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Error Detection and Pattern Prediction Through Phase II Process Monitoring

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Abstract: The continuous monitoring of the machine is beneficial in improving its process reliability through reflected power function distribution. It is substantial for identifying and removing errors at the early stages of production that ultimately benefit the firms in cost-saving and quality improvement. The current study introduces control charts that help the manufacturing concerns to keep the production process in control. It presents an exponentially weighted moving average and extended exponentially weighted moving average and then compared their performance. The percentiles estimator and the modified maximum likelihood estimator are used to constructing the control charts. The findings suggest that an extended exponentially weighted moving average control chart based on the percentiles estimator performs better than exponentially weighted moving average control charts based on the percentiles estimator and modified maximum likelihood estimator. Further, these results will help the firms in the early detection of errors that enhance the process reliability of the telecommunications and financing industry.

Keywords: Reflected power function distribution; exponentially weighted moving average; extended exponentially weighted moving averages; modified maximum likelihood estimator; percentile estimator

1 Introduction

The scholars are anxious to know about the error tendency during the entire manufacturing process to validate the pre-production testing results. It was expected during the machine installation process that the pre-testing results remain valid in the practical life, and also errors remain in control for instance, laptop manufacturing, which passes through several processes. The final product should be error-free when it is delivered to the market for sale as the complaints from distributors and customers can harm the firm's reputation. Therefore, companies follow a rigorous monitoring procedure to identify and handle faults at the early stages. An efficient monitoring system can minimize the likelihood of product failure and improves its quality. That is why organizations paid attention to develop a system that can identify errors at an initial stage and keep them in control overtimes. In this way, it not only increases the efficiency of the product but also enhances its productivity.

However, the underlying situation of the production process may not be as normal in practical life as expected. That is why the error distribution of the manufacturing concerns is not normal and often follows a reflected power function distribution (RPFD). Therefore, there is a need to apply the RPFD under such real-life situations, where normal distribution failed to assess the error patterns. The current study used the RPFD from Zaka et al. [1] in process analysis, reliability testing, and error predicting. We also introduce control charts based on the assumptions that if the number of errors follows the RPFD and there exists a non-random variation in the distribution, then errors can be determined and handled at the initial stage. These control charts make the monitoring process of a machine more reliable and also provides persistent results.

Previously scholars constructed control charts based on the normality assumption. However, few studies like Roberts [2] recommended including exponentially weighted moving average (EWMA). Some studies currently discussed different control charts for real-life situations where normality assumptions do not fulfil [3–15]. These real-life applications of the control charts motivate scholars to explore them in non-normal cases where error patterns are unpredictable, particularly in manufacturing concerns. It helps the practitioners in the early solution to the errors that further lead to continuing the process without any interval, ultimately saving time and cost. It also develops the customers' confidence in the corporations through a continuous quality improvement process.

Generally, statisticians deal with two types of data processing. The first type provides the complete information about the variable of interest, while the second one often depicts misleading or sometimes based on basic details about the study variables. The present study is conducted in the first type of data when the process yields all the observations and follows the RPFD. However, the second type of data is more appropriate for neutrosophic statistics and is used in the literature to make the control charts. The scholars Aslam et al. [9–11], Aslam [12], Aslam et al. [13], and Khan et al. [14,15] have provided empirical support to these arguments.

This study has an added advantage over existing papers on control charts; first, it is applicable in most situations when the normality of any process is in doubt. We have found limited literature [2–15] discussing practical life scenarios. Thus, the current work may be a valuable addition to the literature. Second, it can be extended to neutrosophic statistics by providing the control charts when the process provides indeterminate data without assuming the normality of the process distribution.

We have the following breakage of the manuscript: Section 2.1 provides us with the RPFD and its estimators. Then, we have constructed an EWMA chart and presented it in Section 2.2, and the EEWMA chart is developed in Section 2.6. Simulation study and real-life application are discussed in Section 3, and then finally conclusion is drawn in Section 4.

2 Materials and Methods

2.1 Proposed Process Monitoring for Reflected Power Function Distribution

Using Zaka et al. [1] and assuming $x_1, x_2, x_3, \ldots, x_t$ being independent identically distributed random variables follows the RPFD as given below

$$f(x) = \frac{\gamma(\theta - x)^{\gamma-1}}{\beta^{\gamma}}, \theta - \beta < x < \theta, \text{ and } \beta, \theta, \gamma > 0$$

and

$$F(x) = 1 - \tfrac{(\theta - x)^{\gamma}}{\beta^{\gamma}},$$

where "θ" is the reflecting parameter. Also, γ, β are the shape and scale parameters. From Zaka et al. [1], MMLM and PE estimators are defined below,

$$\hat{\gamma}_{MMLM} = \left(\frac{n(1 + \ln(0.5))}{\left(n\ln\left(\theta - \tilde{x}\right) - \sum_{i=1}^{n} \ln(\theta - x_i)\right)}\right). \tag{1}$$

and

$$\hat{\gamma}_{PE} = \frac{\ln\left(\frac{1-H}{1-L}\right)}{\ln\left(\frac{\theta - P_H}{\theta - P_L}\right)}. \tag{2}$$

where H = maximum percentile, L = minimum percentile and P = percentile

2.2 EWMA Control Chart Using PE

Using $E\left(\hat{\gamma}_{PE}\right) = \gamma$. The shape parameter is estimated through PE, and EWMA statistic is given as

$$EWPE_t = \lambda\hat{\gamma}_{\mathrm{PE(t)}} + (1 - \lambda)\, EWPE_{t-1}, \tag{3}$$

where $EWPE_{t-1}$ represent the EWMA statistic for the preceding time. And λ is a smoothing constant. We refer the Zaka et al. [16] to get the details on generalizing EWMA Statistics,

$$EWPE_t = \lambda\hat{\gamma}_{\mathrm{PE(t)}} + (1 - \lambda)\,\lambda\hat{\gamma}_{\mathrm{PE(t-1)}} + (1 - \lambda)^2\,\hat{\gamma}_{\mathrm{PE(t-2)}} + \ldots + (1 - \lambda)^{t-1}\,\lambda\hat{\gamma}_{\mathrm{PE(1)}} + (1 - \lambda)^t\, EWPE_0,$$

where

$$EWPE_0 = \gamma, \tag{4}$$

Using Zaka et al. [16], we get

$$E(EWPE_t) = \gamma. \tag{5}$$

$$Var(EWPE_t) = \lambda^2 V_{PE}\left(\tfrac{1-(1-\lambda)^{2t}}{1-(1-\lambda)^2}\right).$$

Alternatively, we get

$$Var(EWPE_t) = V_{PE}\left(1 - (1 - \lambda)^{2t}\right)\left(\frac{\lambda}{2 - \lambda}\right). \tag{6}$$

