 SDHM, vol.9, no.2, pp.167-180, 2013

Some Recent Developments on the Application of the Strain Energy Density to Shallow Threaded Plates with Sharp Notches

R. Afshar[1], **F. Berto**[1]

Abstract: In this study, the main advantages of the strain energy density (SED) approach and some recent applications of the SED to the fatigue analysis of welded joints are reviewed. In addition, the paper investigates the scale effect in the threaded plates with sharp notches subjected to tension loading. Some closed form expressions for evaluation of the notch stress intensity factors (NSIFs) of periodic sharp notches, obtained by SED approach, are employed. The new expressions are applicable to narrow notches when the ratio between the notch depth and the plate width, t/W, is lower than 0.025 providing very accurate results. The NSIF ratio of two scaled geometries of periodic sharp notches is a function of averaged SED in the control volume embracing the middle notch tip. The new results are very useful for the assessment under fatigue loading.

Keywords: Periodic notches, Notch Stress Intensity Factor (NSIF), Strain Energy Density (SED), Narrow notches.

1 Introduction

Toothed cutting blades, thread bars and splined shafts play a vital role in various industry applications. They are being used in many facilities across different sectors including: construction, aerospace, wood industry, electronic devices, etc. The variability of the notch stress intensity factors (NSIFs) of periodic blunt and sharp V-notches is investigated by means of the strain energy density (SED) approach in combination with coarse meshes in the finite element method (FEM) (Afshar and Berto, 2011, Berto, Lazzarin and Afshar, 2012, Lazzarin, Afshar and Berto, 2012).

The necessity of a simple criterion for engineering applications led to development of a point-wise SED approach valid for cracks (Sih, 1973, Sih, 1973, Sih, 1974, Sih, 1991) and notches (Sih and Ho, 1991). Factor *S* was defined as the product of the SED by a critical distance from the point of singularity (Sih, 1974). Failure

[1] Department of Management and Engineering, University of Padova, Vicenza, Italy.

was thought of as controlled by a critical value S_c, whereas the direction of crack propagation was determined by imposing a minimum condition on S. The theory was extended to employ the total SED near the notch tip (Sih and Ho, 1991), and the point of reference was chosen to be the location on the surface of the notch, where the maximum tangential stress occurs.

By using the SED concept combined with a coarse mesh in the FE analysis, a fatigue strength assessment of welded joints was carried out (Lazzarin et al., 2008). A procedure for rapid calculations of the NSIFs based on the SED from coarse meshing is drawn in Ref. (Lazzarin, Berto and Zappalorto, 2010). The extension to three-dimensional cases is also possible and very convenient, in particular when edge effects are present or when a narrow spacing between collinear notches is considered. In small bodies a multiscaling and segmentation scheme permits to scale the SED at pico, nano and micro levels (Sih and Tang, 2005, Tang and Sih, 2005, Sih, 2007).

As alternatives of the SED approach, a review of the dual boundary element method for modeling crack growth in two-dimensional and three-dimensional mixed mode problems is presented and compared with available alternative solutions in Ref. (Cisilino and Aliabadi, 2010). In another work, a model based on cohesive crack concept is developed for finite element analysis of quasi-brittle materials (Cendón et al., 2000, Gálvez et al., 2012).

The creation and subsequent shedding of periodic edge cracks is a natural phenomenon which occurs in heat-checked gun tubes, rapidly cooled pressure vessels and rock, dried-out mud flats, paint and concrete and in ceramic coatings and permafrost. Dealing with this topic a complete state of the art together with a simple developed model assessing the shedding behavior is carried out by Parker (Parker, 1999). As discussed in that work, the surface topography of the cracking of ice-wedge polygons in Arctic permafrost, of mud flats in Death Valley and of craze-cracks (heat-checking) at the bore of a gun tube are all strikingly similar, yet they span five orders of magnitude in scale, with the maximum plate dimensions for ice and mud being, respectively, 22 m and 0.25 m and with the minimum plate size for gun tube craze cracking being 0.2 mm. Recently, some new expression for the NSIFs of periodic sharp notches is developed, which is an extended version of that proposed by Tada et al. (Tada, Paris and Irwin, 1985) for edge cracks. It is valid for an infinite plate width, but can be applied with errors within 5 percent if the ratio between the notch depth and plate width remains lower than 0.025. The expressions are suitable both for the direct evaluation of the SED and NSIF as a function of the narrow notch spacing (Berto, Lazzarin and Afshar, 2012).

