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Simplified Method and Influence Factors of Vibration Characteristics of Isolated Curved Girder Bridge

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Abstract: The isolated curved girder bridge's vibration characteristics play a major part in the seismic responses of structures and anti-seismic properties. A clear analytic relationship between design parameters and the system's vibration characteristics could be established by its simplified dynamic analysis model, making it convenient for providing a reference to the optimization of design and safety analysis. A double-mass six-degree-of-freedom model for curved girder bridges with isolation bearings installed at the top of the bridge piers is built and a simplified analysis method for the vibration characteristics of the system is provided. Combined with the Matlab programming, the influences of radius of curvature, central angle, bridge deck width and damping ratio of the isolation layer and circular frequency of the isolation layer of isolated curved girder bridges on the pseudo-undamped natural circular frequency (called pseudo-frequency for short) and system damping ratio are systematically analyzed, and the sensitivity of vibration characteristics of isolated curved girder bridges is studied. The results show that the vibration characteristics of isolated curved girder bridges can be reflected well with this simplified model and calculation method. The pseudo-frequency of curved girder and system damping ratios increases with the increase of the isolation layer. The third-order vibration characteristic is more sensitive to the parameters of a curved girder, and the first-order vibration characteristic is sensitive to both central angle and radius of curvature to some extent while insensitive to the width of the bridge deck. Furthermore, the second-order vibration characteristic is not sensitive to the parameters of a curved girder.

Keywords: Seismic isolation, curved girder bridge, vibration characteristics, sensitivity analysis, simplified analysis method.

1 Introduction

As an important branch of girder bridges, curved girder bridges are widely applied in the area of expressways and interchange ramp bridges because they can overcome topographical restraints and have beautiful line shapes. However, according to existing

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earthquake damage, compared with straight girder bridges, curved girder bridges would be more severely damaged under seismic loading due to the influences of their own curvature. Vibration characteristics are important properties of structure and play a major role in the seismic responses and seismic performance of structures [Li, Yin and Yan (2015); Tang, Zheng and Song (2015); Fang, Zhang and Zhang (2013)]. The vibration period of original structures can be extended by the setting of isolation bearings at the top of bridge piers of curved girder bridges to avoid the highest frequency of seismic excitation and utilize their shock absorption capabilities. With the development and wide application of seismic absorption and isolation technologies, more and more attention is being paid to the vibration characteristics and safety issues of isolated curved girder bridges.

Compared with straight girder bridges, curved girder bridges become more complex in terms of dynamic property and dynamic analysis due to their irregular geometric parameters and uneven distribution of rigidity and mass [Han, Du, Liu et al. (2009); Falamarz-Sheikhabadi and Zerva (2016); Amjadian and Agrawal (2017); Yan (2014)]. The rational dynamical model of curved girder bridges has not been resolved in theory [Han and Wu (2016)]. At present, the major methods for the study of vibration characteristics of curved girder bridges are numerical simulation and model test methods [Yan (2014)].

Markus et al. [Markus and Nasasi (1981)], Laura et al. [Laura and Maurizi (1987)], Childamparam et al. [Childamparam and Leissa (1993)] and Aueiello et al. [Auciello and De Rosa (1994)] summarized the vibration analysis of curved girders. [Viasov and Timoshenko (1961)] were the first to study vibration of thin-walled curved girders. Heins et al. [Heins and Sahin (1979)] reached a series of empirical formulas for determining the natural frequencies of straight bridges and curved bridges, and after verification of the natural frequency of curved bridges obtained with the finite difference method, they studied the parameters of single-span, double-span and three-span curved girder bridges. Kou et al. [Kou, Benzley, Huang et al. (1992)] thought over the analysis theory of free vibration of curved thin-walled box girder bridges under the influence of warping and discussed the mass matrix and rigidity matrix; the results indicate that the rigidity of bridge piers and the curvature of curved beams produced very large influences on the natural frequency of curved girder bridges. Yoon et al. [Yoon, Kang, Choi et al. (2005)] came up with a finite element formula for the analysis of free vibration of curved steel I-girder bridges, where each node had seven degrees of freedom including the degree of freedom of warping, and developed computer programs to analyze the free vibration of various bridges. Through the comparison of frequency with the ABAQUS general program, they verified the validity of numerical formulas. Samaan et al. [Samaan, Kennedy and Sennah (2007)] analyzed the natural frequency and vibration mode of continuous curved composite multiple-box girder bridges with the finite element method and verified the correctness of the finite element method through two model tests, with the influence parameters taken into account including the number of spans, span length, curvature and lane. Ngo-Tran et al. [Ngo-Tran, Hayashikawa and Hirasawa (2007)] analyzed the free vibration characteristics of horizontally curved twin I-girder bridges with the 3-D finite element method of MSC Nastran and put forward measures for rebuilding and reinforcing similar bridges which can improve the vibration characteristics. Sennah et al. [Sennah and Kennedy (2011)] analyzed 120 simply supported curved composite bridges with the finite element method [Lee, Oh and Park (2002)], studied the influences of curvature and the number of lanes and other pairs of design parameters on the natural frequency and vibration mode of curved bridges, deduced the expression of the first-order bend frequency from the analysis of data results and verified the correctness of the model through the test results. Androus et al. [Androus, Afefy and Sennah (2017)] tested the natural frequency of free vibrations through two groups of model tests of steel-concrete composite simply-supported curved box girder bridges and verified the results with the finite element results; the results reveal that the external cross bracing system between the steel boxes had an insignificant effect on the natural frequency in the elastic range of loading. Agrawal et al. [Agrawal and Amjadian (2017)] built a simple three-degree-of-freedom free vibration equation and a dynamic model of a curved girder bridge and conducted model verification with numerical simulation. That model was used to estimate the maximum displacement of curved girder bridges; the results show that the natural frequencies of one-way asymmetric horizontally curved bridges, in general, increase with the increase of the subtended angle of the deck.

The traditional detailed finite element model method, has the advantage of high accuracy, however, the structural modeling and analysis are relatively complicated with heavy a workload, and it reflects the superficial phenomenon of matters, that is improper to propose regular design methods [Li (2015)]. The simplified model could provide a clear analytic relationship and is conducive to the study of the logic relationship between parameters and the system, making it convenient to offer guidance to optimization of design and safety analysis. Wang et al. [Wang, Zhou and Yan (2006)] built a simple curved bridge model with eccentricity of stiffness on the rigid bridge deck system on elastic bearings and provided the simplified computing methods for natural vibration characteristics and seismic responses. But the model could receive the responses of the upper structure only, and it could not receive the relative responses between piers and girders. Similar to the double-mass model for the interlayer seismic isolation of structure, Li [Li (2015)] established a double-mass six-degree-of-freedom control and analysis model for isolated curved girder bridges in consideration of seismic effects of lateral-torsion coupling resulting from the unconformity of the center of rigidity and the center of mass of the upper structure of isolated curved girder bridges, and analyzed the law of changes of vibration mode participation coefficient and the critical direction of earthquakes with the design parameters of curved girders. The results show that when the radius of curvature is small and the bridge deck width is large, the curved bridge will be greatly affected by the input angle, and thus, for such types of bridges, it is essential to determine the most unfavorable seismic responses via multiple angles of earthquake input. Based on Li's double-mass six-degree-of-freedom analysis model, taking no account of the influences of damping devices attached to isolation bearings on the damping matrix, the present paper grasped the principal contradiction and put the emphasis on major parameters reflecting the shock absorption property, and it studied the sensitivities of system pseudo-frequency and system damping ratio to the design parameter of curvature, providing a guidance for optimization of design and safety analysis of isolated curved girder bridges.

2 Model simplification of isolated curved girder bridges

It is necessary to carry out reasonable seismic design so as to guarantee the safety of bridges during and after earthquakes, but due to the effect of bending-torsion coupling of curved girder bridges, the seismic analysis of curved girder bridges becomes complex. Then, it is vitally important to grasp the principal contradiction and obtain major parameters which can reflect isolation performance. In the present paper, the seismic isolation of curved girder bridges is realized by setting isolation bearings at the top of bridge piers, and the double-mass six-degree-of-freedom model can be used to simplify analysis [Li (2015); Wang, Zhou and Yan (2006)].

2.1 Model assumptions

For the isolated curved girder bridge studied in this paper, the following assumptions shall be made during the process of simplification [Li (2015); Wang, Zhou and Yan (2006)].

1. Without regard to the in-plane and out-of-plane coupling of isolated curved girder bridges, since the rigidity of the bridge deck in the horizontal direction is large, it can be simplified as a rigid plate.

