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Boundary Element Analysis of Cracked Thick Plates Repaired with Adhesively Bonded Composite Patches

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Abstract: The fracture analysis of cracked thick plates repaired with adhesively bonded composite patches using a boundary element formulation is presented. The shear deformable cracked isotropic plate was modeled using the dual boundary method. In order to model the repair, a three parameter boundary element formulation was established. This formulation is based on Kirchhoff's theory for symmetric layer composite plates and considers the transversal deflection and two in-plane rotations. Interaction forces and moments between the cracked plate and the composite repair were modeled as distributed loading, and discretized using continuous and semi-discontinuous domain cells. Coupling equations, based on kinematic compatibility and equilibrium considerations for the adhesive, were established. In-plane shear-deformable adhesive model without transversal stiffness was considered in order to modeling the mechanical response of the adhesive. Stress intensity factors in the isotropic Reissner's plate were calculated using crack surface displacements extrapolation. Test problems considering circular composite repair are presented.

Keyword: Fracture mechanics, fracture plates, dual boundary element method, adhesive composite patches, anisotropic repair

1 Introduction

Aeronautic structures are usually constituted of panels and metallic stiffeners. A cracked panel is frequently repaired by bonding, riveting or screwing a metallic patch on the cracked area. The life in fatigue and the residual stresses in the repaired panel are dependent on the efficiency of the load transfer of the cracked panel to the repair. Advanced bonded composite repairs have been used in the aeronautical industry and they are accepted as efficient solutions for the repair of damaged structures. The main advantage, when compared to screwed or riveted repairs is that they supply a load transfer relatively uniform among the structural components that are bonded. The required holes for these fasteners act as stresses concentrators that reduces the useful life of the aeronautical panel.

The Boundary Element Method (BEM) is an atractive numerical alternative to treat fracture problems, mainly to its ability to model continuously high stress gradients without the need of domain discretization. The use of this method in structural analysis has strongly increased since 80s [see Aliabadi (2002)]. The analysis of cracked isotropic plates structures repaired with the application of adhesively bonded anisotropic patches using BEM hasn't been reported in the literature, to the authors knowledge.

Widago and Aliabadi (2001) presented a BEM formulation for the analysis of metallic sheets repaired by screwed composite materials. The cracked sheet is modeled using the dual boundary element technique. Screws are modeled as linear springs whose forces are treated as point forces. The repair is modeled using a boundary element formulation for anisotropic bi-dimensional plates. Later, Wen, Aliabadi and Young (2002) developed a boundary element formulation for the analysis of flat metallic plates with cracks and adhesive isotropic repairs. The effect of the adhesive layer was modeled considering them as distributed forces. A coupled integral formulation for plate with shear deformation and plane

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stress was used to determine bending moments and membrane forces in the adhesive repair.

Boundary element formulations have been applied to plate bending anisotropic problems considering Kirchhoff and shear deformable plate theories. Shi and Bezine (1998) presented a boundary element analysis of plate bending problems based on Kirchhoff's plate bending assumptions. Boundary element method for orthotropic Reisnner's plates was presented by Wang, J. and Huang, M. (1991). Shear deformable cracked plates have been analyzed using boundary element method by Dirgantara and Aliabadi (2001) with the fundamental solution proposed by Vander Weeën (1982). Wen and Aliabadi (2006) presented a displacement discontinuity formulation for modeling cracks in orthotropic Reissner plates. Fundamental solutions for displacement discontinuity were derived for the first time using a Fourier transform method in this reference.

This paper presents the fracture analysis of cracked thick plates repaired with adhesively bonded composite patches using a boundary element formulation. The shear deformable cracked isotropic plate was modeled using the dual boundary method. In order to model the repair, a three parameter boundary element formulation, based on Kirchhoff's theory for symmetric layer composite plates was developed. Coupling equations, based on kinematic compatibility and equilibrium considerations for the adhesive, were established. Stress intensity factors were calculated using crack surface displacements extrapolation. Test problems considering circular composite repair are presented.

