SDHM, vol.4, no.4, pp.221-229, 2008

Evaluation of Concrete Penetration Depth under Impact Loading Employing Empirical Formulae

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Abstract: Empirical formulae based on experimental data to evaluate concrete penetration depth under impact loading are often used due to the complexity of the phenomena. Several design codes employ empirical formulae for the design of protective barriers. This paper presents the details of relative assessment of the use of well known empirical formulae for evaluation of concrete penetration depth under impact loading. The empirical formulae employed in the present studies include Conwep, Army corps, Haldar, Ammann and Whitney, UKAEA, BRL, Modified Petry and modified NDRC. Appropriate expressions and limitations for these empirical formulae have been provided. Performance of these empirical formulae has been studied by conducting numerical studies. The penetration depth values for all the formulae have been compared with selected published experimental observations. From the studies, it is observed that the penetration depth values computed by employing Conwep, Haldar and Ammann formulae are in good agreement with the selected experimental observations. The penetration values obtained by using Modified Petry and BRL formulae are larger compared to experimental observations. Further, it is observed that the penetration depth values computed by using modified NDRC, Army corps and UKAEA are lesser compared to corresponding experimental observations.

Keywords: Concrete; Structural components; Impact loading; Penetration; empirical approach.

1 Introduction

Concrete has been widely used over many years by military and civil engineers in the design and construction of protective structures to resist impact and explosive loads. Potential missiles/projectiles include kinetic munitions, vehicle and aircraft crashes, fragments generated by military and terrorist bombing, fragments generated by accidental explosions and other events (e.g. failure of a pressurized vessel, failure of a turbine blade or other high-speed rotating machines), flying objects due to natural forces (tornados, volcanos, meteoroids), etc. These projectiles vary broadly in their shapes and sizes, impact velocities, hardness, rigidities, impact attitude (i.e. obliquity, yaw, tumbling, etc.) and produce a wide spectrum of damage to the target. Impacting missiles can be classified as either 'hard' or 'soft' depending upon whether the missile deformability is small or large relative to the target deformability. 'Hard' projectile impact results in both local wall damage and in overall dynamic response of the target wall. Local damage consists of spalling of concrete from the front (impacted) face and scabbing of concrete from the rear face of the target together with missile penetration into the target. Overall dynamic response of the target wall consists of flexural deformations. A potential flexural or shear failure will occur if the local strain energy capacity of the wall does not exceed the kinetic energy input to the wall by the striking 'hard' missile.The effects of the impact of a hard projectile on a concrete target have been studied since mid-1700s mainly due to the continuous interest in designing high performance missiles and protective barriers. The more recent requirement to assess the safety of concrete containment vessels for nuclear reactors

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has also contributed considerably to the current understanding of local impact effects on concrete targets. The initial stiffness of target as well as the ultimate strength increases both in compression and tension. Further, the concrete-strain capacity will be increased under dynamic loading due to tension stiffening. When a projectile of certain mass and velocity hits a concrete target, concrete will be generally crushed and cracked and the structure experiences shaking and vibration depending on the relative period of structure and impact pulse duration. The pressure at the front of the nose of the projectile is several times higher than the static uniaxial strength and lateral confining pressure of concrete. In addition, stress waves may propagate from the tip of the nose-of the projectile. Since concrete is very weak in tension, the tensile wave generated when the compressive wave hits the backside of the component may cause scabbing at the backside and cracking in lateral direction. Both the compressive strength and tensile strength of concrete are thus important parameters for evaluating the depth of penetration. The crater size depends on the tensile strength. Both small-scale lab tests and full-scale prototype tests have been used to study impact on concrete targets. These have led to various empirical formulae and analytical models to understand the impact behaviour. The depth of penetration is a function of the impact velocity, angle of inclination of impact, mass and shape of the projectile and target.

There are three important approaches for studying local effects on a concrete target arising from projectile impact, namely experimental, analytical and numerical methods. Experimental data are always important for extending the understanding of impact phenomena and for validating analytical and numerical models. Empirical formulae based on experimental data are especially important due to the easiness and simplicity to represent the complex phenomena. Several design codes employ empirical formulae for the design of protective barriers. Simple and accurate analytical models can be developed when the underpinning mechanics of the local effects of the missile impact are understood. This approach offers the most efficient and economic way of predicting these effects and helps to extend the range of validity of empirical formulae based on experiments.

