Time Series Analysis for Vibration-Based Structural Health Monitoring: A Review

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Abstract: Structural health monitoring (SHM) is a vast, interdisciplinary research field whose literature spans several decades with focusing on condition assessment of different types of structures including aerospace, mechanical and civil structures. The need for quantitative global damage detection methods that can be applied to complex structures has led to vibration-based inspection. Statistical time series methods for SHM form an important and rapidly evolving category within the broader vibration-based methods. In the literature on the structural damage detection, many time series-based methods have been proposed. When a considered time series model approximates the vibration response of a structure and model coefficients or residual error are obtained, any deviations in these coefficients or residual error can be inferred as an indication of a change or damage in the structure. Depending on the technique employed, various damage sensitive features have been proposed to capture the deviations. This paper reviews the application of time series analysis for SHM. The different types of time series analysis are described, and the basic principles are explained in detail. Then, the literature is reviewed based on how a damage sensitive feature is formed. In addition, some investigations that have attempted to modify and/or combine time series analysis with other approaches for better damage identification are presented.

1 Introduction

The science of continuous and online condition monitoring of structures using sensory systems and analysis of measured signals to assess structural operating conditions is known as Structural Health Monitoring (SHM) [Koh, Tee and Quek (2006); Tee (2004); Tee, Koh and Quek (2009)]. In fact, Nondestructive Testing (NDT) is covered by condition monitoring, and condition monitoring is covered by SHM. With advancement in sensor and computer technology as well as the aging of many critical structures, renewed efforts can be seen in the research area of SHM. The need for quantitative global damage detection methods that can be applied to complex structures has led to research into SHM methods that examine changes in the vibration characteristics of the structure [Klepka, Staszewski, Uhl et al. (2012); Koh, Quek and Tee (2002); Tee, Koh and Quek (2003)]. Vibration-based inspection is currently an active area of research in SHM, based

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on examining changes in the characteristics of a structure before and after damage occurrence based on analysis of input and output signals due to dynamic excitation.

The approaches in vibration-based inspection can be roughly classified under time domain, frequency domain and time-frequency domain. One main approach in time domain health monitoring and damage detection methodology is through the use of time series analysis. In the literature on the structural damage detection, many time series-based methods have been proposed by Sohn et al. [Sohn, Worden and Farrar (2002); Gul and Catbas (2011); Tee, Cai and Chen (2013)]. Statistical time series methods for SHM form an important, rapidly evolving, category within the broader vibration-based methods since their implementation for an automated SHM system is relatively more feasible compared with other methodologies such as damage detection based on model updating [Mottershed and Friswell (1993)] which rely on cumbersome finite element modelling processes and/or linear modal properties for damage diagnosis [Chen, Tee and Ni (2012); Wang, Zhang and Tee (2011)]. For practical applications, these methods have been shown to be ineffective because of labour intensive tuning, excessive computational effort and significant uncertainties caused by user interaction and modelling errors [Jenal, Staszewski, Scarpa et al. (2009); Klepka, Staszewski, DiMaio et al. (2013)].

Time series analysis approach for performing SHM is to create a time series predictive model to the vibration data known to be acquired from the structure in its undamaged state. These models are then used to predict subsequent measured data and the residuals (the difference between the model's prediction and the observed value). These methodologies usually make use of AutoRegressive (AR), AR with eXogenous inputs (ARX), AutoRegressive Moving Average (ARMA) models, etc to detect the damage in a statistical manner [Sohn and Farrar (2001); Nair, Kiremidjian, Lei et al. (2003)]. Due to random and systematic variability in measured dynamic response data, statistical approaches are necessary to ensure that changes in a structure's measured dynamic response are a result of damage and not caused by environmental and operational variability. Some of these methodologies directly compare the time series models whereas some of them use the residual errors when the new data are used with the previously created model. These are known as damage sensitive features that are used to check for anomalies.

In this paper, time series analysis for SHM is reviewed. The rest of the paper is organised as follows. The procedure of pre-processing of the data before creating time series models is first discussed. Then, the different types of statistical time series methods for SHM are introduced. Damage sensitive features that discriminate between damaged and non-damaged states of a structure are also reviewed. Investigations that have attempted to modify and/or combine time series analysis with other approaches for better damage identification are also presented.

2 Pre-processing

Before fitting time series models to sensor data, it is important to pre-process the data in order to compare acceleration time histories (at a sensor location) that may have occurred due to different loading conditions (i.e. different magnitudes and directions of loads) and/or different environmental conditions. After pre-processing the features extracted

from the signals from undamaged cases would have similar statistical characteristics and can be compared. The first step is referred to as normalization procedure by Nair et al. [Nair, Kiremidjian, Lei et al. (2003)]. All the time signals are normalized as follows.

$$x_n(t) = \frac{x(t)}{\sqrt{\sum_{j=1}^{n}[x(t)]^2}} \tag{1}$$

where x_n is the normalized signal and n is the number of data points of the signal. Then signal is standardized prior to fitting an AR model such that

$$\hat{x} = \frac{x_n - \mu_x}{\sigma_x} \tag{2}$$

where $\hat{x}$ is the normalized signal, μ_x and β_x are the mean and standard deviation of x_n, respectively. All signals are normalized so that they have zero mean and unit variance. This standardization-normalization procedure should be applied to all signals employed. It is worth noting that the time series data related to some types of damage can contain sharp changes that could be misinterpreted in this standardization and cause incorrect classification. However, this standardization permits the analysis of signals from a structure in different environmental conditions.