The control limits are,

$$LCL_{EWPE_t} = \gamma - L * \sqrt{V_{PE} * \frac{\lambda}{(2-\lambda)}\left(1-(1-\lambda)^{2t}\right)}$$

$$CL_{EWPE_t} = \gamma$$

$$UCL_{EWPE_t} = \gamma + L * \sqrt{V_{PE} * \frac{\lambda}{(2-\lambda)}\left(1-(1-\lambda)^{2t}\right)},$$

where $1-(1-\lambda)^{2t}$ tends to unity if t approach towards larger observation. The control limits are defined below

$$LCL_{EWPE_t} = \gamma - L * \sqrt{V_{PE} * \frac{\lambda}{(2-\lambda)}} \tag{7}$$

$$CL_{EWPE_t} = \gamma \tag{8}$$

$$UCL_{EWPE_t} = \gamma + L * \sqrt{V_{PE} * \frac{\lambda}{(2-\lambda)}}. \tag{9}$$

2.3 EWMA Control Chart Using MMLM

We define the EWMA statisticas

$EW\mathrm{MMLM}_t = \lambda\hat{\gamma}_{\mathrm{MMLM}} + (1-\lambda)\, EW\mathrm{MMLM}_{t-1}$,

where $\hat{\gamma}_{\mathrm{MMLM}}$ is MMLM for the RPFD and $EW\mathrm{MMLM}_{t-1}$ is the statistic from previous time. λ is a smoothing constant. Using (5) and (6), we get

$$LCL_{\mathrm{EWMMLM}_t} = \gamma - L * \sqrt{V_{\mathrm{MMLM}}\left(\frac{\lambda}{2-\lambda}\right)\left(1-(1-\lambda)^{2t}\right)} \tag{10}$$

$$CL_{\mathrm{EWMMLM}_t} = \gamma \tag{11}$$

$$UCL_{\mathrm{EWMMLM}_t} = \gamma + L * \sqrt{V_{\mathrm{MMLM}}\left(\frac{\lambda}{2-\lambda}\right)\left(1-(1-\lambda)^{2t}\right)}. \tag{12}$$

$$Var\left(\hat{\gamma}_{\mathrm{MMLM(t)}}\right) = V_{\mathrm{MMLM}} = E\left(\hat{\gamma}_{\mathrm{MMLM}} - \gamma\right)^2$$

2.4 Algorithm Used for EWMA Control Charts Using PE and MMLM

We generate a random sample from the process following RPFD using the sample size of 150. We then compute the estimate of the shape parameter using PE and MMLM alternatively and get their means and variances. Finally, the control limits using PE and MMLM are computed, and ARL is computed. We have fixed ARL0 = 500. Now we assume that the process parameter is shifted from its true value. We take different shifts and computed ARL for shifted process and called it ARL_1. We have repeated this process 5000 times. Using the algorithm approach above, it is observed that the EWMA chart is helpful in the detection of small shifts at the early stage of the distribution. Here, we apply ARL criteria to compare the efficiency of an EWMA and EEWMA for both estimation methods.

From Tabs. 1–3 and Figs. 1–3, we observe an increasing behavior in ARL for the EWMA control chart with an increase in the value of λ using PE and MMLM to estimate the distribution parameters. We note that as λ gets close to 1, there is less variation in the ARLs. Also, from Figs. 2 to 4, we note that the control chart based on PE performs better than the control chart based on MMLM.

Table 1: ARL_S for EWMA control charts using PE and MMLM ($\lambda = 0.2$)

Estimation methods	Shift								
		0	0.40	1.20	2.40	3.60	4.80	6.00	7.20
PEL = 7.355	ARL	500.38	5.62	1.783	1.016	1	1	1	1
	SDRL	496.529	2.5398	0.5159	0.1255	0	0	0	0
	P10	53.8	3	1	1	1	1	1	1
	P25	139.7	4	1	1	1	1	1	1
	P50	362.0	5	2	1	1	1	1	1
	P75	701.00	7	2	1	1	1	1	1
	P90	1186.80	9	2	1	1	1	1	1
MMLML = 4.55	ARL	500.847	24.155	4.28	2.062	1.523	1.169	1.023	1.002
	SDRL	496.8183	19.10534	1.6193	0.6054	0.46029	0.5000	0.4625	0
	P10	60.00	7	2	1	1	1	1	1
	P25	159.75	10	3	2	2	1	1	1
	P50	345.00	19	4	2	2	2	1	1
	P75	689.75	33	5	3	2	2	1	1
	P90	1102.30	51	6	3	2	2	1	1

Table 2: ARL_S for EWMA control charts for PE and MMLM estimators ($\lambda = 0.6$)

	Shift								
		0	0.40	1.20	2.40	3.60	4.80	6.00	7.20
PEL = 12.15	ARL	500.47	6.199	1.267	1	1	1	1	1
	SDRL	469.28	6.601	0.478	0	0	0	0	0
	P10	54.0	2.0	1	1	1	1	1	1
	P25	157.70	3.0	1	1	1	1	1	1
	P50	356.49	6.0	1	1	1	1	1	1
	P75	707.49	10.0	2	1	1	1	1	1
	P90	1192.01	15.0	2	1	1	1	1	1

(Continued)

Table 2: Continued

	Shift								
		0	0.40	1.20	2.40	3.60	4.80	6.00	7.20
MMLML = 9.95	ARL	500.66	81.948	7.109	1.853	1.199	1.021	1.002	1
	SDR	484.588	76.1125	5.71960	0.8591	0.40194	0.1455	0.044	0
	P10	64.10	10.0	2	1	1	1	1	1
	P25	154.175	26.0	3	1	1	1	1	1
	P50	343.40	61.0	5	2	1	1	1	1
	P75	692.15	114.0	9	2	1	1	1	1
	P90	1174.14	183.1	15	3	2	1	1	1

Table 3: ARLs for P.E and MMLM estimators based EWMA control charts ($\lambda = 0.75$)

Estimation methods	Shift								
		0	0.40	1.20	2.40	3.60	4.80	6.00	7.20
PEL = 6.72	ARL	500.77	8.437	1.233	1	1	1	1	1
	SDRL	470.86	7.122	0.4700	0	0	0	0	0
	P10	54.50	2.010	1	1	1	1	1	1
	P25	156.85	4.020	1	1	1	1	1	1
	P50	356.40	7.0 1	1	1	1	1	1	1
	P75	698.85	13.0 1	1	1	1	1	1	1
	P90	1162.3	20.11	2	1	1	1	1	1
MMLML = 11.80	ARL	500.86	98.398	9.961	1.965	1.188	1.016	1.002	1
	SDR	492.29	93.3436	9.02157	1.070	0.4084	0.1255	0.044	0
	P10	61.00	12.0	2	1	1	1	1	1
	P25	149.75	30.0	4	1	1	1	1	1
	P50	343.50	69.0	7	2	1	1	1	1
	P75	692.25	141.0	14	2	1	1	1	1
	P90	1178.8	215.2	22	3	2	1	1	1

2.5 The Traditional Extended Exponentially Weighted Moving Averages (EEWMA) Control Chart

When the distribution of the process is normal, the EEWMA control chart was introduced by Naveed et al. [17]. The EEWMA control chart by Naveed et al. [17] is given as

$$Z_t = \lambda_1 T_t - \lambda_2 T_{t-1} + (1 - \lambda_1 + \lambda_2) Z_{t-1}$$

where $0 \leq \lambda_1 \leq 1$ and $0 \leq \lambda_2 \leq \lambda_1$. T_{t-1} is represents the previous value of the variable and Z_{t-1} denotes the previous value of the statistic.

