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Electroelastic Problem of Two Anti-Plane Collinear Cracks at the Interface of Two Bonded Dissimilar Piezoelectric Layers

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Abstract: Under the permeable electric boundary condition the problem of two collinear anti-plane shear cracks situated at the interface of two bonded dissimilar piezoelectric layers is considered. It is assumed that applied longitudinal shear stress and electric loading at the layer surfaces are prescribed. By the use of Fourier transforms we reduce the problem to solving a set of triple integral equations with a cosine kernel. The triple integral equations are further reduced to a Fredholm integral equation of the second kind whose iterative solution has been obtained. Analytical expressions for the stress intensity factors are obtained. Numerical results are presented in the form of graphs and compared with the results of stress intensity factors of two collinear cracks in a homogeneous elastic layer.

1 Introduction

Research in the area of piezoelectricity has led to the development of a variety of important electronic and electromechanical devices which are being used in spacecraft launch vehicles and military equipment. When piezoelectric ceramics are subject to mechanical and electrical loads in service, flaws or defects caused by manufacturing may lead to premature failure of these materials (or composites) due to static anti-plane and in plane electric loading which have been studied by several authors. In recent years, many researchers have studied crack problems in piezoelectric materials. Shindo et. al. (14; 15; 16) have made a systematic study on the electro-elastic field of a piezoelectric strip with a central crack parallel to or perpendicular to the strip boundaries. Closed form solutions for one or two cracks in the layers or strip have been obtained by Li and Duan (8), Li (9; 10), Zhong and Li (19) and Li (11; 12). Singh, Rokne and Dhaliwal (17) have solved the problem of two cracks in a layer by obtaining a closed form solution. The problem involving two collinear or parallel anti-plane shear cracks in a piezoelectric layer bonded to two half-spaces has been treated by Zhou et. al (20) and Zhou and Wang (21). Chen and Yu (1), Chen et. al. (2) and Li, X. F. et. al. (7) have considered the problems of a crack at the interface of two dissimilar piezoelectric materials. It is also important to mention some recent work by Haüsler, C. et. al. (6) and Gao, C.L. et. al (4) on the interface cracks in two bonded piezoelectric materials.

In this paper we consider the problem of two anti-plane shear cracks situated at the interface of two bonded infinite layers of different piezoelectric materials subjected to longitudinal shear stresses and electric loading on the layer surfaces. By the use of Fourier transforms we reduce the problem to solving a set of triple integral equations with a cosine kernel which are further reduced to the solution of a Fredholm integral equation of the second kind. Iterative solution of the Fredholm integral equation has been obtained in terms of inverse powers of h_2 by assuming $h_2 >> 1$ and $h_2 \geq h_1$ where h_2 and h_1 are the thicknesses of the two layers. The case $h_1 \geq h_2$ is can be treated in a similar manner. Analytical expressions up to the order of ${h_2}^{-6}$ has been obtained for the stress intensity factors at the edges of the cracks. Numerical results are presented in the form of graphs and compared withe the results of stress intensity factors of two collinear cracks in a homogeneous elastic layer.

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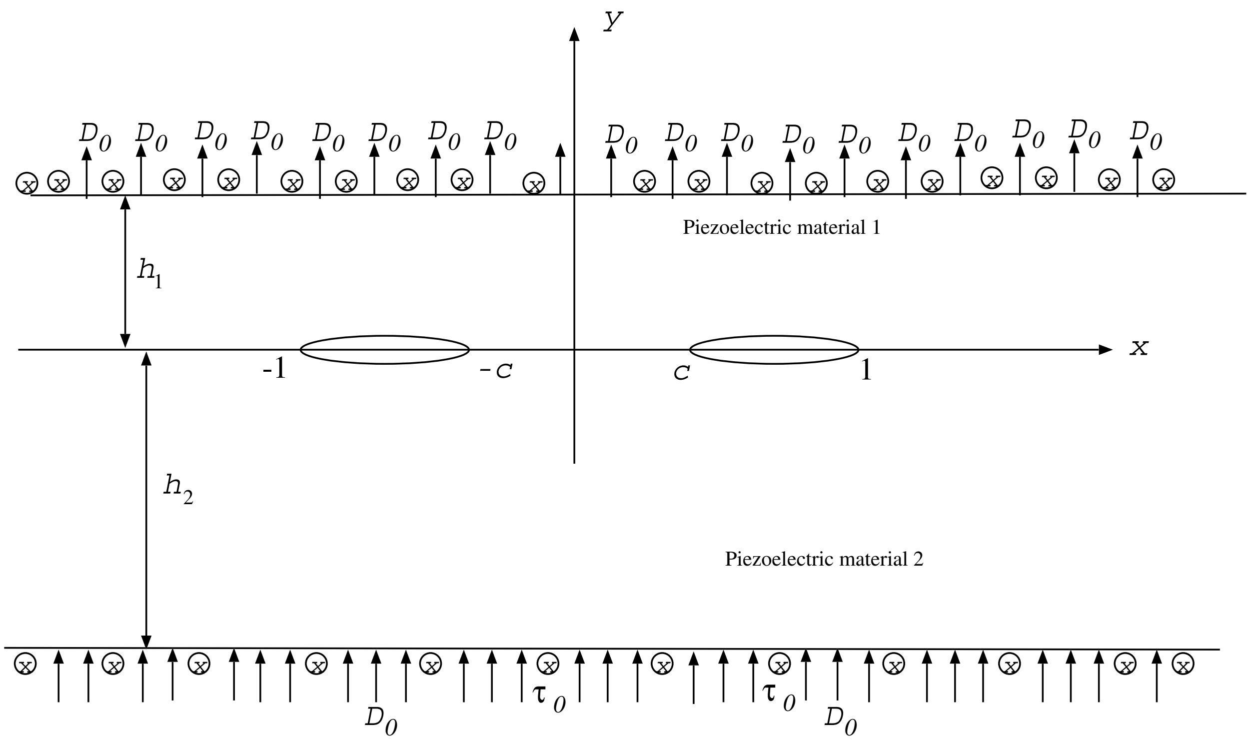