The next step is to check for trends and stationarity in the data [Brockwell and Davis [2002)]. This can be done by observing the autocorrelation function. For detrending the data, a combination of the simple average window and the moving average window can be used. The window sizes are chosen so that the residuals obtained from this process are stationary. A review of the autocorrelation plot or the Ljung-Box statistic provides further test that this condition is preserved. An alternative approach to remove trends using weighted FIR filters is presented in Ganguli [Ganguli (2002)].

Furthermore, the data can also be reduced using Principle Component Analysis (PCA) before it is classified, so that the classification procedure is speeded up considerably [Silva, Junior, Junior et al. (2008)]. PCA is a linear transformation mapping multidimensional data into lower dimensions with minimum loss of information. The procedure compresses the data from multiple measurements by determining the principal components of the dataset using PCA. The m time series vectors $x_t(n)$ are transformed to d vectors $x_d(n)$, where $d \ll m$ by projecting onto the eigenvectors corresponding to the d largest eigenvalues as follows.

$$x_d(n) = T\,x_t(n) = [v_1, \cdots, v_d]^T\,x_t(n) \tag{3}$$

where v_i represents the ith eigenvectors which are the principal components. The loss of information in this mapping can be assessed by remapping the projected data back to the original space.

$$\hat{x}_t(n) = T^T x_d(n) \tag{4}$$

Then, the reconstruction error (residual error) matrix E is defined as

$$E = x_t(n) - \hat{x}_t(n) \tag{5}$$

The matrix T can be found such that the Euclidean norm of the residual matrix, $\|E\|$, is minimized for the given size of d. It is also possible to use nonlinear principal component analysis (NLPCA) which generalizes the linear mapping by allowing arbitrary nonlinear functionalities. NLPCA employs artificial neural networks to generate these arbitrary nonlinear functions.

Random decrement (RD) method is also implemented to eliminate the effects of the exogenous input by normalizing the ambient vibration data before constructing the AR models [Gul and Catbas (2009)]. RD functions are used to obtain pseudo-free vibration response from the ambient vibration data. Being developed by [Cole (1968)], random decrement is used for transformation of random time series into a free decay response, which in essence contains only the information about the structural dynamics properties. RD is used to filter out the effect of the random loading from the signal. The random response of a system at a particular moment contains three components, i.e. the step response due to the initial displacements, the impulse response from initial velocity, and a random part due to the load on the system. Triggering conditions are defined to determine the data windows to be used in the averaging process. If the data are averaged every time the response has an initial displacement bigger than a pre-set trigger level, the random part due to random load will eventually vanish and become negligible. Additionally, since the sign of the initial velocity can be assumed varying randomly in time, the resulting initial velocity will also be zero leaving a pseudo-free response of the system. Eq. (6) shows the formulation for the averaging process as follows.

$$\hat{h}_{rq}(\tau) = \frac{1}{N} \sum_{k=1}^{N} x_r\left(t_k : t_k + \tau\right)\left|\left\{a_1 \leq x_q(t_k) \leq a_2\right\}\right. \tag{6}$$

where the un-scaled impulse responses are shown as $\hat{h}_{rq}$, N is the total number of trigger crossings, $x_r\left(t_k : t_k + \tau\right)$ is the k th time segment of channel r (the values between $x_r(t_k)$ and $x_r(t_k + \tau)$) when the response at channel q is in between the trigger levels (a_1 and a_2).

3 Time series modelling

3.1 AR model

An AR model can be fit to the undamaged sensor output, and the residuals from predictions of subsequent data using this baseline model are then monitored for statistically significant changes that are assumed to be caused by damage. Specifically, an AR model with p autoregressive terms, AR(p), using the time series obtained at the initial condition of the structure (baseline model), can be written as [Box, Jenkins and Reinsel (1994)]

$$x(t) = \sum_{j=1}^{p} \phi_{xj} \; x(t-j) + e_x(t) \tag{7}$$

where $x(t)$ is the representation of the measured signal at discrete time t, ϕ_{xj} are the AR coefficients or model parameters, and $e_x(t)$ is an unobservable noise term or residual term. Thus, an AR model works by fitting a simple linear model to each point with the previous p observed points as dependent variables. Note that an n point time series will yield n-p equations that can be used to generate a least square estimate of the AR coefficients or the Yule-Walker method can be used to solve for the coefficients [Brockwell and Davis (1991)]. In general, the optimal order of the model is not known a priori. There are several ways to estimate it. The two most widely used methods are Akaike's information criteria and Akaike's final prediction error [Ljung (1998)].