With the rapid development of computational tools, computational mechanics and material constitutive models, the numerical simulation of projectile impact effects becomes more reliable and economic. To get a first order approximation of projectile impact effects empirical formula can be useful. In the case of analytical model, representation of projectile as rigid is a major limitation, i.e the deformation and failure of the projectile are negligible. The deformation and damage of the projectile may become important either when the impact velocity is high or when the hardness of the projectile is low. There is scope to improve the analytical model by changing the projectile characteristics. Many material models are used in finite element simulation. Each material model requires special material parameters/constants to conduct analysis. Further, specific limitations are built-in for each material model. Hence, there is scope to improve/develop the gneralised material models. Large numbers of experiments were conducted on concrete structural components, but the numerical studies are limited. Hence, there is ample scope to conduct nonlinear finite element analysis by employing appropriate material model and contact algorithm.

Hanchak et al. (1992) compared the penetration resistance for concrete specimens with unconfined compressive strengths of 40 and 140MPa, showing only minor difference in protective performance for projectiles with L/D = 5.66 and CRH $\approx$ 3.0. The predicted penetration depth values using the empirical formulae were lower compared to experimental values in the case of high strength concrete. Yankelesky (1997) analysed the local response of concrete slabs to low speed missile impact and compared the results with those predicted by the empirical formulae proposed by Petry, the Army corps of Engineers, NDRC, Kar and UKAEA. But the comparison was done for limited experimental studies and inconsistencies in results were observed. Teland and Sjol (1999) predicted penetration depth employing various empirical formulae. It was observed that there are large variations for predicted penetration depth between different formulae when penetration of flat nosed projectiles in concrete is considered. Hanson (2003) conducted studies using Conwep formula to predict penetration depths of projectile in concrete and the predicted values were compared with the experimental observations. Modifications and limitations for the Conwep formula were suggested to consider projectiles with a length to diameter ratio between 6 and 10 and with caliber head radius between 2 and 6. The modified model exhibited a fair agreement with the experimental penetration depth available in the literature. To the best of authors' knowledge, the information on the performance and applicability of widely used empirical formulae is scanty. There is scope and need for studying the performance of widely used empirical formulae for evaluation of penetration depth in concrete.

This paper presents methodologies for evaluation of concrete penetration depth under impact loading by employing well known empirical formulae. Various empirical formulae include, Conwep, Army Corps, Haldar, Ammann and Whitney, UKAEA, BRL, modified Petry and modified NDRC. Appropriate expressions and limitations for all these empirical formulae have been provided. Performance of these empirical formulae has been studied by conducting numerical studies. The predicted penetration depth values for all the formulae have been compared with selected published experimental observations. From the studies, it is observed that the penetration depth values computed by employing Conwep and Haldar formulae exhibit better performance. The penetration depth values obtained by using modified Petry and Ammann formulae are over-estimated compared to corresponding experimental observations. Further, it is observed that the penetration depth values computed by using modified NDRC, Army Corps, BRL and UKAEA are lesser compared to experimental observations.

2 Penetration Depth – Empirical Formulae

Several empirical formulae for evaluation of penetration depth are available in the literature. Widely used formulae with appropriate expressions are presented in Tab. 1.

3 Numerical Studies

Numerical studies have been conducted to validate the use of empirical formulae described in previous section and the predicted values have been compared with the published experimental values available.

3.1 Example 1 – Forrestal et al. (1994)

Forrestal et al. (1994) conducted experiments to find depth of penetration with ogive nose projectiles and concrete targets with unconfined compressive strength of 23 and 39MPa. The projectiles are made of 4340 R_c45 steel. Experiments were conducted for CRH=3.0 and 6.0 and the projectile dimensions for CRH=6.0 are shown in Fig. 1.

Penetration depths are predicted for different velocities, masses, compressive strengths and CRHs. Penetration depths are calculated for all the empirical models described in the previous section. Projectile characteristics are shown in Tab. 2. The variation of calculated penetration depth values for various velocities, masses, compressive strengths is shown in Fig. 2.

3.2 Example 2 – Gran and Frew (1997)

Gran and Frew (1997) conducted penetration experiments of a hardened 4340 steel projectile with length to diameter ratio of 7 into concrete targets having an unconfined compressive strength of 43MPa. Mass and diameter of the projectile are 2.3kg and 50.8mm respectively. Penetration depths are predicted for different velocities. Fig. 3 shows the variation of penetration depth for various velocities and for various formulae.