2. Only the vibration of bridges in the horizontal direction is analyzed.

3. The mass distribution of rigid bridge decks is even, and the inter-coupling between all girders is ignored.

4. The rigidity and damping parameters of isolation layers of equivalent linearization are adopted.

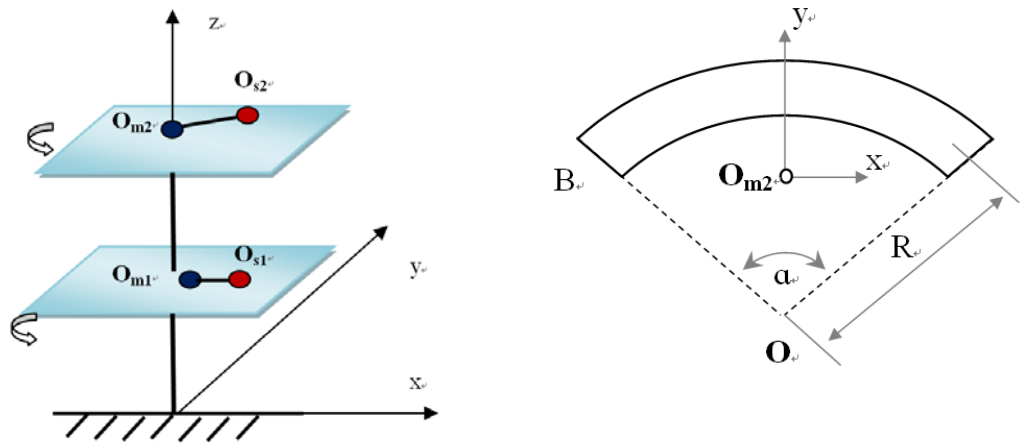
5. All bridge piers have identical mass and rigidity parameters, the vibration responses of all bridge piers are identical when under uniform excitation in the horizontal direction, and equivalent rigid plates can be used for simulation.

6. Regardless of pile-soil interaction, it is assumed that the position of the center of rigidity is unchanged, and the structure is in the small linear elastic range.

Based on the said assumptions, the isolated curved girder bridge can be simulated by two coupled rigid plates with six degrees of freedom in two horizontal directions and torsional direction.

2.2 Description of models of isolated curved girder bridge

The simplified models of isolated curved girder bridge are shown in Fig. 1. Fig. 1(a) is the simplified six-degree-of-freedom model of two rigid plates, respectively having two degrees of freedom of *x* direction and *y* direction in horizontal direction and one degree of freedom twisting around *z* axis. O_{m1} and O_{s1} are the center of mass and the center of rigidity of bridge pier; O_{m2} is the center of mass of curved girder deck (assumed as the origin of coordinates in this paper), while O_{s2} is the center of rigidity corresponding to the upper structure; where, the total mass of all bridge piers is m_1, while the total mass of bridge deck is m_2.



a Two-mass model b Bridge model

Figure 1: Simplified models of isolated curved girder (From Li (2015))

Fig. 1(b) is the geometric plane model of curved bridge. The radius of curved girder is R; the width of bridge deck is B; the corresponding central angle is α. In this paper, the center of mass of curved girder deck O_{m2} is assumed as the origin of plane coordinates for the simplified model analysis, and it can be regarded as the reference point for the coordinates of the center of mass and of the center of rigidity of bridge pier. Where, the distance r_m from the center of mass of curved girder deck to the center O is calculated according to the following formula:

$$r_m = \frac{\int \rho \cos\theta dA}{A} = \frac{\left(12R^2 + B^2\right)\sin\left(\alpha/2\right)}{6R\alpha} \tag{1}$$

The rotational inertia of main girder of curved girder deck relative to the center O is expressed as:

$$I_o = \int \rho^2 dA = \int_{R-B/2}^{R+B/2} \rho^3 \alpha d\rho = \frac{\alpha\left[\left(R+B/2\right)^4 - \left(R-B/2\right)^4\right]}{4} \tag{2}$$

The rotational inertia of main girder of curved girder deck relative to the center of mass O_{m2} is expressed as:

$$I_m = I_o - r_m^2 A \tag{3}$$

The turning radius of main girder of bridge relative to the center of mass is:

$$r_2^2 = \frac{I_m}{A} = \frac{I_o - r_m^2 A}{A} = \left(R^2 + B^2/4\right) - \frac{\left(12R^2 + B^2\right)^2 \sin^2\left(\alpha/2\right)}{36R^2\alpha^2} \tag{4}$$

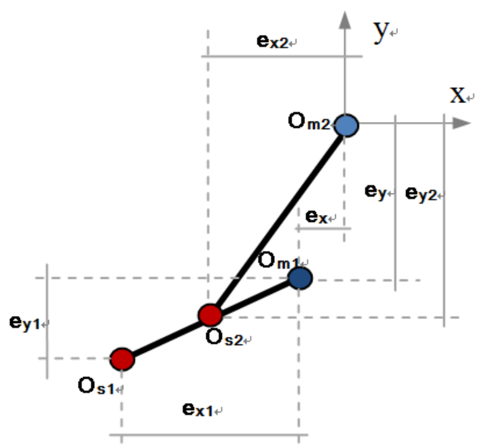


Figure 2: Eccentric distance of curved bridge

The center of mass can be directly expressed as the point of resultant force of gravity in this paper, it shall be calculated according to the following formula.

$$x_{m_1}=\frac{\sum_{i=1}^{n} m_{yi}x_{qi}}{\sum_{i=1}^{n} m_{yi}};\quad y_{m_1}=\frac{\sum_{i=1}^{n} m_{xi}y_{qi}}{\sum_{i=1}^{n} m_{xi}} \tag{5}$$

Where, x_{qi}, y_{qi} is the coordinates of the ith bridge pier in x, y directions; $m_{xi}=m_{yi}$ is the mass of the ith bridge pier in x, y directions; the total mass of bridge pier is $m_1=\sum_{i=1}^{n} m_{xi}=\sum_{i=1}^{n} m_{yi}$. According to the said assumption, the center of mass of curved girder deck is the origin of coordinates, then, $x_{m_2}=0$, $y_{m_2}=0$.

The centers of rigidity of bridge pier and curved girder deck shall be calculated according to the following formula:

$$x_{s_j}=\frac{\sum_{i=1}^{n} k_{y_{ij}}x_{qi}}{\sum_{i=1}^{n} k_{yij}};\, y_{s_j}=\frac{\sum_{i=1}^{n} k_{x_{ij}}y_{qi}}{\sum_{i=1}^{n} k_{x_{ij}}} \tag{6}$$

j=1, 2 is corresponding to curved girder deck and bridge pier; k_{xij}, k_{yij} is the rigidity of the ith bridge pier of the jth part in x and y directions. After the center of mass and the center of rigidity of curved bridge deck and bridge pier are determined, it is very convenient to work out the eccentric distances in Fig. 2. In this paper, the eccentric distance is defined as the distance from the center of mass to the center of rigidity, specifically as follows:

$$e_{x_j}=x_{m_j}-x_{s_j};\, e_{y_j}=y_{m_j}-y_{s_j} \tag{7}$$

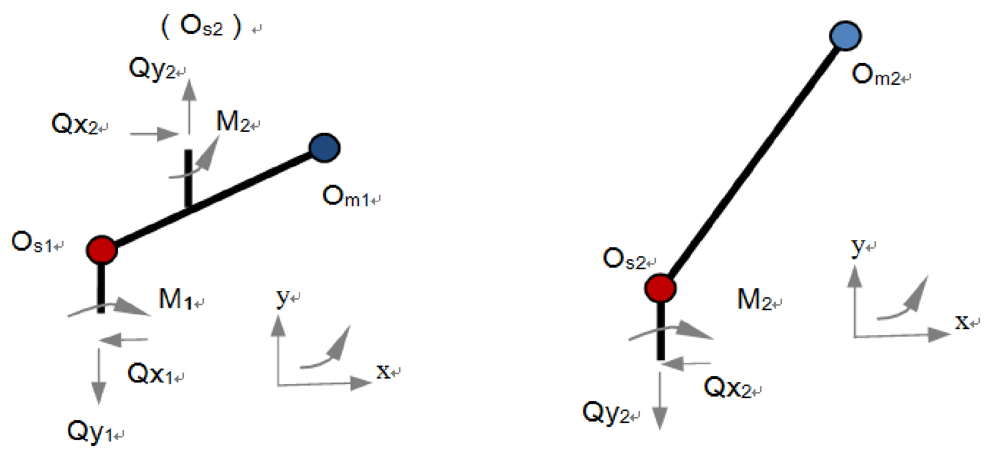
The moment of inertia of curved girder deck relative to the center of mass O_{m2} and the moment of inertia of bridge pier relative to the center of mass Om_1 are:

$$J_2 = m_2 r_2^2 \,;\, J_1 = \sum_{i=1}^{n} m_{x_i} \left(x_{qi} - x_{m1} \right)^2 + m_{y_i} \left(y_{qi} - y_{m1} \right)^2 \tag{8}$$

3 Dynamic model simplifications of curved bridge

Compared with static action, the vibration response of curved bridges to the horizontal seismic excitation may be amplified several times, so it is very important to conduct a dynamic analysis of curved bridges. In the dynamic analysis, the relationship between the key design parameters of main girders and the dynamic characteristics of bridges is the key point of study in this paper. In the following analysis, Rigid Plate 1 is used to express the bridge pier, while Rigid Plate 2 is used to express the curved bridge deck and isolation bearing.