2 Isotropic plate formulation

The two dimensional boundary integral equation for displacements at the boundary point $\mathbf{x}' \in \Gamma$ that describes membrane effects can be written as Aliabadi (2002):

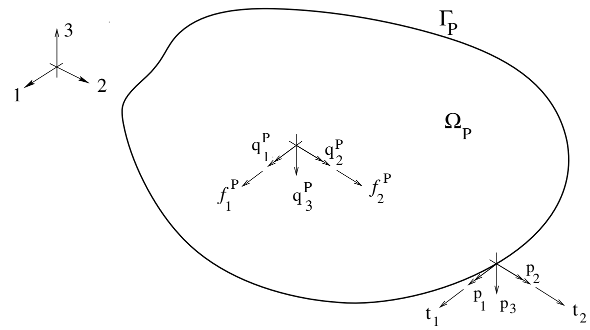

Figure 1: Definition of boundary and distributed body forces and moments at isotropic plate

$$c_{ij}^{P}\left(\mathbf{x}'\right)u_{\beta}\left(\mathbf{x}'\right)=\int\limits_{\Gamma}U_{\alpha\beta}^{P}\left(\mathbf{x}',\mathbf{x}\right)t_{\beta}d\Gamma-\int\limits_{\Gamma}T_{\alpha\beta}^{P}\left(\mathbf{x}',\mathbf{x}\right)u_{\beta}d\Gamma+\frac{1}{h_p}\int\limits_{A}U_{\alpha\beta}^{P}\left(\mathbf{x}',\mathbf{x}\right)f_{\beta}dA \qquad (1)$$

where α, β = 1, 2 and $c_{ij}^{P}(\mathbf{x}')$ is a function of the geometry at the collocation points that can be determinated by considering rigid body movements. The boundary displacements and tractions for the sheet are denoted by u_α and $t_\alpha (= n_\beta \sigma_{\alpha\beta})$, respectively; displacement and traction fundamental solutions for the plane stress condition are $U_{\alpha\beta}^{P}(\mathbf{x}',\mathbf{x})$ and $T_{\alpha\beta}^{P}(\mathbf{x}',\mathbf{x})$ respectively, $f_\beta(\mathbf{x})$ denote two-dimensional body forces por unit area over a region A of patch and h_p is the thickness of the plate. In this work no other in-plane body forces will be considered. The upper index refers to the isotropic plate.

In order to model cracked plates, the Dual Boundary Element Method (DBEM) will be used. In this method, the displacement integral formulation is written for source points on one crack surface and the traction integral equation on the other surface. Then, using the stress and strain relationships for plane stress, the traction integral equation for two-dimensional problems in a smooth boundary can be derived as [see Dirgantara (2002)]:

$$\frac{1}{2}t_\alpha\left(\mathbf{x}'\right) = n_\beta\left(\mathbf{x}'\right)\int_\Gamma U^P_{\alpha\beta\gamma}\left(\mathbf{x}',\mathbf{x}\right)t_\gamma d\Gamma$$
$$-n_\beta\left(\mathbf{x}'\right)\int_\Gamma T^P_{\alpha\beta\gamma}\left(\mathbf{x}',\mathbf{x}\right)u_\gamma d\Gamma$$
$$+n_\beta\left(\mathbf{x}'\right)\frac{1}{h_p}\int_A U^P_{\alpha\beta\gamma}\left(\mathbf{x}',\mathbf{x}\right)f_\beta dA \qquad (2)$$

where $n_\beta(\mathbf{x}')$ is the normal to the boundary evaluated at collocation point. $U^P_{\alpha\beta\gamma}(\mathbf{x}',\mathbf{x})$ and $T^P_{\alpha\beta\gamma}(\mathbf{x}',\mathbf{x})$ are the displacement and traction fundamental solutions for two-dimensional problems.

