 SDHM, vol.8, no.1, pp.31-44, 2012

Application of the Fictitious Notch Rounding Approach to Notches with End-Holes under mode 2 Loading

F. Berto[1], **P. Lazzarin**[1] **and D. Radaj**[2]

Abstract: The fictitious notch rounding concept is applied here for the first time to V-shaped notches with root hole subjected to in-plane shear loading. The fictitious notch radius ρ_f is determined as a function of the real notch radius ρ, the microstructural support length ρ^* and the notch opening angle 2α, at first using the normal stress criterion in combination with the maximum tangential stress criterion for finding the crack propagation angle. An analytical method has been developed resulting in closed form expressions for the multiaxiality factor s in the well known relationship ρ_f=ρ+$s\rho^*$. It expresses the notch stress averaged over the inclined microstructural support length at pointed V-notches by the maximum notch stress of fictitiously rounded V-notches (with root hole), taking advantage of a nearly developed analytical solution. Different failure criteria are considered in combination with two criteria defining the crack propagation angle: maximum tangential stress and minimum strain energy density. Plane stress and plane strain conditions, respectively, are evaluated.

Keywords: Fictitious notch rounding, V-notches with root hole, in-plane shear loading, microstructural support.

1 Introduction

This work extends the fictitious notch rounding (FNR) concept to in-plane shear loading conditions and provides a closed form solution to the problem. This concept takes into account that the theoretical maximum notch stress does *not* characterise the static strength or fatigue strength of pointed or sharply rounded notches (Neuber, 1958, 1968, 1985). The decisive parameter is the notch stress averaged over a short radial distance at pointed notches or over a small distance normal to the notch edge at rounded notches (notch radius ρ). This distance ρ^* is termed 'microstructural support length' (MSL). In the high-cycle fatigue regime, the notch

[1] Department of Management and Engineering, University of Padova, Vicenza (Italy)
[2] Department of Mechanical Engineering, Technical University at Braunschweig (Germany)

stress averaging path should coincide with the initial fatigue crack propagation path. The idea behind the concept FNR is to determine the fatigue-effective notch stress directly (i.e. without notch stress averaging) by performing the stress analysis with the fictitiously enlarged notch radius, ρ_f given by

$$\rho_f = \rho + s\rho* \tag{1}$$

Using the analytic frame given by Filippi, Lazzarin and Tovo (2002) and Neuber (1958), the FNR approach was applied to V-notches subjected to mode 1 and mode 3 loading, respectively (see Berto, Lazzarin and Radaj (2008, 2009)). The factor s, which quantifies multiaxiality effects, was found to be highly dependent on the notch opening angle 2α, resulting in a similar variability of ρ_f. A good correspondence was found between the theoretical stress concentration factors $K_t(\rho_f)$ (evaluated at the fictitiously rounded notch) and the effective stress concentration factors (obtained by integrating the relevant stress over the distance ρ* in the bisector line of the pointed V-notch).

The notch stress averaging method was originally proposed for static loading and brittle fracture by Wiegardt (1907) and later on extended by Weiss (1971). The concept was introduced into notch stress considerations by Neuber (1936) and later on substantiated for fatigue loading (Neuber, 1968) and static loading (Neuber, 1985). There is a vague correspondence to the 'critical distance approach' proposed by Peterson (1950), which was applied many years later to notched thin plates subjected to fatigue loading (Atzori, Lazzarin, Tovo, 1992 and Lazzarin, Tovo, Meneghetti, 1997) using the characteristic length a_0 derived by El-Haddad,Smith and Topper (1979) from the conventional endurance limit and the threshold stress intensity factor.

Referring to Neuber's basic work, Radaj (1969, 1990, 2003) and Radaj, Sonsino and Fricke (2006) proposed to assess the high-cycle fatigue strength of welded joints (toe or root failures) based on the FNR concept. A worst case assessment for low-strength steels, notch radius ρ = 0 mm, MSL ρ* = 0.4 mm and factor s= 2.5, resulted in ρ_f= 1.0 mm. This radius was found to be generally applicable to welded joints in structural steels and aluminium alloys and has become a standardised procedure within the design recommendations of the International Institute of Welding (IIW) (Hobbacher, 2009).