3.1 Undamped free vibration equation



a The lower structure b The upper structure

Figure 3: Schematic diagram of forces of simplified model

It is appropriate to start the derivation of vibration equation of curved bridge from the undamped free vibration analysis. Fig. 3 gives the schematic diagram of forces of simplified model, and the coordinate direction beside is assumed as the positive direction of motion. It shall be noted that the inertia force in horizontal direction and the inertia force in the direction of rotation are not marked in the figure.

According to the force analysis of the lower structure in Fig. 3, the resultant forces (excluding the part of inertia force) in all directions are computed as follows:

x direction, y direction, Twisting around the center of mass O_{m1},

$$\begin{aligned} & f_{x_1} = Q_{x_2} - Q_{x_1} \,;\, f_{y_1} = Q_{y_2} - Q_{y_1} \,; \\ & f_{\theta_1} = R_2 \left(\theta_2 - \theta_1 \right) - R_1 \theta_1 + Q_{x_2} \left(e_{y_2} - e_y \right) + Q_{y_2} \left(e_{x_2} - e_x \right) - Q_{y_1} e_{x_1} - Q_{x_1} e_{y_1} \end{aligned} \tag{9}$$

Where, Q_{x_j} Q_{y_j} , j=1, 2, is the corresponding shear force. Where,

$Q_{x_1} = K_{x_1}\left(x_1 + e_{y_1}\theta_1\right), Q_{y_1} = K_{y_1}\left(y_1 + e_{x_1}\theta_1\right)$,

$Q_{x_2} = K_{x_2}\left\{\left(x_2 + e_{y_2}\theta_2\right) - \left[x_1 + \left(e_{y_2} - e_y\right)\theta_1\right]\right\}$

$Q_{y_2} = K_{y_2}\left\{\left(y_2 + e_{x_2}\theta_2\right) - \left[y_1 + \left(e_{x_2} - e_x\right)\theta_1\right]\right\}$.

K_{x_j} , K_{y_j} , j=1，2，represents the rigidity in X and y directions; $K_{x_j} = \sum_{i=1}^{n} k_{x_{ij}} x_i$, $K_{y_j} = \sum_{i=1}^{n} k_{y_{ij}} y_i$.

R_j , j=1, 2 is the torsional rigidity of the jth rigid plate relative to the center of rigidity, where, $R_j = \sum_{i=1}^{n} k_{x_{ij}}\left(y_{sj} - y_{mj}\right)^2 + k_{y_{ij}}\left(x_{sj} - x_{mj}\right)^2 + k_{\theta_{ij}}$, $k_{\theta_{ij}}$ when j=1, it is the torsional rigidity of the *i*th bridge pier relative to itself, and when j=2, it is the torsional rigidity of the isolation bearing of the *i*th bridge pier relative to itself.

The force analysis of curved girder deck and the resultant forces in all directions are worked out as follows:

x direction, y direction, Rotate around the center of mass O_{m2},

$$f_{x_2} = -Q_{x_2};\ f_{y_2} = -Q_{y_2};\ f_{\theta_2} = -R_2\left(\theta_2 - \theta_1\right) - Q_{x_2}e_{y_2} + Q_{y_2}e_{x_2} \tag{10}$$

The inertia forces of curved girder deck and bridge pier are:

$$\begin{cases} f_{Ix_1} = -m_{x_1}\ddot{x}_1 \\ f_{Iy_1} = -m_{y_1}\ddot{y}_1 \end{cases};\begin{cases} f_{Ix_2} = -m_{x_2}\ddot{x}_2 \\ f_{Iy_2} = -m_{y_2}\ddot{y}_2 \end{cases} \tag{11}$$

Where $m_1 = m_{x_1} = m_{y_1}$ $m_2 = m_{x_2} = m_{y_2}$, according to D'alembert's principle, it is easy to obtain the following from the formulas above:

$$\begin{cases} f_{Ix_1} + f_{x_1} = 0 \\ f_{Iy_1} + f_{y_1} = 0 \\ f_{I\theta_1} + f_{\theta_1} = 0 \end{cases};\begin{cases} f_{Ix_2} + f_{x_2} = 0 \\ f_{Iy_2} + f_{y_2} = 0 \\ f_{I\theta_2} + f_{\theta_2} = 0 \end{cases} \tag{12}$$

After putting them into the formulas, and combining them, the two-rigid-plate six-degree-of-freedom undamped vibration equation of isolated curved bridge structure is obtained:

$$\mathbf{M}\ddot{\mathbf{x}} + \mathbf{K}\mathbf{x} = 0 \tag{13}$$

Where, $\mathbf{x} = [x_1, x_2, y_1, y_2, \theta_1, \theta_2]^T$ is the corresponding vibration response; $\mathbf{M}$, $\mathbf{K}$ are the total mass matrix and the total rigidity matrix, respectively. The specific composition and calculation are as follows:

mass matrix $\mathbf{M} = \begin{bmatrix} \mathbf{m}_x & & \\ & \mathbf{m}_y & \\ & & \mathbf{J} \end{bmatrix}$, where, $\mathbf{m}_x = \mathbf{m}_y = \begin{bmatrix} m_1 & \\ & m_2 \end{bmatrix}$, $\mathbf{J} = \begin{bmatrix} J_1 & \\ & J_2 \end{bmatrix}$.

rigidity matrix $\mathbf{K} = \begin{bmatrix} \mathbf{K}_x & \mathbf{0} & \mathbf{K}_{xc} \\ \mathbf{0} & \mathbf{K}_y & \mathbf{K}_{yc} \\ \mathbf{K}_{cx} & \mathbf{K}_{cy} & \mathbf{K}_{cc} \end{bmatrix}$, where, $\mathbf{K}_x = \begin{bmatrix} K_{x_1} + K_{x_2} & -K_{x_2} \\ -K_{x_2} & K_{x_2} \end{bmatrix}$,

$\mathbf{K}_y = \begin{bmatrix} K_{y_1} + K_{y_2} & -K_{y_2} \\ -K_{y_2} & K_{y_2} \end{bmatrix}$, $\mathbf{K}_{xc} = \begin{bmatrix} K_{x_1} e_{y_1} + K_{x_2}\left(e_{y_2} - e_y\right) & -K_{x_2} e_{y_2} \\ -K_{x_2}\left(e_{y_2} - e_y\right) & K_{x_2} e_{y_2} \end{bmatrix}$, $\mathbf{K}_{cx} = \mathbf{K}_{xc}{}^T$,

$\mathbf{K}_{yc} = \begin{bmatrix} K_{y_1} e_{x_1} + K_{y_2}\left(e_{x_2} - e_x\right) & -K_{y_2} e_{x_2} \\ -K_{y_2}\left(e_{x_2} - e_x\right) & K_{y_2} e_{x_2} \end{bmatrix}$, $\mathbf{K}_{cy} = \mathbf{K}_{yc}{}^T$, $\mathbf{K}_{cc} = \begin{bmatrix} k_{c11} & k_{c12} \\ k_{c21} & k_{c22} \end{bmatrix}$,

$$\begin{cases} k_{c11} = R_1 + R_2 + K_{x_1} e_{y_1}^2 + K_{y_1} e_{x_1}^2 + K_{x_2} e_{y_2}^2 + K_{y_2} e_{x_2}^2 + K_{x_2} e_y\left(e_y - 2e_{y_2}\right) + K_{y_2} e_x\left(e_x - 2e_{x_2}\right) \\ k_{c12} = -\left[R_2 + K_{x_2} e_{y_2}^2 + K_{y_2} e_{x_2}^2 - K_{x_2} e_y e_{y_2} - K_{y_2} e_x e_{x_2}\right] \\ k_{c21} = k_{c12} \\ k_{c22} = R_2 + K_{x_2} e_{y_2}^2 + K_{y_2} e_{x_2}^2 \end{cases}.$$

3.2 The dynamic equation of damped motion

The calculation of damping can be referred to

$$\xi_{x_j} = \frac{\sum_{i=1}^{n} k_{x_{ij}} \xi_{ij}}{\sum_{i=1}^{n} k_{x_{ij}}};\ \xi_{y_j} = \frac{\sum_{i=1}^{n} k_{y_{ij}} \xi_{ij}}{\sum_{i=1}^{n} k_{y_{ij}}};$$
$$\xi_{\theta_j} = \frac{\sum_{i=1}^{n} \left[k_{x_{ij}}\left(y_{sj} - y_{mj}\right)^2 + k_{y_{ij}}\left(x_{sj} - x_{mj}\right)^2 + k_{\theta_{ij}}\right]\xi_{ij}}{\left[k_{x_{ij}}\left(y_{sj} - y_{mj}\right)^2 + k_{y_{ij}}\left(x_{sj} - x_{mj}\right)^2 + k_{\theta_{ij}}\right]} \tag{14}$$

When j=1, ξ_{i1} represents the damping ratio of the *i*th bridge pier, while j=2, ξ_{i2} represents the damping ratio of the *i*th isolation bearing; ξ_{x_j}, ξ_{y_j}, $\xi_{\theta 1}$ are the total damping ratio in x and y directions and the direction of rotation.