For plate bending boundary integral formulation, w_α are defined as rotations in the x_α direction, w_3 is the deflection of the plate along x_3, q^P_α and q^P_3 are the distribution of moments and the out-of-plane body force per unit area, respectively, in the patch area A, and p_o is the pressure force applied in the domain of the plate Ω (see figure 1). Then, the boundary integral formulation for the plate bending problem can be written as:

$$c^P_{ik}\left(\mathbf{x}'\right)w_k\left(\mathbf{x}'\right) = \int_\Gamma W^P_{ik}\left(\mathbf{x}',\mathbf{x}\right)p_k d\Gamma$$
$$-\int_\Gamma P^P_{ik}\left(\mathbf{x}',\mathbf{x}\right)w_k d\Gamma + \int_\Omega W^P_{i3}\left(\mathbf{x}',\mathbf{x}\right)p_o d\Omega$$
$$+\int_A W^P_{ik}\left(\mathbf{x}',\mathbf{x}\right)q^P_k dA \qquad (3)$$

where $k = 1\ldots3$. $W^P_{\alpha\beta}(\mathbf{x}',\mathbf{x})$ and $P^P_{\alpha\beta}(\mathbf{x}',\mathbf{x})$ are the fundamental solutions for Reissner's plate model [see Vander Weeën (1982)] and $p_\alpha = M_{\alpha\beta}n_\beta$, $p_3 = Q_\beta n_\beta$. Constant c^P_{ik} is similar with those at in-plane displacement problem.

In a similar way, fracture mechanics problems involving plate bending can be modeled usign DBEM. In this case, the traction equation can be written as:

$$\frac{1}{2}p_i\left(\mathbf{x}'\right) = n_\beta\int_\Gamma W^P_{i\beta k}\left(\mathbf{x}',\mathbf{x}\right)p_k d\Gamma$$
$$-n_\beta\int_\Gamma P^P_{i\beta k}\left(\mathbf{x}',\mathbf{x}\right)w_k d\Gamma$$
$$+n_\beta\int_\Omega W^P_{i\beta 3}\left(\mathbf{x}',\mathbf{x}\right)p_o d\Omega$$
$$+n_\beta\int_A W^P_{i\beta k}\left(\mathbf{x}',\mathbf{x}\right)q^P_k dA \qquad (4)$$

where $W^P_{i\beta\gamma}(\mathbf{x}',\mathbf{x})$ and $P^P_{i\beta\gamma}(\mathbf{x}',\mathbf{x})$ are the displacement and traction fundamental solutions for isotropic Reissner's plate [see Dirgantara (2002)].

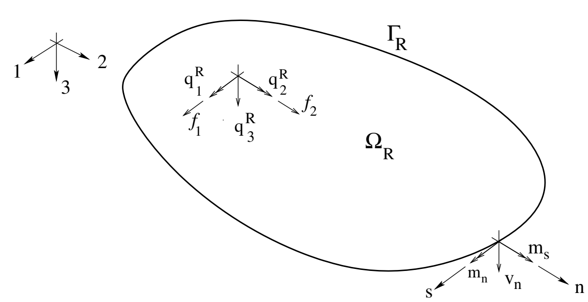

Figure 2: Definition of boundary and distributed body forces and moments at repair

3 Composite patch formulation

Similary to the isotropic case, the in-plane displacements of a point $\mathbf{x}'$ in the anisotropic patch are given by:

$$c^R_{\alpha\beta}\left(\mathbf{x}'\right)u^R_\beta + \int_{\Gamma_R} T^R_{\alpha\beta}\left(\mathbf{x}',\mathbf{x}\right)u^R_\beta d\Gamma$$
$$= \frac{1}{h_R}\int_A U^R_{\alpha\beta}\left(\mathbf{x}',\mathbf{x}\right)f^R_\beta dA \qquad (5)$$

where $T^R_{\alpha\beta}(\mathbf{x}',\mathbf{x})$ and $T^R_{\alpha\beta}(\mathbf{x}',\mathbf{x})$ are the traction and displacements fundamental solutions for anisotropic plane elasticity ploblems and h_R represents the repair thickness [see Albuquerque (2006)]. Others variables have similar meaning to the isotropic case (see figure 2). To model the bending response of the repair, a boundary integral formulation for Kichhoff's plate model with three unknows at every point is used in this work considering the original form of the Betti's theorem for the Kirchhoff plate, as presented by Palermo (2003):

$$\int_\Gamma \left(W_{,n}(\mathbf{x}',\mathbf{x})m_n + W_{,s}(\mathbf{x}',\mathbf{x})t_s - W(\mathbf{x}',\mathbf{x})v_n\right) d\Gamma = \int_\Gamma \left(M_n(\mathbf{x}',\mathbf{x})w_{,n} + T_s(\mathbf{x}',\mathbf{x})w_{,s} - V_n(\mathbf{x}',\mathbf{x})w\right) d\Gamma \tag{6}$$