3.2 ARMA model

The ARMA model is given by

$$x(t) = \sum_{i=1}^{p} \alpha_i \; x(t-i) + \sum_{j=1}^{q} \beta_j \; e_x(t-j) + e_x(t) \tag{8}$$

where α_i and β_j are the kth AR (auto-regressive) and MA (moving-average) coefficient, respectively; p and q are the model orders of the AR and MA processes. The AR part of order p describes the system dynamics while the MA part of order q is related to the external noise as well as to the white noise excitation, and ensures the stationarity of the system response. The Burg algorithm (also known as the maximum entropy method) is used for estimating the coefficients of the ARMA process [Brockwell and Davis (2002)].

3.3 AR-ARX model

Employing a new segment $y(t)$ obtained from an unknown structural condition of the system. Here the new segment $y(t)$ has the same length as the signal $x(t)$ in Eq. (7).

$$y(t) = \sum_{j=1}^{p} \phi_{yj} \; y(t-j) + e_y(t) \tag{9}$$

It is assumed that the error between the measurement and the prediction obtained by the AR model ($e_x(t)$ in Eq. (7)) is mainly caused by the unknown external input. Based on this assumption, an ARX model is employed to reconstruct the input/output relationship between $e_x(t)$ and $x(t)$.

$$x(t) = \sum_{i=1}^{a} \alpha_i \; x(t-i) + \sum_{j=0}^{b} \beta_j \; e_x(t-j) + \varepsilon_x(t) \tag{10}$$

where $\varepsilon_x(t)$ is the residual error after fitting the ARX(*a,b*) model to the $e_x(t)$ and $x(t)$ pair. Note that this AR-ARX modelling is similar to a linear approximation method of an auto-regressive moving average (ARMA) model presented in Ljung [Ljung (1998)]. Here, *a* and *b* values of the ARX model are set rather arbitrarily. However, similar results are obtained for different combinations of *a* and *b* values as long as the sum of *a* and *b* is kept smaller than *p*.

It is investigated how well this ARX(*a,b*) model estimated in Eq. (10) reproduces the input/output relationship of $e_y(t)$ and $y(t)$.

$$\varepsilon_y(t) = y(t) - \sum_{i=1}^{a} \alpha_i\, y(t-i) - \sum_{j=0}^{b} \beta_j\, e_y(t-j) \tag{11}$$

where $e_y(t)$ is considered to be an approximation of the system input estimated from Eq. (9). The α_i and β_j coefficients are associated with $x(t)$ and obtained from Eq. (10). An extension to multivariate models of a two-stage AR-ARX model identification has also been developed [Monroig and Fujino (2006)].

3.4 ARV model

ARV model allows a particular series to be described not only in terms of its own past values, but also in terms of the past values of the other response locations. This provides a thorough description of the interaction between response locations. The general ARV(*p*) model is shown in Eq. (12).

$$x_t = \sum_{k=1}^{p} \varphi_k\, x_{t-k} + e_t \tag{12}$$

where

$$x_t = [x_{1t}, x_{2t}, \cdots, x_{mt}, x_{1t-1}, x_{2t-1}, \cdots, x_{mt-p+1}]^T$$

$$e_t = [e_{1t}, e_{2t}, \cdots, e_{mt}, e_{1t-1}, e_{2t-1}, \cdots, e_{mt-p+1}]^T$$

$$\varphi_k = \{\phi_{ijk}\}; \quad i, j = 1, 2, \cdots, m; k = 1, 2, \cdots, p$$

The x_t and e_t are vectors of responses and residuals, respectively, *m* is the number of output channels and φ_k are (*m* x *m*) matrices of autoregressive parameters.

3.5 ARMAV model

Given a *m*-dimensional time series, the parametric ARMAV(*p,q*) model is described by the following matrix equation [Piombo, Giorcelli, Garibaldi et al. (1993)].

$$x_t = \sum_{k=1}^{p} \alpha_k\, x_{t-k} + \sum_{k=1}^{q} \beta_k\, e_{t-k} + e_t \tag{13}$$

where α_k and β_k are (m x m) matrices of AR and MA coefficients.

4 Damage sensitive features

It is shown that the change in coefficients associated to the input to the ARX models can be attributed directly to the stiffness change in the structure for simple and noise free models. Therefore, these coefficients can be selected as the damage features. This practical analysis gives exact identification, localization, and quantification of damage. However, it is also shown that when there is noise in the data, this approach will not work as it is, and it needs to be modified. Another disadvantage of this damage feature is that it does not work properly for complex models. Considering these shortcomings, it is clear that the methodology will have limitations in real life applications.