The damping ratios of the *j*th rigid plate in x and y directions and the direction of rotation are calculated according to the following formula:

$$C_{x_j} = 2m_{x_1}\omega_{x_1}\xi_{x_j};\ C_{y_j} = 2m_{y_1}\omega_{y_1}\xi_{y_j};\ C_{\theta_j} = 2m_{\theta_1}\omega_{\theta_1}\xi_{\theta_j} \tag{15}$$

Where, $\omega_{x_1} = \sqrt{\dfrac{m_{x_1}}{K_{x_1}}}$, $\omega_{y_1} = \sqrt{\dfrac{m_{y_1}}{K_{y_1}}}$, $\omega_{\theta_1} = \sqrt{\dfrac{m_{\theta_1}}{K_{\theta_1}}}$.

Then, the specific form of C of damping matrix can be referred to the construction mode of rigidity matrix.

$$\mathbf{M}\ddot{\mathbf{x}} + \mathbf{C}\dot{\mathbf{x}} + \mathbf{K}\mathbf{x} = -\mathbf{M}\mathbf{I}\ddot{\mathbf{x}}_g \tag{16}$$

See the undamped vibration equation in Section 3.1 for the specific meaning of the said formula. Where, $\mathbf{I}$ is the indicating matrix of action, $\mathbf{I} = [\mathbf{I}_x, \mathbf{I}_y, \mathbf{I}_\theta]$, $\mathbf{I}_x = \mathbf{I}_y = \mathbf{I}_\theta = [1_{2\times1}, 0_{2\times1}, 0_{2\times1}]^T$, acceleration $\ddot{\mathbf{x}}_g = [\ddot{x}_{xg}, \ddot{x}_{yg}, 0]^T$, since the influence of rotational component on the structure is not studied in this paper, it is set as 0. In the case of unidirectional acceleration excitation, in consideration that the included angle between the unidirectional excitation and x axis of the reference coordinate is ϕ, then, the indicating matrix of action $\mathbf{I} = [\mathbf{I}_x \cos\phi, \mathbf{I}_y \sin\phi, \mathbf{I}_\theta]$; in the view of bidirectional excitation, $\ddot{x}_{xg}$ and $\ddot{x}_{yg}$ are mutually perpendicular and the included angle between $\ddot{x}_{xg}$ and x axis of the reference coordinate is ϕ, then, the indicating matrix $\mathbf{I} = [\mathbf{I}_x(\cos\phi - \sin\phi), \mathbf{I}_y(\cos\phi + \sin\phi), \mathbf{I}_\theta]$.

4 Analysis of influence factors of vibration characteristics of isolated curved girder bridge

The influence factors of vibration characteristics analyzed in this paper mainly include bridge deck width, radius of curved girders and the central angle. Other factors will not be discussed here. The vibration characteristics are mainly pertinent to the pseudo-undamped natural circular frequency (pseudo-frequency for short) and system damping ratio, the structural rigidity of the pseudo-frequency response system, and the two-characteristic corresponding to the frequency and damping conditions of curved girders after seismic isolation.

A new state variable is defined

$$\mathbf{Y} = \begin{Bmatrix} \mathbf{x} \\ \dot{\mathbf{x}} \end{Bmatrix} \tag{17}$$

A new equation is obtained

$$\dot{\mathbf{Y}} = \mathbf{A}\mathbf{Y} + \mathbf{B}\ddot{\mathbf{x}}_g \tag{18}$$

Where, $\mathbf{A} = \begin{bmatrix} \mathbf{0} & \mathbf{I}_1 \\ \text{-}\mathbf{M}^{-1}\mathbf{K} & \text{-}\mathbf{M}^{-1}\mathbf{C} \end{bmatrix}$, $\mathbf{B} = \begin{Bmatrix} \mathbf{0} \\ -\mathbf{I}_2 \end{Bmatrix}$, $\mathbf{Y}_z = \mathbf{C}\mathbf{Y} + \mathbf{D}\ddot{\mathbf{x}}_g$. $\mathbf{I_1}$ and $\mathbf{I_2}$ are the unit matrix and the unit vector; $\mathbf{Y}_z$ is the output response vector; matrixes $\mathbf{C}$ and $\mathbf{D}$ can be determined as required.

In order to obtain the vibration characteristics of isolated curved bridge, the characteristic equation of Eq. (18) can be expressed as

$$\left|\lambda_0 \mathbf{I}_0 - \mathbf{A}\right| = 0 \tag{19}$$

Where, I_0 is the unit matrix; λ_0 is the complex Eigenvalue, and it can be obtained according to the following formula $\lambda_0 = -\xi\omega_0 \pm i\sqrt{1-\xi^2}\omega_0$, where, ω_0 is the pseudo-frequency); ξ is the system damping ratio; they can be worked out according to the following formulas, respectively [Tan, Fang and Zhou (2014)].

$$\omega_0 = \left|\lambda_0\right| \tag{20}$$

$$\xi = -\frac{\mathrm{Re}(\lambda_0)}{\left|\lambda_0\right|} \tag{21}$$

The analysis on vibration characteristics of isolated curved girder bridge can be carried out with ω_0 and ξ .

Analytical example: A three-span curved continuous girder bridge is taken, as shown in Fig. 4, with the radius of curvature *R*=50 m. Span of 3×20 m; the central angle is 69^o; the bridge deck width is 8.5m; the bridge piers are of cylinder type, with the diameter of 1.5m and the pier height of 5 m; the lower structural damping ratio of bridge pier is 0.05; each bridge pier is provided with two lead -core rubber isolation bearings at its top, and the horizontal ratio of isolation layer is 0.15; the isolation bearings with the diameter of 500 mm have the same equivalent rigidity in all directions of cylinder.

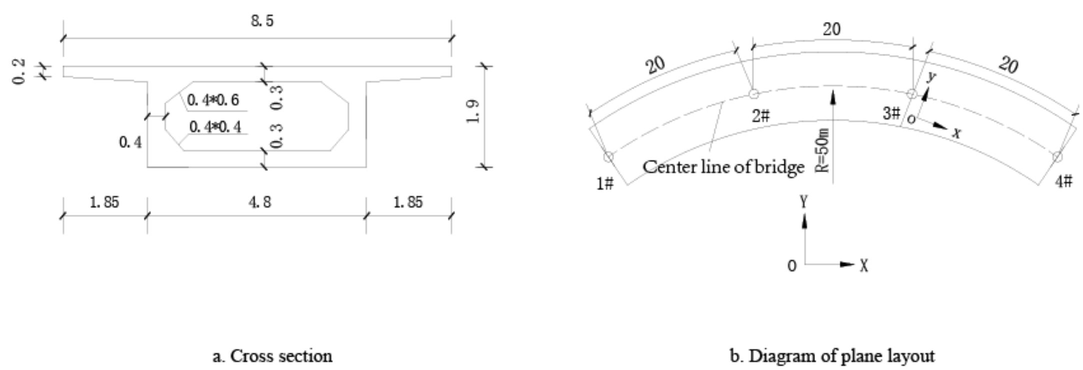


Figure 4: Geometric figure of curved bridge deck

4.1 Relationship between bridge deck width and vibration characteristics influence of bridge deck width on vibration characteristics

Provided that other parameters remain unchanged, the bridge deck width is changed from 5 m to 28 m. e_y/R is defined as the eccentric degree of curved girder bridge, and e_y is the eccentric distance along y axis. The laws of changes of eccentric degree and vibration characteristics with bridge deck width are analyzed.

4.1.1 Influence of the bridge deck width on the eccentric degree

Fig. 5 shows the law of change of eccentric degree with bridge deck width along y axis. It can be seen that with the increase of bridge deck width, the eccentric degree gradually increases, and the property of lateral-torsion coupling gradually increases. The reason is that as the bridge deck width increases, the distance from the center to the center line of bridge increases, and consequently, the eccentric degree increases, and the characteristic of curved bridge becomes more obvious.