3.3 Example 3 – Forrestal et al. (1994)

Forrestal et al. (1994) conducted experiments to find depth of penetration with ogive nose projec-

Table 1: Various empirical formulae

Proposed by	Empirical formulae	
	Imperial units	SI units
Conventional Weapons (Con Wep)[4]	$x = \frac{222\ N\ W\ v^{1.8}}{d^{1.8}\ f_c^{0.5}} + d$ for $x > 2d$	$x = \frac{11.76\ N\ M\ v^{1.8}}{d^{1.8}\ f_c^{0.5}} + d$, for $x > 2d$
Army Corps of Engineers (ACE)[5]	$\frac{x}{d} = \frac{282.6}{\sqrt{f_c}}\left(\frac{M}{d^3}\right)d^{0.215}\left(\frac{v}{1000}\right)^{1.5} + 0.5$	$\frac{x}{d} = \frac{3.5x10^{-4}}{\sqrt{f_c}}\left(\frac{M}{d^3}\right)d^{0.215}v^{1.5} + 0.5$
Ballistic Research Laboratory (BRL)[5]	$\frac{x}{d} = \frac{427}{\sqrt{f_c}}\left(\frac{M}{d^3}\right)d^{0.2}\left(\frac{v}{1000}\right)^{1.33}$	$\frac{x}{d} = \frac{1.33x10^{-3}}{\sqrt{f_c}}\left(\frac{M}{d^3}\right)d^{0.2}v^{1.33}$
Petry [5]	$x = 12K_p\ A_p \log_{10}\left(1+\frac{v^2}{215000}\right)$ K_P = concrete permeability = 0.00799 - massive PC = 0.00426 - normal RC = 0.00284 - special RC	$\frac{x}{d} = K\frac{M}{d^3}\log_{10}\left(1+\frac{v^2}{19974}\right)$ K = 6.36x10-4 - massive PC = 3.39x10-4 - normal RC = 2.26x10-4 - special RC $\frac{x}{d} = -0.0308 + 0.25251\ I, \quad 0.3 \le I \le 4$
Haldar[2]	Not available	$\frac{x}{d} = 0.6740 + 0.567\ I, \ 4 < I \le 21$ $\frac{x}{d} = -0.0308 + 0.25251\ I, \quad 0.3 \le I \le 4$ $\frac{x}{d} = 1.1875 + 0.0299\ I, \ 21 < I \le 455$
Ammann Whitney[5]	$\frac{x}{d} = \frac{282\ N\ W\ d^{0.2}}{d^3\ f_c^{0.5}}\left(\frac{v}{1000}\right)^{1.8}$	$\frac{x}{d} = \frac{6x10^{-4}}{f_c^{0.5}}N(M/d^3)d^{0.2}v^{1.8}$ $I = \frac{WNv^2}{gd^3f_c}$
Modified NDRC[5]	$G = \frac{180NM}{d\sqrt{f_c}}\left(\frac{v}{1000d}\right)^{1.8}$	$G = 3.8x10^{-5}\frac{NM}{d\sqrt{f_c}}\left(\frac{v}{d}\right)^{1.8}$
UKAEA[5]	$G = \frac{180NM}{d\sqrt{f_c}}\left(\frac{v}{1000d}\right)^{1.8}$	$\frac{x}{d} = 0.275 - [0.0756 - G]^{0.5}$ for $G \le 0.0726$ $\frac{x}{d} = [4G - 0.242]^{0.5}$ for $0.0726 \le G \le 1.0605$ $\frac{x}{d} = G + 0.9395$ for $G \ge 1.0605$

Where, N = Nose shape factor or nose performance coefficient =0.72+0.25 (CRH-0.25)0.5; W = Projectile weight; M = Mass of projectile; CRH = Caliber radius head; i.e. ratio of ogive radius and projectile diameter; x = Penetration depth; d = Projectile diameter; f_c = Compressive strength of concrete; v = Velocity of projectile