4.1 Standard deviation of residual error

If the ARX model obtained from the reference signal block pair $x(t)$ and $e_x(t)$ are not a good representation of the newly obtained signal segment pair $y(t)$ and $e_y(t)$, there will be a significant change in the standard deviation of the residual error, $\varepsilon_y(t)$, compared to that of $\varepsilon_x(t)$. In particular, the standard deviation ratio of the residual errors, $h = \sigma(\varepsilon_y)/\sigma(\varepsilon_x)$, is expected to reach its maximum value near the actual damage sources revealing the location of damage. Therefore, this standard deviation ratio or residual error ratio is defined as the damage sensitive feature and the increase of this ratio is monitored to detect system anomalies. For example, if the unknown structural state is undamaged, then the anticipated ratio will be near 1. However, if damage has occurred in the structure, then the ratio is greater than 1. The remaining signals in the reference database lead to a group of h. Thus, the mean value, $\mu(h)$ and standard deviation, $\sigma(h)$ can be evaluated.

Many statistical models have been developed for structural damage detection. It is seen that the introduction of h and its probability distribution provides a more standard test for damage detection and localization since it accounts for the effects of excitation variability as well as the orders a and b in the prediction model. The probability density function of h can be evaluated using kernel density estimation methods described in Scott et al. [Scott and Sain (2004)], and integrated to obtain the cumulative distribution function. From the empirical distribution of h, threshold limits corresponding to appropriate confidence intervals can be ascertained. Damage detection and localization are based on the comparison of these threshold limits with the value of h [Nair, Kiremidjian, Lei et al. (2003)]. For instance, for normally distributed h, the threshold limit may be fixed as $\mu(h) + 1.96\sigma(h)$ for a 97.5% confidence interval. If the value of h lies below this threshold limit, then there is 97.5% confidence in stating that there is no damage to the structure. Similarly, if the value of h is greater than this threshold limit, then there is a 97.5% confidence in stating that there is damage to the structure.

4.2 F-statistic

Another statistical model has been developed with the primary objective to test the null hypothesis, $H_0 : \sigma^2(\varepsilon_x) = \sigma^2(\varepsilon_y)$, against the one-sided alternative $H_1 : \sigma^2(\varepsilon_x) < \sigma^2(\varepsilon_y)$. Here $\sigma^2(\varepsilon_x)$ and $\sigma^2(\varepsilon_y)$ are the variances of ε_x and ε_y, respectively [Sohn and Farrar (2001)]. The null hypothesis H_0 is rejected when the F-statistic in Eq. (14) exceeds the upper 100 x α percentile of the F-distribution as shown in Eq. (15).

$$F = \frac{\sigma^2(\varepsilon_y)}{\sigma^2(\varepsilon_x)} \tag{14}$$

$$\frac{\sigma^2(\varepsilon_y)}{\sigma^2(\varepsilon_x)} > F^{\alpha}_{n_y^*-1, n_x^*-1} \tag{15}$$

where

$$n_x^* - 1 = d(n_x - 1) \qquad n_y^* - 1 = d(n_y - 1)$$

$$d = \left[1 + \frac{1}{2}(b-3) \right]^{-1}$$

$$b = \frac{(n_x + n_y)\left(\sum \varepsilon_x^4(t) + \sum \varepsilon_y^4(t) \right)}{\left(\sum \varepsilon_x^2(t) + \sum \varepsilon_y^2(t) \right)^2}$$

n_x and n_y are the numbers of samples of $\varepsilon_x(t)$ and $\varepsilon_y(t)$, respectively.

4.3 Sequential probability ratio test

A Sequential Probability Ratio Test (SPRT) has also been used in time series analysis which starts with observing a sequence of the residual errors as denoted as follows [Sohn, Worden and Farrar (2002)].

$$E_n = [\varepsilon_y(1), \cdots, \varepsilon_y(n)] \tag{16}$$

A simple two-class damage classifier is then constructed using the standard deviation of the residual errors.

$$H_0 : \sigma(\varepsilon_y) \leq \sigma_0 \qquad H_1 : \sigma(\varepsilon_y) \geq \sigma_1 \qquad 0 < \sigma_0 < \sigma_1 < \infty \tag{17}$$

When the standard deviation of the residual error $\sigma(\varepsilon_y)$ is less than a user specified lower bound σ_0, the system in question is considered undamaged. On the other hand, when $\sigma(\varepsilon_y)$ becomes equal to or larger than the other user specified upper bound σ_1, the system is suspected to be damaged. For the hypothesis test in Eq. (17), a SPRT makes three distinctive decisions as follows [Ghosh (1970)].

Accept H_0 if $Z_n \leq b$

Reject H_0 if $Z_n \geq a$

Continue observing if $b \leq Z_n \leq a$

where the transformed random variable Z_n is the natural logarithm of the probability ratio.

$$Z_n = \ln \frac{f(E_n|H_1)}{f(E_n|H_0)} = \ln \frac{f(E_n|\sigma_1)}{f(E_n|\sigma_0)} \text{ for } n \geq 1 \tag{18}$$

where $f(E_n|H_0)$ or $f(E_n|\sigma_0)$ is the conditional probability of observing the data set E_n given the assumption that the null hypothesis is true. $f(E_n|H_1)$ or $f(E_n|\sigma_1)$ is defined in a similar fashion. Without any loss of generality, Z_n is defined to be zero when $f(E_n|H_1) = f(E_n|H_0) = 0$. b and a are the two stopping bounds for accepting and rejecting H_0, respectively.