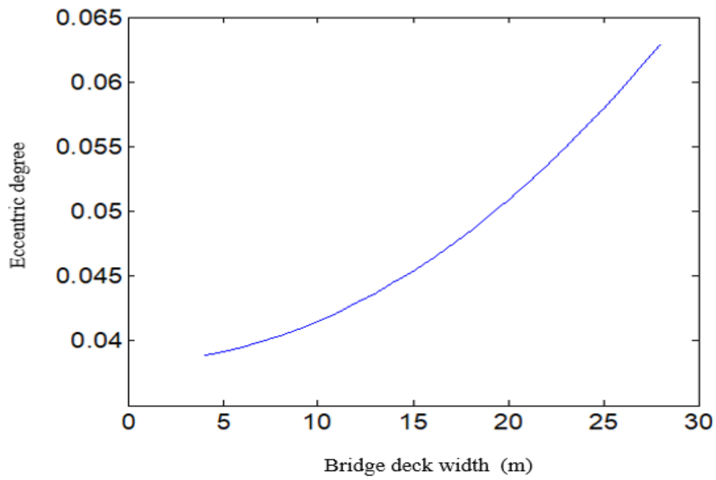
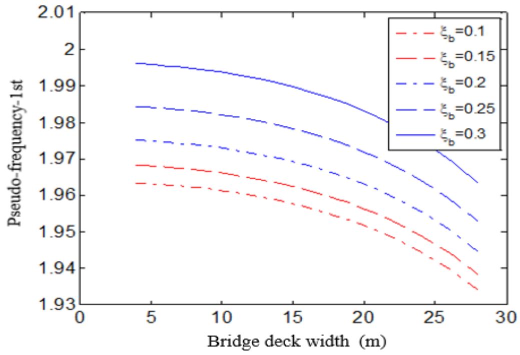


Figure 5: Change of eccentric degree with the bridge deck width

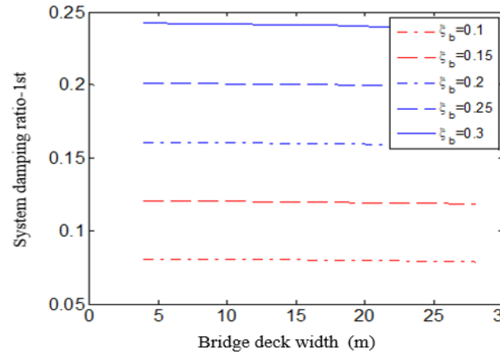
4.1.2 Influences of the bridge deck width on the pseudo-frequency and the system damping ratio

Fig. 6 is the curve of the relationship between the vibration characteristics and the bridge deck width. Fig. 6(a) reveals that the first-order pseudo-frequency of curved girder bridges decreases by a larger and larger margin as the bridge deck width increases. Fig. 6(c) indicates that as the bridge deck width changes, there is no big change in the second-order pseudo-frequency of curved girder bridges, and the smaller the isolation bearing damping ratio is, the smaller the second-order pseudo-frequency of curved girder bridges is, reducing by a gradually smaller margin. Fig. 6(e) reveals that the third-order pseudo-frequency of curved girder bridges decreases as the bridge deck width increases, showing a nearly linear decline. Figs. 6(a), 6(c) and 6(e) reveal that the influence of bridge deck width on the third-order pseudo-frequency of curved girder bridges occupies the first place; that influence on the first-order pseudo-frequency comes second; that influence on the second-order pseudo-frequency is low. Figs. 6(b), 6(d) and 6(f) shows that the bridge deck width has little influence on the first-order to third-order system damping ratios.

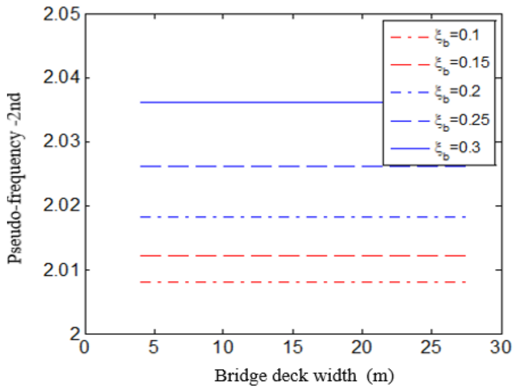
Fig. 6 indicates that the first-order to third-order pseudo-frequencies and system damping ratios of curved girder bridge increase as the damping ratio of isolation layer increases, and the system damping ratio is less than the damping ratio of isolation bearing, while the rate of increase of pseudo-frequency gradually increases as the damping ratio of isolation layer increases.



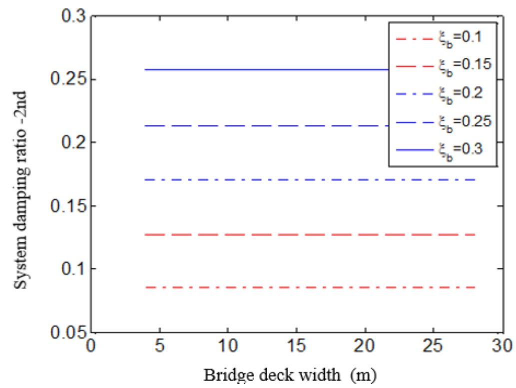
a 1st pseudo-frequency



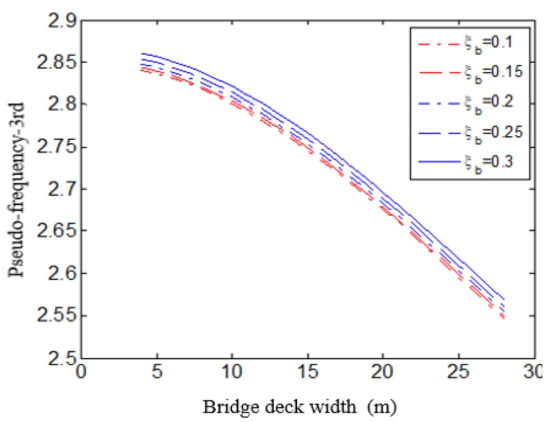
b 1st system damping ratio



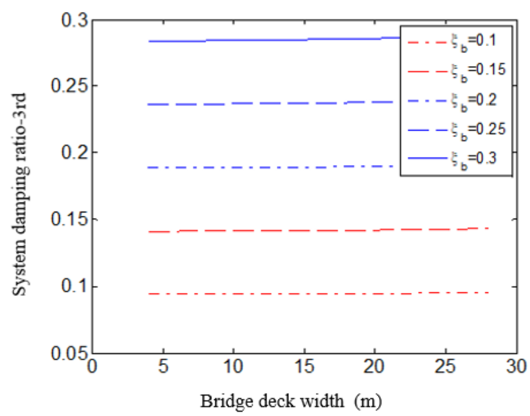
c 2nd pseudo-frequency



d 2nd system damping ratio



e 3rd pseudo-frequency



f 3rd system damping ratio

Figure 6: Relationship between the vibration characteristics and the bridge deck width

4.2 Relationship between the radius of curvature and the vibration characteristics

Provided that other parameters remain unchanged, the radius of curvature R is continuously changed from 20 m to 300 m, when the corresponding central angle is changed from 172° to 12°, respectively. The laws of changes of the eccentric degree and vibration characteristics of curved girder bridge with the radius of curvature are respectively analyzed.

4.2.1 Influences of the radius of curvature on the eccentric degree

Fig. 7 shows the law of change in the eccentric degree of y axis with the radius of curvature. It shows that as the radius of curvature increases, the eccentric degree gradually decreases, and the performance of lateral-torsion coupling gradually declines; when the radius of curvature is larger than about 20 times of the bridge deck width, the eccentric degree is very small, and it can be considered as a straight bridge, which is in line with the provisions of the *Guidelines for Seismic Design of Highway Bridges* (JTG/T B02-01-2008).