where $w(\mathbf{x})$ and $w_{,n}(\mathbf{x})$ are the bending deflexion and the normal rotation, respectively, $v_n(\mathbf{x})$ and $m_n(\mathbf{x})$ are the shear force and the normal moment, respectively, and $t_s(\mathbf{x})$ is the tangent moment. $W(\mathbf{x}',\mathbf{x}), V_n(\mathbf{x}',\mathbf{x}), M_n(\mathbf{x}',\mathbf{x}), T_s(\mathbf{x}',\mathbf{x})$ are the fundamental solutions for Kirchhoff's anisotropic plates [see Shi and Bezine (1988)]. Using the stress-strain relationships for anisotropic Kirchhoff plates, integrating by parts, taking as weight function the Dirac's delta function and considering: $t_s \equiv 0$ we obtain the displacement integral formulation for bending plate with three parameters, two in-plane rotations ($w_{,n}$, $w_{,s}$) and the flexural bending w:

$$\begin{aligned} w &+ \int_\Gamma \left[V_n(\mathbf{x}',\mathbf{x})\, w^R - M_n(\mathbf{x}',\mathbf{x})\, w^R_{,n}\right] d\Gamma \\ &- \int_\Gamma T_s(\mathbf{x}',\mathbf{x})\, w^R_{,s} d\Gamma = \int_\Gamma W(\mathbf{x}',\mathbf{x})\, v_n d\Gamma \\ &- \int_\Gamma W_{,n}(\mathbf{x}',\mathbf{x}) m_n d\Gamma + \int_A W_{,\alpha}(\mathbf{x}',\mathbf{x})\, q^R_\alpha dA \\ &+ \int_A W(\mathbf{x}',\mathbf{x})\, q^R_3 dA \end{aligned} \tag{7}$$

with $\alpha = 1,2$. ${q_\alpha}^R$ and q^R_3 are distributed body moments and out-of-plane body force by unit area, respectively, generated by interaction with the adhesive layer. The upper index R refers to the composite repair.

A second boundary integral equation is obtained by differentiating Equation (7) with respect to point $\mathbf{x}'$ in the tangent direction:

$$\begin{aligned} w_{,s} &+ \int_\Gamma \left[V_{n,s}(\mathbf{x}',\mathbf{x})\, w^R - M_{n,s}(\mathbf{x}',\mathbf{x})\, w^R_{,n}\right] d\Gamma \\ &- \int_\Gamma T_{s,s}(\mathbf{x}',\mathbf{x})\, w^R_{,s} d\Gamma = \int_\Gamma W_{,s}(\mathbf{x}',\mathbf{x})\, v_n d\Gamma \\ &- \int_\Gamma W_{,ns}(\mathbf{x}',\mathbf{x})\, m_n d\Gamma + \int_A W_{,\alpha s}(\mathbf{x}',\mathbf{x})\, q^R_\alpha dA \\ &+ \int_A W_{,s}(\mathbf{x}',\mathbf{x})\, q^R_3 dA \end{aligned} \tag{8}$$

Finally, a third integral equation can be obtained differentiating Equation (7) in the normal direction:

$$\begin{aligned} w_{,n} &+ \int_\Gamma \left[V_{n,n}(\mathbf{x}',\mathbf{x})\, w^R - M_{n,n}(\mathbf{x}',\mathbf{x})\, w^R_{,n}\right] d\Gamma \\ &- \int_\Gamma T_{s,n}(\mathbf{x}',\mathbf{x})\, w^R_{,s} d\Gamma = \int_\Gamma \left[W_{,n}(\mathbf{x}',\mathbf{x})\, v_n\right] d\Gamma \\ &- \int_\Gamma W_{,nn}(\mathbf{x}',\mathbf{x})\, m_n d\Gamma + \int_A W_{,\alpha n}(\mathbf{x}',\mathbf{x})\, q^R_\alpha dA \\ &+ \int_A W_{,n}(\mathbf{x}',\mathbf{x})\, q^R_3 dA \end{aligned} \tag{9}$$