The present work is related to in-plane shear loading (mode 2) which is more complex to analyse than mode 1 and mode 3 loading. The reason for the higher complexity is the fact that out-of-bisector crack propagation occurs, because the maximum notch stress occurs outside the notch bisector line. The mode 2 problem was considered from a theoretical point of view resulting in closed-form solutions for elliptical notches (Radaj and Zhang, 1993). In a recent paper, pointed V-notches subjected to pure mode 2 loading were investigated (Berto and Lazzarin, 2010). Due to the complexity of the analytical developments, the factor s was determined there numerically using the FE method. For the pointed V-notches, the first problem was the choice of the path direction for stress averaging over the MSL. Two criteria available from the literature were used to determine the angle of most probable crack propagation, the maximum tangential stress (MTS) criterion according to Erdogan-Sih (1963) and the minimum strain energy density (MSED) criterion according to Sih (1974). In cracked plates subjected to in-plane shear loading or to superimposed in-plane tensile and shear loading, the crack grows in the radial direction perpendicular to the maximum tangential stress according to the MTS criterion. The MSED criterion determines the crack growth direction from the minimum value of Sih's strain energy density factor S around the so-called 'core region' surrounding the crack tip. The factor S is the strain energy density multiplied a distance from the point of singularity. Failure is thought of as controlled by a critical value of S, whereas the direction of crack propagation is determined by imposing a minimum condition on S. When considering rounded notches in the present paper, the minimum strain energy density will be evaluated at the notch edge.

The FNR concept is applied to V-shaped pointed notches and notches with root hole subjected to in-plane shear loading (see Berto, Lazzarin and Radaj, 2012). An analytical method has been developed for determining the fictitious notch radius ρ_f and therefrom the factor s dependent on the notch opening angle 2α.

2 Pointed versus rounded V-Notch with root hole

The factor s is found from integrating the normal stress along $\rho*$ in the crack propagation direction β determined by the MTS criterion and from equating the averaged stress at the free edge of the V-notch root hole, Fig. 1.

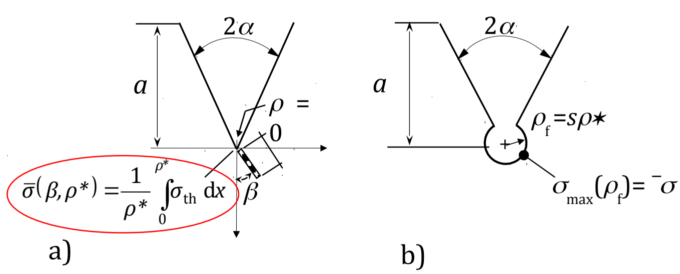

Figure 1: Fictitious notch rounding concept applied to in-plane shear loading: real notch with stress averaged over $\rho*$ (a) and substitute root hole notch with fictitious notch radius ρ_f producing $\sigma_{max} = \bar{\sigma}$ (b).

Considering mode 2 loading, the stress field of pointed V-notches can be expressed as follows (Lazzarin and Tovo, 1996):

$$\begin{bmatrix}\sigma_\theta\\ \sigma_r\\ \tau_{r\theta}\end{bmatrix} = \frac{1}{\sqrt{2\pi}}\frac{r^{\lambda_2-1}K_2}{(1-\lambda_2)+\chi_2(1+\lambda_2)}\begin{bmatrix}-(1+\lambda_2)\sin(1-\lambda_2)\theta\\ -(3-\lambda_2)\sin(1-\lambda_2)\theta\\ (1-\lambda_2)\cos(1-\lambda_2)\theta\end{bmatrix} + \chi_2(1+\lambda_2)\begin{bmatrix}-\sin(1+\lambda_2)\theta\\ \sin(1+\lambda_2)\theta\\ \cos(1+\lambda_2)\theta\end{bmatrix} \quad (2)$$

where K_2 is the mode 2 generalised stress intensity factor defined according to Gross and Mendelson (1972) and λ_2 is Williams' eigenvalue (Williams, 1952) whereas χ_2 is an auxiliar parameter related to the notch opening angle 2α. The parameters of Eq. (2) are summarised in Tab. 1.