The mean and variance are given as

$$E(Z_t) = \mu$$

And

$$var\left(Z_t\right)=\sigma^2\left[\left(\lambda_1^2+\lambda_2^2\right)\left\{\frac{1-\left(1-\lambda_1+\lambda_2\right)^{2t}}{2\left(\lambda_1-\lambda_2\right)-\left(\lambda_1-\lambda_2\right)^2}\right\}-2a\lambda_1\lambda_2\left\{\frac{1-\left(1-\lambda_1+\lambda_2\right)^{2t-2}}{2\left(\lambda_1-\lambda_2\right)-\left(\lambda_1-\lambda_2\right)^2}\right\}\right]$$

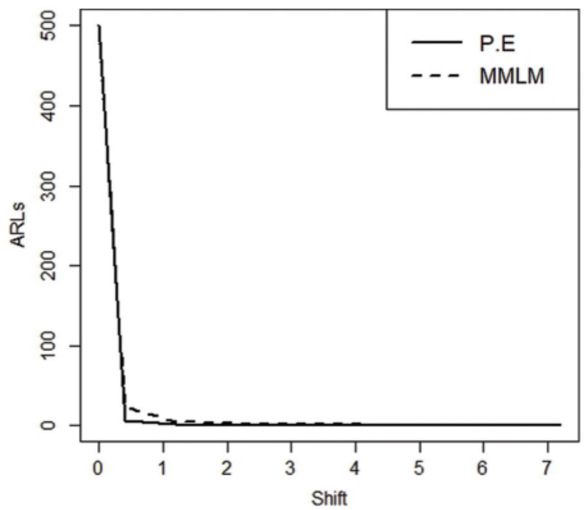

Figure 1: $\mathrm{ARL_S}$ for EWMA control charts $(\lambda = 0.2)$

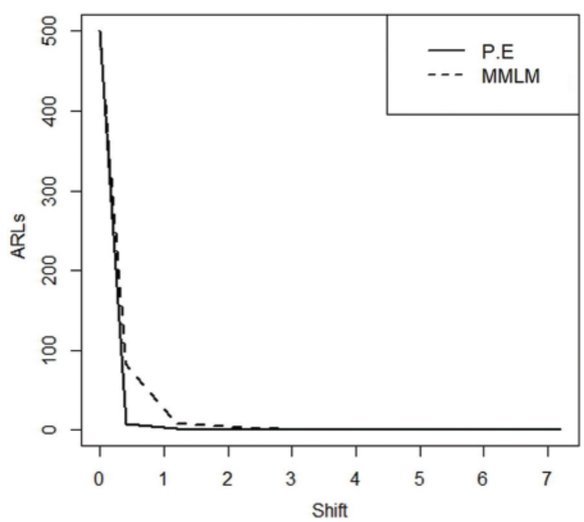

Figure 2: $\mathrm{ARL_S}$ for EWMA control charts taking $\lambda = 0.6$

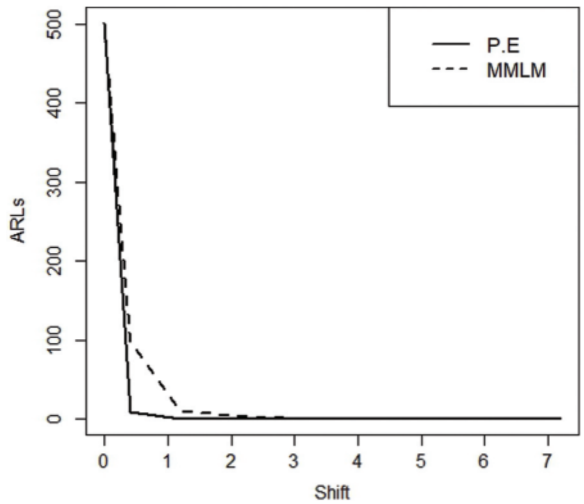

Figure 3: ARL_S for EWMA control charts ($\lambda = 0.75$)

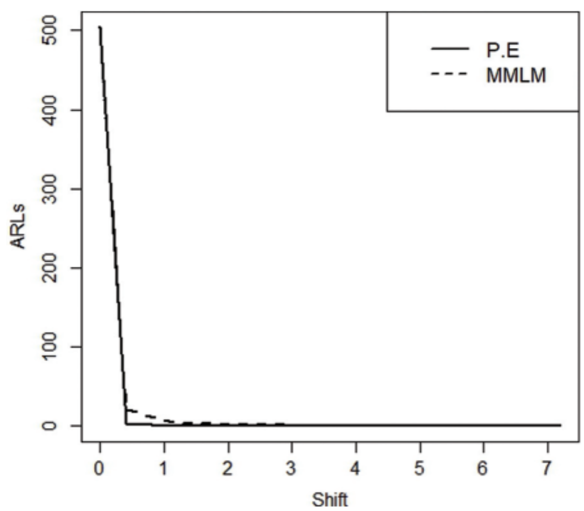

Figure 4: ARL_S for MMLM and PE-based EEWMA control charts taking $\lambda_1 = 0.90, \lambda_2 = 0.20$

2.6 Proposed EEWMA Control Chart Using PE

The EEWMA statistic using PE of the shape parameter of the RPFD using Zaka et al. [1] and Naveed et al. [17] is stated by

$$EEW\text{PE}_t = \lambda_1 \hat{\gamma}_{\text{PE(t)}} - \lambda_2 \hat{\gamma}_{\text{PE(t-1)}} + (1 - \lambda_1 + \lambda_2)\, EEW\text{PE}_{t-1}$$

taking t = 1, 2 and $a = (1 - \lambda_1 + \lambda_2)$, we get

$$EEW\text{PE}_2 = \lambda_1 \hat{\gamma}_{\text{PE(2)}} + (a\lambda_1 - \lambda_2)\, \hat{\gamma}_{\text{PE(1)}} - a\lambda_2 \hat{\gamma}_{\text{PE(0)}} + a^2 EEW\text{PE}_0$$

Let $b = (a\lambda_1 - \lambda_2)$ and solving

$$EEW\text{PE}_3 = \lambda_1 \hat{\gamma}_{\text{PE(3)}} + b\hat{\gamma}_{\text{PE(2)}} + ab\hat{\gamma}_{\text{PE(1)}} - a^2 \lambda_2 \hat{\gamma}_{\text{PE(0)}} + a^3 EEW\text{PE}_0$$

On generalizing above, we get

$$EEW\mathrm{PE}_t = \lambda_1\hat{\gamma}_{\mathrm{PE(t)}} + b\hat{\gamma}_{\mathrm{PE(t-1)}} + ab\hat{\gamma}_{\mathrm{PE(t-2)}} + a^2b\lambda_2\hat{\gamma}_{\mathrm{PE(t-3)}} + a^3\lambda_2\hat{\gamma}_{\mathrm{PE(0)}} + \ldots + a^{t-2}b\hat{\gamma}_{\mathrm{PE(1)}} - a^{t-1}\lambda_2\hat{\gamma}_{\mathrm{PE(0)}} + a^t EEW\mathrm{PE}_0$$

