamma_{d n s}(K \circ H)=\mathrm{n}$.
- Theorem 3: Assume $K$ as a connected graph with order $n_{1} \geq 2$. Then $\gamma_{d n s}\left(K \circ K_{n_{2}}\right)=n_{1} n_{2}$.
- Proof: Let $\mathrm{V}\left(\bar{K}_{n_{2}}\right)=\left\{v_{1}, v_{2}, \ldots, v_{n_{2}}\right\}\left(1 \leq i \leq n_{1}\right)$ and $S_{i}=\left\{v_{i 1}, v_{i 2}, \ldots, v_{i_{2}}\right\}$ be the $\mathrm{i}^{\text {th }}$ copy of $\bar{K}_{n_{2}}$. Then $\mathrm{S}=\cup_{i=1}^{n_{1}} S_{i}$ is the end vertices set of $K \circ \bar{K}_{n_{2}}$. By Observing 3.2.3 (i), $S$ is a subset of every DNSDS of $K \circ \bar{K}_{n_{2}}$ and so $\gamma_{\text {dns }}\left(K \circ \bar{K}_{n_{2}}\right) \geq n_{1} n_{2}$. As S is a DNSDS of G, we have $\gamma_{\text {dns }}($ $\left.K \circ \bar{K}_{n_{2}}\right)=n_{1} n_{2}$.
- Theorem 4: Assume $K$ as a connected graph with order $n_{2} \geq 2$. Then $\gamma_{d n s}\left(K \circ \bar{P}_{n_{2}}\right)=n_{1}$.
- Proof: Let $V\left(\bar{P}_{n_{2}}\right)=\left\{v_{1}, v_{2}, \ldots, v_{n_{2}}\right\}$ and $H_{i}=\left\{v_{i 1}, v_{i 2}, \ldots, v_{i m_{2}}\right\}$ be the ith copy of $\bar{P}_{n_{2}}\left(1 \leq i \leq n_{1}\right)$. By Observation 3.2.3 (ii), every DNSDS of $G$ holds a minimum of one vertex from each Hi and so $\gamma_{d n s}\left(\left(K \circ \bar{P}_{n_{2}}\right) \geq n_{1}\right.$. Then $\mathrm{S}=\left\{v_{11}, v_{21}, \ldots, v_{n_{1} 1}\right\}$ is a DNSDS of G so that $\gamma_{d n s}\left(K \circ \bar{P}_{n_{2}}\right)=n_{1}$.
- Theorem 5: Assume $K$ as a connected graph with order $n_{2} \geq 2$. Then $\gamma_{d n s}\left(K \circ \bar{C}_{n_{2}}\right)=2 n_{1}$, Where $n_{2} \geq 5$.
- Proof: Let $V\left(K \circ \bar{C}_{n_{2}}\right)=\left\{v_{1}, v_{2}, \ldots, v_{n_{2}}\right\}$ and $H_{i}=\left\{v_{i 1}, v_{i 2}, \ldots, v_{i n_{2}}\right.$ be the $\mathrm{i}^{\text {th }}$ copy of $\bar{C}_{n_{2}}$, $($ $1 \leq i \leq n_{2}$ ). In that case, it can be said that every DNSDS of $G$ holds a minimum of two vertices
from each $H_{i}\left(1 \leq i \leq n_{1}\right)$ and so $\gamma_{d n s}(G) \geq 2 n_{1}$. Let $S=\left\{v_{11}, v_{13}, v_{21}, v_{23}, \ldots, v_{n_{1} 1}, v_{n_{1} n_{2}}\right\}$. Then $S$ is a DNSDS of $G$ so that $\gamma_{d n s}(G)=2 n_{1}$.
- Theorem 6: Assume $K$ as a connected graph with order $n_{2} \geq 2$. Then $\gamma_{d n s}\left(K \circ \bar{K}_{n_{2}}\right)=2 n_{1}$, where $2 \leq r \leq s$.
- Proof: Let $U=\left\{u_{1}, u_{2}, \ldots, u_{r}\right\}$ and $W=\left\{w_{1}, w_{2}, \ldots, w_{s}\right\}$ as the bipartite sets of $K_{r, s}$. Let $H_{i}=U_{i} \cup W_{i}=\left\{u_{i 1}, u_{i 2}, \ldots, u_{i r}\right\} \cup\left\{w_{i 1}, w_{i 2}, \ldots, w_{i s}\right\}\left(1 \leq \mathrm{i} \leq n_{1}\right)$ be the $\mathrm{i}^{\text {th }}$ copy of $K_{r, s}$. Since $<\bar{U}_{i}>$ and $<\bar{W}_{i}>$ are complete graphs for all $\mathrm{i}\left(1 \leq \mathrm{i} \leq n_{1}\right)$, every DNSDS of $G$ holds a minimum of two vertices of every $H_{i}\left(1 \leq \mathrm{i} \leq n_{1}\right)$. For this reason $\gamma_{d n s}(G) \geq 2 n_{1}$. Let $S=\left\{u_{11}, u_{21}, \ldots, u_{n_{1} r}, w_{11}, w_{21}, \ldots, w_{n_{1} s}\right\}$. In that case, we conclude $S$ as DNSDS of G with $\gamma_{\text {dns }}(G)$ $=2 n_{1}$.