Figure 1: The two collinear interface cracks between two bonded dissimilar piezoelectric layers.

2 Formulation of the problem and basic equations

Consider two bonded dissimilar piezoelectric layers, of infinite extent in the xz-plane and finite thickness h_1 and h_2 in the y direction. Two infinite cracks of width $2a$ are located at the interface $y = 0$ and the cross-section of the layer is shown in Fig. 1.

Here Cartesian coordinates x, y, z are the principal axes of the material symmetry, while the z axis is oriented in poling direction of two piezoelectric layers. Therefore, in this case there is only non-vanishing out-of-plane displacement $w(x,y)$ and in-plane electric potential $\phi(x,y)$ which satisfy the following basic governing differential equations of anti-plane piezoelectricity:

$$\left.\begin{aligned} c_{44(j)}\nabla^2 w_{(j)} + e_{15(j)}\nabla^2\phi_{(j)} = 0,\\ e_{15(j)}\nabla^2 w_{(j)} - \varepsilon_{11(j)}\nabla^2\phi_{(j)} = 0, \end{aligned}\right\} \tag{1}$$

in the absence of body forces and free charges where $c_{44(j)}, \varepsilon_{11(j)}$ and $e_{15(j)}$ are the shear modulus, the piezoelectric constant and the dielectric constant, respectively. ∇^2 represents the two-dimensional Laplacian operator and subscripts $j = 1$ and $j = 2$ specify the quantities in the upper and lower layers, respectively. Respective components of anti-plane shear stress and in-plane electric displacement in each piezoelectric layer are given by the following equations:

$$\sigma_{zx_{(j)}} = c_{44(j)}\gamma_{zx(j)} - e_{15(j)}E_{x(j)}, \tag{2}$$

$$\sigma_{zy_{(j)}} = c_{44(j)}\gamma_{zy(j)} - e_{15(j)}E_{y(j)}, \tag{3}$$

$$D_{x_{(j)}} = e_{15(j)}\gamma_{zx(j)} + \varepsilon_{11(j)}E_{x(j)}, \tag{4}$$

$$D_{y_{(j)}} = e_{15(j)}\gamma_{zy(j)} + \varepsilon_{11(j)}E_{y(j)} \tag{5}$$

where $(\sigma_{zx_{(j)}}, \sigma_{zy_{(j)}})$ are the shear moduli and $(D_{x_{(j)}}, D_{y_{(j)}})$ are the components of the electrical displacement vector. The where anti-plane strain and electric field in each piezoelectric layer are given as follows

$$\gamma_{zx(j)} = \frac{\partial w_{(j)}}{\partial x}, \ \ \gamma_{zy(j)} = \frac{\partial w_{(j)}}{\partial y}, \tag{6}$$

$$E_{x(j)} = -\frac{\partial \phi_{(j)}}{\partial x}, \quad E_{y(j)} = -\frac{\partial \phi_{(j)}}{\partial y}. \tag{7}$$

3 Boundary conditions

We assume that the longitudinal shear stress and electric displacement are prescribed at the upper and lower surfaces of the layers and hence the boundary conditions are:

$$\sigma_{zy(1)}(x,h_1)=\sigma_{zy(2)}(x,-h_2)=\tau_0,\quad -\infty<x<\infty, \tag{8}$$

$$D_{y(1)}(x,h_1)=D_{y(2)}(x,-h_2)=D_0,\quad -\infty<x<\infty \tag{9}$$

where τ_0 and D_0 are constants.