By taking expectation and Replacing $b = (a\lambda_1 - \lambda_2)$ we get

$$E\left(EEW\mathrm{PE}_t\right) = \gamma\left\{\lambda_1\left(1 + a + a^2 + a^3 + \ldots + a^{t-1}\right) - \lambda_2\left(1 + a + a^2 + a^3 + \ldots + a^{t-1}\right) + a^t\right\}$$

$$E\left(EEW\mathrm{PE}_t\right) = \gamma\left\{(\lambda_1 - \lambda_2)\left(1 + a + a^2 + a^3 + \ldots + a^{t-1}\right) + a^t\right\}$$

By using geometric series, we get

$$E\left(EEW\mathrm{PE}_t\right) = \gamma\left\{(\lambda_1 - \lambda_2)\left(\frac{1-a^t}{1-a}\right) + a^t\right\}$$

So, $E\left(EEW\mathrm{PE}_t\right) = \gamma\left\{1 - a^t + a^t\right\}$

$$E\left(EEW\mathrm{PE}_t\right) = \gamma \tag{13}$$

We know $Var\left(\hat{\gamma}_{\mathrm{PE(t)}}\right) = V_{PE} = E\left(\hat{\gamma}_{\mathrm{PE}} - \gamma\right)^2$ and solving we get

$$Var(EEW\mathrm{PE}_t) = V_{PE}\left\{\lambda_1^2 + a^2\lambda_1^2 - 2a\lambda_1 + \lambda_2^2 + a^4\lambda_1^2 - 2a^3\lambda_1 + a^2\lambda_2^2 + \left(a^6\lambda_1^4 - 2a^5\lambda_1^3 + a^4\lambda_2^4\right) + \ldots + a^{2(t-2)}\lambda_1^2 - 2a^{2t-3}\lambda_1 + a^{2(t-2)}\lambda_2^2 + a^{2(t-1)}\lambda_2^2\right\}$$

$$Var(EEW\mathrm{PE}_t) = V_{PE}\left\{\left(\lambda_1^2 + \lambda_2^2\right)\left(\frac{1-a^{2t}}{1-a^2}\right) - 2a\lambda_1\lambda_2\left(\frac{1-a^{2t-2}}{1-a^2}\right)\right\}$$

$$Var(EEW\mathrm{PE}_t) = V_{PE}\left\{\left(\lambda_1^2 + \lambda_2^2\right)\left(\frac{1-(1-\lambda_1+\lambda_2)^{2t}}{1-(1-\lambda_1+\lambda_2)^2}\right) - 2a\lambda_1\lambda_2\left(\frac{1-(1-\lambda_1+\lambda_2)^{2t-2}}{1-(1-\lambda_1+\lambda_2)^2}\right)\right\} \tag{14}$$

The control limits are given as

$$UCL_{EEW\mathrm{PE}_t} = \gamma + L\sqrt{V_{PE}\left\{\left(\lambda_1^2 + \lambda_2^2\right)\left(\frac{1-(1-\lambda_1+\lambda_2)^{2t}}{1-(1-\lambda_1+\lambda_2)^2}\right) - 2a\lambda_1\lambda_2\left(\frac{1-(1-\lambda_1+\lambda_2)^{2t-2}}{1-(1-\lambda_1+\lambda_2)^2}\right)\right\}}$$

$$CL_{EEW\mathrm{PE}_t} = \gamma$$

$$LCL_{EEW\mathrm{PE}_t} = \gamma - L\sqrt{V_{PE}\left\{\left(\lambda_1^2 + \lambda_2^2\right)\left(\frac{1-(1-\lambda_1+\lambda_2)^{2t}}{1-(1-\lambda_1+\lambda_2)^2}\right) - 2a\lambda_1\lambda_2\left(\frac{1-(1-\lambda_1+\lambda_2)^{2t-2}}{1-(1-\lambda_1+\lambda_2)^2}\right)\right\}}$$

2.7 Proposed EEWMA Control Chart Using MMLM

Assuming $E\left(\hat{\gamma}_{MMLM}\right) = \gamma$. The EEWMA statistic using MMLM of the shape parameter of the RPFD using Zaka et al. [1] and Naveed et al. [17] is stated by

$$EEW\mathrm{MMLM}_t = \lambda_1\hat{\gamma}_{\mathrm{MMLM(t)}} - \lambda_2\hat{\gamma}_{\mathrm{MMLM(t-1)}} + (1 - \lambda_1 + \lambda_2)\, EEW\mathrm{MMLM}_{t-1}$$

The EEWMA statistic using the mean and variance given in (13) and (14). The control limits are given as

$$UCL_{\mathrm{EEWMMLM}_t} = \gamma + L\sqrt{V_{\mathrm{MMLM}}\left\{\left(\lambda_1^2 + \lambda_2^2\right)\left(\frac{1-(1-\lambda_1+\lambda_2)^{2t}}{1-(1-\lambda_1+\lambda_2)^2}\right) - 2a\lambda_1\lambda_2\left(\frac{1-(1-\lambda_1+\lambda_2)^{2t-2}}{1-(1-\lambda_1+\lambda_2)^2}\right)\right\}}$$

$$CL_{\mathrm{EEWMMLM}_t} = \gamma$$

$$LCL_{\mathrm{EEWMMLM}_t} = \gamma - L\sqrt{V_{\mathrm{MMLM}}\left\{\left(\lambda_1^2 + \lambda_2^2\right)\left(\frac{1-(1-\lambda_1+\lambda_2)^{2t}}{1-(1-\lambda_1+\lambda_2)^2}\right) - 2a\lambda_1\lambda_2\left(\frac{1-(1-\lambda_1+\lambda_2)^{2t-2}}{1-(1-\lambda_1+\lambda_2)^2}\right)\right\}}$$

2.8 Algorithm for EEWMA Control Charts Under PE and MMLM

We generate a random sample from the process following RPFD using the sample size of 150. We then compute the estimate of the shape parameter using PE and MMLM alternatively and get their means and variances. Finally, the control limits using PE and MMLM are computed, and ARL is computed. We have fixed $\mathrm{ARL}_0 = 500$. we take different shifts and computed ARL for shifted process and called it ARL_1. We have repeated this process 5000 times.

Using the above algorithm, we observe the clear efficiency of EEWMA over EWMA using PE and MMLM. The ARLs are presented in Tabs. 4 and 5 for both PE and MMLM. We observe that for larger λ_1, we get a large ARL value. We also see that EEWMA control chart increases if the value of λ_1 is increased. Also, we see from Figs. 4–6 that the ARLs for EEWMA under PE remain less than ARLs for MMLM based EEWMA control chart. Additionally, we see that PE provides more efficient control charts compared to the MMLM for monitoring a process that follows RPFD.