Equations (7) to (9) can be presented in matrix form defining:

$$\begin{aligned} &H_{11} = \int_\Gamma V_n d\Gamma; \quad H_{12} = -\int_\Gamma M_n d\Gamma; \\ &H_{21} = \int_\Gamma V_{n,n} d\Gamma; \quad H_{22} = -\int_\Gamma M_{n,n} d\Gamma; \\ &H_{31} = \int_\Gamma V_{n,s} d\Gamma; \quad H_{32} = -\int_\Gamma M_{n,s} d\Gamma; \\ &H_{13} = -\int_\Gamma T_s d\Gamma; \quad H_{23} = -\int_\Gamma T_{s,n} d\Gamma; \\ &H_{33} = -\int_\Gamma T_{s,s} d\Gamma \end{aligned} \tag{10}$$

and,

$$
\begin{aligned}
G_{11} &= \int\limits_{\Gamma} W d\Gamma; \quad G_{12} = -\int\limits_{\Gamma} W_{,n} d\Gamma; \\
G_{21} &= \int\limits_{\Gamma} W_{,n} d\Gamma; \quad G_{22} = -\int\limits_{\Gamma} W_{,nn} d\Gamma; \\
G_{31} &= \int\limits_{\Gamma} W_{,s} d\Gamma; \quad G_{32} = -\int\limits_{\Gamma} W_{,ns} d\Gamma
\end{aligned}
\tag{11}
$$

With similar expressions for domain integrals, equations (7), (8) and (9) can be written in a matrix form as:

$$
\begin{aligned}
&\begin{bmatrix} H_{11} & H_{12} & H_{13} \\ H_{21} & H_{22} & H_{23} \\ H_{31} & H_{32} & H_{33} \end{bmatrix} \begin{Bmatrix} w^R \\ w^R_{,n} \\ w^R_{,s} \end{Bmatrix} \\
&= \begin{bmatrix} G_{11} & G_{12} \\ G_{21} & G_{22} \\ G_{31} & G_{32} \end{bmatrix} \begin{Bmatrix} V_n \\ M_n \end{Bmatrix} \\
&+ \begin{bmatrix} B_{11} & B_{12} & B_{13} \\ B_{21} & B_{22} & B_{23} \\ B_{31} & B_{32} & B_{33} \end{bmatrix} \begin{Bmatrix} q^R \\ q^R_n \\ q^R_s \end{Bmatrix}
\end{aligned}
\tag{12}
$$

4 Coupling equations

Isotropic plate equations have fifteen unknowns variables: five displacements, or tractions, at any boundary points; five unknowns displacements and five interaction body forces at any point in the repair region. In addition, ten unknows appear at the repair: five displacements at boundary and domain and five interacion body forces at domain. In this way we have twenty five unknows in the problem. Expanding equation (1) throught Equation (9) we obtain only fifteen equations. Ten aditional equations must be provided. Aditional equations can be written if displacement compatibility between plate and repair and the equilibrium conditions at adhesive layer, are considered. In this way a total of twenty five equations cand be obtained (See figure 3 for the coupling components).

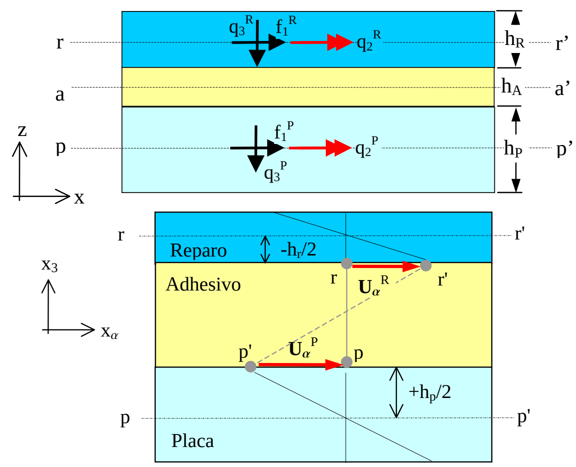

Figure 3: Left: Forces and moments at $x_1 - z$ plane for equilibrium equations. Right: Displacement components at adhesive interfaces for shear stress definition