4.4 First four central moments

The first four central moments, namely the mean, standard deviation, skewness and kurtosis, are investigated as indicators of damage [Mattson and Pandit (2006)]. The variance and the skewness showed well-expressed regular dependence on damage whereas the mean value and the kurtosis demonstrated weak sensitivity and irregular dependence on damage quantity. As mentioned above, an estimate of the autoregressive model residual series standard deviation provides an accurate diagnosis of damage conditions. Similar statistical analysis applied to the raw data necessitates the use of higher-order moments that are more sensitive to disguised outliers, but are also prone to false indications resulting from overemphasising rarely occurring extreme values. Skewness provides the best indication of damage in the raw data. For a multidimensional distribution, these characteristics (variance and skewness) can be defined by the following scalar quantities.

$$\sigma = \frac{1}{M^2} \sum_{i=1}^{n} \sum_{j=1}^{n} \left[(Y_i - \bar{Y})^T S (Y_j - \bar{Y}) \right]^2 \tag{19}$$

$$s = \frac{1}{M^2} \sum_{i=1}^{n} \sum_{j=1}^{n} \left[(Y_i - \bar{Y})^T S (Y_j - \bar{Y}) \right]^3 \tag{20}$$

where M is the number of points in the vectors that characterises the attractor of the response signal $Y_n, n = 1, \cdots, M$, $\bar{Y}$ is the sample mean vector and S is the sample covariance matrix. Instead of using the values for σ and s one can introduce relative changes compared to the non-damaged case.

4.5 First three AR components

Another damage sensitive feature is defined as a function of the first three AR components. The first three AR coefficients appear to be most promising because these coefficients are statistically the most significant among all the coefficients of the model. After testing several different combinations with the first three coefficients, it was found that the first AR coefficient normalized by the square root of the sum of the squares of the first three AR coefficients provides the most robust damage sensitive feature (DSF) as follows [Nair, Kiremidjian and Law (2006)].

$$DSF = \frac{\alpha_1}{\sqrt{\alpha_1^2 + \alpha_2^2 + \alpha_3^2}} \tag{21}$$

where α_1, α_2 and α_3 are the first three AR coefficients. It is found that the mean values of the DSF for the damaged and undamaged signals are different. Thus, to test statistical difference between the means of two groups of data, the standard *t*-test is used [Rice (1999)]. If $\mu_{DSF,damaged}$ and $\mu_{DSF,undamaged}$ are defined as the mean values of the DSFs obtained from the damaged and undamaged case, respectively, then a hypothesis test may be set up as follows to determine if their differences are significant.

$$H_0 : \mu_{DSF,undamaged} = \mu_{DSF,damaged}$$

$$H_1 : \mu_{DSF,undamaged} \neq \mu_{DSF,damaged}$$

H_0 represents the undamaged condition and H_1 represents the damaged condition. The significance level of the test is set at 0.05.

4.6 Fit ratio

Another approach is to use the fit ratios of the models as the damage features [Gul and Catbas (2011)]. The fit ratio (FR) of an ARX model is calculated as follows.

$$FR = \left(1 - \frac{|y - \hat{y}|}{|y - \bar{y}|}\right) \times 100 \tag{22}$$

where y is the measured output, $\hat{y}$ is the predicted output, $\bar{y}$ is the mean of y, and $|y - \hat{y}|$ is the norm of $y - \hat{y}$. The DF is calculated by using the difference between the FRs for healthy and damaged cases as given in Eq. (23).

$$DSF = \frac{FR_{healthy} - FR_{damaged}}{FR_{healthy}} \times 100 \tag{23}$$

4.7 Mahalanobis distance

Mahalanobis distance-based outlier detection has also been used to detect the novelty in the data [Gul and Catbas (2009)]. The outlier detection problem for univariate data is relatively straightforward, e.g. the outliers can be identified from the tails of the

distribution. There are several discordance tests but one of the most common is based on deviation statistics and it is given by the following.

$$z_i = \frac{d_i - \overline{d}}{\sigma} \tag{24}$$

where z_i is the outlier index for univariate data, d_i is the potential outlier and $\overline{d}$ and σ are the sample mean and standard deviation, respectively. The multivariate equivalent of this discordance test for $n \times p$ (where n is the number of the feature vectors and p is the dimension of each vector) data set is known as the Mahalanobis squared distance [Mahalanobis (1936)]. The Mahalanobis squared distance is given as

$$Z_i = (x_i - \overline{x})^T \Sigma^{-1} (x_i - \overline{x}) \tag{25}$$

where Z_i is the outlier index for multivariate data, x_i is the potential outlier vector and $\overline{x}$ is the sample mean vector and Σ is the sample covariance matrix. By using the above equations, the outliers can be detected if the Mahalanobis distance of a data vector is higher than a pre-set threshold level.