Considering the MTS criterion, the equation to determine the crack propagation angle β is obtained from $d\sigma_\theta/d\theta = 0$, resulting in:

$$(1+\lambda_2)\chi_2\cos(1+\lambda_2)\theta - (\lambda_2-1)\chi_2\cos(1-\lambda_2)\theta = 0 \quad (3)$$

The solution of Eq. (3) gives the angle β dependent on the notch opening angle 2α, Tab. 1.

Considering $\sigma_{th} = \sigma_\theta$ along the MTS direction and integrating σ_{th} from 0 to $\rho*$, the expression for the averaged stress is:

$$\bar{\sigma} = \frac{1}{\rho*}\int_0^{\rho*}\sigma_{\mathrm{th}}\,\mathrm{d}x = -K_2\frac{(1+\lambda_2)\left[\chi_2\sin\left(\beta(1+\lambda_2)\right)+\sin\left(\beta(1-\lambda_2)\right)\right]}{\lambda_2\sqrt{2\pi}\left[1+\lambda_2(\chi_2-1)+\chi_2\right]}(\rho^*)^{\lambda_2-1} \quad (4)$$

The maximum principal stress along the free edge of the root hole of a V-notch can be determined from a newly developed theoretical solution by Zappalorto and Lazzarin (2011).

The stress component σ_θ around V-notches with root hole subjected to the anti-symmetric loading, mode 2, is:

$$\sigma_\theta = A_2 r^{\lambda_2-1}\left\{\sin(1-\lambda_2)\theta\left[(\lambda_2+1)-\tilde{\psi}_{21}(\theta)\left(\frac{\rho}{r}\right)^{2\lambda_2}-\tilde{\psi}_{22}(\theta)\tilde{\chi}_{22}(\theta)\left(\frac{\rho}{r}\right)^{2\lambda_2+1}\right] + \phi_2\sin(1+\lambda_2)\theta\left[1+(1-\lambda_2)\left(\frac{\rho}{r}\right)^{2\lambda_2}+(2+\lambda_2)\left(\frac{\rho}{r}\right)^{2(\lambda_2+1)}\right]\right\} \quad (5)$$

where the auxialiar parameter ϕ_2 depends on the notch opening angle 2α, Tab. 2. The closed-form expressions for the other auxiliary terms depend also on the polar co-ordinate θ (Zappalorto and Lazzarin, 2011):

$$\tilde{\psi}_{21}(\theta) = [(1-\lambda_2)\sin(2\lambda_2 - 1)\theta - 2\cos\lambda_2\theta\sin(1-\lambda_2)\theta] / \sin\theta \tag{6}$$

$$\tilde{\psi}_{22}(\theta) \times \chi_{22}(\theta) = 2(2-\lambda_2) / \left[\frac{\tan(\lambda_2 - 1)}{\tan\lambda_2} - 1\right] \tag{7}$$

Table 1: Generalised stress intensity factors K_2 [MPa$\,(\mathrm{mm})^{1-\lambda_2}$] of pointed V-notches in rectangular plate subjected to in-plane shear loading and parameters of the relevant stress fields; crack propagation angle β according to MTS criterion with notch depth a.

2α	β	λ_2	χ_2	K$_2$ a=50 mm)	K$_2$ a=10 mm)
0°	70.56°	0.500	1.000	1220	560
30°	65.23°	0.598	0.921	1070	578
45°	62.42°	0.660	0.814	951	570
60°	59.60°	0.731	0.658	800	541

Table 2: Parameters for determining the maximum tangential stress component σ_θ on the edge of V-notch with root hole; in-plane shear loading; data after Lazzarin, Zappalorto and Berto (2011) in converted form.

2α	$\bar{\theta}/\pi$	$\bar{\theta}$	ξ_θ	ϕ_2
0°	−0.392	−70.6°	4.042	1.500
30°	−0.344	−61.9°	4.000	1.472
45°	−0.329	−59.2°	3.997	1.351
60°	−0.313	−56.3°	3.951	1.140

