

Studies on Methodological Developments in Structural Damage Identification

V. Srinivas¹, Saptarshi Sasmal¹ and K. Ramanjaneyulu²

Abstract: Many advances have taken place in the area of structural damage detection and localization using several approaches. Availability of cost-effective computing memory and speed, improvement in sensor technology including remotely monitored sensors, advancements in the finite element method, adaptation of modal testing and development of non-linear system identification methods bring out immense technical advancements that have contributed to the advancement of modal-based damage detection methods. Advances in modal-based damage detection methods over the last 20-30 years have produced new techniques for examining vibration data for identification of structural damage. In this paper, studies carried out on damage identification methods using model- and nonmodel- based approaches have been presented describing their effectiveness in identification, localisation and quantification of damage. Usefulness of different parameters such as change in frequency, mode shape, modal curvature and strain energy for detection of damage has been studied. Further, advanced nonmodel based techniques have also been studied for damage identification. But, most of these techniques are found to have limitation thereby restricting their usage. Moreover, majority of the approaches need a prior knowledge on the vibration characteristics of undamaged structure which is quite difficult to get in most of the cases. It has been noted that majority of the methodologies show good results in laboratory or analytical investigations and special care is needed in choosing the methodology for damage detection of real structures in field condition.

Keywords: Damage identification, frequency, modeshape, transfer matrix, genetic algorithm, modal strain energy, neural networks.

¹ Scientist, Structural Engineering Research Centre (Council of Scientific & Industrial Research), CSIR Campus, TTTI(Post), Taramani, Chennai-600 113, Email: srinivas@sercm.org; saptarshi@sercm.org

² Deputy Director, Structural Engineering Research Centre (Council of Scientific & Industrial Research), CSIR Campus, TTTI(Post), Taramani, Chennai-600 113, Email: rams@sercm.org

1 Introduction

Structural damage identification is an objective tool for condition assessment of structures. By using structural damage identification and parameter estimation as means of determining the actual state properties, performance, and limit states of a structure, it is possible to gain an improved understanding of a structure's capacity and typical performance during its service. Thus, at any point of time, it would be possible to assess the safety of the structure using the objective results obtained through structural damage identification. The main objective of health monitoring is to detect, localize and quantify the level of structural damage to the civil infrastructure. The objective of the detection and identification of structural damage is to construct the qualitative or quantitative description of the deterioration in a physical structural system from the measured loads and the measured responses of the structure [Chou and Ghaboussi (2001)]. Most researchers have proposed methods for the detection of the existence of structural damage via monitoring the change in structural responses. But, it is generally more difficult to find the location and extent of the damage. Without using any a priori knowledge of modeling real systems, some studies were proposed to detect and identify the damage directly through the change of measured responses.

In recent years, number of works is being reported on damage identification techniques using different approaches which include static and dynamic measurement. It is observed that, the need for development of an efficient procedure for non-destructive structural damage detection is of utmost importance in order to assess the integrity and serviceability of existing structures. Perhaps, first research article on damage detection using vibration measurements was by Lifshitz and Rotem (1969) where the change in the dynamic moduli was related to the frequency shift and proposed as indicator of damage in particle-filled elastomers. Cawley and Adams (1979) are the first researchers to give a formulation for damage detection based on change in frequency of an undamaged and damaged state of a structure. Pape (1993) proposed a technique to identify damaged parts using statistical methods and measured natural frequencies. Slater and Shelley (1993) presented a method based on frequency-shift measurements and Narkis (1994) deduced a closed-form solution for the crack position. A transfer matrix technique was used by Choy, Liang and Xu (1995) to detect damage for beam like structures. Ratcliffe (1997) developed a technique for identifying the location of structural damage in a beam using modified Laplacian Operator on mode shape data. A sensitivity- and statistical- based method to localize structural damage by direct use of incomplete mode shapes was presented by Law, Shi and Zhang (1998) and Shi, Law and Zhang (2000). A numerical study of damage detection using the relationship between damage characteristics and the changes in the dynamic properties was presented

by Abdo and Hori (2002). It was found that the rotation of mode shape is a sensitive indicator of damage localisation. Raghuprasad, Lakshmanan, Gopalakrishnan and Muthumani (2008) presented the formulations for eigen-value sensitivity equations derived from first order perturbation technique for typical structures such as simply supported bridge girder and building frame. Also, they have demonstrated the neural network based damage identification using the known-pairs of damage frequency vector as a trained data set.

Advances in the relative ease of instrumentation and development of new powerful system identification techniques, the nondestructive identification of structural damage through the changes in their vibration characteristics has gained an increasing world-wide attention over the past few years. The analysis categories for damage identification in structures include changes in modal frequency [Cawley and Adams (1979); Salawu (1997); Juneja, Haftka and Cudney (1997)], changes in measured mode shapes and their derivatives [Kam and Lee (1992); Salawu and Williams (1994); Pandey, Biswas and Samman (1991); Pandey and Biswas (1994); Ratcliffe (1997); Stubbs, Kim and Topole (1992)], modal strain energy [Dong, Zheng, Feng and Huang (1994)], changes in measured flexibility coefficients [Pandey and Biswas (1994)] and frequency response function [Rad (1997)] etc. Sinou (2007) proposed a method to identify the size, location and orientation of crack by finding the intersection of the surfaces that correspond to the natural frequency ratios of the lower vertical and horizontal modes obtained from FRFs of the cracked structure. To minimize the discrepancy between the mathematical model and real structural system, several types of formulations have been proposed in structural damage detection and identification problems. Equation error and output error approaches [Hajela (1990)] formulate the problem as an optimization problem. Brasiliano, Souza, Doz and Brito (2008) used the residual error method to identify and quantify the damage in a concrete beam based on the alterations produced by damage in the dynamic properties. A damage identification technique to detect internal damages in large composite structures was proposed by Minakuchi, Mizutani, Tsukamoto, Nishio, Okabe and Takeda (2008) based on Brillouin spectral response acquired from strain profile obtained by running an optical fibre network throughout the structure. Another class of methods, called as model improvement methods, modifies the initial system model so that it can represent the accurate behavior of the current structure. Examples of model improvement methods are optimal matrix modification methods [Kim and Bartkiewicz (1993)], minimum-rank perturbation methods [Doebbling (1996)], sensitivity-based update methods [Doebbling, Hemez, Peterson and Farhat (1997)], and eigenstructure assignment techniques [Cobb and Liebst (1997)].

Recently, artificial intelligence methods have been applied for structural damage

detection and identification. Unlike the mathematical methods, the important characteristics of artificial intelligence methods are their effectiveness and robustness in coping with uncertainty, insufficient information, and noise. In system identification, optimization techniques are used to match the responses of finite element model with that of the damaged structure. Many researchers have recently attempted to solve optimisation problem using neural networks and genetic algorithms [Au, Cheng, Tham and Bai (2003); Barai and Pandey (1995); Choy, Liang and Xu (1995); Friswell, Penny and Garvey (1998); Hao and Xia (2002); Luh and Wu (1999); Mares and Surace (1996); Ricles and Kosmatka (1992); Tsou and Shen (1994)] by studying the variation of localised damage as a function of modal data.

From the above, it is observed that there is a growing awareness in the area of structural damage identification. Number of researchers proposed several methods using different parameters, but many of the methods are restricted by their merits and demerits. Moreover, majority of the approaches need a prior knowledge on vibration characteristics of undamaged structure which is quite difficult to get in most of the cases. Hence, in the present work, studies have been carried out for identification of damage in structures using different traditional and advanced computational methods by using various damage parameters.