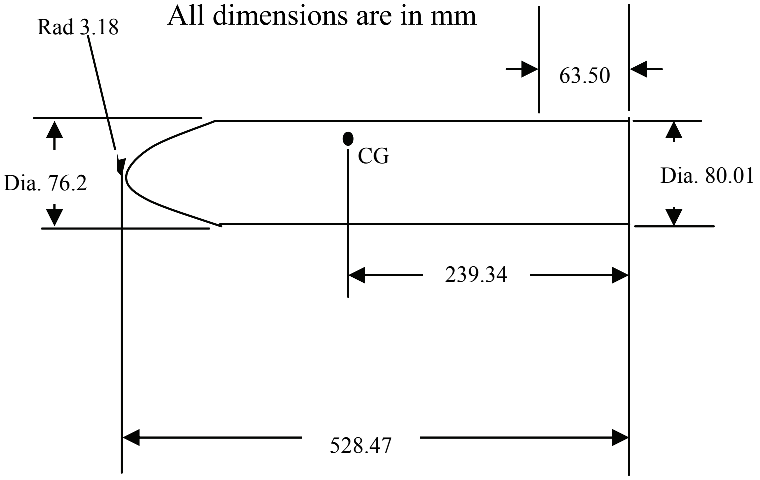

Figure 1: Projectile geometry

Table 2: Projectile characteristics [Forrestal et al. (2003)]

M (kg)	D (mm)	f_c(MPa)	v (m/s)
13.043*	76.2	23	139.3
13.037*	76.2	23	200
13.085*	76.2	23	250
13.158*	76.2	23	283.7
13.080*	76.2	23	336.6
13.119*	76.2	23	378.6
13.061**	76.2	23	238.4
13.064**	76.2	23	378.6
12.923*	76.2	39	238.1
12.900*	76.2	39	275.7
12.910*	76.2	39	314
12.914*	76.2	39	369.5
12.957*	76.2	39	456.4
12.873**	76.2	39	312.5
12.909**	76.2	39	448.5

*- CRH=3.0, **-CRH=6.0

tiles and concrete targets with confined compressive strengths of 14, 35 and 97MPa. Penetration depths are predicted by employing all the empirical formulae for different velocities, masses and compressive strengths. Projectile characteristics are shown in Tab. 3. Fig. 4 shows the variation of penetration depth for various velocities.

3.4 *Example 4- Frew et al. (1998)*

Frew et al. (1998) conducted depth of penetration experiments in concrete targets with 3.0 caliber radius head, steel rod projectiles. The unconfined compressive strength of target is 58.4 MPa. Projectiles are made up of $4340R_c$ 45 steel. Penetration depths are predicted for different velocities and diameter of the projectile by employing all the empirical formulae. Projectile characteristics are shown in Tab. 4. Fig. 5 shows the variation of penetration depth for various velocities.

Table 3: Projectile characteristics [Forrestal et al. (1994)]

S.N	M (kg)	D (mm)	f'_c(MPa)	v (m/s)
1	0.906	26.9	35.2	277
2	0.910	26.9	37.8	410
3	0.907	26.9	38.1	431
4	0.912	26.9	33.5	499
5	0.910	26.9	38.4	567
6	0.905	26.9	36.9	590
7	0.901	26.9	40.1	591
8	0.903	26.9	35.4	631
9	0.905	26.9	34.7	642
10	0.901	26.9	36.0	773
11	0.904	26.9	32.4	800
12	0.907	26.9	90.5	561
13	0.898	26.9	91.0	584
14	0.908	26.9	95.0	608
15	0.905	26.9	101.4	622
16	0.907	26.9	94.0	750
17	0.900	26.9	108.3	793

Table 4: Projectile characteristics [Frew et al. (1998)]

S.No	M (kg)	D (mm)	f'_c(MPa	v (m/s)
1	0.478	20.3	58.4	442
2	0.478	20.3	58.4	610
3	0.478	20.3	58.4	815
4	0.478	20.3	58.4	1009
5	0.478	20.3	58.4	791
6	0.478	20.3	58.4	994
7	0.606	20.3	58.4	797
8	0.734	20.3	58.4	803
9	1.62	30.5	58.4	445
10	1.62	30.5	58.4	584
11	1.62	30.5	58.4	796