By imposing the condition $d\sigma_\theta/d\theta = 0$, it is possible to obtain the maximum stress on the notch edge in the following form (after Lazzarin, Zappalorto and Berto, 2011):

$$\sigma_{\max} = \frac{\xi_\theta}{\sqrt{2\pi}\,\rho_{\mathrm{f}}^{1-\lambda_2}} \tag{8}$$

where ξ_θ has a closed form expression. Here, for the sake of brevity, only some values of the parameter ξ_θ are given in Tab. 2 as a function of the notch opening angles 2α investigated in the present paper. By setting the maximum stress on the root hole edge equal to $\bar{\sigma}$ derived for the pointed V-notch, the following equation is obtained:

$$-\frac{(1+\lambda_2)\,(\rho*)^{\lambda_2}\,(\chi_2\sin[\beta(1+\lambda_2)]+\sin[(1-\lambda_2)\beta])}{\lambda_2\sqrt{2\pi}\rho*(1+\lambda_2(-1+\chi_2)+\chi_2)}=\frac{1}{\sqrt{2\pi}\,(\rho_f)^{1-\lambda_2}}\,\xi_\theta \tag{9}$$

The fictitious notch radius ρ_f derived therefrom is:

$$\rho_f=\left(-\frac{(1+\lambda_2)(\chi_2\sin[\beta(1+\lambda_2)]+\sin[(1-\lambda_2)\beta])}{\xi_\theta\,\lambda_2\,(1+\lambda_2(-1+\chi_2)+\chi_2)}\right)^{\frac{1}{-1+\lambda_2}}\rho* \tag{10}$$

where the first term with the parenthesis on the right hand side represents the factor *s* because of ρ_f = s ρ*. Some values of *s* from Eq. (10) are summarised in Tab. 3. The two equations for the stress concentration factor are as follows:

$$\bar{K}_t=\frac{\bar{\sigma}}{\tau_0}=\frac{1}{\rho*\tau_0}\int_0^{\rho*}\sigma_{th}\,\mathrm{dr} \tag{11}$$

$$\mathrm{K}_t\,(\rho_f)=\frac{\sigma_{\max}(\rho*,s)}{\tau_0} \tag{12}$$

The geometry used for the application of the FE method is shown in Fig. 2 (where *a*= 10 mm and *w* = 90 mm). The results in terms of and $\bar{K}_t$ are compared in terms of the relative deviation Δ defined as follows:

$$\Delta=\frac{K_\mathrm{t}\,(\rho_\mathrm{f})-\bar{K}_\mathrm{t}}{K_\mathrm{t}\,(\rho_\mathrm{f})} \tag{13}$$

For determining the averaged stress $\bar{\sigma}$ from the theoretical (equivalent) stress σ_{th}, mainly the normal stress (Rankine) criterion is used, which gives identical results under plane stress and plane strain conditions. When combined with the MTS criterion, the stress σ_{th} = σ_θ is integrated because this is the maximum principal stress ($\tau_{r\theta}$ = 0). On the other hand, when combined with the MSED criterion, the first principal stress has to be evaluated:

$$\sigma_\mathrm{th}=\sigma_1=\frac{\sigma_\theta+\sigma_\mathrm{r}}{2}+\sqrt{\left(\frac{\sigma_\theta-\sigma_\mathrm{r}}{2}\right)^2+(\tau_{\mathrm{r}\theta})^2} \tag{14}$$

For determining the crack propagation angle β at pointed V-notches, the MTS criterion is used with priority. The MTS occurs where dσ_θ/dθ = 0. The condition

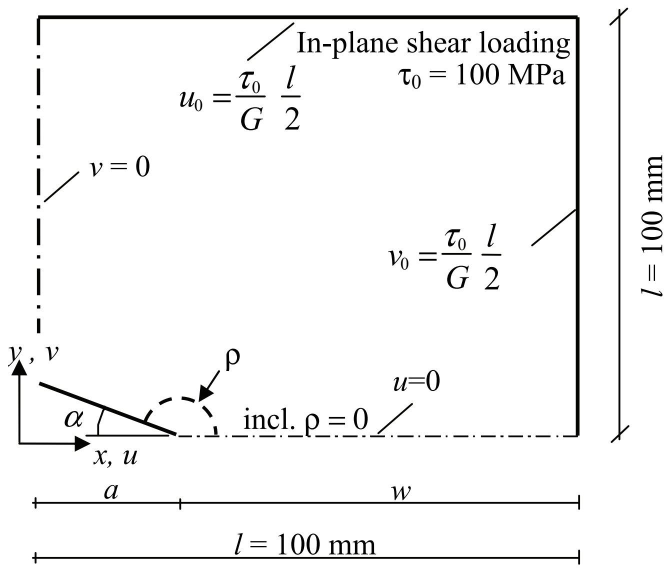

Figure 2: Geometry and dimensions of the V-notched plate specimen (symmetry quarter) considered in the FE analyses; remote loading by prescribed edge displacements.