2 Structural damage identification

For a multi-degree-of-freedom undamped linear dynamic system, the equation of motion is

$$M\ddot{y}(t) + Ky(t) = X(t) \quad (1)$$

where, M is the system mass matrix and K is the system stiffness matrix with initial conditions $y(t)=0$ and $\dot{y}(t)=0$. Both mass and stiffness matrices are of the order $(n \times n)$. $y(t)$ and $X(t)$ are the physical displacement and applied load vectors of order $(n \times 1)$ respectively, where n is the number of degrees of freedom. The associated j -th eigen value equation is

$$K\phi_j - \lambda_j M\phi_j = 0 \text{ for } j = 1, \dots, m_u \quad (2)$$

where, ϕ_j is eigen vector (mode shape) and λ_j is eigenvalue (natural frequency) of the structure; and m_u is the total number of mode shapes obtained for the undamaged structure. In the finite element model of the structure, the global stiffness matrix can be represented as an assemblage of element stiffness matrices, i.e.,

$$K = \sum_{i=1}^m k_i \quad (3)$$

where k_i represents the stiffness matrix of the i -th element; m is the total number of elements and ‘ Σ ’ represents the assembly of elemental stiffness matrices based on nodal connectivity and the associated degrees of freedom. Structural damage often causes a loss of stiffness in one or more elements of a structure, but without any loss in the mass. In the theoretical development, the skeletal structure is modeled in an FEM, and damage is assumed to affect only the stiffness matrix of the system. Small changes in the stiffness of the system produce small changes in the square of the modal frequencies λ_i and the mode shapes ϕ_i . When damage occurs in the structure, it can be represented as a small perturbation in the original system. Thus, the stiffness matrix K^d , the i -th modal eigenvalue λ_i^d , and the i -th mode shape ϕ_i^d of the damaged system can be expressed as

$$K^d = K + \sum_{j=1}^L \Delta K_j = K + \sum_{j=1}^L \alpha_j K_j \quad (-1 < \alpha_j \leq 0) \quad (4)$$

$$\lambda_i^d = \lambda_i + \Delta \lambda_i \quad (5)$$

$$\phi_i^d = \phi_i + \Delta \phi_i \quad (6)$$

where superscript d denotes the damaged case, α_j is the coefficient defining a fractional reduction in the j -th elemental stiffness matrices; and L is the total number of elements in the system. In other way, the stiffness matrix of the damaged structure [K^d] can be represented as an assemblage of element stiffness matrices multiplied by reduction factors β_j ($j=1,2,\dots,m$) associated with each of the ‘ m ’ elements, i.e.,

$$K^d = \sum_{j=1}^m \beta_j k_j \quad (7)$$

The values of the parameters β_j fall in the range between 0 and 1; the value of unity for a particular element would indicate that the element is undamaged whereas a damaged element would cause the stiffness reduction factor for that location to take a fractional value or zero. If it is assumed that the experimental natural frequencies and mode shapes of the damaged structure continue to satisfy the eigenvalue equation, the i -th mode of the damaged structure can be rewritten as

$$K^d \phi_i^d - \lambda_i^d M \phi_i^d = 0 \quad (8)$$

where, λ_i^d and ϕ_i^d are the experimentally determined eigenvalue and eigenvector corresponding to the i -th mode of damaged structure. Furthermore, the stiffness matrix is directly affected by the damage, whereas the mass matrix M is assumed

to be unaltered. The equation (8) can be satisfied by knowing the damaged stiffness matrix of the actual structure, and incorporating the identified eigenvalues and eigenvectors obtained from the damaged structure. The damaged stiffness matrix can be calculated by identifying the stiffness reduction factors for the elements.

3 Model based approaches (Traditional methods)

From the review of literature, it is found that the vibration data such as frequency and mode shape are very important parameters for detecting the damage in structure. Though, changes in mode shape are much more sensitive to local damage compared to changes in frequency, use of mode shape information is restricted because i) lower modes (usually measured from vibration tests of large structure) may not significantly reflect the damage which is a local phenomenon, ii) extracted mode shapes are prone to be affected by environmental noise and iii) number of sensors and the choice of sensor coordinate may have a crucial effect on accuracy of damage detection.

Many research publications can be found on various methods which were employed for structural damage identification of linear systems in frequency domain by comparing the changes in dynamic parameters. These methods are categorized based on the type of dynamic parameters used, viz., frequency changes, mode shape changes, mode shape curvature changes, frequency response function (FRF) changes, modal flexibility changes, and modal strain energy changes etc. A comparative review of several methods proposed by different researchers can be found in ref [Yan, Cheng, Wu and Yam (2007)]. In this section, a brief review on some of the methodologies developed by the present authors are described for damage identification using frequency and mode shape based techniques to show the efficacy of the model/modal based approaches.

3.1 Detection of damage based on changes in mode shapes

From the review of literature on detection of damage using frequency or mode shape, it is found that a large number of research works has been carried out in this area. But, there is no confirmation on the superiority of any method over the others. Hence, a detailed study has been carried out to identify the efficacy of mode shape information from a damaged structure for identification and localisation of damage in a structure. The change in three mode shapes with the change in flexural rigidity of a beam with 11 segments (elements) for an assumed damage in 5th element has been shown in Figure 1. The study has been restricted to first three modes because, in practice, it is difficult to realize the next higher modes information from vibration testing.

Number of observations can be made based on the above results: i) It is observed that displacement mode shapes are sensitive to damage and the mode shape changes with damage, ii) Higher modes are more predominant in showing the shift in mode shape displacements due to damage in the structure whereas lower modes (usually measured from vibration tests of large structure) may not significantly reflect the damage which is local phenomenon, iii) Shift in mode shape largely depends on the location of damage and the mode considered. Higher mode will magnify the shift in mode shape, if the damage location does not fall near the zero-displacement points, iv) Any shift in mode shape of a damaged structure with respect to the mode shape of undamaged mode shape may lead to an erroneous interpretation of damage in that location. v) It is also important to mention that extracted mode shapes are affected by environmental noise and number of sensors and the choice of sensor coordinate may have a bearing on accuracy of damage detection, and vi) it is very difficult to quantify damage accurately from mode shape information alone. On the other hand, though significant damage causes very small change in natural frequency (particularly for large structures), natural frequencies are easy to be measured and are less influenced by environmental noise. In this study, a methodology for detection of damage in structures has been proposed based on change in natural frequency.

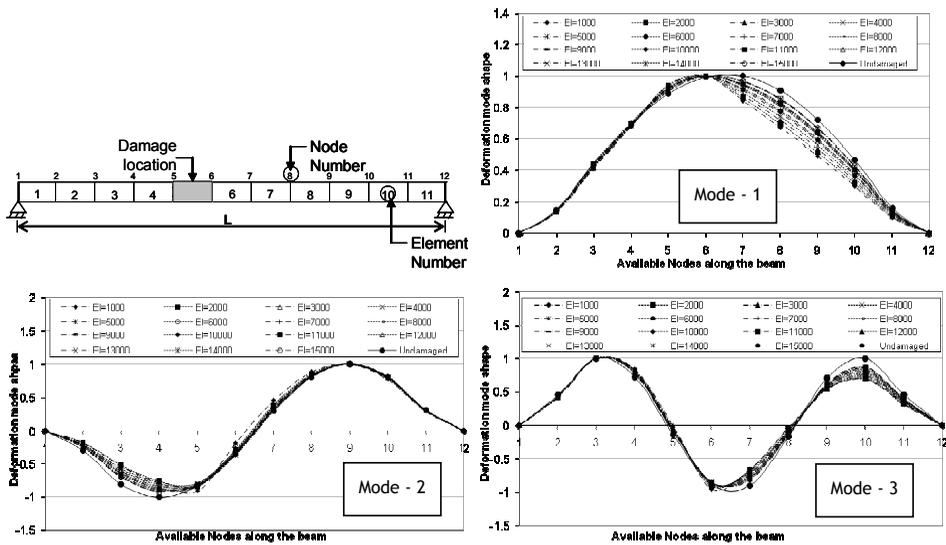


Figure 1: Difference in the mode shape of the beam for different degrees of damage in 5th element

3.2 Detection of damage based on changes in frequencies

It has been noted that amount of research work towards quantification of damage is considerably fewer than localisation of damage. In view of this, a methodology has been developed for detection and quantification of damage using modal frequencies obtained from a damaged structure using transfer matrix method. Transfer matrix method [Pestel and Leckie (1963)] is used in this study because of its versatility and ease with which it can be applied to a structure of either uniform or non-uniform cross section and under a variety of boundary conditions. Moreover, for a methodology based on an iterative algorithm developed for this study, transfer matrix method is much useful and easy to handle compared to finite element (FE) formulation. Since the theory and procedure of transfer matrix method for dynamic analysis are well established, it is not presented.