The equilibrium of forces acting in the adhesive layer can be written as [see Wen, Aliabadi and Young (2002)]:

$$f_\alpha^P + f_\alpha^R = 0$$

$$q_3^P + q_3^R = 0$$

$$q_\alpha^P + q_\alpha^R + f_\alpha^R \left(h_A + \frac{h_P + h_R}{2} \right) = 0 \tag{13}$$

Where h_A is the thickness of the adhesive. The shear force $\tau_{3\alpha}^A$, acting at interior of the adhesive layer can be written as:

$$\tau_{3\alpha}^A = f_\alpha^R = \frac{\mu_A}{h_A} \left(u_\alpha^R - \frac{h_R}{2} w_\alpha^R \right) - \left(u_\alpha^P + \frac{h_P}{2} w_\alpha^P \right) \tag{14}$$

where μ_A is the shear modulus of the adhesive. Finally, we can consider that deflexion and rotation angles at coincident points at plate and repair related as:

$$w_3^P = w_3^R$$

$$q_\alpha^P = C\left(w_\alpha^R + w_\alpha^P \right) \tag{15}$$

where, $C = D(1-\nu)\lambda^2/2$ for the isotropic plate. D is the bending stiffness of the plate, ν is the

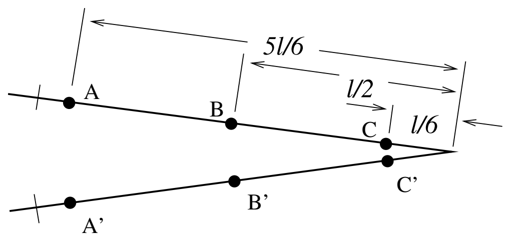

Figure 4: Crack tip discontinuous element definition used to evaluate the stress intensity factors.

Poisson ratio and $\lambda = \sqrt{10}/h_P$. In this way, equations (13) through equations (15) represent ten aditional equations obtained considering equilibrium and compatibility conditions in the adhesive layer. These equations can be written in matrix form as:

$$\mathbf{I}\mathbf{q}^P + \mathbf{I}\mathbf{q}^R - \mathbf{C}^{-1}\mathbf{f}^P = \mathbf{0}$$
$$\mathbf{I}\mathbf{u}^R - \mathbf{I}\mathbf{u}^P - \mathbf{C}_2\mathbf{q}^P - (\mathbf{C}_1)^T\mathbf{w}^P + \mathbf{A}\mathbf{f}^P = \mathbf{0} \qquad (16)$$

where the constant $1/2(h_R + h_P)$ was included into matrix $(\mathbf{C}_1)^T$, $\mathbf{C}_2 = (h_R/2)(\mathbf{C}_1)^T\mathbf{C}^{-1}$ and $\mathbf{A} = h_A/\mu_A\mathbf{I}$. In these equations, $\mathbf{I}$ represents the identity matrix, $\mathbf{q}^P$ and $\mathbf{q}^R$ are vectors of bending moments and transverse loads for plate and the repair, respectively; $\mathbf{u}^P$ and $\mathbf{u}^R$ contains the in-plane displacement components for plate and the repair; $\mathbf{f}^P$ is the vector of in-plane loads for the plate; $\mathbf{w}^P$ is the vector of transversal displacements for the plate and $\mathbf{C}_1$ is a matrix of constants.

5 Boundary element formulation

5.1 *Isotropic plate equations*

Using the boundary element method, the discretized version of Equations (1) and (2) for the in-plane elasticity for the isotropic cracked plate, can be written in matrix form as:

$$^M\mathbf{H}^P\mathbf{u}^P = {}^M\mathbf{G}^P\mathbf{t}^P + {}^M\mathbf{B}^P\mathbf{f}^P \qquad (17)$$

where the upper index M refers to membrane response. $\mathbf{H}$, $\mathbf{G}$ and $\mathbf{B}$ are the usual matrix founded in BEM formulation. Using the cell method, the matrix $^M\mathbf{B}^P$ is given by,

$$\int_A U^P_{\alpha\beta} f_\beta dA = \left[\sum_{i=1}^{Ncell} \int_{\Omega cell} U^P_{\alpha\beta} N_{k\beta} d\Omega_{cell}\right] f_\beta = {}^M\mathbf{B}^P\mathbf{f}^P \qquad (18)$$

where N_{cell} is the number of cells, Ω_{cell} refers to the cell's domain and ND are the number of collocation points at the repair area.