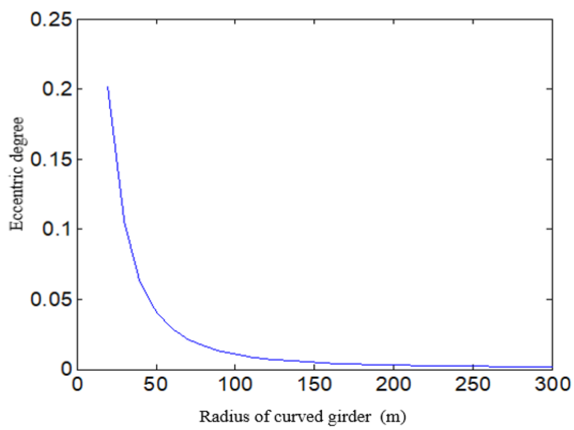


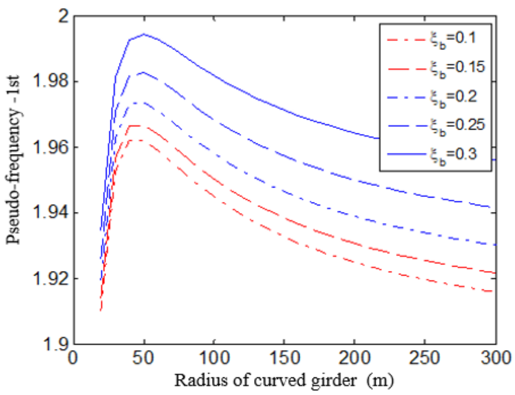
Figure 7: Change of the eccentric degree with the radius of curvature

4.2.2 Influences of the radius of curvature on the pseudo-frequency and the system damping ratio

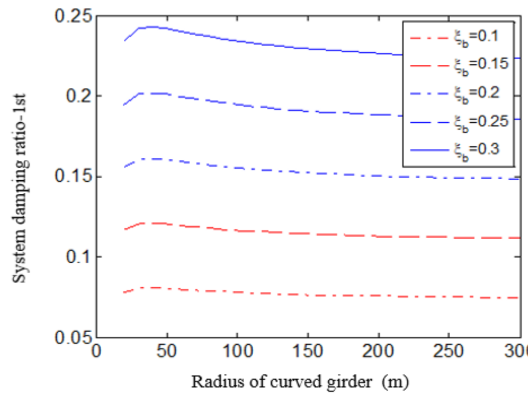
Fig. 8 is the curve of the relationship between the vibration characteristics of curved girder bridge and the radius of the curvature. It can be seen from Fig. 8 that when the radius of the curvature changes, the law of changes of the pseudo-frequency ω_0 and the system damping ratio ξ with the isolation bearing damping ratio ξ_b are similar.

Figs. 8(a), 8(c) and 8(e) show that when the radius of curvature is within the range of 20 m to 50 m (when the radius of curvature is approximately smaller than 6 times of the range), as the radius of curvature increases, the first-order pseudo-frequency, second-order pseudo-frequency and the third-order pseudo-frequency of curved girder bridges increase rapidly; when the radius of curvature is larger than 50 m, as the radius of curvature increases, the first-order and the second-order pseudo-frequencies gradually decline, and the decline rate of the first-order pseudo-frequency is larger than that of the second-order pseudo-frequency, while the third-order pseudo-frequency maintains a stable value. They indicate that when the radius of curvature is 6 times of the bridge deck width, the pseudo-frequency reaches the maximum value; when it is larger than 6 times of the bridge deck width, the influence of the radius of curvature on the first-order pseudo-frequency of curved girder bridge occupies the first place; that influence on the second-order pseudo-frequency comes second; that influence on the third-order is the minimum.

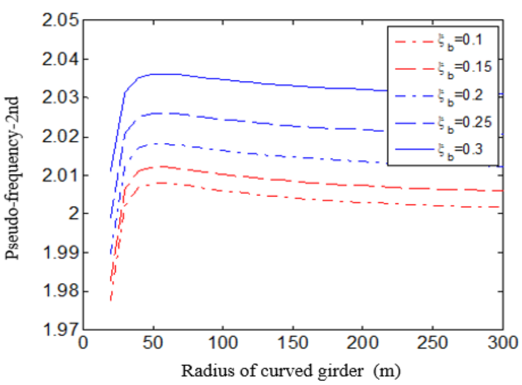
Figs. 8(b), 8(d) and 8(f) show that except that the first-order system damping ratio, the second-order system damping ratio and the third-order system damping ratio of curved girder bridge slightly change when the radius of curvature is 20 m at the very beginning, the radius of curvature produces few influences on the system damping ratio.



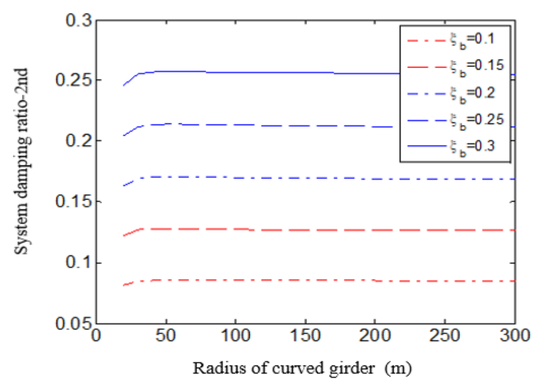
a 1st pseudo-frequency



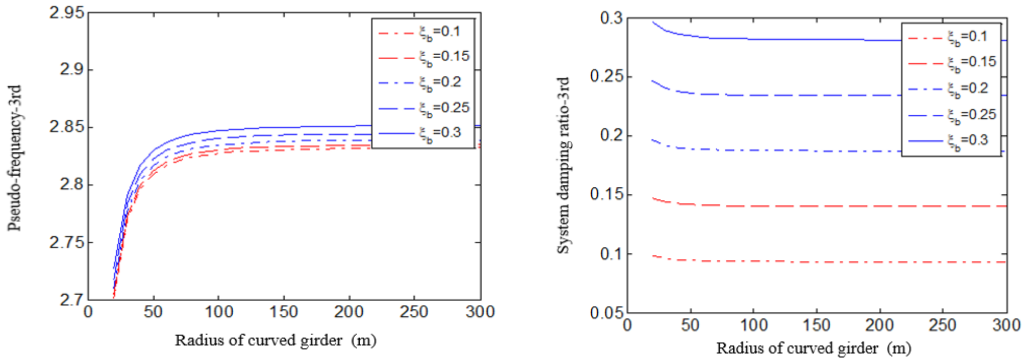
b 1st system damping ratio



c 2nd pseudo-frequency



d 2nd system damping ratio



e 3rd pseudo-frequency f 3rd system damping ratio

Figure 8: Relationship between the vibration characteristics and the radius of curvature

4.3 Relationship between the central angle and the vibration characteristics

It can be learned from the analysis above that the radius of curvature of curved girder bridge is a very important factor affecting the vibration characteristics of curved bridge, and under the circumstance that the bridge length is not changed, the change in the radius of curvature is equivalent to the change in the central angle. The calculation results from Section 4.2 are taken as the horizontal ordinates of the central angle, and the curve of law of change of eccentric degree and vibration characteristics with the central angle is drawn.

4.3.1 Influences of the central angle on the eccentric degree

Fig. 9 shows the law of change of y-axis eccentric degree of curved girder bridge with the central angle. It can be seen that with the increase of central angle, the eccentric degree of curved girder bridge gradually increases, showing a tendency approximating to a linearly progressive increase, and the lateral-torsion coupling property gradually increases, and the characteristic of curved bridge becomes more significant; when the central angle is very small, the bridge is close to a straight girder bridge, when the eccentric degree is also extremely small.

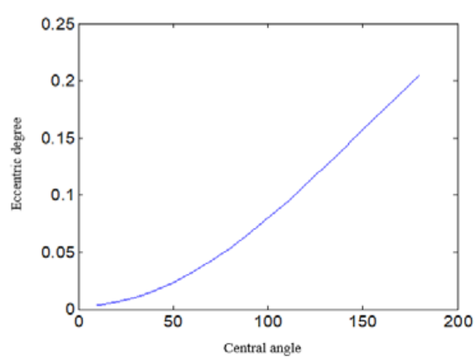
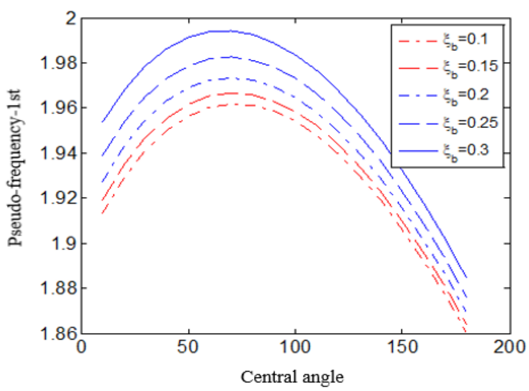


Figure 9: Change of the eccentric degree with the central angle

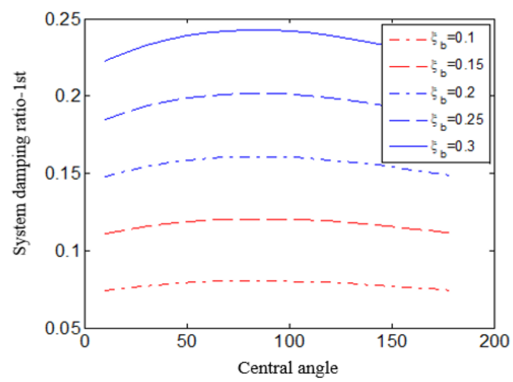
4.3.2 Influences of the central angle on the pseudo-frequency and the system damping ratio

Fig. 10 is the curve of relationship between the vibration characteristics of curved girder bridge and the central angle. Fig. 10(a) and 10(c) show that as the central angle increases, the first-order pseudo-frequency and the second-order pseudo-frequency of curved girder bridge increase first and then decrease, and reach the maximum value when the central angle is 70^o; the first-order pseudo-frequency changes by a larger margin than the second-order pseudo-frequency. Fig. 10(e) shows that as the central angle increases, the third-order pseudo-frequency first sharply declines and then slowly declines.