3 Results and Discussion

The 50 observations are generated using RPFD. Half of which is generated for the process to be in control and the other half are created by a shift of 0.40. We presented the estimates for the shape parameter of RPFD using MMLM and PE are calculated in Tab. 6, Figs. 7 and 8, illustrate the estimates from EEWMA. We see that PE-based on the EEWMA control chart detects an out-of-control state earlier than MMLM.

Table 4: ARL_S for MMLM based EEWMA control charts

Estimation methods	Shift								
		0	0.40	1.20	2.40	3.60	4.8	6.0	7.2
MMLM	ARL	507.848	14.388	3.363	1.571	1.11	1.006	1.001	1
$\lambda_1 = 0.30$,	SDRL	483.7063	8.5163	1.4089	0.5807	0.3130	0.0772	0.03162	0
$\lambda_2 = 0.20$	P10	53.90	5	2	1	1	1	1	1
L = 4.30	P25	157.50	9	2	1	1	1	1	1
	P50	358.50	13	3	2	1	1	1	1
	P75	712.25	19	4	2	1	1	1	1
	P90	1117.50	25	5	2	2	1	1	1
MMLM	ARL	503.865	23.533	3.197	1.388	1.045	1.002	1	1
$\lambda_1 = 0.30$,	SDRL	511.57	20.6742	1.605	0.5307	0.2074	0.0446	0	0
$\lambda_2 = 0.60$	P10	46.00	5	1	1	1	1	1	1
L = 3.98	P25	149.75	9	2	1	1	1	1	1
	P50	334.50	17	3	1	1	1	1	1
	P75	712.25	33	4	2	1	1	1	1
	P90	1159.00	52	5	2	1	1	1	1
MMLM	ARL	503.618	26.877	3.387	1.419	1.051	1.002	1.001	1
$\lambda_1 = 0.30$,	SDRL	512.5012	24.15486	1.768	0.55112	0.2201	0.0446	0.031	0
$\lambda_2 = 0.75$	P10	46.90	5	1	1	1	1	1	1
L = 3.98	P25	144.25	10	2	1	1	1	1	1
	P50	329.5	19	3	1	1	1	1	1
	P75	713.25	37	4	1	1	1	1	1
	P90	1164.6	59	6	1	1	1	1	1
MMLM	ARL	506.836	14.863	3.211	1.464	1.067	1.003	1.001	1
$\lambda_1 = 0.5$,	SDRL	492.8587	9.447194	1.4527	0.5558	0.2501	0.0547	0.0316	0
$\lambda_2 = 0.20$	P10	56.80	5	1	1	1	1	1	1
L = 8.80	P25	153.75	8	2	1	1	1	1	1
	P50	358.50	13	3	1	1	1	1	1
	P75	707.00	19	4	2	1	1	1	1
	P90	1124.10	27	5	2	1	1	1	1
MMLM	ARL	502.748	36.614	3.661	1.453	1.059	1.002	1.001	1
$\lambda_1 = 0.5$,	SDRL	496.0317	34.778	2.1005	0.5693	0.2357	0.04469	0.0316	0
$\lambda_2 = 0.60$	P10	53.90	6.0	1	1	1	1	1	1
	P25	146.75	11.0	2	1	1	1	1	1
	P50	337.50	25.0	3	1	1	1	1	1
	P75	708.75	52.0	5	2	1	1	1	1
	P90	1156.20	82.1	6	2	1	1	1	1

(Continued)

Table 4: Continued

Estimation methods	Shift								
		0	0.40	1.20	2.40	3.60	4.8	6.0	7.2
MMLM	ARL	505.222	46.909	4.239	1.526	1.077	1.005	1.001	1
$\lambda_1 = 0.5$,	SDRL	495.0974	45.73822	2.846	0.6047	0.26672	0.07056	1.001	0
$\lambda_2 = 0.75$,	P10	60.0	6	1	1	1	1	1	1
L= 10.65	P25	147.0	14	2	1	1	1	1	1
	P50	337.5	33	4	1	1	1	1	1
	P75	711.0	64	5	2	1	1	1	1
	P90	1217.7	107	8	2	1	1	1	1
MMLM	ARL	505.382	21.432	3.361	1.424	1.053	1.002	1.001	1
$\lambda_1 = 0.90$,	SDRL	515.024	17.76437	1.6695	0.5572	0.2241	0.0446	0.0316	0
$\lambda_2 = 0.20$,	P10	46.00	5	1	1	1	1	1	1
L = 9.725	P25	142.00	9	2	1	1	0	1	1
	P50	332.00	16	3	1	1	1	1	1
	P75	712.25	29	4	2	1	1	1	1
	P90	1160.10	46	6	2	1	1	1	1
MMLM	ARL	503.865	72.193	5.892	1.666	1.12	1.006	1.001	1
$\lambda_1 = 0.90$,	SDRL	486.214	66.673	4.607	0.7368	0.3281	0.0772	0.03162	0
$\lambda_2 = 0.60$,	P10	64.9	9.0	2	1	1	1	1	1
L = 4.14	P25	154.0	22.0	3	1	1	1	1	1
	P50	343.5	53.0	5	2	1	1	1	1
	P75	688.0	101.0	8	2	1	1	1	1
	P90	1174.4	160.1	12	3	2	1	1	1
MMLM	ARL	506.2	92.447	8.418	1.816	1.146	1.007	1.001	1
$\lambda_1 = 0.90$,	SDRL	490.6586	86.438	7.653	0.9214	0.3561	0.08341	0.0316	0
$\lambda_2 = 0.75$	P10	63.9	11.00	2.0	1	1	1	1	1
L = 12.00	P25	155.0	29.00	3.0	1	1	1	1	1
	P50	346.0	66.00	6.0	2	1	1	1	1
	P75	693.0	132.25	11.0	2	1	1	1	1
	P90	1178.7	206.00	18.10	3	2	1	1	1

Table 5: ARL_S for PE-based EEWMA control charts

Estimation method	Shift								
		0	0.40	1.20	2.40	3.60	4.8	6.0	7.20
PE	ARL	500.291	4.191	1.41	1	1	1	1	1
$\lambda_1 = 0.30$,	SDRL	493.5543	2.246	0.5120	0	0	0	0	0
$\lambda_2 = 0.20$	P10	61.00	2	1	1	1	1	1	1
L = 6.83	P25	139.75	3	1	1	1	1	1	1
	P50	341.00	5	1	1	1	1	1	1
	P75	666.50	6	2	1	1	1	1	1
	P90	1190.50	8	2	1	1	1	1	1

(Continued)