We assume that the electric potential and the normal electric displacement are continuous across the crack surfaces at $y=0$ and that the crack surfaces are free of longitudinal shear stress. Hence we have

$$\left.\begin{array}{l}\sigma_{zy(1)}(x,0^+)=\sigma_{zy(2)}(x,0^-)=0,\\ \phi_{(1)}(x,0^+)=\phi_{(2)}(x,0^-),\\ D_{y(1)}(x,0^+)=D_{y(2)}(x,0^-),\end{array}\right\} c<|x|<1. \tag{10}$$

In addition, the electric and elastic fields should fulfill the following continuity conditions along the interface:

$$\left.\begin{array}{l}\sigma_{zy(1)}(x,0^+)=\sigma_{zy(2)}(x,0^-),\\ D_{y(1)}(x,0^+)=D_{y(2)}(x,0^-),\end{array}\right\} \quad |x|>1,\ 0<|x|<c. \tag{11}$$

$$\left.\begin{array}{l}w_{(1)}(x,0^+)=w_{(2)}(x,0^-),\\ \phi_{(1)}(x,0^+)=\phi_{(2)}(x,0^-),\end{array}\right\} \quad |x|>1,\ 0<|x|<c. \tag{12}$$

4 Solution of the problem

Due to the symmetry of the problem under consideration, it is sufficient to analyze the right-half portion $x\geq 0$. Using the Fourier cosine transforms, it is easy to obtain an appropriate solution of equations (1) in the form

$$w_j(x,y)=\int_0^\infty [A_j(\xi)\cosh(y\xi)+B_j(\xi)\sinh(y\xi)]\cos(x\xi)d\xi + a_j y, \tag{13}$$

$$\phi_j(x,y)=\int_0^\infty [C_j(\xi)\cosh(y\xi)+D_j(\xi)\sinh(y\xi)]\cos(x\xi)d\xi + b_j y, \tag{14}$$

$x\geq 0,\ 0\leq y\leq h_1$ for $j=1$,
$-h_2\leq y\leq 0$ for $j=2$,

where $A_j(\xi),\cdots,D_j(\xi)$ $(j=1,2)$ are unknown functions to be determined. For the convenience of satisfying the conditions (8) and (9) we assume that

$$a_j=\left[\frac{\tau_0\varepsilon_{11(j)}+D_0e_{15(j)}}{e_{15(j)}^2+c_{44(j)}\varepsilon_{11(j)}}\right], \tag{15}$$

$$b_j=\left[\frac{\tau_0e_{15(j)}-D_0c_{44(j)}}{e_{15(j)}^2+c_{44(j)}\varepsilon_{11(j)}}\right]. \tag{16}$$

From equations (3), (5) and (13)-(16) we find that

$$\sigma_{zy(j)}=\tau_0+\int_0^\infty \xi[(c_{44(j)}A_j+e_{15(j)}C_j)\sinh(y\xi) + (c_{44(j)}B_j+e_{15(j)}D_j)\cosh(y\xi)]\cos(x\xi)d\xi, \tag{17}$$

$$D_{y(j)}=D_0+\int_0^\infty \xi[(e_{15(j)}A_j-\varepsilon_{11(j)}C_j)\sinh(y\xi) + (e_{15(j)}B_j-\varepsilon_{11(j)}D_j)\cosh(y\xi)]\cos(x\xi)d\xi. \tag{18}$$

Now the boundary conditions (8) and (9) will be satisfied if

$$B_1=-A_1\tanh(h_1\xi),\ D_1=-C_1\tanh(h_1\xi), \tag{19}$$

$$B_2=A_2\tanh(h_2\xi),\ D_2=C_2\tanh(h_2\xi). \tag{20}$$

With the help of conditions (10), (11) and $(12)_2$ at $y=0$ we find that

$$B_2=a_{11}B_1+a_{12}D_1, \tag{21}$$

$$D_2=a_{21}B_1+a_{22}D_1, \tag{22}$$

$$C_1=C_2, \tag{23}$$

where

$$a_{11}=\frac{c_{44(1)}\varepsilon_{11(2)}+e_{15(1)}e_{15(2)}}{e^2_{15(2)}+c_{44(2)}\varepsilon_{11(2)}}, \tag{24}$$

$$a_{12}=\frac{e_{15(1)}\varepsilon_{11(2)}-\varepsilon_{11(1)}e_{15(2)}}{e^2_{15(2)}+c_{44(2)}\varepsilon_{11(2)}}, \tag{25}$$

$$a_{21}=\frac{c_{44(1)}e_{15(2)}-c_{44(2)}e_{15(1)}}{e^2_{15(2)}+c_{44(2)}\varepsilon_{11(2)}}, \tag{26}$$

$$a_{22}=\frac{e_{15(1)}e_{15(2)}+c_{44(2)}\varepsilon_{11(1)}}{e^2_{15(2)}+c_{44(2)}\varepsilon_{11(2)}}. \tag{27}$$

