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Imperfect Premise Matching Controller Design for Interval Type-2 Fuzzy Systems under Network Environments

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Abstract: The interval type-2 fuzzy sets can describe nonlinear plants with uncertain parameters. It exists in nonlinearity. The parameter uncertainties extensively exist in the nonlinear practical Networked Control Systems (NCSs), and it is paramount to investigate the stabilization of the NCSs on account of the section type-2 fuzzy systems. Notice that most of the existing research work is only on account of the convention Parallel Distribution Compensation (PDC). For overcoming the weak point of the PDC and acquire certain guard stability conditions, the state tickling regulator under imperfect premise matching can be constructed to steady the NCSs using the section type-2 indistinct muster, where the fuzzy plant and fuzzy regulator may enjoy together not the same membership functions. By leading into the message of the up and down membership functions of both the fuzzy pattern and fuzzy regulator, we build a new composed term of the section type-2 network systems. The new results we obtained can provide a larger area of stability than the conventional membership unconcerned stability results. Moreover, a novel unmatching state feedback controller design method for the interval type-2 networked systems is explored in our work. The proposed approach can significantly improve the design flexibility of the fuzzy controllers, because their membership functions can be arbitrarily selected. Two numerical examples are further used to demonstrate the less-conservativeness and effectiveness of the novel technique.

Keywords: Network control systems; stability analysis; interval type-2 fuzzy systems; controller design; imperfect premise matching

1 Introduction

As we know, the Takagi-Sugeno can efficacious express non-linear dynamics, and often is called a type-1 system [1]. Unfortunately, the type-1 fuzzy sets have some difficulties in describing the nonlinear plants with uncertain parameters. In order to handle this drawback, Zadeh [2] introduced the type-2 fuzzy sets as an extension of the type-1 fuzzy sets. In view of the type-2 fuzzy sets, Mendel et al. [3] proposed the section Type-2 (IT2) fuzzy sets. During the past decades, the IT2 fuzzy model has been widely applied in practice [4,5]. For example, in Lam et al. [6], the stability area and the fuzzy regulator for the IT2 fuzzy model-based system were examined, in which the parameter non-determinacy are captured by both the up and down grades of fuzzy membership. Most of the controller design methods reported in the literature are inspired by the PDC scheme, and the fuzzy pattern and fuzzy regulator are provided with the identical membership functions. Nevertheless, Lam developed the concept of "imperfect premise matching" in Lam et al. [7], i.e., the membership functions of the fuzzy regulator probable be distinct from fuzzy model. The stabilization issue of the type-1 systems was investigated in Zhang et al. [8–11]. Li et al. [12,13] generalized these research results to the IT2 fuzzy model with the imperfect premise matching, thus improving the flexibility in highway design and cutting down the complexity of implementation. More work on the Interval type-2 fuzzy model stability conditions and controller design was presented in Wu et al. [14–16].

The Networked Control Systems (NCSs) have been got a lot of attention owing to their theory and reality purport [17]. Compared with the conventional point-point connections, the network systems reduce the heavy expenses of cables and facility maintenances. The network systems have found numerous applications in the fields of automobile, aerospace, industrial manufacturing, etc. [18–21]. Moreover, It is difficult if not impossible to guarantee the steadiness of the NCSs. what to analyze and devise the stability requirement for the T-S under NCSs has been a popular research topic [22,23]. Such as in Chi et al. [24], the networked H_∞ filtration with many output and many transducer nonsynchronous take sample for the T-S fuzzy systems was explored. By drawing into Laypunov-Krasovskii function to analysis the robust stability and the project of the state feedback for the NCSs in Rouamel et al. [25]. Actually, the above issues were received in view of the type-1 fuzzy set theory. NCSs also have parameter uncertainties, and considerable research work concerning the sector type-2 fuzzy pattern has been carried out [26,27]. However, the current efforts on the controller design are mainly inspired by the convention PDC idea. For the sake of overcoming the shortcomings of the PDC and receive a number of less conservative stability conditions, a novel devise way was put forward to manage the unmatching premise of the type-1 systems [28]. It is necessary to further investigate the controller design scheme under the imperfect premise matching for the Interval type-2 T-S fuzzy systems under NCSs.

In this article, the innovations of the paper are as follows:

- A less conservative stability conditions are obtained.
- The unmatching regulator is designed for the nonlinear NCSs in view of the Interval typ-2 T-S, which makes the regulator design simpler and more flexible.
- The proposed method can be generalized to other types of nonlinear control systems.

The rest of our article is arranged as follows. In Section 2, the controller design problem under consideration is represented in details. In Section 3, the synthesis of the state-feedback type-2 fuzzy controller under the imperfect premise matching is presented. In Section 4, for explaining the practicability and validity of the recommend technique, we give two numerical examples. Ultimately, In Section 5, we summarize this paper with some remarks and conclusions

2 Preparatory Knowledge

Here the interval type-2 T-S fuzzy model can be described as follows:

The ith rule can be represented as follows:

IF $\rho_1(\chi(t))$ is Λ_1^i … $\rho_\kappa(\chi(t))$ is Λ_κ^i THEN

$$\dot{\chi}(t) = \mathrm{A}_i\chi(t) + \mathrm{B}_i\mu(t), \tag{1}$$

Λ_α^i, $\alpha = 1, 2, \ldots, \kappa; i = 1, 2, \ldots, l$ l is the number of the fuzzy rules. A_i, B_i are the constant matrices. $\mu(t) \in R^m$ is the input vector, $\chi(t) \in R^n$ is the state vector.

We have:

$$\theta_i(\chi(t)) = \left[\underline{\theta}_i(\chi(t)) \quad \bar{\theta}_i(\chi(t))\right], i = 1, 2, \cdots, l, \tag{2}$$

where

$$\underline{\theta}_i(\chi(t)) = \prod_{\alpha=1}^{\kappa} \underline{u}_{\Lambda_\alpha^i}(\rho_\alpha(\chi(t))) \geq 0, \quad \bar{\theta}_i(\chi(t)) = \prod_{\alpha=1}^{\kappa} \bar{u}_{\Lambda_\alpha^i}(\rho_\alpha(\chi(t))) \geq 0,$$

They content the character of $1 \geq \bar{u}_{\Lambda_\alpha^i}(\rho_\alpha(\chi(t))) \geq \underline{u}_{\Lambda_\alpha^i}(\rho_\alpha(\chi(t))) \geq 0$, and $\underline{\theta}_i(\chi(t))$ and $\bar{\theta}_i(\chi(t))$ show the down and up grades of the membership. Then we have

$$\dot{\chi}(t) = \sum_{i=1}^{l} \theta_i(\chi(t))[A_i\chi(t) + B_i\mu(t)], \tag{3}$$

where

$$\theta_i(\chi(t)) = \frac{\underline{\theta}_i(\chi(t))\underline{\upsilon}_i(\chi(t)) + \bar{\theta}_i(\chi(t))\bar{\upsilon}_i(\chi(t))}{\sum_{i=1}^{l}\left[\underline{\theta}_i(\chi(t))\underline{\upsilon}_i(\chi(t)) + \bar{\theta}_i(\chi(t))\bar{\upsilon}_i(\chi(t))\right]},$$

in which $\sum_{i=1}^{l} \theta_i(\chi(t)) = 1$, and $\underline{\upsilon}_i$, $\bar{\upsilon}_i \in [0, 1]$ are the nonlinear functions with $\underline{\upsilon}_i(\chi(t)) + \bar{\upsilon}(\chi(t)) = 1$.

In this article, The system in Eq. (1) through IP based network control, system status can be used for feedback, as shown in Fig. 1 [29].