## 4 Proposed System Model

The scavenging behavior of ants excites ACO. When ants walk, they leave a pheromone trail in each node they pass through. The pheromone likelihood, which is provided on every node, aids in determining the shortest path of food from source to destination. A DNSDS, which is rich in energy, is produced in our suggested study. We use two rules in the ACO algorithm to do this: (i) the pheromone updating rule (which signals the updated for each node and is handled in Eq. (4)) and (ii) the state transition rule (which assists with choosing the next node based on the probability value and is addressed in Eq. (1) [22].
$P_{i}^{k}=\frac{\tau_{i}^{\alpha}}{\sum_{i \in A_{k}} \tau_{i}^{\alpha} n_{i}^{\beta}}$
In our algorithm, we start with $\tau 0$ in each node of the graph. Ants wander throughout the graph randomly by dropping pheromones on all nodes. In this fashion, the ant iterates $\mathbf{N}$ times. During this each chosen node with a high P and E based on the probabilistic state transition criteria is added to the DNSDS.
$P_{i}^{k}=\frac{\tau_{i}^{\alpha} n_{i}^{\beta} E_{i}^{\gamma}}{\sum_{i \in A_{k}} \tau_{i}^{\alpha} n_{i}^{\beta} E_{i}^{\gamma}}$
$E_{i}=\frac{R E}{E_{\text {initial }}}$
The $\tau$ of the nodes which are in the DNSDS is refreshed by the pheromone updating rule.
$\tau_{i}=(1-\rho) . \tau_{i}+\rho . \tau_{o}$

In Eq. (4), the value of $\rho$ is always $0 \leq \rho \leq 1$. The $\tau$ of the nodes that are unavailable in the DNSDS is evaporated by Eq. (5).
$\tau_{i}=\tau_{i} \times \rho$

Notations utilized in the work are specified in Tab. 1.

Table 1: Notations and their description

| Notation | Description |
| :--- | :--- |
| $P$ | Pheromone probability |
| $\tau_{i}$ | Pheromone value of an $\mathrm{i}^{\text {th }}$ node |
| $E_{i}$ | Initial energy |
| $\rho$ | Pheromone persistence |
| $\tau_{o}$ | Initial pheromone value |
| $\mathrm{A}_{\mathrm{k}}$ | Accessible nodes to ant |
| $R E$ | Residual energy |

## 5 Experimental Evaluation

In this section, we have illustrated and investigated fewer limitations, namely the experimental parameters and performance indicators, before presenting the assessment results.

### 5.1 Simulation Setup

The simulation is completed by assuming the sensor field to be $1000 \times 1000$. During its execution, the following suspicions are taken into account:

- Sensor nodes are homogeneous and fixed.
- The region is both constrained and consistent.

The energy and degree of the node are taken into account during the DNSDS creation process. Using ACO and the GA, we led large-scale replications to create the DNSDS. We have evaluated the effectiveness of both strategies. Tab. 2 specifies the simulated limitations used to raise the EE-DNSDS using the ACO approach.

Table 2: Simulation constraints

| Channel type | Wireless channel Radio |
| :--- | :--- |
| Propagation Model | Two-way ground |
| Transmission Area | $1000 \times 1000\left(\mathrm{~m}^{2}\right)$ |
| Transmission range | $20(\mathrm{~m})$ |
| $\tau_{o}$ | 10 |
| Ant count | 10 |
| $\alpha, \beta, \gamma$ | 1 |
| $\rho$ | 0.985 |

### 5.2 Performance Evaluation

The evaluation of the performance is completed by considering two measurements. To be specific the construction time and the size of the DNSDS between the ACO technique and the GA are as follows:

DNSDS Construction Time: Fig. 4 addresses the DNSDS construction time exploited by GA and the ACO technique. While contrasting the ACO and GA, the ACO utilized not as much DNSDS construction time as GA. When the node gets tally to build, the ACO technique performs better when compared to GA.


Node Count
Figure 4: Comparison of DNSDS construction time
DNSDS Size: Fig. 5 addresses the simulation yield of DNSDS size for different quantities of nodes. We have considered the DNSDS is off to be in average size. In the projected system, the DNSDS size is low in ACO than the GA for the more modest number of nodes. As the node count gets expanded, the average DNSDS size of ACO is lower than the GA.


Figure 5: Comparison of DNSDS size

## 6 Conclusion

By modifying the ACO approach, we created an Energy Efficient Detour Non-Split Dominating Set (EE-DNSDS) in this paper. The scavenging behavior of ants inspires the ACO process. The DNSDS created an abundance of energy. The correlation between two algorithms, specifically the ACO and the GA, is performed here. When comparing the ACO and GA, the ACO used less DNSDS build time than the GA. As the number of nodes increases, the average DNSDS size of ACO is roughly
equal to that of GA. This is accomplished by employing the network's energy-efficient DNSDS nodes. Furthermore, the number of standard graphs is resolved and a fraction of its overall qualities are considered in the detour non-split domination. Other optimization approaches can be used in the future, and performance measures can be checked and compared.

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