In this paper, the scale effect of periodic narrow-sharp notches by using the aforementioned closed form expressions in the macro-meso scale range is studied. The theoretical results reported here can be directly applied to the fatigue design.

2 Advantages of the SED for fatigue assessment

As opposed to the direct evaluation of the NSIFs, which needs very refined meshes, the mean value of the elastic SED on the control volume can be determined with high accuracy by using coarse meshes ...(Lazzarin et al., 2008, Berto and Lazzarin, 2009, Lazzarin, Berto and Zappalorto, 2010).Very refined meshes are necessary to directly determine the NSIFs from the local stress distributions. Refined meshes are not necessary when the aim of the FE analysis is to determine the mean value of the local SED on a control volume surrounding the points of stress singularity. The SED in fact can be derived directly from nodal displacements, so that also coarse meshes are able to give sufficiently accurate values for it. Some recent contributions document the slight variability of the SED as determined from very refined meshes and coarse meshes, considering some typical welded joint geometries and provide a theoretical justification to the weak dependence exhibited by the mean value of the local SED when evaluated over a control volume centered at the weld root or the weld toe. On the contrary singular stress distributions are strongly mesh dependent. The NSIFs can be estimated from the local SED value of pointed V-notches in plates subjected to mode I, Mode II or a mixed mode loading. Taking advantage of some closed-form relationships linking the local stress distributions ahead of the notch to the maximum elastic stresses at the notch tip the coarse mesh SED-based procedure is used to estimate the relevant theoretical stress concentration factor K_t for blunt notches considering, in particular, a circular hole and a U-shaped notch, the former in mode I loading, the latter also in mixed, I + II, mode ...(Livieri and Lazzarin, 2005, Berto and Lazzarin, 2009). Other important advantages can be achieved by using the SED approach. The most important advantages of SED method are as follows:

- It permits consideration of the scale effect, which is fully included in the NSIF Approach
- It permits consideration of the contribution of different Modes.
- It permits consideration of the cycle nominal load ratio .
- It overcomes the complex problem tied to the different NSIF units of measure in the case of different notch opening angles (i.e crack initiation at the toe (2α=135°) or root (2α=0°) in a welded joint)
- It overcomes the complex problem of multiple crack initiation and their interaction on different planes.

- It directly takes into account the T-stress and this aspect becomes fundamental when thin structures are analysed.
- It directly includes three-dimensional effects and out-of-plane singularities not assessed by Williams' theory.

2.1 Synthesis of fatigue analysis based on SED in a control volume

The mean value of the SED in a circular sector of radius R_0 located at the fatigue crack initiation sites has been used to summarise fatigue strength data from steel welded joints of complex geometry (Fig. 1).

Local SED ($\Delta\bar{W}$), averaged in a finite size volume surrounding weld toes and roots is a scalar quantity which can be given as a function of mode I-II NSIFs in plane problems and mode I-II-III NSIFs in three dimensional problems. The evaluation of the local SED needs precise information about the control volume size. From a theoretical point of view, the material properties in the vicinity of the weld toes and the weld roots depend on a number of parameters as residual stresses and distortions, heterogeneous metallurgical micro-structures, weld thermal cycles, heat source characteristics, load histories and so on. To device a model capable of predicting R_0 and fatigue life of welded components on the basis of all these parameters is really a task too complex. Thus, the spirit of the approach is to give a simplified method able to summarize the fatigue life of components only on the basis of geometrical information, treating all the other effects only in statistical terms, with reference to a well-defined group of welded materials and, for the time being, to arc welding processes.

In a plane problem all stress and strain components in the highly stressed region are correlated to mode I and mode II NSIFs.