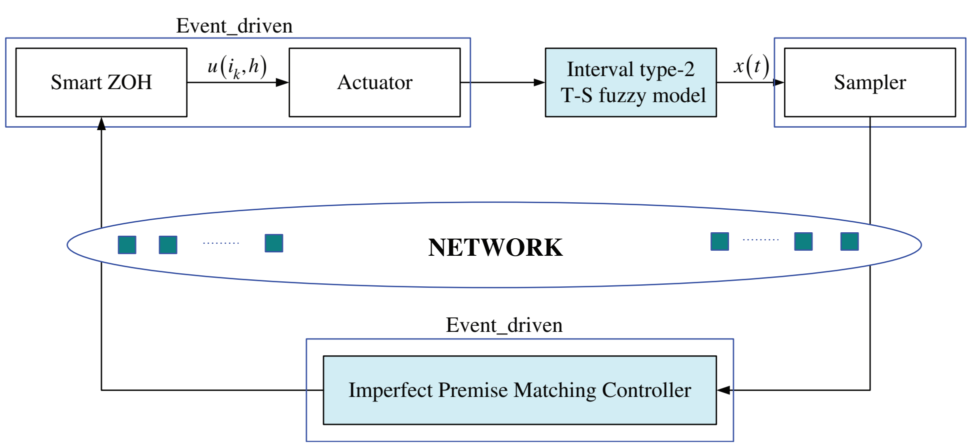


Figure 1: The interval type-2 networking system diagram

Distinct of the famous Parallel Distributed Compensation (PDC) plan means, we adopt a new unmatching fuzzy control law to make the interval type-2 fuzzy systems in Eq. (3) stable. For the NCSs, the transducer is clock-driven and $h(h > 0)$ express the sampling period is constant. The regulator and actuator are eventdriven. Suppose that $h(h > 0)$ express the sampling cycle of the semaphore, and the $\Delta h(\Delta = 1, 2, 3 \cdots)$ express the Δth acquisition time.

Under imperfect premise matching, we have the following fuzzy regulator:

Rule j: IF $\sigma_1(\chi(\Delta h))$ is $\Gamma_1^j \ldots$ $\sigma_\lambda(\chi(\Delta h))$ is Γ_λ^j THEN

$$\mu(t) = Z_j \chi(\Delta h), \qquad j = 1, 2, \ldots, l,\ t \in [\Delta h + \tau_k, (\Delta + 1)h + \tau_{k+1}], \Delta = 1, 2, \cdots, \tag{4}$$

where Z_j are the unknown tickling gains to be pending. On the other hand,

$$\varepsilon_j(\chi(t)) = \left[\underline{\varepsilon}_j(\chi(t)) \quad \bar{\varepsilon}_j(\chi(t))\right], \quad t \in [\Delta h + \tau_k, (\Delta + 1)h + \tau_{k+1}], \Delta = 1, 2, \cdots, \tag{5}$$

where

$$\underline{\varepsilon}_j(\chi(t)) = \prod_{\beta=1}^{\lambda} \underline{u}_{\Gamma_\beta^j}\left(\sigma_\beta(\chi(t))\right) \geq 0, \quad \bar{\varepsilon}_j(\chi(t)) = \prod_{\beta=1}^{\lambda} \bar{\mu}_{\Gamma_\beta^j}\left(\sigma_\beta(\chi(t))\right) \geq 0.$$

For $t \in [\Delta h + \tau_k, (\Delta + 1)h + \tau_{k+1}]$, we have

$$\mu(t) = \sum_{j=1}^{l} \left[\underline{\varepsilon}_j(\chi(t)) + \bar{\varepsilon}_j(\chi(t))\right] Z_j \chi(\Delta h), \tag{6}$$

Here the $\underline{u}_{\Gamma_\beta^j}\left(\sigma_\beta(\chi(t))\right)$ and $\bar{u}_{\Gamma_\beta^j}\left(\sigma_\beta(\chi(t))\right)$ express the down and up membership functions. They satisfy the character $0 \leq \underline{u}_{\Gamma_\beta^j}\left(\sigma_\beta(\chi(t))\right) \leq \bar{u}_{\Gamma_\beta^j}\left(\sigma_\beta(\chi(t))\right) \leq 1$. $\underline{\varepsilon}_j(\chi(t))$ and $\bar{\varepsilon}_j(\chi(t))$ express the down and up grades of the membership functions. We have

$$\sum_{j=1}^{l} \left[\underline{\varepsilon}_j(\chi(t)) + \bar{\varepsilon}_j(\chi(t))\right] = 1. \tag{7}$$

Owing to the network slowdown is invariably bounded, The input delay is:

$$\tau(t) = t - \Delta h, \quad 0 \leq \tau(t) \leq c. \tag{8}$$

where c is the upper limit of the lag. Putting Eq. (8) into Eq. (6), the following regulator can be shown:

$$\mu(t) = \sum_{j=1}^{l} \left[\underline{\varepsilon}_j(\chi(t)) + \bar{\varepsilon}_j(\chi(t))\right] Z_j \chi(t - \tau(t)). \tag{9}$$

We have the following closed-loop networked system:

$$\dot{\chi}(t) = \sum_{i=1}^{l} \sum_{j=1}^{l} \theta_i(\chi(t))\left(\bar{\varepsilon}_j(\chi(t)) + \underline{\varepsilon}_j(\chi(t))\right) \times \left[\mathrm{A}_i \chi(t) + \mathrm{B}_i Z_j \chi(t - \tau(t))\right]. \tag{10}$$

Lemma 1 [30] For $\varphi(v) \in R^n, v \geq 0$, $D = D^T \in R^{n \times n}$, supposing that there has $\chi(\tau) \in R^n, \tau \in [v_1, v_2]$, for $\varphi(v) \in R^n, v \geq 0$, $D = D^T \in R^{n \times n}$, we have:

$$-2\varphi^T(v) \int_{v_1}^{v_2} \dot{\chi}(\tau) d\tau \leq (v_2 - v_1)\varphi^T(v) D^{-1} \varphi(v) + \int_{v_1}^{v_2} \dot{\chi}^T(\tau) D \dot{\chi}(\tau) ds. \tag{11}$$

Lemma 2 [31] Ξ_1, Ξ_2, and Ω are constant matrices, and $0 \leq \tau(t) \leq d$. We have

$$\tau(t)\Xi_1 + (d - \tau(t))\Xi_2 + \Omega < 0. \tag{12}$$

if and only if $d\Xi_1 + \Omega < 0$ and $d\Xi_2 + \Omega < 0$ set up.

Lemma 3 [32] The $\begin{bmatrix} \alpha & \beta^T \\ \beta & \delta \end{bmatrix} > 0$ is equivalent to $\delta > 0$, $\alpha - \beta^T \delta^{-1} \beta > 0$, where $\alpha = \alpha^T$, $\delta = \delta^T$, and β is a matrix .

3 Main Results

Theorem 1 When the membership functions satisfy $\underline{\varepsilon}_j(\chi(t)) - \rho_j \bar{\theta}_j(\chi(t)) \geq 0$ and $\bar{\varepsilon}_j(\chi(t)) - \rho_j \underline{\theta}_j(\chi(t)) \geq 0$ for all $j, \chi(t)$, where $0 < d_j < 1$, and for given invariant $c > 0$ and matrix Z_j, and there exist matrices $P > 0, R > 0, Q > 0, g_i = g_i^T \in R^{4n \times 4n} > 0 (i = 1, 2, \cdots, l), T_i (i = 1, 2, 3, 4)$ and $O_{ij} = [O_{1ij} \quad O_{2ij} \quad 0 \quad 0]^T, U_{ij} = [0 \quad U_{1ij} \quad U_{2ij} \quad 0]^T, Y_{ij} = [Y_{1ij} \quad Y_{2ij} \quad 0 \quad 0]^T, S_{ij} = [0 \quad S_{1ij} \quad S_{2ij} \quad 0]^T$ of such that the following LMIs hold:

$$\begin{bmatrix} \Phi_{ij} - g_i & cO_{ij} \\ * & -cR \end{bmatrix} < 0, \tag{13}$$

$$\begin{bmatrix} \Phi_{ij} - g_i & cU_{ij} \\ * & -cR \end{bmatrix} < 0, \tag{14}$$

$$\begin{bmatrix} d_i\Phi_{ii} - d_i g_i + g_i & cd_iO_{ij} \\ * & -cR \end{bmatrix} < 0 \tag{15}$$

$$\begin{bmatrix} d_i\Phi_{ii} - d_i g_i + g_i & cd_iU_{ij} \\ * & -cR \end{bmatrix} < 0, \tag{16}$$