The central philosophy of detection of damage of beam like structure using transfer matrix formulation employed here is to determine the reduction in flexural rigidity of one or more elements of the beam which will signify the existence of damage in the structure. A detailed study has been carried out to evaluate the frequency determinant by changing the magnitude and locations of the damaged element(s) to evaluate the influence of damage (both magnitude and location) on frequency of a structure. It is noticed that the frequencies corresponding to higher modes are influenced predominantly by the change in flexural rigidity of one or more elements of the beam. For a better presentation and clarity, the changes in determinant values only for first two frequencies are shown in Figure 2. It is observed from the figure that by reducing flexural rigidity in a particular element of the beam considered in this study, the second mode frequency varies over a wider range than that of the first mode. This signifies that the shift in second mode frequency due to damage is more predominant than the first mode frequency. It is also noticed from the study that the trend is valid for next higher modes.

The accuracy of the initial flexural rigidity can be checked by comparing the measured and calculated natural frequencies of the original undamaged structure. When a fault occurs in a certain beam segment, it can be detected through the changes in the system natural frequencies. The location and magnitude of the damage in the structure can be identified by the intersection of various rigidity versus damaged beam element location curves. The intersection of the curves obtained for different modes represent fault locations and magnitudes (flexural rigidity) which caused the changes in the system natural frequencies.

Using the proposed methodology and computer program developed, an iterative study has been carried out for satisfying the measured frequency of a damaged beam for different modes. Final flexural rigidities of each element along the length

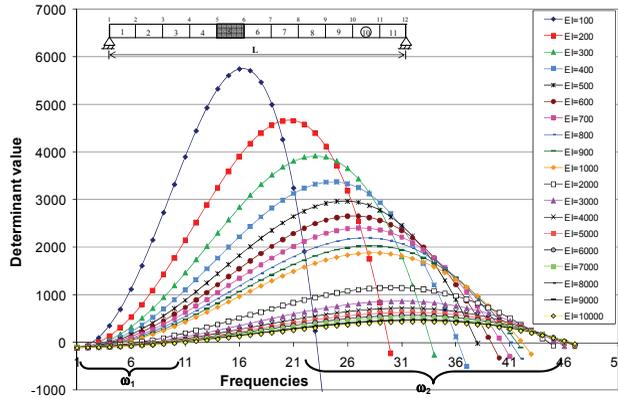


Figure 2: Variation of determinant with degree of damage (EI in kNm^2)

of the beam are obtained from the computer program and plotted for few damage cases as shown in Figures 3a and 3b respectively. It is observed that the true location and magnitude of the damages for both the cases are identified by the intersections of the various rigidity versus element location curves.

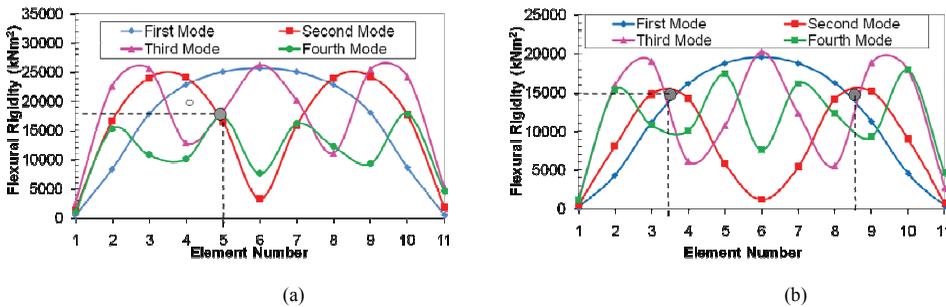


Figure 3: (a) Plot of flexural rigidity versus element diagram for a single damage case (b) Plot of flexural rigidity versus element diagram for a multiple damage case

From Figure 3a, it is observed that, intersection of curves for different modes indicates damage in 5th element with a remaining flexural rigidity of 17500 kNm^2 (i.e. 47.5% damage) which is considerably accurate with the simulated damage of 50%. It is worthy to mention (from Figure 3a) that the plot corresponding to first mode has not converged with other modes considered in this study. It may be due to error or lack of accuracy in frequency measurement for first mode. Therefore, it is significant to note that the consideration of more number of modes would provide

a more correct and reliable result. Result for second case as shown in Figure 3b, depicts that it is likely to have damage in either 3rd or 8th element.

This is due to the fact that reduction of flexural rigidity (damage) in symmetric locations would lead to same frequency (or reduction in frequency) of the damaged structure, and this can be solved by detailed inspection. This procedure can be adopted for damage detection of structures provided that the natural frequencies of the damaged structure for first few modes are measured with considerable accuracy.

3.3 Detection of damage from shift in measured frequency

From the above study, it is noticed that the measured frequency of a damaged structure can be used for detection and localisation of damage in the structure only when the previous history of the structure is known. In other words, the geometrical and mechanical properties of existing and the undamaged structure must be known prior to its detection of damage. But, in many cases, it may not be possible to get the previous information to compare the change or evaluate the shift in frequencies of an undamaged structure with that of a damaged one. In view of this, a procedure has been adopted in this study where the normalised frequencies of an existing (damaged) structure are used for detection of damage. Thus, the information about the undamaged structure need not be known. In this regard, it is to be mentioned that the frequency ratios (of different modes) are considered as the normalised frequencies. In view of this, a study has been carried out considering a beam with damage at $1/11^{th}$, $2/11^{th}$, $3/11^{th}$, $4/11^{th}$ and $5/11^{th}$ location of the beam. For this particular beam, first 5 modes are considered for detection and localisation of damage prevailed in the beam. As discussed above, normalisation of frequencies has been carried out as ω_{i+1}/ω_i where ω_i is the frequency of the structure in i^{th} mode. In this study, normalisation has been carried out with respect to first mode frequency (ω_1) to obtain the possible maximum values. Figures 4a to 4d reveal that, the frequency ratios change significantly with the change (reduction) in flexural rigidity, in turn the damage in structure.

Procedure for detecting damage using normalised frequencies is illustrated by two cases, as *i*) a beam with a damage with a magnitude of 80% at 3rd element and *ii*) a beam with a damage with a magnitude of 60% at 5th element of the beam (the beam is shown in Figure 1). For both cases, natural frequencies of the damaged beam are evaluated and normalised frequencies with respect to first fundamental frequency are given in Table 1.