above is represented by Eq. (3). There is no difference in β under plane stress and plane strain conditions. In some cases, the MSED criterion is additionally evaluated. Here, slight differences in β occur between plane stress and plane strain conditions. The crack propagation angle β is obtained from dW/dθg 0, with W being the strain energy density. The following two equations are derived for plane stress and plane strain conditions, respectively:

$$(1+\lambda_2)(1+\nu)\chi_2 \sin(2\theta) + (\nu - 1)\sin 2(1-\lambda_2)\theta = 0 \text{ plane stress} \tag{15}$$

$$(1+\lambda_2)\chi_2 \sin(2\theta) + (2\nu - 1)\sin 2(1-\lambda_2)\theta = 0 \text{ plane strain}$$

The values of β according to the MSED criterion are listed in Tab. 3.

3 Validation of the FNR approach

The FNR approach applied to in-plane shear loading conditions is validated by comparisons based on the FE method considering pointed V-notches with different notch opening angles. Geometry and dimensions of the V-notched plate are shown in Fig. 2 together with the applied remote boundary conditions. The values of the

Table 3: Crack propagation angle β according to the MSED criterion for different notch opening angles 2α; in-plane shear loading.

2α	β (plane strain)	β (plane stress)
0°	82.38°	79.70°
30°	82.82°	80.39°
45°	82.91°	80.54°
60°	82.91°	80.57°

generalised stress intensity factor K_2 for the considered geometries are reported in Tab. 1 for different notch opening angles. The following parameters are considered in the numerical investigation:

- The normal stress criterion (Rankine) for $\bar{\sigma}$ in combination with the MTS criterion for β,
- the notch opening angles $2\alpha = 0$, 30, 45 and 60°,
- the MSL $\rho*$ varying between 0.01 mm and 0.4 mm.

As the first step of the evaluation, the crack propagation angle β has been determined according to the MTS criterion, which gives the same angle under plane stress and plane strain conditions. The values of βgepend on the notch opening angle and vary from $\beta = 70.5°$ for $2\alpha = 0°$ to $\beta = 59.6°$ for $2\alpha = 60°$, see Tab. 1.

It was shown by the numerical investigation that the values of s found based on the MTS criterion for β in combination with the normal stress criterion for $\bar{\sigma}$ can be used with sufficiently small errors in a limited range of $\rho*$ values. This validation implies the generality of the values of s summarised in Tab. 4, and it is of major importance for the applicability of the method proposed when considering different materials. To this end, a large number of FE analyses have been performed. Different values of $\rho*$ have been considered, with $\rho*$ ranging from 0.05 to 0.4 mm.

The validation is performed with restriction to pointed notches.

In Tab. 6, the values of the stress concentration factors, K_t (ρ_f) versus $\bar{K}_t$, for values of $\rho*$ ranging from 0.05 to 0.4 mm, are summarised.

The results in the table are discussed based on the stated Δ values:

- Positive and negative Δ values occur with approximately the same frequency.

- The Δ values change continuously between small and large $\rho*$ values with a tendency of larger Δ values for larger $\rho*$ values.

Table 4: Factor s for pointed V-notches from according to Method 1; different failure criteria, plane stress and plane strain conditions; β values determined according to MTS criterion; in-plane shear loading.

	Factor s				
2α	Normal stress	von Mises plane stress	von Mises plane strain	Beltrami plane stress	Beltrami plane strain
0°	3.06	3.06	3.88	3.06	3.37
30°	6.34	5.05	5.93	5.44	5.83
45°	11.87	7.60	9.27	9.02	9.62
60°	33.03	17.75	20.44	21.03	22.43

Table 5: Factor s for pointed V-notches according to Method 1; different failure criteria, plane stress and plane strain conditions; values of β determined according to MSED criterion; in-plane shear loading.