$$\begin{bmatrix} d_j\Phi_{ij} - d_j g_i + d_i\Phi_{ji} & c\left(d_jO_{ij} + d_iO_{ji}\right) \\ +g_i + d_i g_j + g_j & \\ * & -cR \end{bmatrix} < 0, \tag{17}$$

$$\begin{bmatrix} d_j\Phi_{ij} + g_i + d_i\Phi_{ji} & c\left(d_jU_{ij} + d_iU_{ji}\right) \\ +g_i - d_i g_j + g_j & \\ * & -cR \end{bmatrix} < 0, \tag{18}$$

$$\begin{bmatrix} \Theta_{ij} - g_i & cY_{ij} \\ * & -cR \end{bmatrix} < 0, \tag{19}$$

$$\begin{bmatrix} \Theta_{ij} - g_i & cS_{ij} \\ * & -cR \end{bmatrix} < 0, \tag{20}$$

$$\begin{bmatrix} d_i\Theta_{ii} - d_i g_i + g_i & cd_iY_{ij} \\ * & -cR \end{bmatrix} < 0, \tag{21}$$

$$\begin{bmatrix} d_i\Theta_{ii} - d_i g_i + g_i & cd_iS_{ij} \\ * & -cR \end{bmatrix} < 0, \tag{22}$$

$$\begin{bmatrix} d_j\Theta_{ij} - d_j g_i + d_i\Theta_{ji} & c\left(d_jY_{ij} + d_iY_{ji}\right) \\ +g_i - d_i g_j + g_j & \\ * & -cR \end{bmatrix} < 0, \tag{23}$$

$$\begin{bmatrix} d_j\Theta_{ij} - d_j g_i + d_i\Theta_{ji} & c\left(d_jS_{ij} + d_iS_{ji}\right) \\ +g_i - d_i g_j + g_i + g_j & \\ * & -cR \end{bmatrix} < 0. \tag{24}$$

where

$$\Phi_{ij} = \begin{bmatrix} \Phi_{ij}^{11} & \Phi_{ij}^{12} & 0 & P + T_1 + \mathrm{A}_i^T T_4^T \\ * & \Phi_{ij}^{22} & -U_{1ij} + U_{2ij}^T & -T_2 - Z_j^T \mathrm{B}_i^T T_4^T \\ * & * & -U_{2ij} - U_{2ij}^T - Q & -T_3 \\ * & * & * & cR - T_4 - T_4^T \end{bmatrix},$$

$\Phi_{ij}^{11} = O_{1ij} + O_{1ij}^T + Q + T\mathrm{A}_i + \mathrm{A}_i^T T_1^T,$

$\Phi_{ij}^{12} = -O_{1ij} + O_{2ij}^T - T_1 \mathrm{B}_i Z_j + \mathrm{A}_i^T T_2^T,$

$\Phi_{ij}^{22} = -O_{2ij} - O_{2ij}^T + U_{1ij} + U_{1ij}^T - T_2 \mathrm{B}_i Z_j - Z_j^T \mathrm{B}_i^T T_2^T,$

$$\Theta_{ij} = \begin{bmatrix} \Theta_{ij}^{11} & \Theta_{ij}^{12} & 0 & P + T_1 + \mathrm{A}_i^T T_4^T \\ * & \Theta_{ij}^{22} & -S_{1ij} + S_{2ij}^T & -T_2 - Z_j^T \mathrm{B}_i^T T_4^T \\ * & * & -S_{2ij} - S_{2ij}^T - Q & -T_3 \\ * & * & * & cR - T_4 - T_4^T \end{bmatrix},$$

$\Theta_{ij}^{11} = Y_{1ij} + Y_{1ij}^T + Q + T_1 \mathrm{A}_i + \mathrm{A}_i^T T_1^T,$

$\Theta_{ij}^{12} = -Y_{1ij} + Y_{2ij}^T - T_1 \mathrm{B}_i Z_j + \mathrm{A}_i^T T_2^T,$

$\Theta_{ij}^{22} = -Y_{2ij} - Y_{2ij}^T + S_{1ij} + S_{1ij}^T - T_2 \mathrm{B}_i Z_j - Z_j^T \mathrm{B}_i^T T_2^T.$

Here, "*" represents the transposition of symmetric position elements, then the fuzzy type-2 NCSs described by Eq. (10) is asymptotically stable.

Proof. Select the following Lyapunov function:

$$\mathrm{T}(\chi(t)) = \mathrm{T}_1(\chi(t)) + \mathrm{T}_2(\chi(t)) + \mathrm{T}_3(\chi(t)), \tag{25}$$

where

$$\mathrm{T}_1(\chi(t)) = \chi^T(t)P\chi(t), \quad \mathrm{T}_2(\chi(t)) = \int_{t-c}^{t} \chi^T(s)Q\chi(s)ds, \quad \mathrm{T}_3(\chi(t)) = \int_{-c}^{0}\int_{t+\theta}^{t} \dot{\chi}^T(s)R\dot{\chi}(s)dsd\theta$$

We take the derivative of $T(\chi(t))$:

$$\dot{\mathrm{T}}(\chi(t)) = \dot{\mathrm{T}}_1(\chi(t)) + \dot{\mathrm{T}}_2(\chi(t)) + \dot{\mathrm{T}}_3(\chi(t)), \tag{26}$$

where

$\dot{\mathrm{T}}_1(\chi(t)) = \dot{\chi}^T(t)P\chi(t) + \chi^T(t)P\dot{\chi}(t),$

$\dot{\mathrm{T}}_2(\chi(t)) = \chi^T(t)Q\chi(t) - \chi^T(t-c)Q\chi(t-c),$

$$\dot{\mathrm{T}}_3(\chi(t)) = c\dot{\chi}^T(t)R\dot{\chi}(t) - \int_{t-c}^{t} \dot{\chi}^T(s)R\chi(s)ds = c\dot{\chi}^T(t)R\dot{\chi}(t) - \int_{t-\tau(t)}^{t} \dot{\chi}^T(s)R\chi(s)ds - \int_{t-c}^{t-\tau(t)} \dot{\chi}^T(s)R\chi(s)ds$$

There exist some matrices $O_{ij} = [O_{1ij} \quad O_{2ij} \quad 0 \quad 0]^T, U_{ij} = [0 \quad U_{1ij} \quad U_{2ij} \quad 0]^T, Y_{ij} = [Y_{1ij} \quad Y_{2ij} \quad 0 \quad 0]^T, S_{ij} = [0 \quad S_{1ij} \quad S_{2ij} \quad 0]^T$, here we use Newton-Leibniz, we have

$$\sum_{i=1}^{l}\sum_{j=1}^{l}\theta_i(\chi(t))\underline{\varepsilon}_j(\chi(t))2\xi(t)^T O_{ij}\left[\chi(t)-\chi(t-\tau(t))-\int_{t-\tau(t)}^{t}\dot{\chi}(s)ds\right]=0, \tag{27}$$

$$\sum_{i=1}^{l}\sum_{j=1}^{l}\theta_i(\chi(t))\underline{\varepsilon}_j(\chi(t))2\xi(t)^T U_{ij}\left[\chi(t-\tau(t))-\chi(t-c)-\int_{t-c}^{t-\tau(t)}\dot{\chi}(s)ds\right]=0, \tag{28}$$

$$\sum_{i=1}^{l}\sum_{j=1}^{l}\theta_i(\chi(t))\bar{\varepsilon}_j(\chi(t))2\xi(t)^T Y_{ij}\left[\chi(t)-\chi(t-\tau(t))-\int_{t-\tau(t)}^{t}\dot{\chi}(s)ds\right]=0, \tag{29}$$

$$\sum_{i=1}^{l}\sum_{j=1}^{l}\theta_i(\chi(t))\bar{\varepsilon}_j(\chi(t))2\xi(t)^T S_{ij}\left[\chi(t-\tau(t))-\chi(t-c)-\int_{t-c}^{t-\tau(t)}\dot{\chi}(s)ds\right]=0, \tag{30}$$

where

$$\xi(t)=\begin{bmatrix}\chi^T(t) & \chi^T(t-\tau(t)) & \chi^T(t-c) & \dot{\chi}^T(t)\end{bmatrix}^T.$$