The material parameter R_0 can be estimated by using the fatigue strength $\Delta\sigma_A$ of the butt ground welded joints (in order to quantify the influence of the welding process, in the absence of any stress concentration effect) and the NSIF-based fatigue strength of welded joints having a V-notch angle at the weld toe constant and large enough to ensure the non singularity of mode II stress distributions.

The mean value of the SED, ΔW, as determined on a control volume embracing the weld toe or the weld root was directly used as parameter for fatigue strength assessments of welded joints (see Refs. (Livieri and Lazzarin, 2005, Berto and Lazzarin, 2009)). The relevant ΔW-N scatterband is shown in Fig. 1, where a large bulk of fatigue test data (about 900 data) from different welded joints with a main plate thickness ranging from 6 to 100 mm are plotted. Details on welded joint geometries, materials and welding technologies are reported in some previous papers.

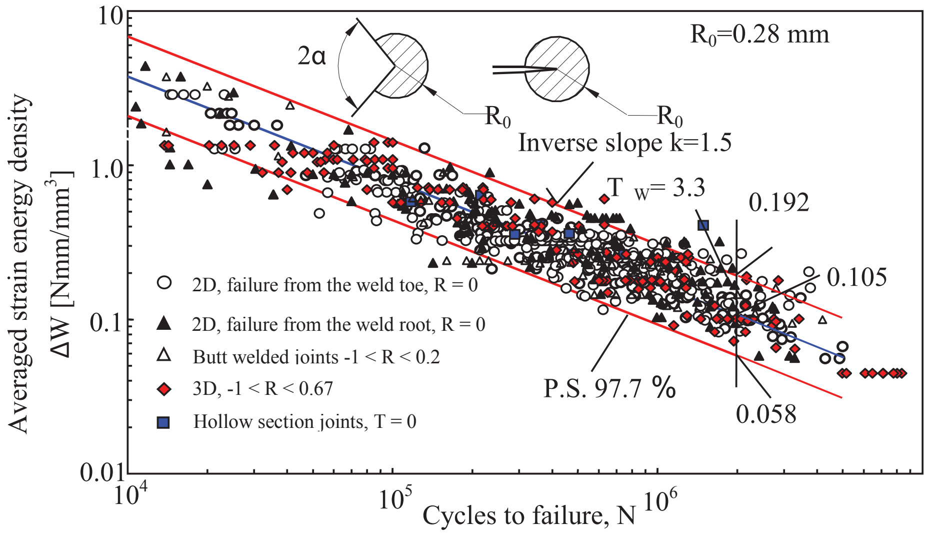

Figure 1: Fatigue strength of the welded joints as a function of the averaged local SED; R is the nominal load ratio (Berto and Lazzarin, 2009).

3 SED in a control volume applied to sharp notches

Dealing with a sharp V-notch subjected to Mode I loading (see Fig. 3), assuming a plane strain condition, the mean value of the SED in the semi-circular sector of radius R_0 is (Lazzarin and Zambardi, 2001):

$$\bar{W}_1 = \frac{e_1}{E}\left(\frac{K_1}{R_0^{1-\lambda_1}}\right)^2 \tag{1}$$

where K_1 is the NSIF, λ_1 is Williams'eigenvalue (Williams, 1952), e_1 is a shape function that depends both on the notch angle 2α and the Poisson's ratio ν and E is the Young modulus.

By inverting Eq.(1) K_1 can be easily defined as follows:

$$K_1 = R_0 1 - \lambda_1 \sqrt{\frac{E\bar{W}_1}{e_1}} \tag{2}$$

Due to the linearity of the problem, the NSIF value can be computed, similarly to stress intensity factors (SIFs) in linear elastic fracture mechanics (LEFM) as:

$$K_1 = k_1\, \sigma_n\, t^{1-\lambda_1} \tag{3}$$

where k_1 is a non dimensional parameter that depends on the overall geometry and can be seen as an extension of the shape factor used in the LEFM, σ_n is the reference stress (e.g., the remote tensile stress); $t^{1-\lambda 1}$quantifies the influence of the specimen size and in particular of the notch depth.