For case-(*i*), possible damages with location can be determined from Figures 4a to 4d as (a) 3rd element with 80% damage, (b) 3rd element with 80% damage, (c) 3rd element with 80% damage or 4th element with 68% damage, and (d) 3rd element with 80% damage or 4th element with 52% damage, respectively. Thus,

Table 1: Normalised frequency of the damaged structure

Normalised Frequency (subscript denotes mode number)	Case-i	Case-ii
ω_2/ω_1	3.724	4.567
ω_3/ω_1	9.416	9.812
ω_4/ω_1	17.626	17.979
ω_5/ω_1	26.948	28.271

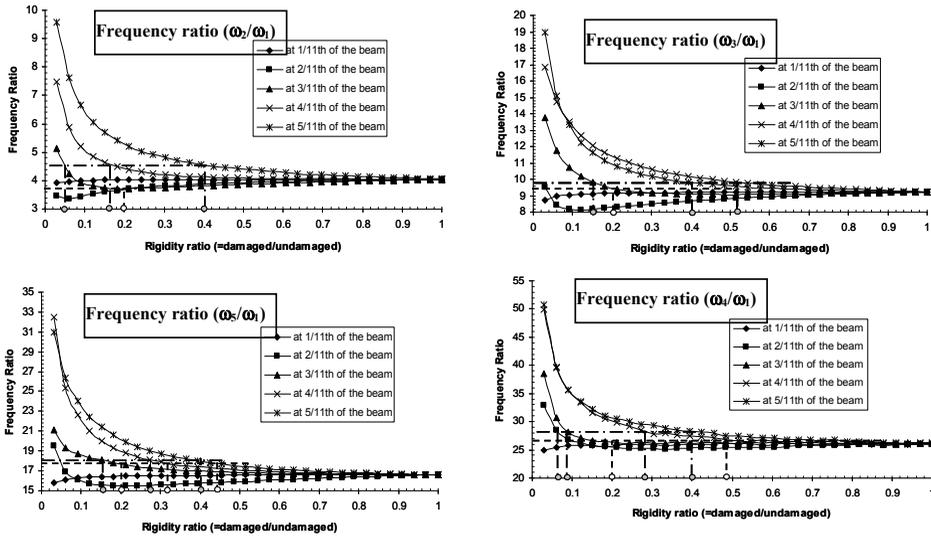


Figure 4: Identification of damage from frequency ratio

it shows the strong confirmation of damage in 3rd element of the beam with 80% magnitude. Similarly, for case-(ii) possible damages with location can be evaluated from Figures 4a to 4d as (a) 3rd element with 95% damage or 4th element 84% damage, or 5th element with 60% damage, (b) 3rd element with 85% damage or 4th element 48% damage, or 5th element with 60% damage, (c) 3rd element with 85% damage or 4th element 72% damage, or 5th element with 60% damage, and (d) 2nd element with 94% damage, or 3rd element with 92% damage or 4th element 72% damage, or 5th element with 60% damage, respectively. Thus, it shows a strong possibility of damage in 5th element with 60% magnitude. Therefore, it can be mentioned that the procedure is able to detect the magnitude and location of damage in the beam with considerable accuracy. But, it is worthy to mention that this procedure is capable of detecting and localising the damage when the damage is prominent.

3.4 Damage detection using Uniform Moment Surface curvatures

The uniform moment surface (UMS) curvature obtained from rotational mode shape has been used here as a new local damage indicator due to its high sensitivity to damage with less experimental error effect. The curvature has been calculated using Chebyshev polynomial approximation. Numerical simulations are performed using transfer matrix method. Frequency and mode shape information obtained from a damaged structure was used and no information from undamaged structure is required.

This approach is based on flexibility matrix and uniform moment surface concept where both the mode shape and natural frequency for few lower modes are utilised. Usage of number of modes would minimise any experimental error from a particular mode and application of both frequency and mode shape certainly claims its generality.

By making use of the orthogonal property of Chebyshev polynomial, the curvature of the UMS can be approximated by the second derivatives of the Chebyshev polynomials as

$$\text{Curvature} = u_{xx}(x) = \sum_{i=1}^N c_i \frac{\partial T_i^2(x)}{\partial x^2} \quad (9)$$

Therefore, the formulation of the damage index can be calculated as

$$d(x_i) = u_{xx}^D - u_{xx} \quad (10)$$

where superscript D denotes the parameters for damaged structure.

Curvature of modal flexibility using displacement mode shapes for first three modes of a beam with 30% damage has been shown in Figure 5. Curvature of UMS using three modeshapes of a beam with various percentages of damage has been shown in Figure 6. From the figures, it is observed that curvature of rotation flexibility is considerably more sensitive than the curvature of displacement flexibility to detect the location of damage(s). It is noted that the curvature of displacement flexibility can identify the case with 30% damage. It is also to note that curvature of displacement flexibility is capable of identifying a low damage case (say, 10%) provided that the damage occurs near the mid of the beam, whereas the curvature of rotation flexibility can even identify and locate very small damage (1%) cases.

It is significant to mention that curvature of rotational mode shape for individual modes and the UMS is more sensitive and superior to displacement modes. On the other hand, use of number of available lower modes in UMS assures its stability. But, it is important to note that UMS may not provide sufficient indication

of damage when any damage falls near mid zones. But, it can be solved easily because curvature of first rotational mode itself is sufficiently sensitive and capable in indicating any damage. Hence, it is worthy to mention that the curvature of rotational flexibility and UMS is capable of identifying damage(s) in a structure. So, by judicious instrumentation and from near free-from-error experimental data, the proposed methodology using information from a few lower modes (or even first mode information alone) from a damaged structure can identify at very low damage cases.

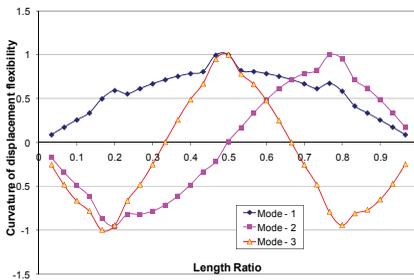


Figure 5: Curvature of modal flexibility (displacement) of a beam with damages (30%) for 3-modes

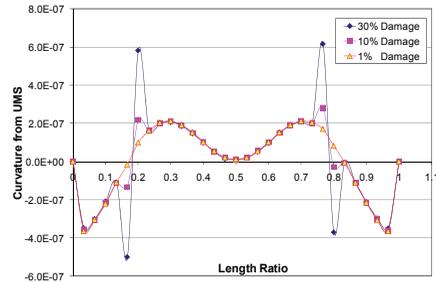


Figure 6: Curvature of Uniform Moment Surface of a beam with different damage magnitudes

4 Non-model based approaches (New developments)

Many methods have been developed and applied in recent years using artificial intelligence methods for structural damage detection and identification. Compared to the traditional model based methods, these methods have the advantage of dealing with uncertain, insufficient and noise contaminated information effectively. These methods also called intelligent diagnosis methods mainly take modern signal-processing techniques and artificial intelligence (AI) as analysis tools with less dependence on structural shape. The representative methods include Wavelet analysis, Genetic algorithm (GA) and Artificial Neural Network (NN), Pattern Recognition etc. Many studies have been reported in the literature on damage detection for large and complex structures using these advanced methods.

4.1 Application of Artificial Neural Networks for damage detection of structures

Most of the quantitative global damage detection methods that can be applied to complex structures examine the changes in the vibration characteristics of a struc-

ture. Artificial neural network (ANN) has been used in the present study to detect damage in the structure by the use of changes in natural frequencies between damaged and undamaged structures. In this, an ANN was trained using the vibration data obtained by simulating the different degrees of damage in finite element model. The trained neural network will be tested for the unforeseen data for validation and further used for identification of damage by using the frequency data obtained from the field tests.