For any matrices $T_i, i=1,2,3,4$, we have :

$$\begin{aligned}&2\sum_{i=1}^{l}\sum_{j=1}^{l}\theta_i(\chi(t))\left(\bar{\varepsilon}_j(\chi(t))+\underline{\varepsilon}_j(\chi(t))\right)\begin{bmatrix}\chi^T(t)T_1+\chi^T(t-\tau(t))T_2+\\ \chi^T(t-c)T_3+\dot{\chi}^T(t)T_4\end{bmatrix}\\&\left[\mathrm{A}_i\chi(t)-\mathrm{B}_iZ_j\chi(t-\tau(t))-\dot{\chi}(t)\right]=0.\end{aligned} \tag{31}$$

By Lemma 1, we get

$$-2\xi(t)^T O_{ij}\int_{t-\tau(t)}^{t}\dot{\chi}(s)ds\le\tau(t)\xi^T(t)O_{ij}R^{-1}O_{ij}^T\xi(t)+\int_{t-\tau(t)}^{t}\dot{\chi}^T(s)R\chi(s)ds, \tag{32}$$

$$-2\xi(t)^T U_{ij}\int_{t-c}^{t-\tau(t)}\dot{x}(s)ds\le(d-\tau(t))\xi^T(t)U_{ij}R^{-1}U_{ij}^T\xi(t)+\int_{t-c}^{t-\tau(t)}\dot{x}^T(s)Rx(s)ds, \tag{33}$$

$$-2\xi(t)^T Y_{ij}\int_{t-\tau(t)}^{t}\dot{x}(s)ds\le\tau(t)\xi^T(t)Y_{ij}R^{-1}Y_{ij}^T\xi(t)+\int_{t-\tau(t)}^{t}\dot{\chi}^T(s)R\chi(s)ds, \tag{34}$$

$$-2\xi(t)^T S_{ij}\int_{t-c}^{t-\tau(t)}\dot{x}(s)ds\le(d-\tau(t))\xi^T(t)S_{ij}R^{-1}S_{ij}^T\xi(t)+\int_{t-c}^{t-\tau(t)}\dot{\chi}^T(s)R\chi(s)ds. \tag{35}$$

With the above equation, we have

$$\begin{aligned}\dot{\Gamma}(\chi(t))\le{}&\xi^T(t)\left[\sum_{i=1}^{r}\sum_{j=1}^{r}\theta_i\underline{\varepsilon}_j\left(\Phi_{ij}+\tau(t)O_{ij}R^{-1}O_{ij}^T+(d-\tau(t))U_{ij}R^{-1}U_{ij}^T\right)\right]\xi(t)\\&+\xi^T(t)\left[\sum_{i=1}^{r}\sum_{j=1}^{r}\theta_i\bar{\varepsilon}_j\left(\Theta_{ij}+\tau(t)Y_{ij}R^{-1}Y_{ij}^T+(d-\tau(t))S_{ij}R^{-1}S_{ij}^T\right)\right]\xi(t).\end{aligned} \tag{36}$$

Let

$$\Psi_{ij}^1=\Phi_{ij}+\tau(t)O_{ij}R^{-1}O_{ij}^T+(c-\tau(t))U_{ij}R^{-1}U_{ij}^T, \tag{37}$$

$$\Psi_{ij}^2=\Theta_{ij}+\tau(t)Y_{ij}R^{-1}Y_{ij}^T+(c-\tau(t))S_{ij}R^{-1}S_{ij}^T, \tag{38}$$

where

$$\Phi_{ij} = \begin{bmatrix} \Phi_{ij}^{11} & \Phi_{ij}^{12} & 0 & P + T_1 + \mathrm{A}_i^T T_4^T \\ * & \Phi_{ij}^{22} & -U_{1ij} + U_{2ij}^T & -T_2 - Z_j^T \mathrm{B}_i^T T_4^T \\ * & * & -U_{2ij} - U_{2ij}^T - Q & -T_3 \\ * & * & * & cR - T_4 - T_4^T \end{bmatrix},$$

$$\Phi_{ij}^{11} = O_{1ij} + O_{1ij}^T + Q + T_1 \mathrm{A}_i + \mathrm{A}_i^T T_1^T,$$

$$\Phi_{ij}^{12} = -O_{1ij} + O_{2ij}^T - T_1 \mathrm{B}_i Z_j + \mathrm{A}_i^T T_2^T,$$

$$\Phi_{ij}^{22} = -O_{2ij} - O_{2ij}^T + U_{1ij} + U_{1ij}^T - T_2 \mathrm{B}_i Z_j - Z_j^T \mathrm{B}_i^T T_2^T,$$

$$\Theta_{ij} = \begin{bmatrix} \Theta_{ij}^{11} & \Theta_{ij}^{12} & 0 & P + T_1 + \mathrm{A}_i^T T_4^T \\ * & \Theta_{ij}^{22} & -O_{1ij} + O_{2ij}^T & -T_2 - Z_j^T \mathrm{B}_i^T T_4^T \\ * & * & -O_{2ij} - O_{2ij}^T - Q & -T_3 \\ * & * & * & cR - T_4 - T_4^T \end{bmatrix},$$

$$\Theta_{ij}^{11} = Y_{1ij} + Y_{1ij}^T + Q + T_1 \mathrm{A}_i + \mathrm{A}_i^T T_1^T,$$

$$\Theta_{ij}^{12} = -Y_{1ij} + Y_{2ij}^T - T_1 \mathrm{B}_i Z_j + \mathrm{A}_i^T T_2^T,$$

$$\Theta_{ij}^{22} = -Y_{2ij} - Y_{2ij}^T + S_{1ij} + S_{1ij}^T - T_2 \mathrm{B}_i K_j - Z_j^T \mathrm{B}_i^T T_2^T.$$

From Eq. (34), it is obvious that if

$$\sum_{i=1}^{l} \sum_{j=1}^{l} \theta_i \underline{\varepsilon}_j \Psi_{ij}^1 < 0, \tag{39}$$

$$\sum_{i=1}^{l} \sum_{j=1}^{l} \theta_i \bar{\varepsilon}_j \Psi_{ij}^2 < 0. \tag{40}$$

we have $\dot{\mathrm{T}}(\chi(t)) < 0$. Next, we will consider the following equation

$$\sum_{i=1}^{l} \sum_{j=1}^{l} \theta_i \big(\theta_j - \underline{\varepsilon}_j\big) g_i = \sum_{i=1}^{l} \theta_i \left(\sum_{j=1}^{l} \theta_j - \sum_{j=1}^{l} \underline{\varepsilon}_j \right) g_i = \sum_{i=1}^{l} \theta_i (1-1) g_i = 0, \tag{41}$$

$$\sum_{i=1}^{l} \sum_{j=1}^{l} \theta_i \big(\theta_j - \bar{\varepsilon}_j\big) g_i = 0. \tag{42}$$

where $g_i = g_i^T \in R^{4n \times 4n} > 0, i = 1, 2, \ldots, l$. For reducing the conservativeness, the above equations are added to Eqs. (39) and (40), we have

$$\begin{aligned}\bar{\Psi}_{ij}^{1} &= \sum_{i=1}^{l}\sum_{j=1}^{l}\theta_{i}\underline{\varepsilon}_{j}\Psi_{ij}^{1} = \sum_{i=1}^{l}\sum_{j=1}^{l}\theta_{i}\underline{\varepsilon}_{j}\Psi_{ij}^{1} + \sum_{i=1}^{l}\sum_{j=1}^{l}\theta_{i}\left(\theta_{j} - \underline{\varepsilon}_{j} + d_{j}\theta_{j} - d_{j}\theta_{j}\right)g_{i} \\ &= \sum_{i=1}^{l}\sum_{j=1}^{l}\theta_{i}\theta_{j}\left(d_{j}\Psi_{ij}^{1} - d_{j}g_{i} + g_{i}\right) + \sum_{i=1}^{l}\sum_{j=1}^{l}\theta_{i}\left(\underline{\varepsilon}_{j} - g_{j}\theta_{j}\right)\left(\Psi_{ij}^{1} - g_{i}\right) \\ &\leq \sum_{i=1}^{l}\theta_{i}^{2}\left(d_{i}\Psi_{ii}^{1} - d_{i}g_{i} + g_{i}\right) + \sum_{i=1}^{l}\sum_{j=1}^{l}\theta_{i}\left(\underline{\varepsilon}_{j} - d_{j}\theta_{j}\right)\left(\Psi_{ij}^{1} - g_{i}\right) \\ &+ \sum_{i=1}^{l}\sum_{i<j}\theta_{i}\underline{\varepsilon}_{j}\left(d_{j}\Psi_{ij}^{1} + d_{i}\Psi_{ji}^{1} - d_{j}g_{i} - d_{i}g_{j} + g_{i} + g_{j}\right),\end{aligned} \tag{43}$$