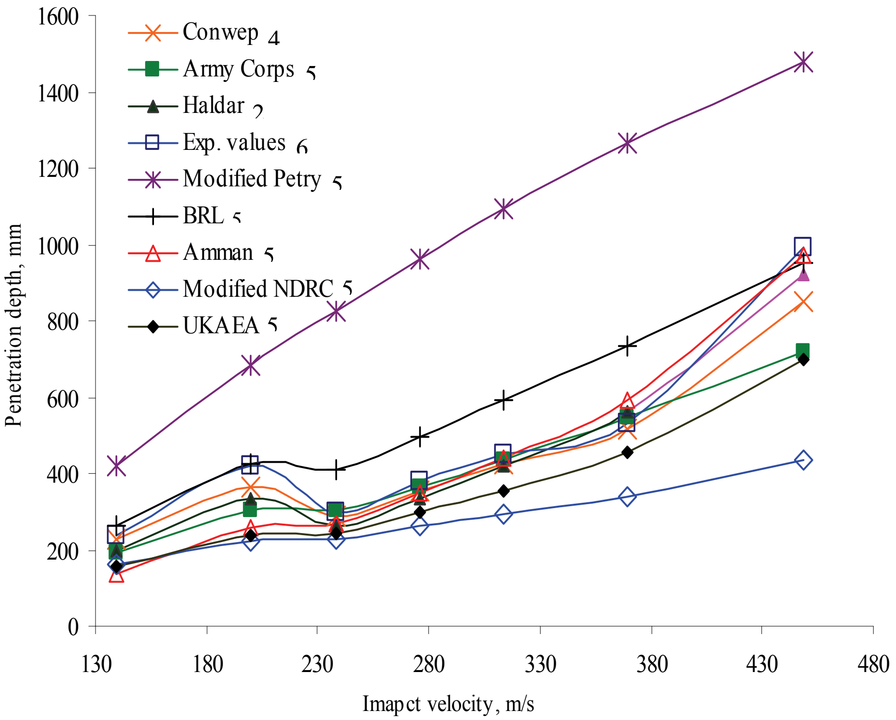

Figure 2: Impact velocity vs penetration depth (Forrestal et al. (2003))

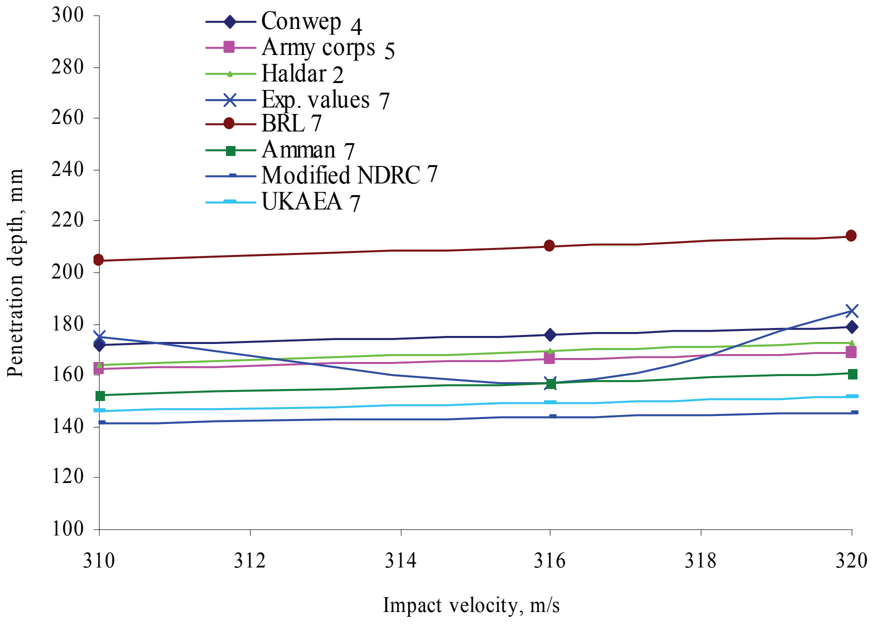

Figure 3: Impact velocity vs penetration depth (Gran and Frew (1997))

3.5 Observations

From Figs. 2 to 5, it can be observed that the penetration depth values by employing Conwep, Haldar and Ammann formulae are in good agreement with the chosen experimental observations