To demonstrate the applicability of AI in damage detection, a cantilever plate shown in Figure 7a was considered for detection of the location and evaluation of magnitude of the damage. The experimental results of intact and damaged structure which was excited using impact hammer was considered for validation of the trained ANN. A feedforward back-propagation neural network has been used in the present study for the damage detection in the cantilever plate. The number of hidden layers and the number of processing elements (PEs) for each of them were determined by the Root Mean Square (RMS) error associated with the output layers as a decision quantity. The final architecture of the ANN is shown in Figure 7b.

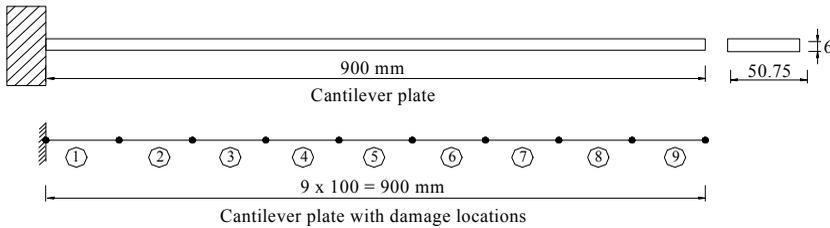


Figure 7a: Cantilever plate for simulation

The damage was defined as the fractional loss of the second moment of area over one location. The damage function used is a vector of six values containing the fractional difference of the resonant frequencies of undamaged and damaged structures as shown below.

$$z_i = \frac{f_{u_i} - f_{d_i}}{f_{u_i}} \quad (11)$$

where, z_i is the fractional change in the i -th mode, and f_u and f_d are the frequencies of undamaged and damaged structure. Figure 8 shows the damage detection results in three damage states which are obtained from ANN as well as from finite element analysis for the test data set using the network trained with fractional changes of the frequencies in undamaged and damaged case.

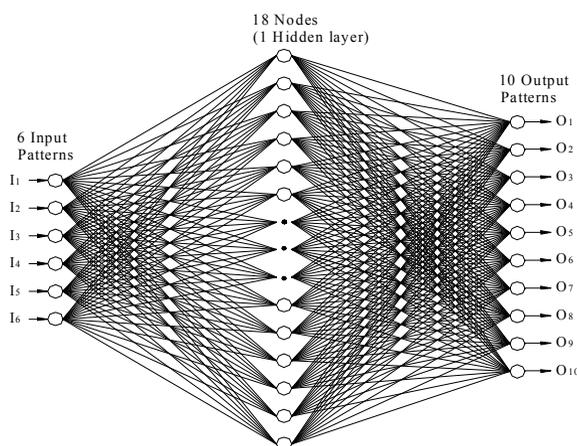


Figure 7b: ANN Architecture (6-18-10 network)

From the bar charts, it can be observed that the normalized values of the frequencies which were predicted by ANN are in close proximity to the values obtained from FE analysis. Figure 9 shows the comparison of results obtained from the ANN and from the impact tests. It was observed that the neural network is capable of predicting the location and magnitude of the damage with better accuracy when fractional difference of frequencies of undamaged and damaged structures is used as the input instead of actual frequencies.

4.2 Damage identification using genetic algorithm approach by updating the stiffness matrix

The damage can be identified and quantified by minimizing the errors between the measured data and numerical results using genetic algorithm. The evaluation of errors may use either the frequencies only or mode shapes only or a combination of both frequencies and mode shapes. In the application of genetic algorithm for detection of damage in structures, an objective function in terms of parameters related to the physical properties and state of the structure has to be formulated. When the optimization procedure arrives at the solution, the values of the parameters indicate the location and level of damage. No matrix condensation is needed even if measurements are only made at a few DOFs.

Furthermore, GA uses multiple points to search for the solution compared to a single point search in the traditional gradient based optimization method. Different objective functions were formulated using the changes in frequencies, mode shapes

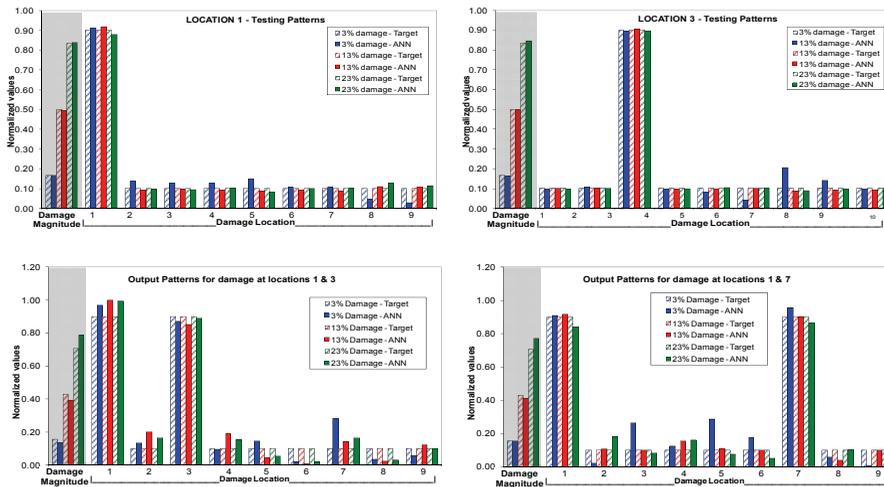


Figure 8: Comparison of Actual and ANN predicted damage magnitude at different locations

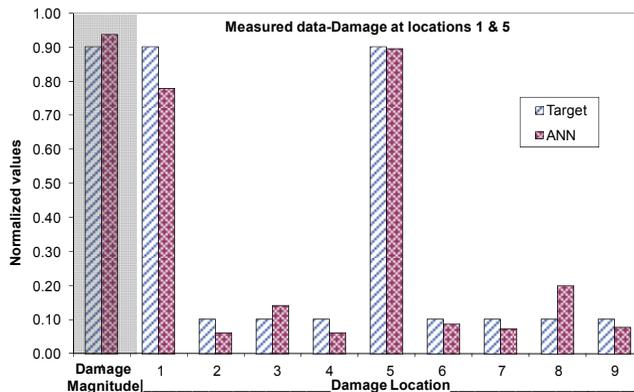


Figure 9: Damage at locations 1 & 5 for measured frequencies of cantilever beam (Target values taken from Brownjohn et al., report 2001) (6-18-10 ANN)

and combination of frequencies and modeshapes for identification of damage using genetic search algorithm as described below.

For representing damage identification in terms of an optimization problem with the genetic algorithms, it is necessary to specify an objective function to be maximized. Using the general concept of residual forces, and taking into account several practical considerations, one can formulate the appropriate objective function that

can be maximized. The matrix form of the residual force equation considering all the modes of vibration can be written in full as follows:

$$\begin{bmatrix} R_{1j} \\ R_{2j} \\ \dots \\ R_{nj} \end{bmatrix} = \begin{bmatrix} (k_{d11} - \lambda_{jd}m_{11}) & (k_{d12} - \lambda_{jd}m_{12}) & \dots & (k_{d1n} - \lambda_{jd}m_{1n}) \\ (k_{d21} - \lambda_{jd}m_{21}) & (k_{d22} - \lambda_{jd}m_{22}) & \dots & (k_{d2n} - \lambda_{jd}m_{2n}) \\ \dots & \dots & \dots & \dots \\ \dots & \dots & \dots & \dots \\ (k_{dj1} - \lambda_{jd}m_{j1}) & (k_{dj2} - \lambda_{jd}m_{j2}) & \dots & (k_{djn} - \lambda_{jd}m_{jn}) \end{bmatrix} \times \begin{Bmatrix} \phi_{1jd} \\ \phi_{2jd} \\ \dots \\ \phi_{njd} \end{Bmatrix},$$

$$j = 1, 2, \dots, m \quad (12)$$

where R_{1j} ($I = 1, 2, \dots, n$) are the residual forces and for all m modes, this forms a residual force matrix $[R]$ of size $n \times m$. If $[K_d]$ and $[M]$ are real symmetric matrices, it can be shown that the diagonal terms of matrix $[R]$ are 0s, when a correct set of λ_d and ϕ_d are substituted in those equations. Hence the objective function chosen for this case is as follows:

$$f(\beta_1, \beta_2, \dots, \beta_n) = \sqrt{R_{11}^2 + R_{22}^2 + \dots + R_{nm}^2} \quad (13)$$