$$\begin{aligned}\bar{\Psi}_{ij}^{2} &\leq \sum_{i=1}^{l}\theta_{i}^{2}\left(d_{j}\Psi_{ij}^{2} - d_{j}g_{i} + g_{i}\right) + \sum_{i=1}^{l}\sum_{j=1}^{l}\theta_{i}\left(\varepsilon_{j} - d_{j}\theta_{j}\right)\left(\Psi_{ij}^{2} - g_{i}\right) + \\ &\sum_{i=1}^{l}\sum_{i<j}\theta_{i}\bar{\varepsilon}_{j}\left(d_{j}\Psi_{ij}^{2} + d_{i}\Psi_{ji}^{2} - d_{j}g_{i} - d_{i}g_{j} + g_{i} + g_{j}\right).\end{aligned} \tag{44}$$

If $\underline{\varepsilon}_j - d_j\bar{\theta}_j \geq 0$ and $\bar{\varepsilon}_j - d_j\underline{\theta}_j \geq 0$, $\underline{\varepsilon}_j - d_j\theta_j \geq 0$and $\bar{\theta}_j - d_j\theta_j \geq 0$ hold. With $\underline{\varepsilon}_j - d_j\bar{\theta}_j \geq 0$ and $\bar{\varepsilon}_j - d_j\underline{\theta}_j \geq 0$, let

$$\Psi_{ij}^{1} - g_i < 0, \tag{45}$$

$$d_i\Psi_{ii}^{1} - d_ig_i + g_i < 0, \tag{46}$$

$$d_j\Psi_{ij}^{1} + d_i\Psi_{ji}^{1} - d_jg_i - d_ig_j + g_i + g_j < 0, i < j, \tag{47}$$

$$\Psi_{ij}^{2} - g_i < 0, \tag{48}$$

$$d_i\Psi_{ii}^{2} - d_ig_i + g_i < 0, \tag{49}$$

$$d_j\Psi_{ij}^{2} + d_i\Psi_{ji}^{2} - d_jg_i - d_ig_j + g_i + g_j < 0, i < j. \tag{50}$$

The Eqs. (45)–(50) are equivalent to the following inequalities on the basis of Eq. (37), Eq. (38) and Lemma 2:

$$\Phi_{ij} - g_i + cO_{ij}R^{-1}O_{ij}^{T} < 0, \tag{51}$$

$$\Phi_{ij} - g_i + cU_{ij}R^{-1}U_{ij}^{T} < 0, \tag{52}$$

$$d_i\Phi_{ii} - d_ig_i + g_i + cd_iO_{ij}R^{-1}O_{ij}^{T} < 0, \tag{53}$$

$$d_i\Phi_{ii} - d_ig_i + g_i + cd_iU_{ij}R^{-1}U_{ij}^{T} < 0, \tag{54}$$

$$d_j\Phi_{ij} - d_jg_i + d_i\Phi_{ji} - d_ig_j + cd_jO_{ij}R^{-1}O_{ij}^{T} + g_i + cd_iO_{ji}R^{-1}O_{ji}^{T} + g_j < 0, \tag{55}$$

$$d_j\Phi_{ij} - d_jg_i + d_i\Phi_{ji} - d_ig_j + cd_jU_{ij}R^{-1}U_{ij}^{T} + d_i + cd_iU_{ji}R^{-1}U_{ji}^{T} + d_j < 0, \tag{56}$$

$$\Theta_{ij} - g_i + cY_{ij}R^{-1}Y_{ij}^{T} < 0, \tag{57}$$

$$\Theta_{ij} - g_i + cS_{ij}R^{-1}S_{ij}^T < 0, \tag{58}$$

$$d_i\Theta_{ii} - d_ig_i + cd_iY_{ij}R^{-1}Y_{ij}^T + g_i < 0, \tag{59}$$

$$d_i\Theta_{ii} - d_ig_i + cd_iS_{ij}R^{-1}S_{ij}^T + g_i < 0, \tag{60}$$

$$d_j\Theta_{ij} - d_jg_i + d_i\Theta_{ji} - d_ig_j + cd_jY_{ij}R^{-1}Y_{ij}^T + g_i + cd_iY_{ji}R^{-1}Y_{ji}^T + g_j < 0, \tag{61}$$

$$d_j\Theta_{ij} - d_jg_i + d_i\Theta_{ji} - d_ig_j + cd_jS_{ij}R^{-1}S_{ij}^T + g_i + cd_iS_{ji}R^{-1}S_{ji}^T + g_j < 0, \tag{62}$$

We can see from Shur complement, the above inequalities Eqs. (51)–(62) are equivalent to Eqs. (13)–(24). Since $\dot{T}(x(t)) < 0$, the system described in Eq. (10) is asymptotically stable.

Remark 1 We consider the relationship between the membership and the fuzzy regulator in Theorem 1, which is more relaxed than the stability condition that does not consider the information of the membership (Corollary 1). When $\theta_i(\chi(t)) = \varepsilon_i(\chi(t))$, Theorem 1 can be reduced to Corollary 1 below, i.e., the membership function independent stability condition.

Corollary 1. The scalars c and the matrix Z_j are given, the Eq. (10) is asymptotically stable, if there exist matrices $\tilde{P} > 0, \tilde{R} > 0, \tilde{Q} > 0, \tilde{T}_i (i = 1, 2, 3, 4)$ and $\tilde{O}_{ij} = \begin{bmatrix} \tilde{O}_{1ij} & \tilde{O}_{2ij} & 0 & 0 \end{bmatrix}^T, \tilde{U}_{ij} = \begin{bmatrix} 0 & \tilde{U}_{1ij} & \tilde{U}_{2ij} & 0 \end{bmatrix}, \tilde{Y}_{ij} = \begin{bmatrix} \tilde{Y}_{1ij} & \tilde{Y}_{2ij} & 0 & 0 \end{bmatrix}^T, \tilde{S}_{ij} = \begin{bmatrix} 0 & \tilde{S}_{1ij} & \tilde{S}_{2ij} & 0 \end{bmatrix}^T$ make the under non-equality set up:

$$\begin{bmatrix} \tilde{\Phi}_{ij}^{11} & \tilde{\Phi}_{ij}^{12} & 0 & \tilde{P} + \tilde{T}_1 + \mathrm{A}_i^T\tilde{T}_4^T & c\tilde{O}_{1ij} \\ * & \tilde{\Phi}_{ij}^{22} & -\tilde{E}_{1ij} + \tilde{E}_{2ij}^T & -\tilde{T}_2 - \tilde{K}_j^T\mathrm{B}_i^T\tilde{T}_4^T & c\tilde{O}_{2ij} \\ * & * & -\tilde{E}_{2ij} - \tilde{E}_{2ij}^T - \tilde{Q} & -\tilde{T}_3 & 0 \\ * & * & * & c\tilde{R} - \tilde{T}_4 - \tilde{T}_4^T & 0 \\ * & * & * & * & c\tilde{R} \end{bmatrix} < 0, \tag{63}$$