Results of narrow V-notches with a/t=0, the most critical geometry, are reported in Table 1. Here the comparison is between the normalized NSIFs obtained from coarse and fine meshes. The maximum detected error is less than 2%.

Table 1: Comparison between fine and coarse mesh-based results; SED approach applied to the models with narrow periodic notches, a/t=0.0, (t/d=0.05).

2α (^{o})	F_I^V (coarse mesh)	F_I^V (fine mesh)	Δ%
30	0.329	0.333	-1.06
45	0.405	0.411	-1.56
60	0.498	0.498	-0.17
90	0.734	0.730	0.58
120	1.092	1.090	0.14
135	1.322	1.323	-0.07

3.1 Application of the SED to periodic narrow notches: an extension of Tada Paris diagram

A system of multiple, equal length edge cracks of depth t and spacing 2h has been considered by Tada et al. (Tada, Paris and Irwin, 1985) in a plate of infinite width. The variation of the SIF normalized by $K_0 = \sigma\sqrt{h}$ is presented as a function of s=t/(t+h). It is shown that in the case of narrow cracks and having a s value greater than 0.3, the SIF (K_I) can be simply estimated by using the following equation, which is valid for an infinite plate width:

$$K_I = K_0 = \sigma_n\sqrt{h} \tag{4}$$

Eq. (4) can be directly applied without requiring numerical simulations and is independent of the notch depth. The only parameter involved is the crack spacing, h, which can be easily measured.

Consider now multiple, equal-length double symmetric V-notches of depth t under Mode I loading and plane strain conditions (see Fig. 3).

An extension of Eq. (4) is proposed in (Berto, Lazzarin and Afshar, 2012) only for narrow notches to matches the Tada Paris diagram valid for the crack case. The

NSIF can be expressed as follows:

$$K_1 = K_0 = \kappa_0\ \sigma_n h^{\lambda_1} \tag{5}$$

In Eq. (5) κ_0 is a dimensional parameter that depends on the notch opening angle and λ_1 is Williams'eigenvalue (Williams, 1952). Equation (5) collapses into Equation (4) when the crack case is considered (κ_0=1 and λ_1=0.5). For an infinite plate width W>>t and narrow notches, being t/(t+a) close to 1 the stain energy averaged over a control volume of radius R_0 can be written as follows:

$$\bar{W}_1 = \frac{\kappa_0^2 e_1}{E}\left(\frac{\sigma_n h^{\lambda_1}}{R_0^{1-\lambda_1}}\right)^2 = \frac{\tilde{e}_1}{E}\left(\frac{\sigma_n h^{\lambda_1}}{R_0^{1-\lambda_1}}\right)^2 \tag{6}$$

By inverting Eq. (6) it is easy to determine parameters κ_0 and $\tilde{e}_1$ that can be expressed according to the following equations:

$$\tilde{e}_1 = \kappa_0^2 e_1 = \frac{E\bar{W}_1}{\sigma_n^2}\left(\frac{R_0^{1-\lambda_1}}{h^{\lambda_1}}\right)^2 \tag{7}$$

$$\kappa_0 = \sqrt{\frac{E\bar{W}_1}{e_1\sigma_n^2}}\left(\frac{R_0^{1-\lambda_1}}{h^{\lambda_1}}\right) \tag{8}$$

The values of κ_0 and $\tilde{e}_1$ are given in Ref. (Berto, Lazzarin and Afshar, 2012) for the five different characteristic notch opening angles (2α=30, 60, 90, 120 and 135°) both for symmetric and edge notches. By means of κ_0 and $\tilde{e}_1$ it is possible to straightforwardly evaluate the NSIF and the SED over an area of radius R_0 by using Eq. (5) and (6), respectively. It is also possible to express the ratio between K_1 and K_0 in a general form by combining Eqs (3) and (5). It yields to the final expression:

$$\frac{K_1}{K_0} = \frac{k_1}{\kappa_0}\left(\frac{t^{1-\lambda_1}}{h^{\lambda_1}}\right) \tag{9}$$