4.8 Anomaly measure

The behavioural changes from nominal condition are described as anomalies which can also be characterized by a scalar called anomaly measure ($\wp$). The anomaly measure at slow-time epoch t_k is obtained as [Khatkhate, Gupta, Ray et al. (2008)]

$$\wp_k \equiv \left[\sum_{j=1}^{n} \left(|p_k(j) - p_0(j)| \right)^2 \right]^{1/2} \tag{26}$$

The above distance function is the standard Euclidean norm of the difference between the state probability vectors, $p_1, p_2, \cdots, p_k, \cdots$ which are obtained at slow-time epochs $t_1, t_2, \cdots, t_k, \cdots$ based on the respective time series data.

4.9 Relative state sequence histogram error

An index, the relative state sequence histogram error (RSSHe) has also been developed to measure the distance between histograms [Li, Mita and Zhou (2013)].

$$RSSHe = \sqrt{\frac{\sum_{i=1}^{i=m} \left(d_b^i - d_a^i \right)^2}{\sum_{i=1}^{i=m} \left(d_a^i \right)^2}} \tag{27}$$

where $d_{a/b}^i$ is the frequency of state i in state sequence histogram $SSH_a = [d_a^1, d_a^2, \cdots, d_a^m]$ or $SSH_b = [d_b^1, d_b^2, \cdots, d_b^m]$. The elements in SSH_a and SSH_b are obtained by symbolic time series analysis using raw acceleration data from a healthy and unknown structure, respectively.

4.10 Poincare map

A way to analyse the nonlinear time domain vibration response in a state space is to use its Poincare map to extract damage sensitive features [Trendafilova and Manoach (2008)]. Poincare maps contain data for the displacements and the velocities of the structure in a compact form, and since these two parameters are expected to be sensitive to damage, these diagrams can be used to detect damage. Even when the damage is small, and the responses of the damaged and the healthy structure are close to each other, the points from the Poincare map are easier to use for comparison and identification purposes because the number of these points is not comparable to the enormous number of points in the time history. Accordingly, the following damage index can be introduced:

$$I_i^d = \frac{S_i^u - S_i^d}{S_i^u} \tag{28}$$

where $S_i^u = \sum_{j=1}^{N_p-1} \sqrt{\left(w_{i,j+1}^u - w_{i,j}^u\right)^2 + \left(\dot{w}_{i,j+1}^u - \dot{w}_{i,j}^u\right)^2}$, $S_i^d = \sum_{j=1}^{N_p-1} \sqrt{\left(w_{i,j+1}^d - w_{i,j}^d\right)^2 + \left(\dot{w}_{i,j+1}^d - \dot{w}_{i,j}^d\right)^2}$

for $i = 1, 2, \cdots, N_{node}$, N_{node} is the number of nodes, N_p is the number of points in the Poincare map, $\left(w_{i,j}^u, \dot{w}_{i,j}^u\right)$ and $\left(w_{i,j}^d, \dot{w}_{i,j}^d\right)$ denote the jth point in the Poincare map for the undamaged and damaged states, respectively. A small (close to 0) damage index will indicate no damage, while a big damage index will indicate the presence of a fault at the corresponding location. The above damage index depends on the location of the point on the structure and consequently it is a function of the structure coordinates x and y. One can expect that the maximums of the surface I_i^d will represent the location of damage in the structure $\left(x_d, y_d\right)$.

4.11 Statistical process control

Statistical process control also provides a framework for monitoring future extracted data features and for identifying new data that are inconsistent with past data. The X-bar and S control charts can be employed to monitor the mean and variance of the selected features [Fugate, Sohn and Farrar (2001)]. Control limits for the control charts are constructed based on the features obtained from the initial undamaged structure. To detect a change in the mean of residuals, a rational approach is to form subgroups of size *n*. The centreline of the chart is the sample mean of the residuals and after the normalization is 0. The sample variance of each subgroup is determined, and these variances are then averaged to give a pooled estimate of variance. The square root of the pooled variance, s_p is used as an estimate of the population standard deviation. Control limits are drawn at $0 \pm z_{\tau/2} s_p / \sqrt{n}$ where z_τ represents the τ quantile of the standard normal distribution. To monitor variability within each subgroup, an S control chart can be used. For each subgroup the sample standard deviation of the (normalized) residuals is computed, s_j. The upper and lower control limits are

$$UCL = \overline{S}\sqrt{\frac{\chi_{1-\tau/2,n-1}}{n-1}} \qquad LCL = \overline{S}\sqrt{\frac{\chi_{\tau/2,n-1}}{n-1}} \tag{29}$$

where $\chi_{p,n}^2$ denotes the *p*th quantile of a Chi-square random variable and $\overline{S}$ is the average of s_j. A statically significant number of error terms outside the control limits indicate a system transit from a healthy state to a damage state.