$$\begin{bmatrix} \tilde{\Phi}_{ij}^{11} & \tilde{\Phi}_{ij}^{12} & 0 & P + \tilde{T}_1 + \mathrm{A}_i^T\tilde{T}_4^T & 0 \\ * & \tilde{\Phi}_{ij}^{22} & -\tilde{E}_{1ij} + \tilde{E}_{2ij}^T & -T_2 - Z_j^T\mathrm{B}_i^T\tilde{T}_4^T & c\tilde{E}_{1ij} \\ * & * & -\tilde{E}_{2ij} - \tilde{E}_{2ij}^T - Q & -\tilde{T}_3 & \tilde{E}_{2ij} \\ * & * & * & c\tilde{R} - \tilde{T}_4 - \tilde{T}_4^T & 0 \\ * & * & * & * & c\tilde{R} \end{bmatrix} < 0, \tag{64}$$

$$\tilde{\Phi}_{ij}^{11} = \tilde{D}_{1ij} + \tilde{D}_{1ij}^T + \tilde{Q} + \tilde{T}_1\mathrm{A}_i + \mathrm{A}_i^T\tilde{T}_1^T, \tag{65}$$

$$\tilde{\Phi}_{ij}^{12} = -\tilde{D}_{1ij} + \tilde{D}_{2ij}^T - \tilde{T}_1\mathrm{B}_i\tilde{Z}_j + \mathrm{A}_i^T\tilde{T}_2^T, \tag{66}$$

$$\tilde{\Phi}_{ij}^{22} = -\tilde{D}_{2ij} - \tilde{D}_{2ij}^T + \tilde{E}_{1ij} + \tilde{E}_{1ij}^T - \tilde{T}_2\mathrm{B}_i\tilde{Z}_j - \tilde{Z}_j^T\mathrm{B}_i^T\tilde{T}_2^T, \tag{67}$$

$$\begin{bmatrix} \tilde{\Theta}_{ij}^{11} & \tilde{\Theta}_{ij}^{12} & 0 & \tilde{P} + \tilde{T}_1 + \mathrm{A}_i^T\tilde{T}_4^T & c\tilde{F}_{1ij} \\ * & \tilde{\Theta}_{ij}^{22} & -\tilde{G}_{1ij} + \tilde{G}_{2ij}^T & -\tilde{T}_2 - \tilde{Z}_j^T\mathrm{B}_i^T\tilde{T}_4^T & c\tilde{F}_{2ij} \\ * & * & -\tilde{G}_{2ij} - \tilde{G}_{2ij}^T - \tilde{Q} & -\tilde{T}_3 & 0 \\ * & * & * & c\tilde{R} - \tilde{T}_4 - \tilde{T}_4^T & 0 \\ * & * & * & * & -c\tilde{R} \end{bmatrix} < 0, \tag{68}$$

$$\begin{bmatrix} \tilde{\Theta}_{ij}^{11} & \tilde{\Theta}_{ij}^{12} & 0 & \tilde{P} + \tilde{T}_1 + \mathrm{A}_i^T \tilde{T}_4^T & 0 \\ * & \tilde{\Theta}_{ij}^{22} & -\tilde{G}_{1ij} + \tilde{G}_{2ij}^T & -\tilde{T}_2 - \tilde{Z}_j^T \mathrm{B}_i^T \tilde{T}_4^T & c\tilde{G}_{1ij} \\ * & * & -\tilde{G}_{2ij} - \tilde{G}_{2ij}^T - \tilde{Q} & -\tilde{T}_3 & \tilde{G}_{2ij} \\ * & * & * & C\tilde{R} - \tilde{T}_4 - \tilde{T}_4^T & 0 \\ * & * & * & * & -c\tilde{R} \end{bmatrix} < 0, \tag{69}$$

$$\tilde{\Theta}_{ij}^{11} = \tilde{F}_{1ij} + \tilde{F}_{1ij}^T + \tilde{Q} + \tilde{T}_1 \mathrm{A}_i + \mathrm{A}_i^T \tilde{T}_1^T, \tag{70}$$

$$\tilde{\Theta}_{ij}^{12} = -\tilde{F}_{1ij} + \tilde{F}_{2ij}^T - \tilde{T}_1 \mathrm{B}_i Z_j + \mathrm{A}_i^T \tilde{T}_2^T, \tag{71}$$

$$\tilde{\Theta}_{ij}^{22} = -\tilde{F}_{2ij} - \tilde{F}_{2ij}^T + \tilde{G}_{1ij} + \tilde{G}_{1ij}^T - \tilde{T}_2 \mathrm{B}_i \tilde{Z}_j - \tilde{Z}_j^T \mathrm{B}_i^T \tilde{T}_2^T, \tag{72}$$

$$\begin{bmatrix} d_j\Theta_{ij} - d_j g_i + d_i \Theta_{ji} + g_i - d_i g_j + g_j & c\left(d_j F_{ij} + d_i F_{ji}\right) \\ * & -cR \end{bmatrix} < 0, \tag{73}$$

$$\begin{bmatrix} d_j\Theta_{ij} + g_i + d_i \Theta_{ji} - d_j g_i + g_j - d_i g_j & c\left(d_j G_{ij} + d_i G_{ji}\right) \\ * & -cR \end{bmatrix} < 0. \tag{74}$$

Based on Theorem 1, The following Theorem 2 will give the controller design method.

Theorem 2. The scalars $c > 0$ are given, the Eq. (10) is asymptotically stable with feedback gains $Z_j = \Upsilon_j X^{-T}$, if there satisfy satisfy $\underline{\varepsilon}_j(\chi(t)) - d_j\bar{\theta}_j(\chi(t)) \geq 0$ and $\bar{\varepsilon}_j(\chi(t)) - d_j\underline{\theta}_j(\chi(t)) \geq 0$ for all $j, \chi(t)$, where $0 < d_j < 1$, and the following matrices $\hat{P} > 0, \hat{R} > 0, \hat{Q} > 0, \hat{g}_i = \hat{g}_i^T \in R^{4n \times 4n} > 0 (i = 1, 2, \cdots, l)$ and $\hat{O}_{ij} = \begin{bmatrix} \hat{O}_{1ij} & \hat{O}_{2ij} & 0 & 0 \end{bmatrix}^T, \hat{U}_{ij} = \begin{bmatrix} 0 & \hat{U}_{1ij} & \hat{U}_{2ij} & 0 \end{bmatrix}, \hat{Y}_{ij} = \begin{bmatrix} \hat{Y}_{1ij} & \hat{Y}_{2ij} & 0 & 0 \end{bmatrix}^T, \hat{S}_{ij} = \begin{bmatrix} 0 & \hat{S}_{1ij} & \hat{S}_{2ij} & 0 \end{bmatrix}^T$ exist, and make the following LMIs set up:

$$\begin{bmatrix} \hat{\Phi}_{ij} - \hat{g}_i & c\hat{O}_{ij} \\ * & -c\hat{R} \end{bmatrix} < 0, \tag{75}$$

$$\begin{bmatrix} \hat{\Phi}_{ij} - \hat{g}_i & c\hat{U}_{ij} \\ * & -c\hat{R} \end{bmatrix} < 0, \tag{76}$$

$$\begin{bmatrix} d_i\hat{\Phi}_{ii} - d_i\hat{g}_i + \hat{g}_i & cd_i\hat{O}_{ij} \\ * & -c\hat{R} \end{bmatrix} < 0, \tag{77}$$

$$\begin{bmatrix} d_i\hat{\Phi}_{ii} - d_i\hat{g}_i + \hat{g}_i & cd_i\hat{U}_{ij} \\ * & -c\hat{R} \end{bmatrix} < 0, \tag{78}$$

$$\begin{bmatrix} d_j\hat{\Phi}_{ij} + g_j + d_i\hat{\Phi}_{ji} + g_j - d_i\hat{g}_j + \hat{g}_j & c\left(d_j\hat{O}_{ij} + d_i\hat{O}_{ji}\right) \\ * & -C\hat{R} \end{bmatrix} < 0, \tag{79}$$

$$\begin{bmatrix} d_j\hat{\Phi}_{ij} + g_j + d_i\hat{\Phi}_{ji} - d_j\hat{g}_i + \hat{g}_j - d_i\hat{g}_j & c\left(d_j\hat{U}_{ij} + d_i\hat{U}_{ji}\right) \\ * & -c\hat{R} \end{bmatrix} < 0, \tag{80}$$