Table 5: Continued

Estimation method	Shift								
		0	0.40	1.20	2.40	3.60	4.8	6.0	7.20
PE	ARL	500.291	4.575	1.161	1	1	1	1	1
$\lambda_1 = 0.30$,	SDRL	515.416	2.604	0.3757	0	0	0	0	0
$\lambda_2 = 0.60$,	P10	51.00	2	1	1	1	1	1	1
L = 7.144	P25	141.75	3	1	1	1	1	1	1
	P50	349.50	4	1	1	1	1	1	1
	P75	686.75	6	1	1	1	1	1	1
	P90	1158.00	8	2	1	1	1	1	1
PE	ARL	498.948	4.783	1.164	1	1	1	1	1
$\lambda_1 = 0.30$,	SDRL	505.03	2.938	0.375	0	0	0	0	0
$\lambda_2 = 0.75$,	P10	51.90	2	1	1	1	1	1	1
L = 7.27	P25	141.75	3	1	1	1	1	1	1
	P50	349.0	4	1	1	1	1	1	1
	P75	681.0	6	1	1	1	1	1	1
	P90	1170.4	9	2	1	1	1	1	1
PE	ARL	506.48	4.814	1.28	1	1	1	1	1
$\lambda_1 = 0.5$,	SDRL	493.1145	2.4391	0.4602	0	0	0	0	0
$\lambda_2 = 0.20$,	P10	65.00	2	1	1	1	1	1	1
L = 7.00	P25	145.75	3	1	1	1	1	1	1
	P50	363.50	5	1	1	1	1	1	1
	P75	673.50	6	2	1	1	1	1	1
	P90	1232.20	8	2	1	1	1	1	1
PE	ARL	502.326	5.104	1.294	1	1	1	1	1
$\lambda_1 = 0.5$,	SDRL	496.0043	3.5320	0.375	0	0	0	0	0
$\lambda_2 = 0.60$,	P10	60	2	1	1	1	1	1	1
L = 7.35	P25	161	3	1	1	1	1	1	1
	P50	349	4	1	1	1	1	1	1
	P75	648	7	1	1	1	1	1	1
	P90	1195	10	2	1	1	1	1	1

(Continued)

Table 5: Continued

Estimation method	Shift								
		0	0.40	1.20	2.40	3.60	4.8	6.0	7.20
PE	ARL	503.405	5.456	1.166	1	1	1	1	1
$\lambda_1 = 0.5$,	SDRL	480.0373	4.0849	0.37760	0	0	0	0	0
$\lambda_2 = 0.75$,	P10	52.00	2	1	1	1	1	1	1
L = 7.44	P25	150	3	1	1	1	1	1	1
	P50	361.00	4	1	1	1	1	1	1
	P75	703.25	7	1	1	1	1	1	1
	P90	1195.20	10	2	1	1	1	1	1
PE	ARL	503.174	4	1.228	1	1	1	1	1
$\lambda_1 = 0.90$,	SDRL	490.6912	2.519	0.429	0	0	0	0	0
$\lambda_2 = 0.20$,	P10	56.00	2	1	1	1	1	1	1
L = 7.25	P25	142.75	3	1	1	1	1	1	1
	P50	363.00	4	1	1	1	1	1	1
	P75	708.25	6	1	1	1	1	1	1
	P90	1185.40	8	2	1	1	1	1	1
PE	ARL	503.452	6.589	1.18	1	1	1	1	1
$\lambda_1 = 0.90$,	SDRL	471.5114	5.013	0.3996	0	0	0	0	0
$\lambda_2 = 0.60$,	P10	50.90	2	1	1	1	1	1	1
L = 7.59	P25	150.75	3	1	1	1	1	1	1
	P50	366.50	5	1	1	1	1	1	1
	P75	708.50	9	1	1	1	1	1	1
	P90	1172.60	13	2	1	1	1	1	1
PE	ARL	505.005	8.112	1.191	1	1	1	1	1
$\lambda_1 = 0.90$,	SDRL	478.2393	6.746	0.4274	0	0	0	0	0
$\lambda_2 = 0.75$,	P10	52.0	2	1	1	1	1	1	1
L = 7.66	P25	155.0	3	1	1	1	1	1	1
	P50	353.0	6	1	1	1	1	1	1
	P75	712.50	11	1	1	1	1	1	1
	P90	1192.0	17	2	1	1	1	1	1

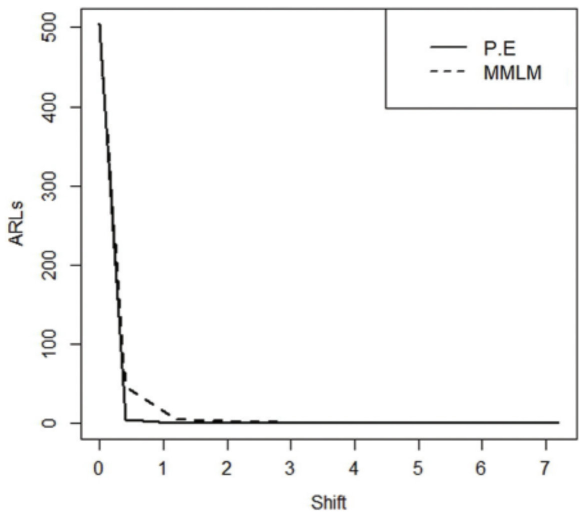

Figure 5: ARL_S EEWMA control charts($\lambda_1 = 0.50, \lambda_2 = 0.75$)

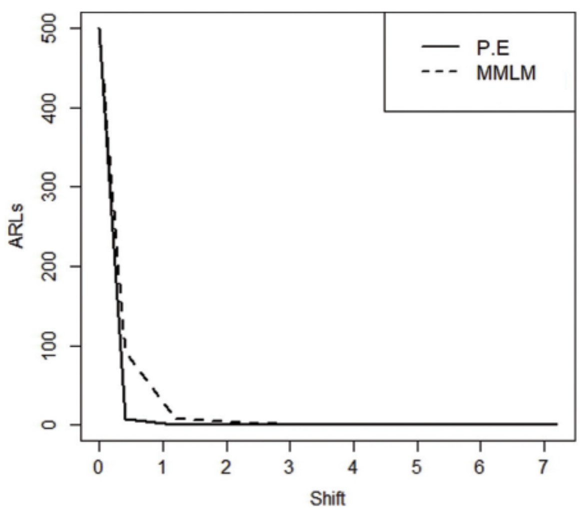

Figure 6: ARL_S EEWMA control charts ($\lambda_1 = 0.90, \lambda_2 = 0.75$)

Table 6: Simulated results for LCL and UCL based on proposed EEWMA

MMLM based EEWMA			PE-based EEWMA		
$\lambda_1 = 0.90$		$\lambda_2 = 0.75$	$\lambda_1 = 0.90$		$\lambda_2 = 0.75$
L = 12			L = 7.66		
$EEW\text{MMLM}_t$	LCL	UCL	$EEW\text{PE}_t$	LCL	UCL
2.792763	0.2380000	3.802000	2.174939	1.389540	2.630460
2.757910	0.1320048	3.907995	2.055011	1.352634	2.667366

(Continued)