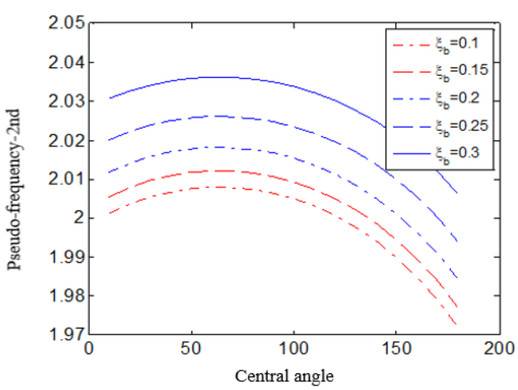
Figs. 10(b), 10(d) and 10(f) show that the first-order system damping ratio and the third-order system damping ratio of curved girder bridge change slightly as the central angle changes; the first-order system damping ratio first increases and then decreases; the third-order system damping ratio decreases as the central angle increases; the second-order system damping ratio changes a little with the central angle.



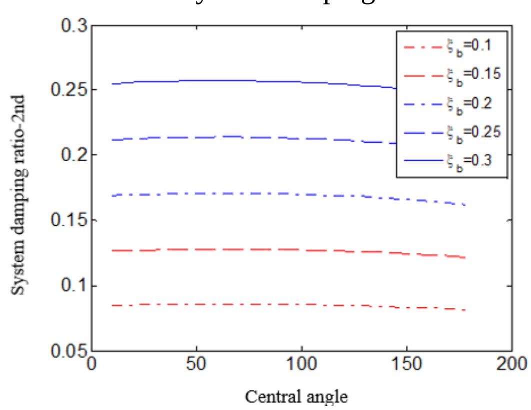
a 1st pseudo-frequency



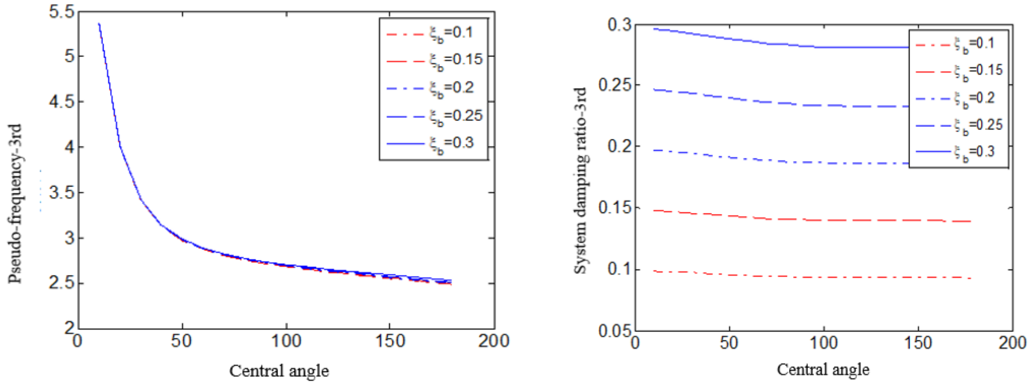
b 1st system damping ratio



c 2nd pseudo-frequency



d 2nd system damping ratio



e 3rd pseudo-frequency f 3rd system damping ratio

Figure 10: Relationship between the vibration characteristics and the radius of curvature

5 Sensitivity analysis of vibration characteristics of isolated curved girder bridge

The sensitivity of system represents the degree of change in a specific quantity of system when a system parameter changes, i.e. level of sensitivity. The sensitivity study of system parameter is conductive to distinguishing major parameters affecting the system, consequently offering more favorable information to the seismic design for the structural system. This paper mainly studies the sensitivity of vibration characteristics (pseudo-frequency and system damping ratio) of curved girder bridge, when a parameter "$*$" of isolated curved girder bridge changes, specifically as follows:

The sensitivity of pseudo-frequency of curved girder bridge

$$\Delta\omega_0 = \frac{\partial\omega_0}{\partial *}\Delta * \tag{22}$$

The sensitivity of system damping ratio of curved girder bridge

$$\Delta\xi = \frac{\partial\xi}{\partial *}\Delta * \tag{23}$$

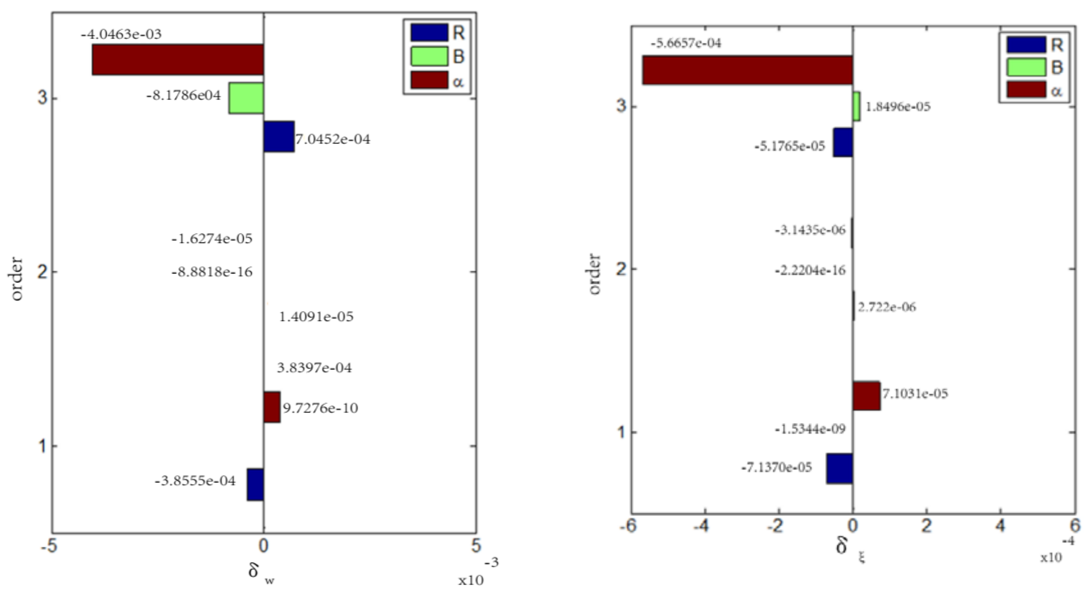
Since the computation process of pseudo-frequency and system damping ratio of curved girder bridge is complicated, when the sensitivity is determined, it can be assumed that the research object (a parameter) has a minor incremental change (e.g. with the change of 1%), which is expressed by the numerical difference between the pseudo-frequency and the system damping ratio of the curved girder bridge after and before the change. If the numerical difference is positive, it means that the sensitivity caused is positive; if the numerical difference is negative, it means that the sensitivity caused is negative.

5.1 Parameter sensitivity of curved girder bridge

Fig. 11 is the diagram of levels of sensitivities of vibration characteristics of isolated curved girder bridge to the radius of curvature R, the bridge deck width B and the central angle α of curved girder bridge.

Fig. 11 shows that the levels of sensitivities of the first-order pseudo-frequency and the first-order system damping ratio of curved girder bridge to the radius of curved girder and the central angle are close, but the sensitivity to the central angle is positive, while the sensitivity to the radius of curvature is negative; the sensitivity to the bridge deck width is very small, differing by five orders of magnitude with the sensitivity to the central angle and that to the radius of curvature, which can be ignored. The sensitivities of the second-order pseudo-frequency and the second-order system damping of curved girder bridge to the radius of curved girder, the central angle and the bridge deck width are very small, and they can be ignored. The third-order pseudo-frequency and the third-order system damping ratio of curved girder bridge have a higher sensitivity to the design parameters of curved girder, among which the sensitivity to the central angle is obviously higher than the sensitivity to the radius of curvature and the bridge deck width, by one order of magnitude.

Generally, it indicates that the third-order vibration characteristic of curved girder bridge is more sensitive to the design parameters, among which the sensitivity to the central angle is stronger than that to the radius of curvature and the bridge deck width; the first-order vibration characteristic of curved girder bridge has almost the same sensitivity to the central angle and the radius of curvature, having no sensitivity to the bridge deck width; the second-order vibration characteristic of curved girder bridge is not sensitive to the three design parameters.