$$\begin{bmatrix} \hat{\Theta}_{ij} - \hat{g}_i & c\hat{Y}_{ij} \\ * & -c\hat{R} \end{bmatrix} < 0, \tag{81}$$

$$\begin{bmatrix} \hat{\Theta}_{ij} - \hat{g}_i & c\hat{S}_{ij} \\ * & -c\hat{R} \end{bmatrix} < 0, \tag{82}$$

$$\begin{bmatrix} d_i\hat{\Theta}_{ii} - d_i\hat{g}_i + \hat{g}_i & cd_i\hat{Y}_{ij} \\ * & -c\hat{R} \end{bmatrix} < 0, \tag{83}$$

$$\begin{bmatrix} d_i\hat{\Theta}_{ii} - d_i\hat{g}_i + \hat{g}_i & cd_i\hat{S}_{ij} \\ * & -c\hat{R} \end{bmatrix} < 0, \tag{84}$$

$$\begin{bmatrix} d_j\hat{\Theta}_{ij} + \hat{g}_i + d_i\hat{\Theta}_{ji} - d_j\hat{g}_i + \hat{g}_j - d_i\hat{g}_j & c\left(d_j\hat{Y}_{ij} + d_i\hat{Y}_{ji}\right) \\ * & -c\hat{R} \end{bmatrix} < 0, \tag{85}$$

$$\begin{bmatrix} d_j\hat{\Theta}_{ij} + \hat{g}_i + d_i\hat{\Theta}_{ji} - d_j\hat{g}_i + \hat{g}_i - d_i\hat{g}_j & c\left(d_j\hat{S}_{ij} + d_i\hat{S}_{ji}\right) \\ * & -c\hat{R} \end{bmatrix} < 0, \tag{86}$$

where

$$\hat{\Phi}_{ij} = \begin{bmatrix} \hat{\Phi}_{ij}^{11} & \hat{\Phi}_{ij}^{12} & 0 & \hat{P} + X^T + t_4 X \mathrm{A}_i^T \\ * & \hat{\Phi}_{ij}^{22} & -\hat{U}_{1ij} + \hat{U}_{2ij}^T & -t_2 X^T - t_4 \Upsilon_j^T \mathrm{B}_i^T \\ * & * & -\hat{U}_{2ij} - \hat{U}_{2ij}^T - \hat{Q} & -t_3 X^T \\ * & * & * & c\hat{R} - t_4 X^T - t_4 X \end{bmatrix},$$

$$\hat{\Phi}_{ij}^{11} = \hat{O}_{1ij} + \hat{O}_{1ij}^T + \hat{Q} + \mathrm{A}_i X + X\mathrm{A}_i^T,$$

$$\hat{\Phi}_{ij}^{12} = -\hat{O}_{1ij} + \hat{O}_{2ij}^T - \mathrm{B}_i\Upsilon_j + X\mathrm{A}_i^T,$$

$$\hat{\Phi}_{ij}^{22} = -\hat{O}_{2ij} - \hat{O}_{2ij}^T + \hat{U}_{1ij} + \hat{U}_{1ij}^T - t_2\mathrm{B}_i\Upsilon_j - t_2\Upsilon_j^T\mathrm{B}_i^T,$$

$$\hat{\Theta}_{ij} = \begin{bmatrix} \hat{\Theta}_{ij}^{11} & \hat{\Theta}_{ij}^{12} & 0 & \hat{P} + X^T + t_4 X \mathrm{A}_i^T \\ * & \hat{\Theta}_{ij}^{22} & -\hat{S}_{1ij} + \hat{S}_{2ij}^T & -t_2 X^T - t_4 \Upsilon_j^T \mathrm{B}_i^T \\ * & * & -\hat{S}_{2ij} - \hat{S}_{2ij}^T - \hat{Q} & -t_3 X^T \\ * & * & * & c\hat{R} - t_4 X^T - t_4 X \end{bmatrix},$$

$$\hat{\Theta}_{ij}^{11} = \hat{Y}_{1ij} + \hat{Y}_{1ij}^T + \hat{Q} + \mathrm{A}_i X + X\mathrm{A}_i^T,$$

$$\hat{\Theta}_{ij}^{12} = -\hat{Y}_{1ij} + \hat{Y}_{2ij}^T - \mathrm{B}_i\Upsilon_j + X\mathrm{A}_i^T,$$

$$\hat{\Theta}_{ij}^{22} = -\hat{Y}_{2ij} - \hat{Y}_{2ij}^T + \hat{S}_{1ij} + \hat{S}_{1ij}^T - t_2\mathrm{B}_i\Upsilon_j - t_2\Upsilon_j^T\mathrm{B}_i^T.$$

Proof. Pre- and post-multiply the diag $[X \quad X \quad X \quad X \quad X]$ and its transpose to both sides of Eqs. (13)–(24). Let $T_i = t_i T_1 (i = 2, 3, 4)$, and the new variables are as follows: $X = T_1^{-1}$,$\hat{P} = XPX^T$,$\hat{Q} = XQX^T$, $\hat{O}_{1ij} = XO_{1ij}X^T$, $\hat{O}_{2ij} = XO_{2ij}X^T$, $\hat{U}_{1ij} = XU_{1ij}X^T$, $\hat{U}_{2ij} = X\hat{U}_{2ij}X^T$, $\hat{Y}_{1ij} = XY_{1ij}X^T$, $\hat{Y}_{2ij} = XY_{2ij}X^T$, $\hat{S}_{1ij} = XS_{1ij}X^T$, $\hat{S}_{2ij} = X\hat{S}_{2ij}X^T$. With $Z_j = \Upsilon_j X^{-T}, j = 1, 2, \cdots, l$, the inequalities Eqs. (75)–(86) can be obtained.

4 Numerical Examples

Example 1 [33]. In order to compare the results of Theorem 1 and Corollary 1, stability regions are shown in Figs. 2(a) and 2(b).

Rule i: $\chi_1(t)$ is M_1^i,

$$\dot{\chi}(t) = A_i\chi(t) + B_iu(t), i = 1, 2, \chi \in [\chi_1, \chi_2], \chi_1 \in [-10, 10] \tag{87}$$

where

$$A_1 = \begin{bmatrix} 2.78 & -5.63 \\ 0.01 & 0.33 \end{bmatrix},\ A_2 = \begin{bmatrix} -a & -4.33 \\ 0 & 0.05 \end{bmatrix}(0 \le a \le 45),\ B_1 = [\,2 \quad -1\,]^T,$$

$$B_2 = [\,-b+6 \quad -1\,]^T(0 \le b \le 25).$$

We choose the following function as the membership functions of Eq. (87)

$$\theta_1(\chi_1) = 1 - 1\Big/\left(1 - 4e^{-(\chi_1+4+\eta(t))}\right),\ \theta_2(\chi_1) = 1 - \theta_1(\chi_1),\ \eta(t) \in [-0.25, 0.25].$$

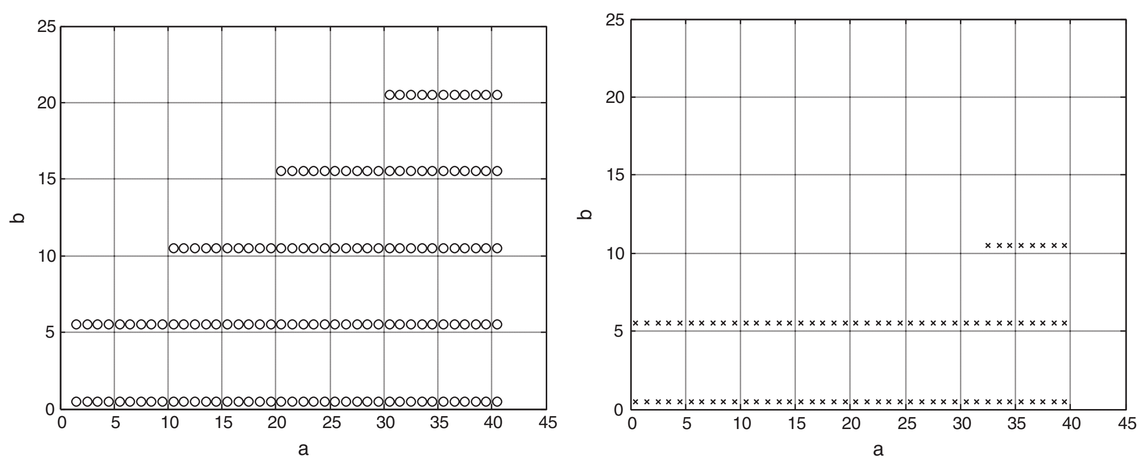
$$\underline{\varepsilon}_1(\chi_1) = 1 - 1\Big/\left(1 - 4e^{-(\chi_1+4+0.25)}\right),\ \bar{\varepsilon}_1(\chi_1) = 1 - 1\Big/\left(1 - 4e^{-(\chi_1+4-0.25)}\right),$$

$$\underline{\theta}_2(\chi_1) = 1 - \bar{\theta}_1(\chi_1),\ \bar{\theta}_2(\chi_1) = 1 - \underline{\theta}_1(\chi_1).$$