Similarly, bending equations for the isotropic plate are given by,

$$^B\mathbf{H}^P\mathbf{w}^P = {}^B\mathbf{G}^P\mathbf{p}^P + {}^B\mathbf{B}^P\mathbf{q}^P + \mathbf{q}^0 \qquad (19)$$

where upper index B refers to bending response. In this equation, $\mathbf{q^0}$ represents pressure loads acting on the surface of the plate. Equations (17) and (18) are the BEM equations for the isotropic plate.

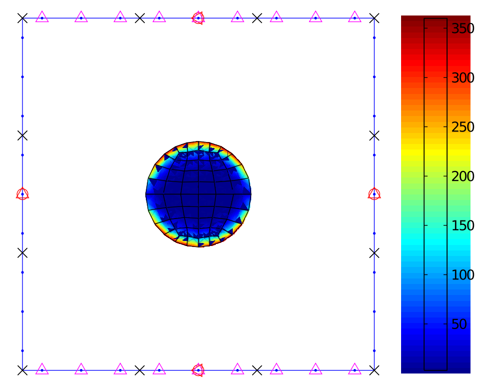

Figure 5: Normalized shear stress distribution in the adhesive layer.

5.2 *Patch repair equations*

In a similar way, the discretized BEM equations for the in-plane elasticity for the repair plate are given by,

$$^M\mathbf{H}^R\mathbf{u}^R = {}^M\mathbf{B}^R\mathbf{f}^R \qquad (20)$$

Bending equations for the repair plate is given by equation (12) treated using the BEM. These equations can be written in matrix form as:

$$^P\mathbf{H}^R\mathbf{w}^R = {}^P\mathbf{B}^R\mathbf{q}^R \qquad (21)$$

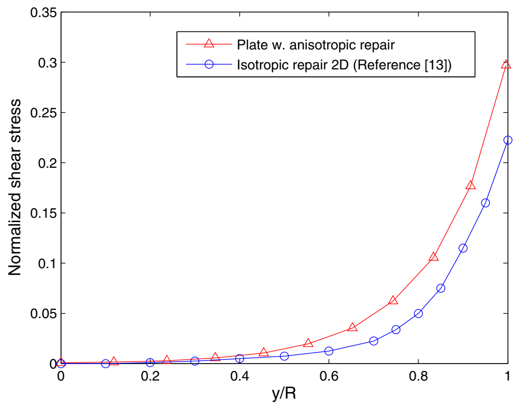

Figure 6: Normalized shear stress in the adhesive layer along x_2-axis normalized with respect to radius of the repair.

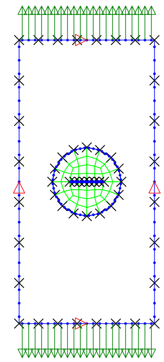

Figure 7: Boundary element model for cracked isotropic plate with composite circular patch.

Equations (17) through (21) constitutes the complete system of equations for the cracked plate repair problem.

6 Stress intensity factors evaluation

For plate problems, considering bending and plane stress, the stress intensity factors can be represented by superposition of five stress intensity factors (SIF's), two due to membrane loads and three due to bending and shear loads. In terms of displacements on the crack surfaces they can be written as [see Dirgantara (2002)]:

$$\{K\} = \frac{1}{\sqrt{r}}\mathbf{C}\{\Delta w\} \tag{22}$$

where $\{K\}$ is a vector containing the five stress intensity factors, $\mathbf{C}$ is a matrix containing the elasticity material properties and plate thickness and r is the distance from crack tip to the specific $\{K\}$ evaluation point. Using the displacement extrapolation technique and discontinuous quadratic boundary elements for modeling crack surfaces, SIF can be evaluated as (see figure 4):

$$\{K\}^{tip} = \frac{r_{AA'}}{r_{AA'} - r_{BB'}}\left[\{K\}^{BB'} - \frac{r_{BB'}}{r_{AA'}}\{K\}^{AA'}\right] \tag{23}$$