Based on frequencies, the objective function can be written as minimization of ratio of change in natural frequency between measured and analytical values to the experimentally measured frequency for ‘ m ’ number of modes.

$$\text{Objective function} = \min_{\beta} F_f; \quad \beta = \{\beta_1 \quad \beta_2 \quad \dots \quad \beta_n\}^T \quad (14)$$

where,

$$F_f = \sum_{i=1}^m \left(\frac{\lambda_{dei} - \lambda_{dai}}{\lambda_{dei}} \right)^2$$

in which λ_{dei} and λ_{dai} are the experimentally measured and analytically calculated natural frequencies (eigenvalues) of i^{th} mode of the damaged structure. The above equation leads to zero, if both the measured and calculated eigenvalues of the structure are equal. Hence, the objective function for the maximization problem can be written as:

$$F_{freq} = \max_{\beta} \frac{c_1}{c_2 + F_f} \quad (15)$$

where c_1 represents a constant used to control the value of the objective function; c_2 represents a constant used to build a well defined function for the ideal case, i.e., with no experimental errors. In the present case, both c_1 and c_2 are taken as unity.

The frequency criterion is practical in real-time monitoring of structures, as the natural frequencies can be easily and accurately evaluated even in the case of single point measurement. But, when the locations of damage are at symmetrical locations in a symmetric structure, the damage cannot be differentiated, because the frequencies are not much sensitive to structural damage, especially to localized damages having small magnitudes. In such cases, the change in mode shapes can be used for better identification of damage. One of the criteria which can be used based on the changes in mode shapes is Modal Assurance Criterion (MAC) value which uses both analytical and experimental mode shape vectors.

MAC criterion is given in the following form,

$$MAC(\phi_e, \phi_a) = \frac{|\phi_e^T \phi_a|^2}{(\phi_e^T \phi_e)(\phi_a^T \phi_a)} \quad (16)$$

where, ϕ_e and ϕ_a denote the measured and analytically calculated eigenvectors respectively for the i^{th} mode. The MAC takes the value between 0 and 1; the value of 1 represents exact correlation between analytical and experimental mode shapes.

Eventhough vibration mode shapes are sensitive to structural damage, in practice, the measured mode shapes usually have relatively larger errors than measured frequencies. If mode shape changes alone are considered in damage detection process, this might lead to unsatisfactory results. The sensitivity of mode shapes, as measured by the MAC, depends very much on the nature of the damage. If the damage is distributed, such as widespread cracking in concrete, the mode shape may change little, although there is a much change in frequency. Localised damage, on the other hand, may result in larger reductions in the MAC values. Hence, both frequency and MAC values have been combined to formulate a new objective function to get the advantage of both. Thus, the objective function considering both frequency and MAC values for m number of modes can be written as

$$\max F = \max_{\beta} \frac{c_1}{\beta} + \max_{\beta} C \cdot F_m \quad (17)$$

where,

$$F_m = \sum_{i=1}^m MAC(\phi_e, \phi_a) \quad (18)$$

4.2.1 Damage identification using objective functions

The results obtained for a simply supported beam based on frequency based criteria in terms of stiffness reduction factors (β_i^*) are presented in Table-2.

Table 2: Stiffness reduction factors (β_i^*) for beam considering frequency based criteria

Elem. No.	D_0^0	D_6^{25}	D_6^{50}	D_6^{75}	$D_{2,8}^{25,25}$	$D_{2,8}^{50,50}$	$D_{2,8}^{75,75}$	D_3^{75}
1	0.9998	0.9988	0.9073	0.9945	0.8510	0.7953	0.8554	1.0000
2	0.9993	0.9980	0.9466	0.9998	0.9774	0.6980	0.7897	0.9999
3	1.0000	0.9989	0.9099	0.9991	0.9176	0.9083	0.7926	0.5000
4	1.0000	1.0000	0.9665	1.0000	0.9421	0.7546	0.7949	0.8981
5	1.0000	0.9990	0.9454	0.9954	0.9509	0.9957	0.8374	1.0000
6	0.9999	0.7510	0.5000	0.2505	1.0000	0.9919	0.9496	0.9996
7	0.9999	0.9993	0.8712	0.9945	0.9997	0.9352	0.9861	0.9992
8	0.9999	0.9995	0.9381	0.9996	0.8448	0.7051	0.5004	0.7607
9	1.0000	0.9994	0.8925	0.9998	0.9759	0.7728	0.2692	0.5011
10	0.9993	0.9999	0.9992	0.9998	0.8665	0.8726	0.5253	1.0000
11	0.9994	0.9980	0.8362	0.9993	0.9420	0.9779	0.9510	0.9999

D_i^a represents the SRF for element i with a damage magnitude of $a\%$,
 D_0^0 represents undamaged case

Table 3: Stiffness reduction factors (β_i^*) for beam considering MAC based criteria

Elem. No. ↓	D_0^0	D_6^{25}	D_6^{50}	D_6^{75}	$D_{2,8}^{25,25}$	$D_{2,8}^{50,50}$	$D_{2,8}^{75,75}$	$D_{2,8}^{40,25}$	$D_{2,8}^{75,25}$	$D_{2,6,8}^{25,25,25}$	$D_{2,6,8}^{50,30,50}$	$D_{2,6,8}^{75,75,75}$	$D_{2,6,8}^{50,75,25}$
1	0.9383	0.8722	0.8730	0.9698	0.8103	0.8652	0.9524	0.9395	0.9212	0.9685	0.7476	0.9727	0.9660
2	0.9411	0.8827	0.8724	0.9702	0.5932	0.4375	0.2412	0.5519	0.2186	0.8126	0.3730	0.2340	0.4881
3	0.9385	0.8618	0.8783	0.9713	0.8751	0.8708	0.9651	0.9383	0.9312	0.9708	0.8697	0.9440	0.9863
4	0.9284	0.8631	0.8778	0.9570	0.8370	0.8751	0.9707	0.9458	0.9261	0.9663	0.8790	0.9756	0.9846
5	0.9335	0.8802	0.8776	0.9763	0.8145	0.8681	0.9624	0.8727	0.9059	0.9988	0.8145	0.9001	0.9958
6	0.9419	0.7581	0.4363	0.2506	0.8207	0.8668	0.9594	0.9141	0.9071	0.7528	0.3873	0.2502	0.2505
7	0.9393	0.8737	0.8759	0.9505	0.8045	0.8672	0.9728	0.8951	0.9057	0.9986	0.8762	0.9487	0.9810
8	0.9366	0.8579	0.8737	0.9620	0.6045	0.4415	0.2395	0.6907	0.6910	0.7499	0.4117	0.2501	0.7667
9	0.9395	0.8589	0.8764	0.9752	0.8112	0.8775	0.9690	0.9314	0.8750	0.9974	0.7439	0.9381	0.9771
10	0.9330	0.8708	0.8743	0.9905	0.8196	0.8588	0.9630	0.9475	0.9063	1.0000	0.8134	0.9842	0.9922
11	0.9422	0.8933	0.8739	0.9375	0.8222	0.8744	0.9728	0.9409	0.9080	0.9999	0.7975	0.9321	0.9848

From Table 2, it can be observed that by using the frequency based objective function, the damage can be detected in a single damage scenario (eg. damage in element $i=6$) accurately, but it is not able to identify the damage when it is at symmetrical locations, (eg. at element $i=3$) in symmetrical type of structures. Also, it is very difficult to identify the damage correctly in multiple damage scenarios, because, frequencies are not sensitive to the location of damage.