4 Scale effect for not narrow periodic notches

Since the NSIF depends on the absolute dimension of the notch and the plates, it is surely convenient to normalize its value as a function of the notch size. Some preliminary considerations might be useful. It is well known that two notched plates scaled in geometrical proportion have the same theoretical stress concentration since it simply depends on the geometrical ratios, i.e. notch depth to plate width ratio. Consider now two plates weakened by sharp V-notches, plates and notches in geometric proportion, the corresponding NSIF is different and can be quantified by means of the simple expression ...(Dunn, Suwito and Cunningham, 1997, Dunn, Suwito and Cunningham, 1997, Lazzarin and Tovo, 1998):

$$K_1 = k_1\ \sigma_0\ t^{1-\lambda_1} \tag{10}$$

where σ_0 is the reference stress (e.g., the remote tensile stress), 1-λ_1 is the stress singularity in the close vicinity of the notch tip and k_1 is a nondimensional shape factor. In the case of periodic notches the shape factor k_1 does depend on the geometrical ratios t/d and a/t. Table 2 gives the NSIFs and the shape factor k_1 for some plates with a/t=0.1 and t/d=0.025. In addition, the normalized NSIFs of the data presented in Table 2 as a function of depth of the notch (t) is shown in Fig. 2, where the shape factor k_i clearly appears as the coefficient of the power function.

Table 2: NSIFs as a function of depth of the notch (t) with constant relative depth (t/d=0.025) for a plate with periodic notches of 2α=60° and a/t=0.1 (1-λ_1=0.4878) (Lazzarin, Afshar and Berto, 2012).

t (mm)	d (mm)	t/d	K_1 MPa $mm^{0.4878}$	Shape factor k_i
0.250	10	0.025	46.664	0.9176
0.625	25	0.025	72.962	0.9176
2.5	100	0.025	143.476	0.9176
10	400	0.025	282.140	0.9176
25	1000	0.025	441.144	0.9176

Following the guidelines given in (Lazzarin and Tovo, 1996), scaling the geometrical dimensions by a factor of "n", the NSIF value can be obtained by using the following equation:

$$K_{i,b} = K_{i,a} \cdot n^{1-\lambda_i} \tag{11}$$

where the first index "i" is 1 or 2 according to the loading mode and the second index is a or b representing the former or later geometry.

Applying the similar concept to the periodic notches under mode I loading (Fig. 3) it is possible to give the ratio of the two NSIFs as follows:

$$\frac{K_{1,b}}{K_{1,a}} = \sqrt{\frac{\bar{W}_{1,b}}{\bar{W}_{1,a}}} = \left(\frac{h_b}{h_a}\right)^{1-\lambda_1} = \left(\frac{t_b}{t_a}\right)^{1-\lambda_1} = \left(\frac{W_b}{W_a}\right)^{1-\lambda_1} \tag{12}$$

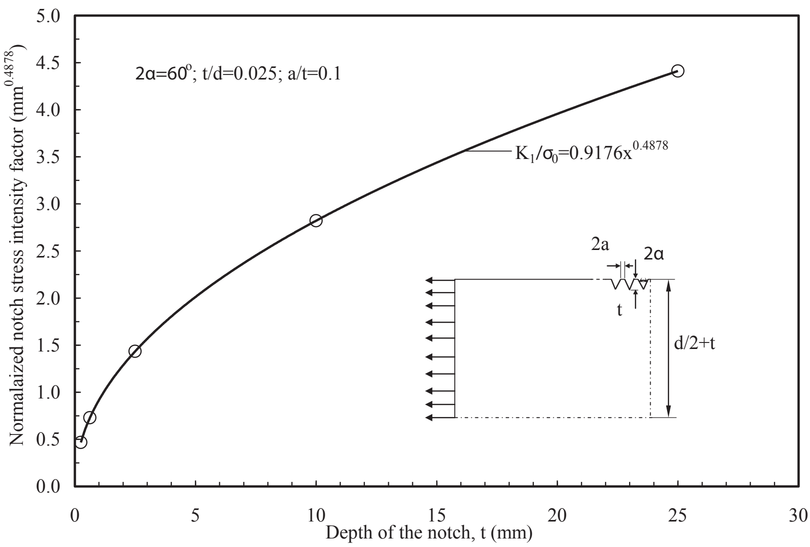

Figure 2: Normalized NSIFs as a function of notch depth with constant relative depth (t/d=0.025); plate with periodic notches with 2α=60^o and a/t=0.1 (scale effect) (Lazzarin, Afshar and Berto, 2012).