4.12 Sum of squares of residuals

A statistical classification algorithm has also been proposed. [Box, Jenkins and Reinsel (1994)] demonstrated that approximate $1 - \varepsilon$ confidence region of the ARMA parameters are bounded by a contour on the surface of the sum of the squares of the residuals given by Carden et al. [Carden and Brownjohn (2008)]

$$S(\beta) < S(\hat{\beta})\left[1 + \frac{\chi_\varepsilon^2(k)}{n}\right] \tag{30}$$

where $S(\cdot)$ is the sum of the squares of the residuals function, β are the ARMA parameters estimated from a time series of length *n* and $\hat{\beta}$ are the previously known ARMA parameters from a particular process. Generally, only an estimate of $\hat{\beta}$ is known. $\chi_\varepsilon^2(k)$ is the significance point exceeded by a proportion ε of the χ^2 distribution and *k* is equal to the order of the ARMA model. Eq. (30) allows the comparison of all the ARMA parameters simultaneously. If it is postulated that the time series is generated by a process with true ARMA parameters $\hat{\beta}$, which means it is a structure in a healthy state, then Eq. (30) tests whether the parameters β are significantly statistically similar or different to $\hat{\beta}$, in order words healthy or not healthy. Eq. (30) will be used as the basis for a classification algorithm for SHM assuming that ARMA models can be fitted to the responses of the structure being measured.

4.13 Distance between centers of damaged and undamaged clouds

Based on the damaged and undamaged clouds in the AR coefficient space, two damage localization indices can be defined as follows [Nair, Kiremidjian and Law (2006)].

$$LI_1 = \frac{d_{mean}}{d_{undam\,cloud}} \qquad LI_2 = \frac{d_{dam\,cloud}}{d_{undam\,cloud}} \tag{31}$$

where d_{mean} is the distance between the centers of the damaged and undamaged clouds, $d_{dam\,cloud}$ is the distance from the origin to the center of the damaged cloud and $d_{undam\,cloud}$ is the distance from the origin to the undamaged cloud. At the sensor locations where damage is introduced, the values of LI_1 and LI_2 appear to increase from their values obtained at the undamaged baseline state.

5 Variations and improvement of the existing time series models

5.1 Output-only problem

In many cases, damage detection and identification have to be based on vibration response-only measurement (the output-only problem). This is so because the force excitation may be due to various sources that are difficult or impossible to precisely isolate and measure. TARMA models resemble their conventional, stationary ARMA counterparts with the significant difference being that they allow their parameters to depend on time. Depending on the nature of the mathematical structure imposed on the time evolution of their parameters, TARMA models may be classified as unstructured parameter evolution, stochastic parameter evolution and deterministic parameter evolution.

A TARMA (n_a, n_c) model with n_a, n_c designating its autoregressive (AR) and moving average (MA) orders, respectively is given as follows [Spiridonakos, Poulimenos and Fassois (2010)].

$$x(t) + \sum_{i=1}^{n_a} a_i(t)\, x(t-i) = e_x(t) + \sum_{j=1}^{n_c} c_j(t)\, e_x(t-j), \quad e_x(t) \sim NID\big(0, \sigma_{e_x}^2(t)\big) \tag{32}$$

where t designating normalized discrete time, $x(t)$ is the nonstationary vibration response signal, $e_x(t)$ is an unobservable uncorrelated (white) nonstationary residual signal characterized by zero mean and time-varying variance $\sigma_{e_x}^2(t)$, and $a_i(t), c_j(t)$ are the model's time varying AR and MA parameters, respectively.

5.2 Sensor cluster

To identify local damage, it is important that the models correspond to dynamic properties at the level of structural members. In this respect, it seems that exchanging time histories between adjacent sensors could improve the results, assuming that multivariate models more accurately describe the structural behaviour in the members between sensors. A local physical model method has been developed by rewriting the second order equation of dynamics of a node as an ARX (2, 2) model. This model can be identified from acceleration measurements of the nodes that are connected by a structural member to the current node.

The dynamics of node i are ruled by the following Eq. (33) [Monroig and Fujino (2006)].

$$m_i \ddot{x}_i + c_i\big(\dot{x}_i - \dot{x}_{i-1}\big) + c_{i+1}\big(\dot{x}_i - \dot{x}_{i+1}\big) + k_i\big(x_i - x_{i-1}\big) + k_{i+1}\big(x_i - x_{i+1}\big) = f_i \tag{33}$$

After differentiating twice Eq. (33), the first and second derivatives of the accelerations are approximated by finite differences and Eq. (34) can be thought of as a multivariate ARX (2, 2) model.

$$y(t) + A_1 y(t-1) + A_2 y(t-2) = B_1 u(t-1) + B_2 u(t-2) + e(t) \tag{34}$$

The core of the methodology is to create different ARX models for different sensor clusters and then extract damage sensitive features from these models to detect the damage.

5.3 Symbolic time series analysis

While classical data analysis focuses on individuals, symbolic data analysis deals with concepts, a less specific type of information. The original time series signals are converted into sequences of discrete symbols, and the statistical features of the symbols can be used to describe the dynamic statuses of a system. Data symbolization by using symbolic TSA alleviates the effects of harmful noise in raw acceleration data [Li, Mita and Zhou (2013); Khatkhate, Gupta, Ray et al. (2008); Rajagopalan and Ray (2006)].