Table 3 shows the results obtained considering mode shape criteria using MAC values. As observed from Table-3, by using the objective function based on MAC values, location of damage can be identified even in multiple damage cases but, unable to quantify exactly in lower level damage scenarios and the error observed at other locations is more in this case. The results obtained by using the combined objective function is presented in Table 4 for different damage scenarios which

Table 4: Stiffness reduction factors (β_i^*) for beam considering combined frequency and MAC based criteria

Elem. No. ↓	D_0^0	D_6^{20}	$D_{2,8}^{25,25}$	$D_{2,8}^{50,50}$	$D_{2,8}^{75,25}$	$D_{2,6,8}^{25,25,25}$	$D_{2,6,8}^{50,50,50}$	$D_{2,6,8}^{75,75,75}$	$D_{2,6,8}^{50,75,25}$	$D_{4,5,6,7,8}^{10,20,30,20,10}$
1	0.9995	0.9977	0.9996	0.9980	0.9972	0.9964	0.9968	0.9996	0.9903	0.9989
2	0.9993	0.9989	0.7501	0.5000	0.2520	0.7530	0.5006	0.2502	0.5156	0.9996
3	0.9999	0.9991	0.9988	0.9990	0.9956	0.9998	0.9913	0.9999	0.9822	0.9948
4	0.9997	0.9990	0.9998	0.9922	0.9975	1.0000	0.9980	0.9883	0.9919	0.8747
5	1.0000	0.9999	0.9951	0.9995	0.9988	0.9991	0.9980	0.9994	0.9971	0.8126
6	0.9984	0.8011	0.9991	0.9997	0.9996	0.7511	0.4998	0.2507	0.2480	0.7501
7	0.9999	0.9988	0.9999	0.9987	0.9961	0.9975	0.9915	0.9995	1.0000	0.7953
8	0.9998	0.9995	0.7515	0.5011	0.7499	0.7500	0.5005	0.2500	0.7582	0.8749
9	0.9994	0.9996	1.0000	0.9998	1.0000	0.9960	0.9998	0.9974	0.9999	0.9999
10	0.9995	0.9961	0.9961	0.9984	1.0000	0.9990	0.9998	0.9969	1.0000	1.0000
11	0.9993	0.9995	0.9999	0.9985	1.0000	0.9993	0.9914	0.9964	1.0000	0.9998

shows the clear indication of damage in all the cases. This method is also effective when incorporating the simulated noise in the data used for analysis.

Studies similar to that of simply supported beam have been carried out on a warren type plane truss structure shown in Figure 10. The results obtained from the studies for noise-free data are presented in Table 5.

Table 5: Stiffness reduction factors (β_i^*) for plane truss considering combined frequency and MAC based criteria

Elem. No. ↓	D_0^0	D_{15}^{25}	D_{15}^{50}	D_3^{30}	$D_{3,10}^{50,50}$	$D_{3,10}^{25,25}$	$D_{7,14,15}^{25,25,25}$	$D_{7,14,15}^{50,50,50}$	$D_{7,14,15}^{75,75,75}$	$D_{1,8,15,16}^{25,25,25,25}$	$D_{1,8,15,16}^{50,50,50,50}$
1	0.9971	0.9948	0.9998	0.9933	0.9798	0.9973	0.9951	0.9989	0.9624	0.7598	0.4984
2	1.0000	0.9101	0.9816	0.9962	0.9931	0.9990	0.9897	0.9917	0.9680	0.9990	0.9926
3	0.9806	0.9489	0.9666	0.5079	0.5031	0.7501	0.9955	0.9739	0.9470	0.9867	0.9839
4	0.9966	0.9807	0.9806	0.9954	0.9973	0.9994	0.9990	0.9815	0.9748	0.9522	0.9982
5	0.9957	0.9410	0.9823	0.9916	0.9899	0.9897	0.9922	0.9999	0.9966	0.9819	1.0000
6	0.9923	0.9539	0.9217	0.9974	0.9950	0.9989	0.9830	0.9800	0.9597	0.9950	0.9906
7	0.9991	0.9590	0.9748	0.9843	0.9840	0.9936	0.7520	0.4997	0.2658	0.9998	0.9975
8	0.9994	0.9749	0.9747	0.9774	0.9685	0.9998	0.9915	1.0000	0.9789	0.7491	0.4991
9	0.9944	0.9386	0.9980	0.9977	0.9774	0.9997	0.9996	0.9938	0.9960	0.9822	0.9978
10	0.9887	0.9742	0.9784	0.9911	0.4973	0.7509	0.9953	0.9682	0.9445	0.9910	0.9919
11	0.9999	0.9849	0.9877	0.9973	0.9916	0.9993	1.0000	0.9947	0.9821	0.9982	0.9919
12	0.9909	0.9368	0.9995	0.9991	0.9973	0.9820	0.9902	0.9958	0.9872	0.9990	0.9988
13	0.9949	0.9696	0.9797	0.9889	0.9908	0.9858	0.9958	0.9720	0.9979	0.9998	0.9995
14	0.9947	0.9962	0.9990	0.9875	0.9826	0.9828	0.7592	0.5040	0.2497	0.9958	0.9941
15	0.9932	0.2718	0.5014	0.9980	0.9974	0.9784	0.7400	0.5005	0.2623	0.7384	0.5006
16	0.9985	0.9797	0.9774	0.9987	0.9989	0.9929	0.9959	0.9921	0.9926	0.7538	0.4967
17	0.9684	0.9748	0.9336	0.9985	0.9982	0.9836	0.9976	0.9665	0.9728	0.9963	0.9995
18	0.9988	0.9428	0.9918	0.9933	0.9991	1.0000	0.9854	0.9921	0.9901	0.9955	0.9963
19	0.9841	0.9273	0.9439	0.9975	0.9997	0.9897	0.9950	0.9629	0.9359	0.9920	0.9969
% error (avg.)	0.6984	4.2053	2.2721	3.8558	0.9000	0.6686	0.6709	1.2926	2.7742	0.8968	0.4384

The proposed methodology gives better identification of damage in this case also, but error percentage is more because of large structure. Because of increased number of elements, convergence in GA process takes more time and also convergence is not accurate in some of the cases even though it is able to locate and quantify the damage with reasonable accuracy in multiple damage cases, it is slightly less accurate in single damage cases.

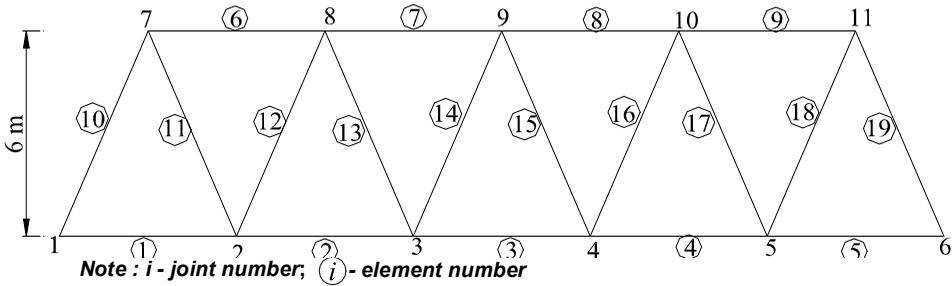


Figure 10: Plane truss structure

4.3 Damage identification using Modal Strain Energy criteria

In order to make the damage detection method more effective especially for large scale structures, a two stage approach has been used. The approach followed is in two stages, in the first stage, modal strain energy is used to approximately identify the damaged elements in finite element model of the structure and in the second stage, the identified damaged elements are used as solution space (solution parameters) in GA based optimization approach for further exact identification and quantification of damage.