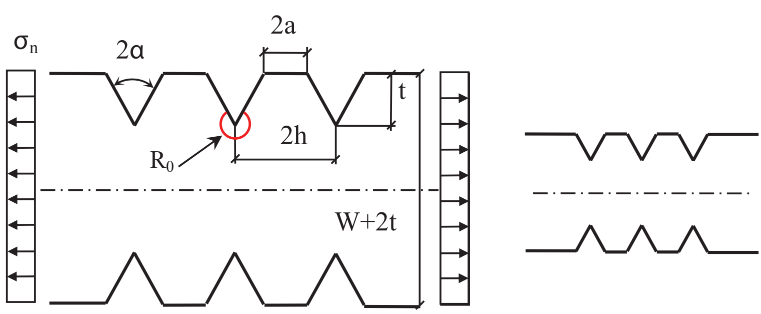

Figure 3: Scaling of threaded plate with double symmetric notches.

According to Eq. (12) there is the need to redefine K_0 according to the following expression to re-establish non dimensionality of k_0 and to obtain an expression very similar to Eq. (3) being in this case the spacing between the notches, h, the main parameter:

$$K_0 = k_0\ \sigma_n h^{1-\lambda_1} \tag{13}$$

The non-dimensional parameter k_0 will be provided for a large range of periodic notches from the narrow to the deep case. Differently from Eq. (5) which provides a κ_0 equal to 1 for the opening angles ranging from 0° to 90°, the new defined k_0 will be substantially different from 1 just when the case 2α=90° will be considered.

By plotting Eq. (9) an extended version of the Tada et al. (Tada, Paris and Irwin, 1985) diagram is obtained for double symmetric notches and edge notches (Berto, Lazzarin and Afshar, 2012). The trend of the NSIFs for a threaded plate with sharp notches in an infinite plate (t/W=0.01) is shown in Fig. 4 (shallow notches). On the left hand side of the diagram the notches are very far from each other and the results collapse into the case of a single notch. On the other hand, on the right side the diagram matches the case of narrow notches (a=0).

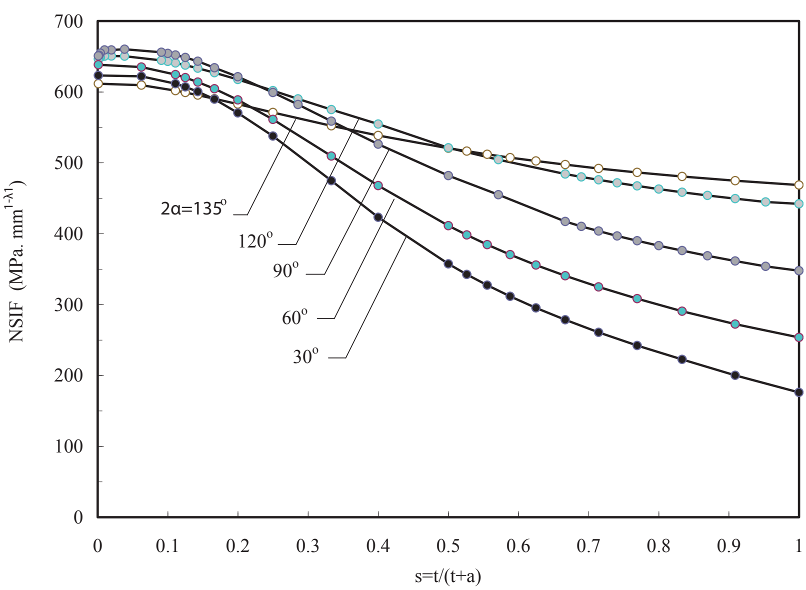

Figure 4: New generalised diagram for edge and double symmetric periodic notches (Berto, Lazzarin and Afshar, 2012).