	Factor s					
2α	Normal Str. Pl.stress	Normal str. Pl. strain	von Mises Pl.stress	von Mises Pl. strain	Beltrami Pl.stress	Beltrami Pl. strain
0°	3.03	2.99	3.31	4.75	3.17	3.60
30°	6.15	6.12	5.97	8.10	6.01	6.75
45°	11.32	11.31	10.19	13.87	10.54	11.89
60°	31.00	31.29	25.17	35.61	26.82	30.79

4 Application of the method combined with different failure criteria

With the aim to complete the present investigation on pointed V-notches with root hole, two additional failure criteria have been considered. In particular the distortional strain energy criterion (von Mises) and the total strain energy criterion (Beltrami) have been applied both under plane stress and plane strain conditions in addition to the normal stress criterion discussed in the previous sections. The MTS and MSED criteria have been used again to determine the crack propagation angle at the pointed V-notch tip. In Tab. 4 and 5, the factor s for the different failure criteria with the angle β according to the MTS and the MSED criterion are summarised. The values related to the normal stress criterion are also repeated in order to facilitate comparisons. The great influence of the notch opening angle 2α on the value of s is evident. The Beltrami criterion under plane strain conditions in combination with the MSED criterion correlate best with the normal stress criterion in combination with the MTS criterion. The Beltrami values of s range from 3.60 (for $2\alpha = 0^\circ$) to 30.79 (for 2α= 60°) to be compared with the normal stress data

Table 6: FNR results according to the normal stress failure criterion combined with the MTS criterion for β; different values of $\rho*$ and notch depth a;in-plane-shear loading.

Notch depth			a=10 mm		
	$\rho*$	ρ_f[mm]	$K_t(\rho_f)$	$\bar{K}_t$	Δ
$2\alpha=0^\circ$	0.05	0.153	22.97	23.07	-0.45
s=3.06	0.10	0.306	16.40	16.31	0.52
	0.20	0.612	11.80	11.54	2.23
	0.30	0.918	9.82	9.42	4.12
	0.40	1.224	8.68	8.16	6.07
$2\alpha=30^\circ$	0.05	0.32	14.67	14.63	0.25
s=6.40	0.10	0.64	11.14	11.07	0.58
	0.20	1.28	8.50	8.38	1.43
	0.30	1.92	7.32	7.12	2.77
	0.40	2.56	6.64	6.34	4.40
$2\alpha=45^\circ$	0.05	0.60	10.76	10.80	-0.36
s=11.97	0.10	1.20	8.50	8.53	-0.37
	0.20	2.39	6.75	6.74	0.15
	0.30	3.59	5.97	5.87	1.62
	0.40	4.79	5.53	5.32	3.64

ranging from 3.03 to 31.00.

In Tab. 7 and 8, results of the application of the FNR approach to the V-notched plate shown in Fig. 2, considering the notch depth a= 50 mm and the MSL $\rho* =$ 0.2 mm, are summarised. The stress concentration factors are shown as well as the relative deviation Δ. The correspondence between $K_t(\rho_f)$ and $\bar{K}_t$ is satisfactory in all cases, the maximum relative deviation amounting to 2.6%.

5 Conclusions

Based on FNR concept used in combination with the normal stress criterion for $\bar{\sigma}$ and the MTS criterion for β, the multiaxiality factor s has been evaluated for V-notches with root hole subjected to in-plane shear loading (mode 2). Taking advantage of a recently conceived analytical frame for V-notches with root holes, the original Neuber procedure for determining the fictitious notch radius ρ_f and the factor s was applied out-of-bisector, the propagation direction being determined by the MTS criterion.

The values of s for pointed V-notches have been found to be almost coincident and

Table 7: FNR concept combined with the MTS criterion applied with different failure criteria; ρ*=0.2 mm, *a*=50 mm and *w*=50 mm; in-plane shear loading. Abbreviations: vM von Mises criterion, B Beltrami criterion, ps plane stress, pn plane strain. Results for the normal stress criterion in Tab. 6.

			2α=0°		
ρ*= 0.2 mm	s	$\rho_f = s\rho^*$	$K_t(\rho_f)$	$\bar{K}_t$	Δ %
vM, ps	3.06	0.612	25.60	25.14	1.80
vM, pn	3.88	0.776	22.75	22.35	1.76
B, ps	3.06	0.612	25.60	25.14	1.80
B, pn	3.37	0.674	24.39	23.98	1.68
			2α=45°		
ρ* = 0.2 mm	s	$\rho_f = s\rho^*$	$K_t(\rho_f)$	$\bar{K}_t$	Δ %
vM, ps	7.60	1.520	13.02	12.84	1.38
vM, pn	9.27	1.854	12.16	12.23	-0.58
B, ps	9.02	1.804	12.26	12.35	-0.73
B, pn	9.62	1.924	12.00	12.08	-0.67

mainly dependent on the notch opening angle.