Elemental modal strain energy (MSE) is defined as the product of the elemental stiffness matrix and the second power of its mode shape component. For the j -th element and the i -th mode, the MSE before and after the occurrence of damage is given as

$$MSE_{ij} = \frac{1}{2} \phi_i^T K_j \phi_i \tag{19a}$$

$$MSE_{ij}^d = \frac{1}{2} \phi_i^{d^T} K_j \phi_i^d \tag{19b}$$

where MSE_{ij} and MSE_{ij}^d are the MSE of the j -th element for the i -th mode shape for undamaged and damaged cases respectively. Because the damaged elements

are not known, the undamaged elemental stiffness matrix K_j is used instead of the damaged one as an approximation in MSE_{ij}^d , and the modal strain energy change ratio (MSECR), defined as follows, has been found to be a good indicator for damage localization [Law, Shi and Zhang (1998); Shi, Law and Zhang (1998); Shi, Law and Zhang (2000); Shi, Law and Zhang (2002)].

$$MSECR_{ij} = \frac{|MSE_{ij}^d - MSE_{ij}|}{MSE_{ij}} \quad (20)$$

The MSECR obtained using the above formulations is shown in Figure 11 for identifying damages in different locations. The damaged elements identified using this method were used as solution parameters in genetic search optimization method described in the previous section for quantifying the damage by obtaining the stiffness reduction factors of the damaged elements. Damage localization using this method is very useful for detection of damage in large scale structures as number of solution parameters required are significantly reduced when using the genetic search optimization for detecting the damage.

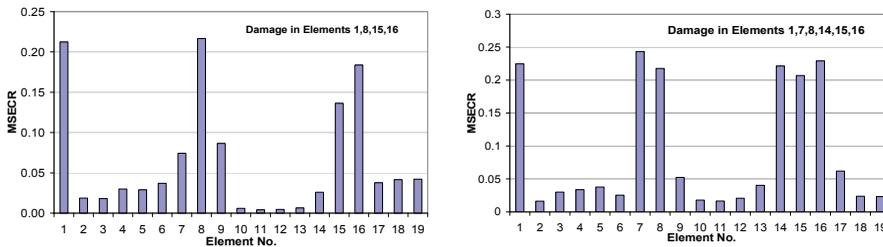


Figure 11: MSECR for damage cases

It is observed that the present methodology of damage detection based on genetic search procedure using frequency and MAC values is capable of identifying the damage with good accuracy. Studies show that the present approach simplifies the identification of damage for large scale structures by using the vibrational characteristics and reduces the computational effort significantly.

4.4 Signature analysis techniques

Wavelets and pattern recognition (PR) approach/ techniques, that utilize the response signatures of a structure, are gaining importance in the damage identification in structures. The signals (signatures) obtained from dynamic tests will have

to be transformed from one form to another to characterize, analyze, and establish relation among digital signals.

Recent development in signal processing made it possible to perform on-line health monitoring and damage assessment which has several advantages over current expensive and time-consuming inspections. During the last decade, damage detection methods using wavelets have been studied and wavelet-based methods have been applied to various civil and mechanical structures. Some of such methods may not be feasible for in situ conditions but the accomplishments achieved at this time will eventually lead to practical developments. On-line structural health monitoring will be possible because the moment at which modal properties of the structure experience a sudden change has appeared to show an unusual sharp trend when the response curve is analyzed by DWT. Moreover, wavelet analysis can also be used to perform denoising of the response signals, which will result in better detection of the damage time and location. However, the measurement of damage severity is still a concern.

The use of pattern recognition in the process of damage detection and localization has been used and demonstrated by many researchers in the recent years. Among the various frameworks in which pattern recognition has been traditionally formulated, the statistical approach has been most intensively studied and used in practice. More recently, neural network techniques and methods imported from statistical learning theory have been receiving increased attention. The design of a recognition system requires careful attention to the following issues: definition of pattern classes, sensing environment, pattern representation, feature extraction and selection, cluster analysis, classifier design and learning, selection of training and test samples, and performance evaluation. To date, all vibration based-damage detection methods reported in the technical literature can be described by the statistical PR paradigm with the vast majority of this literature focused on the identification of damage sensitive features. However, few of these studies apply statistical pattern recognition procedures to the damage-sensitive features. This lack of statistical analysis presents some potential problems for the development of vibration-based damage detection technology. From the review of literature it is found that, if one or more common forms of damage occur, it may be possible to not only determine that a system is damaged but to determine which form of damage has occurred. To advance the state of the art in vibration-based damage detection, developments of non-model based pattern recognition methods will be needed to supplement the existing model-based techniques. It is anticipated that such methods will be particularly effective when analyzing a structure where the damage changes the structure from a predominantly linear system to a predominantly nonlinear system. Also, the success of structural health monitoring will depend crucially on the selected

damage-sensitive features. If the distribution of the chosen features does not change in a meaningful way when the structure is damaged, then it is doubtful if monitoring these features will yield useful information regarding the structural health of the system no matter what statistical pattern recognition algorithm is applied.

5 Concluding Remarks

Most of the structures need assessment, maintenance and monitoring to insure their integrity. Recent development in signal processing made it possible to perform on-line health monitoring and damage assessment which has several advantages over current expensive and time-consuming inspections. In this paper, several studies have been carried out on damage identification of structures using different approaches based on traditional model and advanced non-model based methods. The damage identification studies were carried out based on the dynamic properties of the structure such as frequencies, modeshapes and their derivatives and also using advanced techniques such as neural networks and genetic algorithms. Each method has its own merits and demerits. Few observations made on these methods are discussed here. The traditional methods for structural damage detection utilize the mechanics characteristics of structures, such as natural frequencies, modal damping, modal strain energy or modal shapes, etc. However, these methods generally require experimental modal analysis or transfer function measurements, which is not very much convenient for online detection of structures in service because these experimental measures often need multiple instruments and manual operation. The disadvantages of traditional model based methods are: (i) they are more dependent on experiments (measurement of frequency, mode shape and damping) which are time-consuming and expensive, and are not suitable for online damage detection for in-service structures; (ii) these methods are more dependent on the properties of the individual structures to be detected and hence it is difficult to establish a universal methodology for various structures and (iii) they are generally not sensitive to small damage in structures. The modern methods which utilize the signatures of measurement profiles have some advantages compared to traditional methods which can be summarised as: (i) these methods are less dependent on experiments and only require to measure vibration responses at few points in the servicing structure to be detected and the responses can be measured in operating structural conditions; (ii) these methods do not depend on the structural properties and geometry and hence they can be applied universally to any type of structure of similar nature. (iii) it is possible to detect the smaller damage magnitudes by constructing and extracting better characteristic information from structural dynamic response signals. But, still there are some problems to be investigated and solved in utilizing the advanced methods for damage detection, such as: (i) these methods have to rely on the type

of excitation to the structure; (ii) the problem of noise elimination in measurement has to be taken care properly which is very important in detecting small damage magnitudes; and (iii) the features which are being used as damage indicators are to be chosen carefully.

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