Fig. 5 shows the variation of the dimensionless NSIF as a function of the notch opening angle varying the relative distance a/t. Here the relative notch depth assumes its minimum values, t/W=0.025, in order to make possible a comparison in engineering terms with the results reported in Ref. (Savruk and Kazberuk, 2008), all related to the case of periodic notches in a semi-infinite plate (a/t→0).

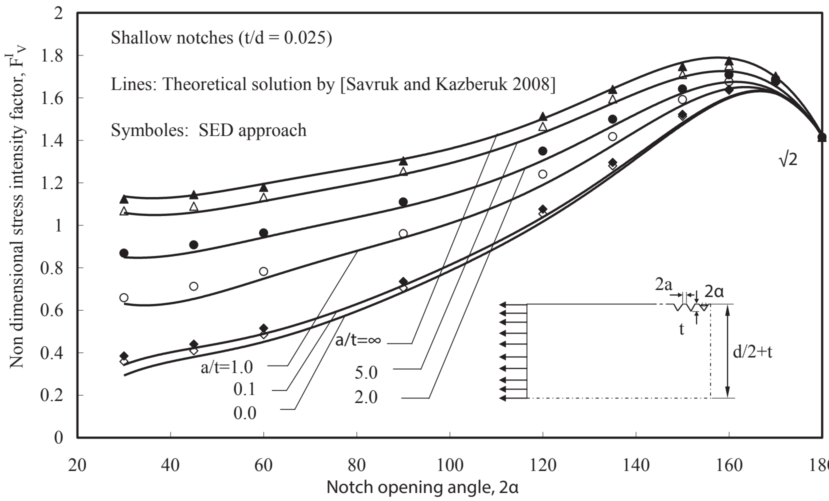

Figure 5: Variation of dimensionless NSIF with notch opening angle for the modes with t/d=0.025; comparison with data reported in Ref. (Savruk and Kazberuk, 2008) for periodic notches in infinite plate (Lazzarin, Afshar and Berto, 2012).

It can be observed from Fig. 5 that there is a very good agreement between the present results and those reported in the literature for the infinite plate problem. The mean relative deviations for different distances between periodic notches with a/t=0.0, 0.1, 1.0, 2.0, 5.0 and ∞ are: 3.81±6.89, 3.54±3.23, 3.07±0.70, 2.39±0.35, 1.62±0.48 and -0.3±0.29, respectively. It is clear that the relative deviations decrease as the notch opening angle and the a/t ratio increases.

5 Conclusions

By using the SED concept combined with a coarse mesh in the FE analysis, fatigue strength assessment of some welded components is addressed .The NSIF of a narrow threaded plate with an infinite width is only a function of the notch spacing raised to William's eigenvalue. The equation gives accurate results when the ratio between the notch depth and the plate width is lower than 0.025 and collapses into Tada et *al*., solution in the crack case. The problem of the NSIF ratio of two scaled geometries of periodic sharp notches is also addressed.

References

Afshar, R.; F. Berto (2011): Stress concentration factors of periodic notches determined from the strain energy density. *Theoretical and Applied Fracture Mechanics,* vol. 56, pp. 127-139.

Berto, F.; P. Lazzarin (2009): A review of the volume-based strain energy density approach applied to V-notches and welded structures. *Theoretical and Applied Fracture Mechanics,* vol. 52, pp. 183-194.

Berto, F.; P. Lazzarin;R. Afshar (2012): Simple new expressions for the notch stress intensity factors in an array of narrow V-notches under tension. *International Journal of Fracture,* vol. 176, pp. 237-244.

Cendón, D. A.; J. C. Gálvez; M. Elices; J. Planas (2000): Modelling the fracture of concrete under mixed loading. *International Journal of Fracture,* vol. 103, pp. 293-310.

Cisilino, A. P.; M. H. Aliabadi (2010): Crack growth modeling for mixed-mode problems. *Structural Durability and Health Monitoring,* vol. 6, pp. 213-238.

Dunn, M. L.; W. Suwito; S. Cunningham (1997): Fracture initiation at sharp notches: Correlation using critical stress intensities. *International Journal of Solids and Structures,* vol. 34, pp. 3873-3883.