Table 6: Continued

MMLM based EEWMA			PE-based EEWMA		
$\lambda_1 = 0.90$		$\lambda_2 = 0.75$	$\lambda_1 = 0.90$		$\lambda_2 = 0.75$
L = 12			L = 7.66		
$EEW\text{MMLM}_t$	LCL	UCL	$EEW\text{PE}_t$	LCL	UCL
2.890396	0.1240272	3.915973	1.884097	1.349857	2.670143
2.031693	0.1234880	3.916512	1.851996	1.349669	2.670331
1.607408	0.1234533	3.916547	1.845604	1.349657	2.670343
2.457326	0.1234511	3.916549	2.209770	1.349656	2.670344
2.706825	0.1234509	3.916549	2.468172	1.349656	2.670344
2.578589	0.1234509	3.916549	2.109744	1.349656	2.670344
2.277098	0.1234509	3.916549	2.124111	1.349656	2.670344
2.033351	0.1234509	3.916549	2.082389	1.349656	2.670344
2.089141	0.1234509	3.916549	2.135626	1.349656	2.670344
2.007338	0.1234509	3.916549	1.958650	1.349656	2.670344
2.080522	0.1234509	3.916549	1.994993	1.349656	2.670344
1.995012	0.1234509	3.916549	2.039405	1.349656	2.670344
1.985600	0.1234509	3.916549	1.999159	1.349656	2.670344
2.404916	0.1234509	3.916549	1.954418	1.349656	2.670344
1.881724	0.1234509	3.916549	2.070869	1.349656	2.670344
2.660258	0.1234509	3.916549	2.120229	1.349656	2.670344
2.194834	0.1234509	3.916549	2.278065	1.349656	2.670344
2.066108	0.1234509	3.916549	2.277322	1.349656	2.670344
2.170824	0.1234509	3.916549	2.277124	1.349656	2.670344
1.925193	0.1234509	3.916549	2.230325	1.349656	2.670344
1.711615	0.1234509	3.916549	2.109841	1.349656	2.670344
2.086559	0.1234509	3.916549	2.112682	1.349656	2.670344
2.466933	0.1234509	3.916549	2.196142	1.349656	2.670344
3.217173	0.1234509	3.916549	2.478410	1.349656	2.670344
3.268923	0.1234509	3.916549	2.428640	1.349656	2.670344
3.453694	0.1234509	3.916549	2.249904	1.349656	2.670344
2.431717	0.1234509	3.916549	2.218243	1.349656	2.670344
1.925132	0.1234509	3.916549	2.212262	1.349656	2.670344
2.943783	0.1234509	3.916549	2.649364	1.349656	2.670344
3.242825	0.1234509	3.916549	2.959301	1.349656	2.670344
3.089214	0.1234509	3.916549	2.529532	1.349656	2.670344
2.728023	0.1234509	3.916549	2.546798	1.349656	2.670344

(Continued)

Table 6: Continued

MMLM based EEWMA			PE-based EEWMA		
$\lambda_1 = 0.90$		$\lambda_2 = 0.75$	$\lambda_1 = 0.90$		$\lambda_2 = 0.75$
L = 12			L = 7.66		
$EEW\text{MMLM}_t$	LCL	UCL	$EEW\text{PE}_t$	LCL	UCL
2.436000	0.1234509	3.916549	2.496781	1.349656	2.670344
2.502835	0.1234509	3.916549	2.560622	1.349656	2.670344
2.404841	0.1234509	3.916549	2.348429	1.349656	2.670344
2.492511	0.1234509	3.916549	2.391968	1.349656	2.670344
2.390080	0.1234509	3.916549	2.445245	1.349656	2.670344
2.378799	0.1234509	3.916549	2.396851	1.349656	2.670344
2.881183	0.1234509	3.916549	2.343292	1.349656	2.670344
2.254360	0.1234509	3.916549	2.482961	1.349656	2.670344
3.187066	0.1234509	3.916549	2.542158	1.349656	2.670344
2.629467	0.1234509	3.916549	2.731320	1.349656	2.670344
2.475245	0.1234509	3.916549	2.730486	1.349656	2.670344
2.600721	0.1234509	3.916549	2.730273	1.349656	2.670344
2.306430	0.1234509	3.916549	2.674169	1.349656	2.670344
2.050553	0.1234509	3.916549	2.529708	1.349656	2.670344
2.499746	0.1234509	3.916549	2.533056	1.349656	2.670344
2.955474	0.1234509	3.916549	2.633164	1.349656	2.670344

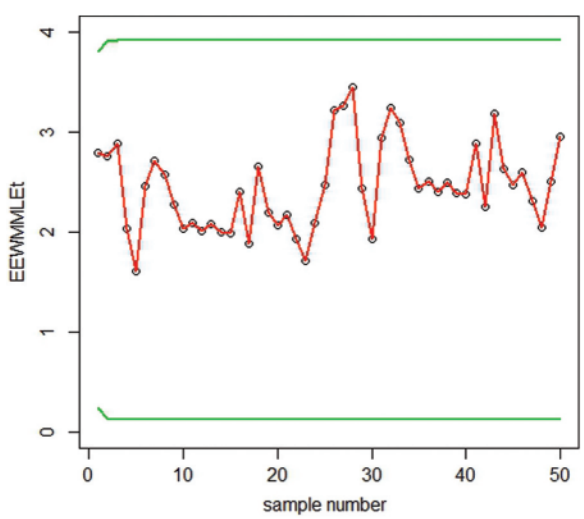

Figure 7: Proposed EEWMA control chart using MMLM (L = 12, $\lambda_1 = 0.90$ and $\lambda_2 = 0.75$)

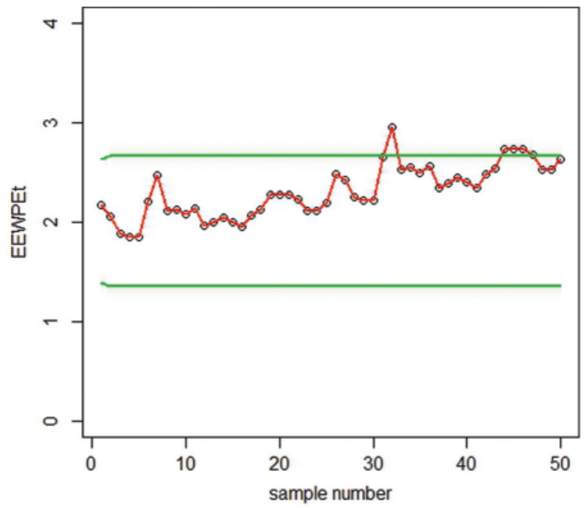

Figure 8: Proposed EEWMA control chart using PE (L = 7.66, $\lambda_1 = 0.90$ and $\lambda_2 = 0.75$)

3.1 Real-Life Application

The Real-life data is taken from the operation side of the telecommunication industry, where errors in the software frequently occurred regarding the billing amount, dispatch issue, contact and login details. The per-day frequency of errors in the software is reported below: 1124, 1013, 1187, 1153, 1141, 1051, 1178, 1145, 1124, 1132, 1141, 1136, 1241,1301, 1214, 1421, 1258, 1109, 1321, 1121,1114, 1021, 1131, 1142, 1165, 1184. The data followed the RPFD and plotted for EEWMA control charts under MMLM and PE, as shown in Figs. 9 and 10.