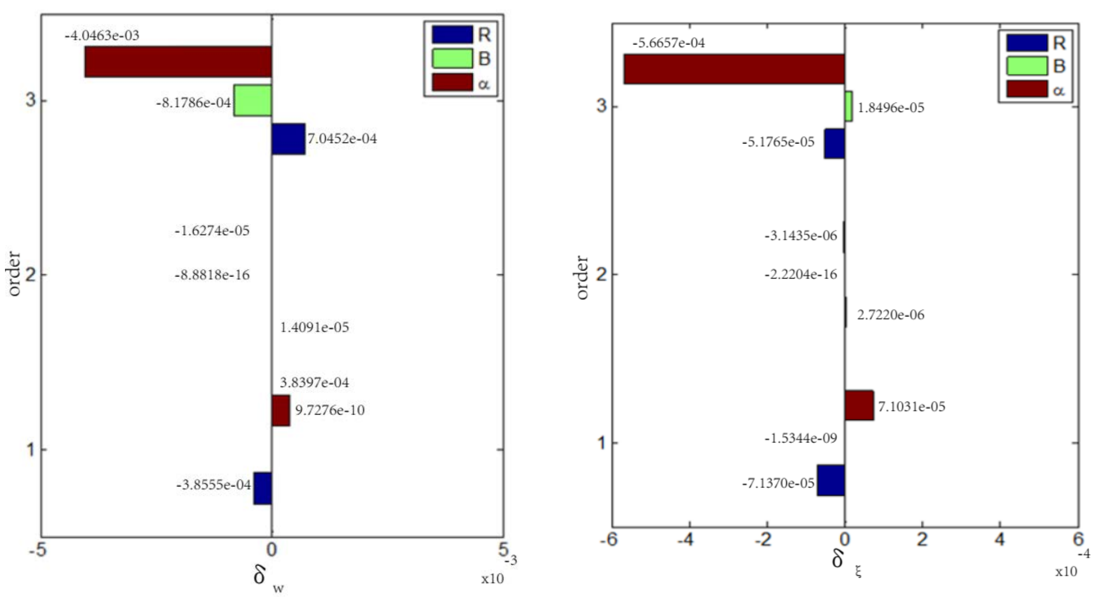


a Sensitivity of pseudo-frequency   b Sensitivity of system damping ratio

Figure 11: Sensitivity of the vibration characteristic to all parameters of curved girder

5.2 Sensitivity to the damping of isolation layer

Fig. 12 is the diagram of sensitivity of vibration characteristics of isolated curved girder bridge to the damping ratios of isolation layer in all directions. It can be seen from Fig. 12 that the first-order pseudo-frequency and the first-order system damping ratio of curved girder bridge are sensitive to the x-direction damping ratio ξ_x of isolation layer, and the sensitivity is positive; the sensitivity to the y-direction damping ratio ξ_y and torsional damping ratio ξ_c of isolation layer is tiny, and it can be ignored. Similarly, the second-order pseudo-frequency and the second-order system damping ratio of curved girder bridge are sensitive to the y-direction damping ratio ξ_y of isolation layer only, while the third-order pseudo-frequency and the third-order system damping ratio of curved girder bridge are sensitive to the torsional damping ratio ξ_c of isolation layer only, and the sensitivity is positive, but the sensitivities to the damping ratios in other two directions are tiny, which can be ignored.



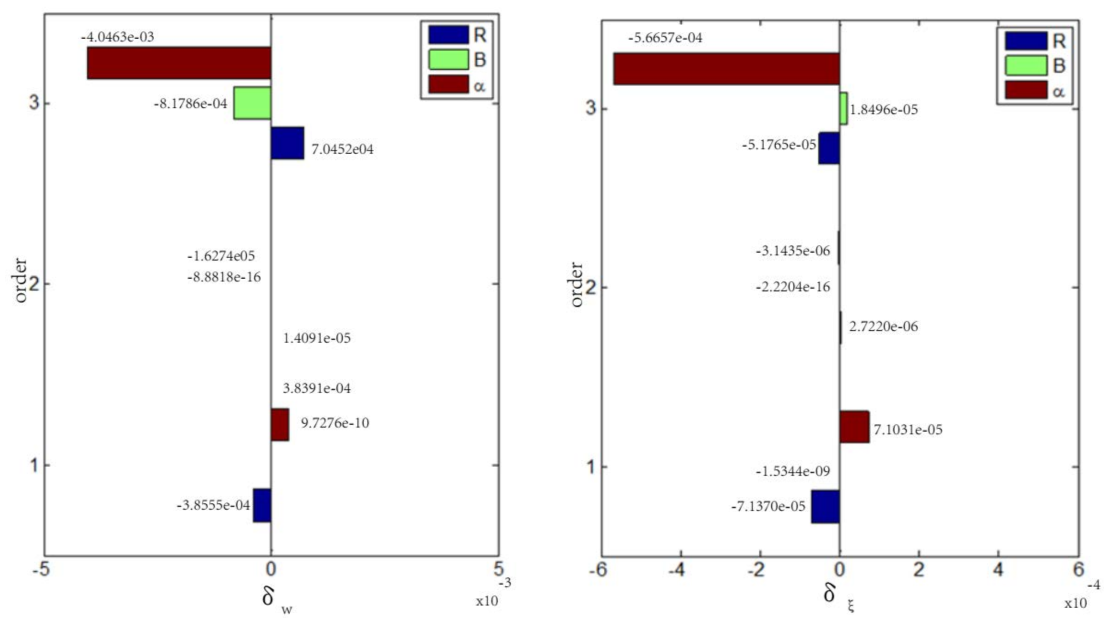
a Sensitivity of pseudo-frequency b Sensitivity of system damping ratio

Figure 12: Sensitivity of vibration characteristics to x-direction, y-direction and torsional damping ratios of isolation layer

5.3 Sensitivity to the frequency of isolation layer

Fig. 13 is the diagram of levels of sensitivities of vibration characteristic of isolated curved girder bridge to the frequencies of isolation layer in all directions. It can be seen from Fig. 13 that the first-order pseudo-frequency and the first-order system damping ratio curved girder bridge are sensitive to the x-direction circular frequency ω_x of isolation layer, and the sensitivity is positive, while the sensitivities to y-direction frequency ω_y and torsional frequency ω_c of isolation layer are tiny, which can be ignored. Similarly, the second-order pseudo-frequency and the second-order system damping ratio of curved girder bridge are sensitive to the y-direction frequency ω_y of isolation layer only; the third-order pseudo-frequency and the third-order system damping ratio of curved girder bridge are sensitive to the torsional frequency ω_c of isolation layer only, and the sensitivity is positive, while the sensitivities to the frequencies in other two directions are tiny, which can be ignored.

Fig. 12 and Fig. 13 show that the first-order vibration mode of curved girder bridge gives priority to x-direction motion, while the second-order vibration mode gives priority to y-direction motion, and the third-order vibration mode gives priority to torsional motion. The results are exactly identical to the numerical simulation results.



a Sensitivity of pseudo-frequency b Sensitivity of system damping ratio

Figure 13: Sensitivities of vibration characteristic to x-direction, y-direction and torsional frequencies of isolation layer

6 Conclusion

In this paper, the factors affecting the vibration characteristic of isolated curved girder bridges are analyzed by the simplified theoretical and computational method, and the sensitivities of all affecting factors are studied. The results show that:

1. The simplified model and calculation method can reflect the vibration characteristics of isolated curved girder bridges very well and be conductive to proposing regular design

methods, guaranteeing the safety of structure and reaching the objective of seismic isolation.

2. Under the condition that other design parameters of curved girders are not changed, the eccentric degree of curved girders along y axis gradually declines with the increase of the radius of curvature, while gradually increasing with the increase of the central angle, and gradually increases with the increase of the bridge deck width.

3. The first-order to the third-order pseudo-frequency and system damping ratio of curved girder bridges increases with the increasing of the damping ratio of the isolation layer, while the system damping ratio is less than the damping ratio of the isolation bearing.

4. The first-order vibration mode of curved girder bridges gives priority to x-direction motion; and the second-order vibration mode gives priority to y-direction motion; and the third-order vibration mode gives priority to torsional motion.

5. In this example, the third-order vibration characteristic of curved girder bridges is more sensitive to the design parameters of curved girders. Its sensitivity to the central angle is higher than its sensitivity to the radius of curvature and the bridge deck width. The first-order vibration characteristics have almost the same sensitivity to the central angle and the radius of curvature, while has no sensitivity to the bridge deck width; furthermore, the second-order vibration characteristic is not very sensitive to the three design parameters.

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