A number of FE analyses have been carried out in order to compare the theoretical stress concentration factor to the effective stress concentration factor, the former obtained by considering a fictitiously rounded notch using the new values of s, the latter obtained by evaluating the stress rise at the pointed V-notch over the MSL, aligned in the most critical direction. Different values of the microstructural support length, the notch depth and the notch opening angle have been considered. The obtained values of s are well suited for engineering usage in structural strength assessments.

Finally, the distortional strain energy criterion (von Mises) and the total strain energy criterion (Beltrami) have been applied under plane stress and plane strain conditions, in addition to the normal stress (Rankine) criterion. The MTS and MSED criteria have been used to determine the crack propagation angle at the pointed V-notch. The values of the multiaxiality factor s have been validated by means of a large amount of FE analysis results.

When considering the significance of the encouraging results presented in the paper, it should be kept in mind that FNR is a procedure well suited only for the engineers' preliminary strength assessments, because the effects of nearby boundaries, loading and support conditions, cross-sectional weakening among them, may deteriorate the results. As soon as FE models are available, direct notch stress averaging over the microstructural support length in the critical directions or strain energy density evaluation over a control volume should be preferred.

Table 8: FNR concept combined with the MSED criterion applied to different failure criteria; ρ*=0.2 mm, a=50 mm and W=50 mm; In-plane shear loading. Abbreviations: NS normal stress criterion, vM von Mises criterion, B Beltrami criterion, ps plane stress, pn plane strain

			$2\alpha=0^\circ$		
ρ*= 0.2 mm	s	ρ_f=sρ*	$K_t(\rho_f)$	$\bar{K}_t$	Δ %
NS, ps	3.03	0.606	25.71	25.25	1.79
NS, pn	2.99	0.598	25.88	25.31	2.20
vM, ps	3.31	0.662	24.61	24.17	1.79
vM, pn	4.75	0.950	20.59	20.17	2.04
B, ps	3.17	0.634	25.14	24.70	1.75
B, pn	3.60	0.720	23.61	23.19	1.78
			$2\alpha=45^\circ$		
ρ*= 0.2 mm	s	ρ_f=sρ*	$K_t(\rho_f)$	$\bar{K}_t$	Δ %
NS, ps	11.32	2.264	11.35	11.43	-0.71
NS, pn	11.31	2.262	11.35	11.43	-0.71
vM, ps	10.19	2.038	11.77	11.84	-0.60
vM, pn	13.87	2.774	10.57	10.66	-0.85
B, ps	10.54	2.108	11.63	11.71	-0.69
B, pn	11.89	2.378	11.16	11.24	-0.72

References

Atzori, B.; Lazzarin, P.; Tovo, R. (1992): Evaluation of the fatigue strength of a deep drawing steel. *Österr Ing. Arch. Z.* vol. 137, pp. 556-561.

Berto, F.; Lazzarin, P.; Radaj, D. (2008): Fictitious notch rounding concept applied to sharp V-notches: evaluation of the microstructural support factor for different failure hypotheses. Part I: Basic stress equations. *Eng. Fract. Mech.*, vol. 75, pp. 3060-3072.

Berto, F.; Lazzarin, P.; Radaj, R. (2009): Fictitious notch rounding concept applied to sharp V-notches: evaluation of the microstructural support factor for different failure hypotheses. Part II: Microstructural support analysis. *Eng. Fract. Mech.*, vol. 76, pp. 1151-1175.

Berto, F.; Lazzarin P. (2010): Fictitious notch rounding approach of pointed V-notch under in-plane shear. *Theor. Appl. Fract. Mech.*, vol. 53, pp. 127-135.

Berto, F.; Lazzarin, P.; Radaj, D. (2012): Fictitious notch rounding concept applied to V-notches with root holes subjected to in-plane shear loading, *Eng. Fract. Mech.*, vol. 79, pp. 281-294.