7 Numerical results

7.1 Plate with bonded anisotropic circular patch

In order to test the formulation presented here, a square isotropic plate with adhesively bonded anisotropic circular patch will be analyzed and shear stress distribution in the repair zone will be compared with the analytical solution given by Rose and Wang (2002) for two-dimensional isotropic repair. The width of the plate is 180 mm, thickness 1.5 mm and it is subject to an in-plane traction load σ_0 = 1000 MPa applied at $x_2 = \pm 90mm$. The isotropic plate elastic properties are chosen as E = 70 GPa, $\nu = 0.3$. A circular anisotropic patch of radius R = 30 mm is bonded to the plate over the region $R^2 = x_1^2 + x_2^2$. The mechanical properties considered for the patch are: $E_1 = 25$ GPa, E_2 = 208 GPa, G_{12} = 72.4 GPa and $\nu = 0.02$. Twelve boundary element were used to discretize the plate border and 24 quadratic discontinuous boundary elements in the boundary repair.

Figure 5 shows the normalized shear stress distribution in the adhesive layer. A similar distribution with that founded for the isotropic repair is observed. Figure 6 shows the normalized shear stress in the adhesive along x_2-axis compared with analytic solution given by Rose and Wang (2002) for two-dimensional isotropic repair.

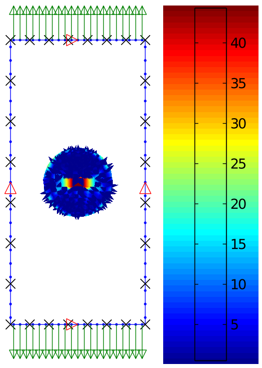

Figure 8: Shear stress distribution in the adhesive layer

7.2 *Rectangular cracked plate with bonded composite patch*

The plate is 248 mm × 118 mm, thickness h_P =2.0 mm and it is subject to in-plane load σ_0 =79.4 MPa. The material constants are chosen as E = 72.39 GPa, ν =0.33. A circular anisotropic patch of radius R = 25 mm and thickness h_R = 3.2 mm is bonded to the plate (see Figure **??**). The mechanical properties are of patch are: E_1 = 11.38 GPa, E_2 = 37.35 GPa, G_{12} = 5.97 GPa and ν = 0.38. The adhesive layer has thickness h_a = 0.1 mm and shear modulus μ_a = 0.44 GPa. The same problem was analyzed by Sekine, Yan and Yasuho (2005) where the cracked plate is modeled using a 3D BEM model and the repair using a finite element plate model.

Table 1: Stress intensity factors for cracked plate with bonded composite patch

z(mm)	K_{Imax} **BEM** $MPa.m^{1/2}$	K_{Imax}**-Sekine et. al.** $MPa.m^{1/2}$
0.40	13.82	12.60
0.80	11.56	11.09
1.20	9.89	9.52
1.60	8.15	7.84

A total of 28 quadratic discontinuous boundary elements were used to discretize the boundary of the isotropic cracked plate. Meshes from 4 to 16 quadratic discontinuous boundary elements were used to discretize the crack faces. Patch domain was discretized using 128 cells and 24 quadratic discontinuos boundary elements has been used, [see figure 7]. Simply supported conditions were applyed to all sides. The resultant shear stress distribution in the adhesive layer is showed in figure 8.

Table 1 compares values for the maximum stress intensity factor: $K_{Imax} = K_{Im} + 6/h_P^2 K_I^b$ calculed along plate thickness with those K_I reported by Sekine, Yan and Yasuho (2005). In this equation, K_{Im} represents the stress intensity factor in mode I for membrane response and K_I^b represents the stress intensity factor in mode I for bending response.

8 Conclusions

The analysis of cracked isotropic thick plates repaired with symmetrical laminate composite plates using the boundary element method, was presented. The equations for the repair is based on boundary integral formulation considering three parameters, based on the theory of Kirchhoff's plates as a generalization of the integral formulation of thin plates. The linear isotropic model proposed for the adhesive, considers shear forces and bending moments acting on it. This way, equations for kinematic coupling for displacements and rotations, as well as, a system of equations that describe the equilibrium of forces and moments that act on the adhesive, were stablished. Domain integrals containing forces and moments in the repair's area were threated with using the cell method. The examples shows that the new formulation can be used with reasonable accuracy to study the mechanical behaviour of cracked plates repaired with adhesively bonded composite repairs under in-plane load actions.

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