Distinct of the above membership, the down and up membership functions of controller are

$$\underline{\varepsilon}_1(\chi_1) = 1 - 1\Big/e^{-(\chi_1+0.15)/2},\ \bar{\varepsilon}_2(\chi_1) = 1 - \underline{\varepsilon}_1(\chi_1),\ \bar{\varepsilon}_1(\chi_1) = 1 - 1\Big/e^{-(\chi_1-0.15)/2},\ \underline{\varepsilon}_2(\chi_1) = 1 - \bar{\varepsilon}_1(\chi_1).$$

Here, we select $c = 0.19, d_1 = 0.75$, and $d_2 = 0.85$, and the feedback gains of $Z_j, j = 1, 2$ take the value when the characteristic root is -5, and we obtain the stability area which is shown in Figs. 2(a) and 2(b) based on Theorem 1 and Corollary 1. we can see larger stability area can be obtained from Theorem 1 than that from Corollary 1(based on the PDC method), which means that the stability condition under the incomplete premise matching is less conservative.



(a) (b)

Figure 2: (a) Stable area of Theorem 1 with $c = 0.19$, $d_1 = 0.75$, $d_2 = 0.85$ (denoted by "o"); (b) Stable area of Corollary 1 with $c = 0.19$ (denoted by "×")

Example 2 Consider the following mechanical system [27]

$$\varepsilon\ddot{\eta} + \sigma\dot{\eta} + \omega\eta + \omega a^2\eta^3 = \delta(t), \tag{88}$$

Let $\eta(t) = [\eta_1(t) \quad \eta_2(t)]^T = [\eta \quad \dot{\eta}]$, $\eta_1(t) \in [-2, 2], \varepsilon = 1, \sigma = 2, a = 0.3$, and $\omega \in [5, 8]$. $q = \dfrac{-\omega - \omega a^2\eta_1^2(t)}{\varepsilon}$, $q_{\min} = -10.88, q_{\max} = -5$. The mass-spring-damper system in Eq. (88) is described as:

$$\dot{x}(t) = \sum_{i=1}^{2} \theta_i(x_1(t))[\mathrm{A}_i(x(t)) + \mathrm{B}_i\mu(t)], \tag{89}$$

where

$$\mathrm{A}_1 = \begin{bmatrix} 0 & 1 \\ q_{\min} & -\dfrac{c}{m} \end{bmatrix}, \mathrm{A}_2 = \begin{bmatrix} 0 & 1 \\ q_{\max} & -\dfrac{c}{m} \end{bmatrix}, \mathrm{B}_1 = \begin{bmatrix} 0 \\ \dfrac{1}{m} \end{bmatrix}, \mathrm{B}_2 = \begin{bmatrix} 0 \\ \dfrac{1}{m} \end{bmatrix}.$$

In this example, the lower and upper bounds of the membership functions of the plant in Eq. (89) can be selected as the same as those in Example 1. However, according to Theorem 2, the membership functions of the controller cannot be chosen as $\theta_i(x_1), i = 1, 2$. For simplification, they are set to be $\bar{\varepsilon}_j(x_1), \underline{\varepsilon}_j(x_1) j = 1, 2$, which satisfy $\underline{\varepsilon}_j - d_j\bar{\theta}_j \geq 0$ and $\bar{\varepsilon}_j - d_j\underline{\theta}_j \geq 0, j = 1, 2$. Using Theorem 2 with $d_1 = 0.75$ and $d_2 = 0.85$, we can obtain the maximum value of delay $c = 0.12$, and the controller gains are:

$$Z_1 = [-6.5123 \quad -3.3432], Z_2 = [-7.0032 \quad -4.1432].$$

With the controller gains and membership functions $\bar{\varepsilon}_j(x_1), \underline{\varepsilon}_j(x_1) j = 1, 2$, the controller in Eq. (6) is applied to control the mass-spring-damper mechanical system in Eq. (89). The system responses and control inputs are shown in Figs. 3 and 4, respectively under the initial condition of $x(0) = [-1 \quad -1]^T$, which demonstrate that the proposed controller design method is indeed effective and efficient.

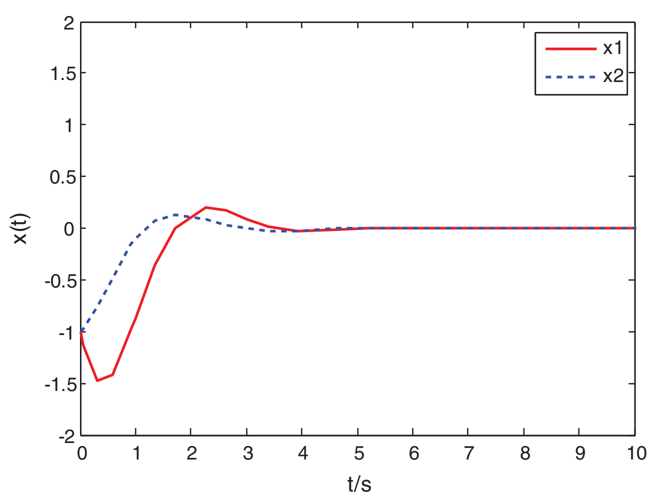


Figure 3: Answers of the control system in Eq. (10) with $c = 0.12$

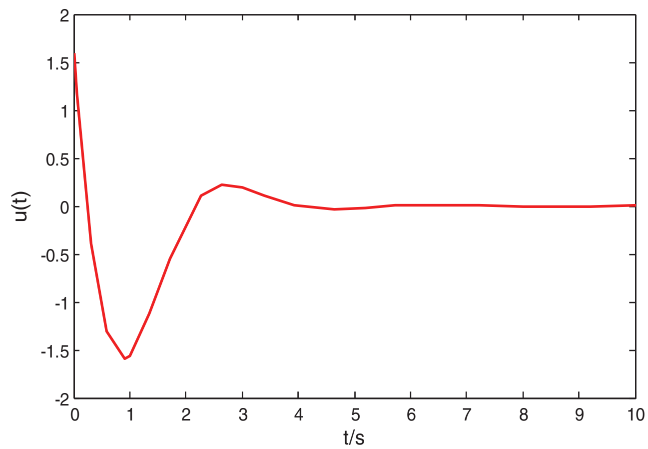


Figure 4: System control input

Remark 2 Compared with conventional controller design techniques proposed in Lam et al. [6,14–16,26,27] where the fuzzy regulator is bear on the up and down membership functions of the interval type-2 model, the membership functions of our networked fuzzy controller are much simpler, thus significantly enhancing the controller design flexibility.

5 Conclusions

In this article, we investigate the stability and synthesis for the interval type-2 fuzzy systems. A less conservative stability criterion is got. The stability condition can offer larger stability regions than those obtained from the traditional PDC scheme. Furthermore, a new design technique under the imperfect premise matching is developed in order to stabilize the control systems. Distinct of the PDC controller design method, some simple and certain functions as the membership functions of the fuzzy controller can be selected. Two numerical examples are used to demonstrate the less conservativeness of the above methods. They can significantly improve the design flexibility as well as reduce the implementation complexity of the fuzzy controller.

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