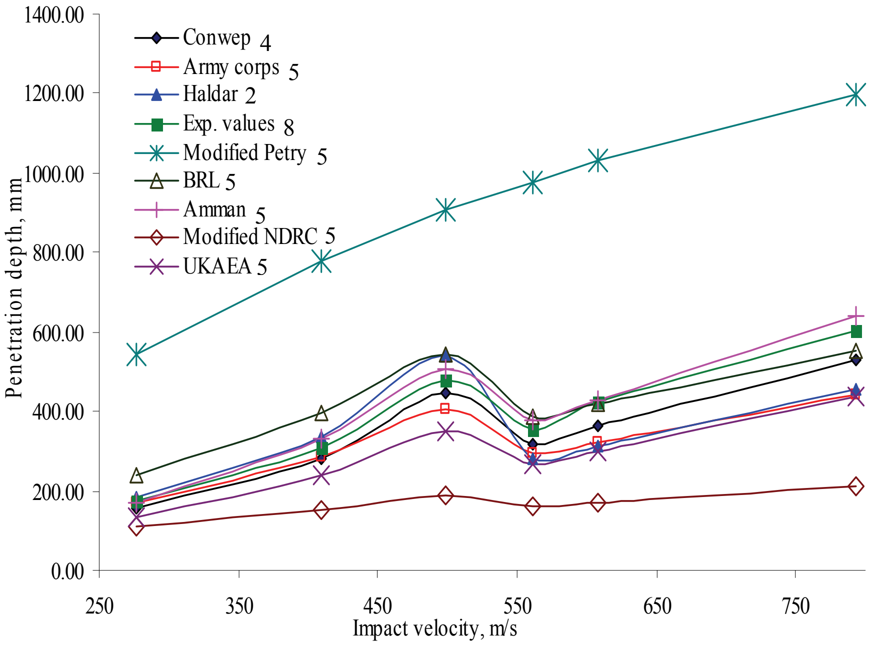

Figure 4: Impact velocity vs penetration depth [Forrestal et al. (1994)]

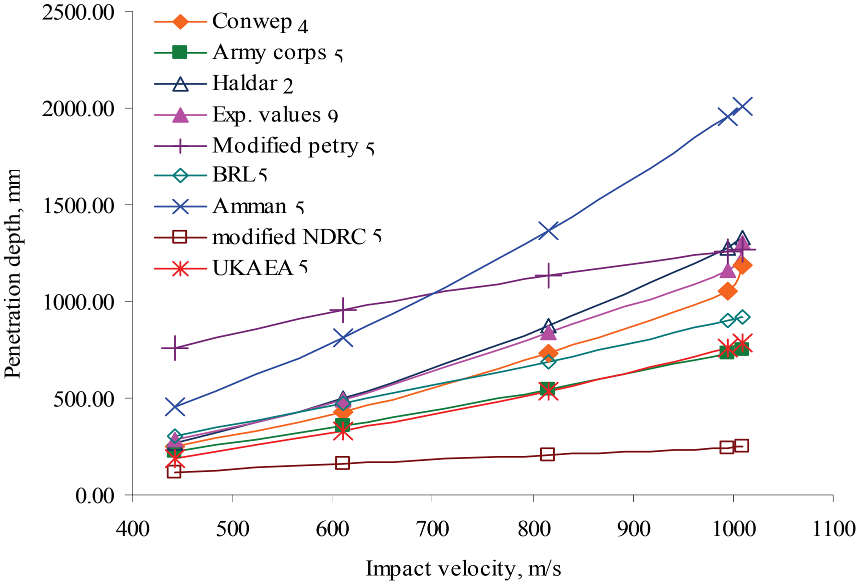

Figure 5: Impact velocity vs penetration depth [Frew et al. (1998)]

for all the example problems solved. The penetration depth values obtained by using modified Petry and BRL are larger compared to the experimental observations. Further, it can be observed that the penetration values obtained by using Army Corps, modified NDRC and UKAEA are lesser compared to experimental observations.

4 Summary and Concluding Remarks

The performance of widely used empirical formulae for evaluation of concrete penetration depth under impact loading has been studied. Various empirical formulae include Conwep, Army corps, Haldar, Ammann and Whitney, UKAEA, BRL, modified Petry and modified NDRC. Appropriate expressions for all these empirical formulae have been presented. Performance of these empirical formulae has been studied by conducting numerical studies. The penetration depth values for all the above formulae have been compared with the corresponding experimental values. Based on these comparisons, the following conclusions are drawn:

- conwep and Haldar formulae exhibit better performance
- modified Petry and Ammann formulae generally overestimate
- modified NDRC, BRL, Army Corps and UKAEA formulae generally underestimate

Acknowledgement: We acknowledge with thanks the valuable suggestions provided by our colleagues Mr J. Rajasankar, Ms Smitha Gopinath, Ms P. Kamatchi and Ms A. Cinitha, Scientists, during the course of this investigation. This paper is being published with the kind permission of the Director, SERC, Chennai, India.

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