The first step is to transform the raw acceleration data into a binary symbol series. A binary code needs to be transformed into the decimal domain. Alternatively, symbolic time series analysis can also take advantage of the information generated by partitioning the time series data in its wavelet domain. A data sequence (time series data) is converted to a symbol sequence by partitioning a compact region S of the phase space, over which the trajectory evolves into finitely many discrete blocks. The occurrence number of certain states in the state series varies. Then the statistics of the symbolic state can be derived, i.e. compute the vector of the observed state frequencies. A bar graph used to plot the occurrence number of every state in a state series is called "state sequence histogram" $SSH = [d^1, d^2, \cdots, d^m]$ where d^i is the number of occurrences (Eq. (27)).

5.4 Nonlinear time series analysis

As the physical long-term behaviour of a dynamic system is on the attractor, the system is thereby characterized by its attractor. Therefore, the steady-state converging trajectories, which represent the attractor, are very sensitive to any changes in the system. A number of the nonlinear invariants are introduced as damage sensitive features from the state space representation of the attractor of a vibrating system. The average mutual information (AMI) can be used as a measure of correlation between two measurements [Trendafilova and Manoach (2008); Trendafilova (2006)]. It has been found for some investigated cases that the AMI increases with the introduction and with the increase of damage. Thus, the relative change of AMI referred to the undamaged case can be used to form a damage feature. The AMI is a quantity which is easy and straightforward to compute directly from the measured acceleration data, which make it an attractive candidate for damage diagnosis.

The correlation dimension is another invariant of the motion of a dynamic system which can be estimated from data [Trendafilova (2006)]. The correlation dimension decreases with the introduction of damage. Therefore, considering its sensitivity to damage and its invariance for smooth changes of the coordinate system, the relative change of correlation dimension can be introduced as a possible damage feature (index). Unfortunately, the estimation of the correlation dimension is not an easy and straightforward process. Thus, it is very difficult or rather impossible to find a true and reliable estimate for the correlation dimension. Due to these difficulties, it should not be considered a good candidate for a damage feature.

For a linear vibrating system, the Lyapunov exponents (LEs) are determined by the real parts of the eigenvalues of the state space matrix of the system [Trendafilova (2006)]. Previous research has shown that damage will affect the eigen state of the structure, and thus they are expected to affect the structure's LEs and the geometry of the attractor. The relative change in the largest LE can be suggested as a possible damage feature. Although there is evidence that in some cases the Lyapunov spectrum changes with damage, in many practical cases the eigenvalues of the structure remain rather insensitive to damage. An alternative way to characterize the attractor of a dynamic system is to use its probability density. The introduction of damage will change the vectors that characterises the attractor of the response signal and its distribution. The new distribution will be represented by a different set of coefficients. Thus, the average root mean square difference between the coefficients representing the distribution of points on the undamaged and unknown attractor can be used as a possible damage feature.

5.5 NARMAX

Another study is to make use of NARMAX (Nonlinear Auto-Regressive Moving Average with eXogenous Inputs) modelling [Peng, Lang, Wolters et al. (2011)]. A study to combine the NARX modelling and nonlinear output frequency response function (NOFRF) based analysis was conducted to perform damage detection for structural systems to overcome the problems and difficulties associated with the time domain modelling and residual analysis based techniques. The technique first applies the NARMAX modelling method to establish a NARX model from an inspected structural system. Then, the NOFRFs and an associated index for the inspected structure are determined from the established NARX model. Finally, structural damage detection is conducted by comparing the values of the NOFRF index of the inspected structure and the values of the index for an undamaged structure.

5.6 Combination with immune algorithm

A hybrid methodology combining immune algorithm and symbolic time series analysis has been developed for structural health monitoring [Li, Mita and Zhou (2013)]. Real-valued negative selection (RNS) is used to detect structural damages and adaptive immune clonal selection algorithm (AICSA) is used to localise and quantify the damages by minimizing the Euclidean distance between the SSH (Eq. (27)). The negative selection algorithm is inspired by observation of the activity of the human immune system. RNS tries to alleviate some of the drawbacks of NS while using the higher level of representation real space to speed up the detector generation process. Inspired by the clonal selection principle, the clonal selection algorithm (CSA) has been used to deal with optimization problems. AICSA embodies three strategies: Secondary response, adaptive mutation regulation and vaccination to speed up CSA's convergence and ability to find the global optimum.

6 Conclusions

This paper reviews the application of time series analysis for SHM. The different types of time series analysis are described, and the basic principles are explained in detail.

Depending on the technique employed, various damage sensitive features have been proposed to capture the deviations. Damage sensitive features that discriminate between damaged and non-damaged states of a structure are reviewed including residual error ratio, hypothesis testing, SPRT, first four central moments, the first AR coefficient normalized by the square root of the sum of the squares of the first three AR coefficients, fit ratio, Mahalanobis distance, anomaly measure, RSSHe, Poincare map, statistical process control, and statistical classification algorithm. In addition, some investigations that have attempted to modify and/or combine time series analysis with other approaches for better damage identification are presented including TARMA for solving output-only problem, sensor cluster, symbolic TSA, nonlinear TSA, NARMAX, and combination with immune algorithm.

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