El Haddad, M.H.; Topper, T.H.; Smith, K.N. (1979): Prediction of non-propagating cracks. *Eng. Fract. Mech.*,vol. 11, pp. 573-584.

Erdogan, F.; Sih C.G. (1963): On the crack extension in plates under plane loading and transverse shear. *J. Basic Eng.*, vol. 85, pp. 519-525.

Filippi, S.; Lazzarin, P.; Tovo, R. (2002): Developments of some explicit formulas useful to describe elastic stress fields ahead of notches in plates. *Int. J. Solids Struct.*, vol. 39, pp. 4543-4565.

Gross, R.; Mendelson, A. (1972): Plane Elastostatic Analysis of V-notched plates. *Int. J. Fract. Mech.*, vol. 8, pp. 267-276.

Hobbacher, A. (2009): Fatigue Design of Welded Joints and Components. Abington Publishing, Cambridge, UK, 1996 (IIW Doc. XIII-1539/XV-845-96) and update 2008 (IIW Doc. XIII-2151-07/XV-1254-07). Welding Research Council, New York, Bulletin 520, 2009.

Lazzarin, P.; Tovo, R. (1996): A unified approach to the evaluation of linear elastic fields in the neighbourhood of cracks and notches. *Int. J. Fract.*, vol. 78, pp. 3-19.

Lazzarin, P.; Tovo, R.; Meneghetti, G. (1997): Fatigue crack initiation and propagation phases near notches in metals with low notch sensitivity. *Int. J. Fatigue*, vol. 19, pp. 647-657.

Lazzarin, P.; Zappalorto, M.; Berto, F. (2011): Generalised Stress Intensity Factors for rounded notches in plates under in-plane shear loading. *Int. J. Fract.*, vol. 170, pp. 123-144.

Neuber, H. (1936): Zur Theorie des technischen Formzalhl. *Forzschg Ing. Wes.*, Vol. 7, pp. 271-274.

Neuber, H. (1958): *Kerbspannungslehre*, 2nd ed. Berlin, Springer-Verlag.

Neuber, H. (1968): Über die Berücksichtigung der Spannungskonzentration bei Festigkeitsberechnungen. *Konstruktion*, vol. 20, pp. 245-251.

Neuber, H. (1985): *Kerbspannungslehre*, 3rd ed. Berlin, Springer-Verlag.

Peterson, R.E. (1950): Relation between stress analysis and fatigue of metals. Proc. SESA 11, 199-206.

Radaj, D. (1969): Näherungsweise Berechnung der Formzahl von Schweiβnähten. *Schw. Schn.*, vol. 21, pp. 97-105, and vol. 21, pp. 151-158.

Radaj, D. (1990): *Design and Analysis of Fatigue Resistant Welded Structures*, Cambridge, Abington Publishing.

Radaj, D.; Zhang, S. (1993): On the relations between notch stress and crack stress intensity in plane shear and mixed mode loading. *Eng. Fract. Mech.*, vol. 44, pp. 691-704.

Radaj, D. (2003): *Ermüdungsfestigkeit*, 2nd ed. Berlin, Springer Verlag.

Radaj, D.; Sonsino, C.M.; Fricke, W. (2006): *Fatigue Assessment of Welded Joints by Local Approaches*. 2nd ed. Cambridge, Woodhead Publishing; Boca Raton, F.L. C.R.C. Press.

Sih G.C. (1974): Strain-energy-density factor applied to mixed mode crack problems. *Int. J. Fract.*, vol. 10, pp. 305-321.

Weiss, V. (1971): Eine Bruchmechanik für Kerben. *Schweiz Arch. Angew Wiss. Techn.*, Vol. 37, pp. 1-7.

Wieghardt, K. (1907): Über das Spalten und Zerreiβen elastischer Körper. Z. *Mathematik u. Physik*, vol. 55, pp. 60-103; translated in English by Rossmanith, H.P. (1995), *Fatigue Fract. Engng Mater. Struct.*, vol. 18, pp. 1371-1405.

Williams, M.L. (1952): Stress singularities resulting from various boundary conditions in angular corners on plates in tension. *J. Appl. Mech.*, vol. 19, 526-528.

Zappalorto, M.; Lazzarin, P. (2011): In-plane and out-of-plane stress field solutions for V-notches with end holes. *Int. J. Fract.* vol. 168, pp. 167-180.