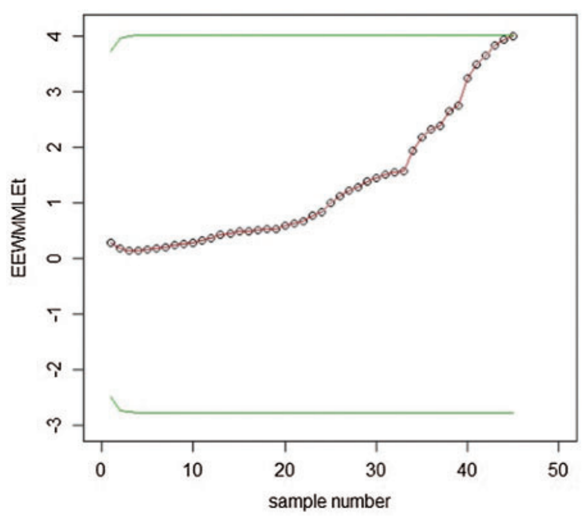

Figure 9: Graph of real data of the EEWMA control chart under MMLM when L = 12, $\lambda_1 = 0.90$ and $\lambda_2 = 0.75$

We have constructed EEWMA control charts on real-life data under MMLM and PE. For example, in Figs. 9 and 10, we see that EEWMA under PE predicts the process shift at early levels compared to EEWMA under MMLM, which indicates that EEWMA under PE provides a better explanation of the distributions when the underlying process is based on RPFD.

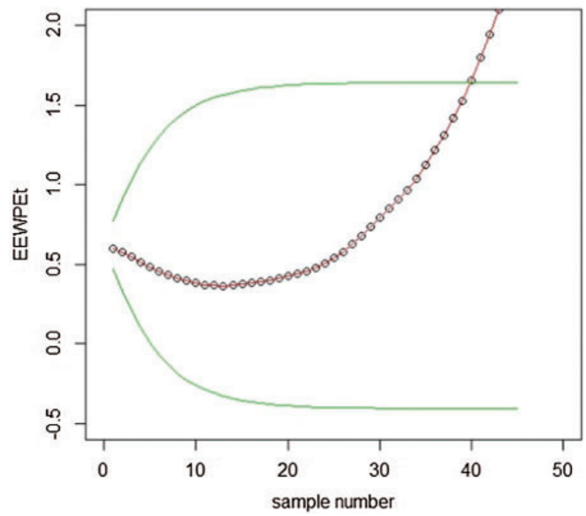

Figure 10: Graph of real data of the EEWMA control chart under PE when L = 7.66, $\lambda_1 = 0.90$ and $\lambda_2 = 0.75$

3.2 Earnings Per Share (EPS) of the National Refinery Ltd

Real-life data for earnings per share (EPS) of the National Refinery Ltd. were taken from the State Bank of Pakistan (SBP) report for non-financial companies from the year 1984–2019. The data in Tab. 7 followed the RPFD and plotted for EEWMA control charts under MMLM and PE, as shown in Figs. 11 and 12.

Table 7: State Bank of Pakistan (SBP) report for non-financial companies from the year 1984–2019

Years	1984	1985	1986	1987	1988	1989	1990	1991	1992	1993	1994	1995	1996
EPS	0.8	1.8	1.8	1.8	1.8	6.96	3.09	4.3	2.9	4.5	4.9	3	2.2
Years	1997	1998	1999	2000	2001	2002	2003	2004	2005	2006	2007	2008	2009
EPS	7.4	7.3	10.3	10.8	11.2	12.4	16.9	26.7	30.2	52.8	61.4	73.96	22.37
Years	2010	2011	2012	2013	2014	2015	2016	2017	2018	2019			
EPS	37.81	88.16	34.68	33.96	12.03	46.38	96.14	100.61	22.14	108.7			

We have constructed EEWMA control charts on real-life data under MMLM and PE. For example, in Figs. 11 and 12,

We see that EEWMA using PE better predicts the process shifts using less samples as compare to MMLM. So it can be used effectively for Earning per share data.

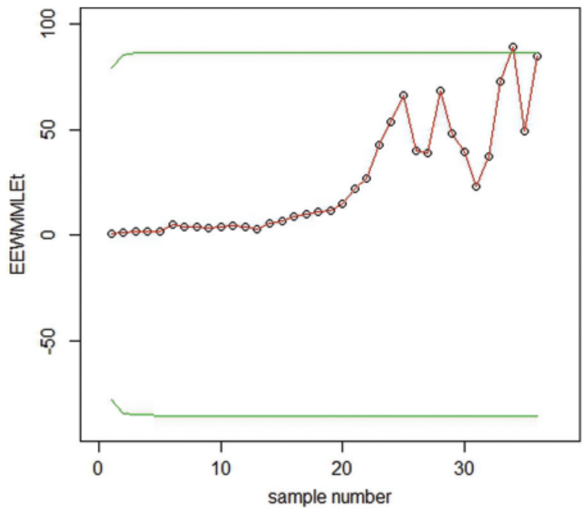

Figure 11: Graph of real data of the EEWMA control chart under MMLM when L = 12, $\lambda_1 = 0.90$ and $\lambda_2 = 0.75$

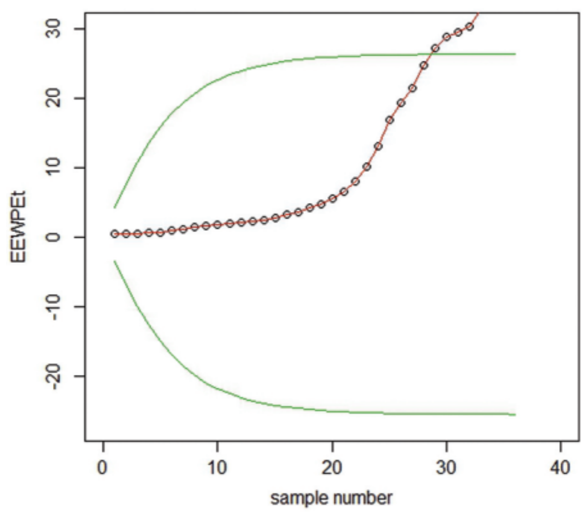

Figure 12: Graph of real data of the EEWMA control chart under PE when L = 7.66, $\lambda_1 = 0.90$ and $\lambda_2 = 0.75$

4 Conclusions

This study is conducted to develop the control charts in a real-life situation that does not follow a normal distribution by employing statistical methods. The main findings of the study are:

- It discussed the application of the RPED in the production process of telecommunications and finance.
- It has applied two memory-based control charts and suggested that EWMA and EEWMA based on PE and MMLM are better estimators when the underlying distribution follows the RPFD.
- Further, we compared the performance of each control chart with a PE-based EEWMA control chart. We concluded that PE-based control charts are an effective estimator for the

early deduction of machine errors. It can even identify minor shift errors more efficiently than MMLM-based estimators if a distribution works under the RPFD function.

- These results can be beneficial for scholars and practitioners of the diversified field of management and applied sciences. It will help them design the strategies to cope with the errors during the production process that improves product quality and eventually cause high returns.

Data Availability: The data used to support the findings of this study are included in the article.

Funding Statement: The authors received no specific funding for this study.

Conflicts of Interest: The authors declare that they have no conflicts of interest to report regarding the present study.

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