Dunn, M. L.; W. Suwito; S. Cunningham (1997): Stress intensities at notch singularities. *Engineering Fracture Mechanics,* vol. 57, pp. 417-430.

Gálvez, J. C., et al. (2012): An embedded cohesive crack model for finite element analysis of quasi-brittle materials. *Engineering Fracture Mechanics*.

Lazzarin, P.; R. Afshar; F. Berto (2012): Notch stress intensity factors of flat plates with periodic sharp notches by using the strain energy density. *Theoretical and Applied Fracture Mechanics,* vol. 60, pp. 38-50.

Lazzarin, P.; F. Berto; F. J. Gomez; M. Zappalorto (2008): Some advantages derived from the use of the strain energy density over a control volume in fatigue strength assessments of welded joints. *International Journal of Fatigue,* vol. 30, pp. 1345-1357.

Lazzarin, P.; F. Berto; M. Zappalorto (2010): Rapid calculations of notch stress intensity factors based on averaged strain energy density from coarse meshes: Theoretical bases and applications. *International Journal of Fatigue,* vol. 32, pp. 1559-1567.

Lazzarin, P.; R. Tovo (1996): A unified approach to the evaluation of linear elastic stress fields in the neighborhood of cracks and notches. *International Journal of Fracture,* vol. 78, pp. 3-19.

Lazzarin, P.; R. Tovo (1998): A notch intensity factor approach to the stress analysis of welds. *Fatigue and Fracture of Engineering Materials and Structures,* vol. 21, pp. 1089-1103.

Lazzarin, P.; R. Zambardi (2001): A finite-volume-energy based approach to predict the static and fatigue behavior of components with sharp V-shaped notches. *International Journal of Fracture,* vol. 112, pp. 275-298.

Livieri, P.; P. Lazzarin (2005): Fatigue strength of steel and aluminium welded joints based on generalised stress intensity factors and local strain energy values. *International Journal of Fracture,* vol. 133, pp. 247-276.

Parker, A. P. (1999): Stability of arrays of multiple edge cracks. *Engineering Fracture Mechanics,* vol. 62, pp. 577-591.

Savruk, M.; A. Kazberuk (2008): A plane periodic boundary-value problem of elasticity theory for a half-plane with curvilinear edge. *Materials Science,* vol. 44, pp. 461-470.

Sih, G. C. (1973): Energy-density concept in fracture mechanics. *Engineering Fracture Mechanics,* vol. 5, pp. 1037-1040.

Sih, G. C. (1973): Some basic problems in fracture mechanics and new concepts. *Engineering Fracture Mechanics,* vol. 5, pp. 365-377.

Sih, G. C. (1974): Strain-energy-density factor applied to mixed-mode crack problems. *International Journal of Fracture,* vol. 10, pp. 305-321.

Sih, G. C. (1991): *Surface and volume energy density applied as failure criterion*. Dordrecht, Kluwer Academic Publisher.

Sih, G. C. (2007): *Multiscaling in molecular and continuum mechanics: interaction of time and size from macro to nano*, Springer.

Sih, G. C.; J. W. Ho (1991): Sharp notch fracture strength characterized by critical energy density. *Theoretical and Applied Fracture Mechanics,* vol. 16, pp. 179-214.

Sih, G. C.; X. S. Tang (2005): Scaling of volume energy density function reflecting damage by singularities at macro-, meso- And microscopic level. *Theoretical and Applied Fracture Mechanics,* vol. 43, pp. 211-231.

Tada, H.; P. C. Paris; G. Irwin (1985): *The stress analysis of cracks handbook*. St Louis, Paris Productions Incorporated and Del Research Corporate.

Tang, X. S.; G. C. Sih (2005): Weak and strong singularities reflecting multiscale damage: Micro-boundary conditions for free-free, fixed-fixed and free-fixed constraints. *Theoretical and Applied Fracture Mechanics,* vol. 43, pp. 5-62.

Williams, M. L. (1952): Stress singularities resulting from various boundary conditions in angular corners of plates in extension. *J. Appl. Mech.,* vol. 19, pp. 526-528.