From the above result we can easily find that

$$C_1=C_2=-\frac{a_{21}A_1J_{12}(\xi)}{1+a_{22}J_{12}(\xi)}, \tag{28}$$

$$A_2=-\left[a_{11}-\frac{a_{12}a_{21}J_{12}(\xi)}{1+a_{22}J_{12}(\xi)}\right]J_{12}(\xi)A_1, \tag{29}$$

where

$$J_{12}(\xi)=\frac{\tanh(h_1\xi)}{\tanh(h_2\xi)}. \tag{30}$$

From the equations (13) and (17) we find that

$$w_2(x,0^-)-w_1(x,0^+)=\int_0^\infty[A_2-A_1]\cos(x\xi)d\xi, \tag{31}$$

$$\sigma_{zy(1)}(x,0^+)=\tau_0+\int_0^\infty\xi[c_{44(1)}B_1+e_{15(1)}D_1]\cos(x\xi)d\xi. \tag{32}$$

With the help of equation (29) we find from equation (31) that

$$w_2(x,0^-)-w_1(x,0^+)=-\int_0^\infty\left[1+a_{11}J_{12}(\xi)-\frac{a_{12}a_{21}J^2_{12}(\xi)}{1+a_{22}J_{12}(\xi)}\right]A_1(\xi)\cos(x\xi)d\xi. \tag{33}$$

Making use of equations (19) and (28), the equation (32) can be written in the form

$$\sigma_{zy(1)}(x,0^+)=\tau_0-\int_0^\infty\left[c_{44(1)}-\frac{e_{15(1)}a_{21}J_{12}(\xi)}{1+a_{22}J_{12}(\xi)}\right]\xi A_1(\xi)\tanh(h_1\xi)\cos(x\xi)d\xi. \tag{34}$$

Using equations (33) and (34), the boundary conditions $(10)_1$ and $(12)_1$ lead to the following integral equations

$$\int_0^\infty M(\xi)\cos(x\xi)d\xi=0,\ 0<x<c,\ x>1, \tag{35}$$

$$\int_0^\infty \xi M(\xi)[R_0+R(\xi,h_1,h_2)]\cos(x\xi)d\xi=\frac{\pi}{2}\tau_0,\quad c<x<1, \tag{36}$$

where

$$\frac{2}{\pi}M(\xi)=\left[\frac{(1+a_{11}J_{12}(\xi))(1+a_{22}J_{12}(\xi))-a_{12}a_{21}J^2_{12}(\xi)}{1+a_{22}J_{12}(\xi)}\right]A_1(\xi), \tag{37}$$

$$R(\xi,h_1,h_2)=\tanh(h_1\xi)\left[\frac{c_{44(1)}(1+a_{22}J_{12}(\xi))-a_{21}e_{15(1)}J_{12}(\xi)}{(1+a_{11}J_{12}(\xi))(1+a_{22}J_{12}(\xi))-a_{12}a_{21}J^2_{12}(\xi)}\right]-R_0, \tag{38}$$

$$R_0=\left[\frac{c_{44(1)}(1+a_{22})-a_{21}e_{15(1)}}{(1+a_{11})(1+a_{22})-a_{12}a_{21}}\right]. \tag{39}$$

We note that when

$$\xi\to\infty,\ J_{12}(\xi)\to 1. \tag{40}$$

We assume that

$$M(\xi)=\frac{1}{\xi R_0}\int_c^1 h(t^2)\sin(\xi t)dt, \tag{41}$$

and then following Srivastava and Lowengrub (18) it is found that the solution of the triple integral equations equations (35) and (36) leads to the following Fredholm integral equation of the second kind for the determination of $h(x^2)$:

$$h(x^2)+\int_c^1 h(t^2)K(x^2,t)dt=F(x^2),\ c<x<1, \tag{42}$$

satisfying the condition

$$\int_c^1 h(x^2)dx=0, \tag{43}$$

where

$$K(x^2,t) = -\frac{4}{\pi^2}\left(\frac{x^2-c^2}{1-x^2}\right)^{\frac{1}{2}}\int_c^1\left(\frac{1-y^2}{y^2-c^2}\right)^{\frac{1}{2}}\frac{yK_1(y,t)dy}{y^2-x^2}, \tag{44}$$

$$K_1(y,t) = \frac{1}{R_0}\int_0^\infty R(\xi,h_1,h_2)\cos(\xi y)\sin(\xi t)d\xi, \tag{45}$$

$$F(x^2) = -\frac{2\tau_0}{\pi}\left(\frac{x^2-c^2}{1-x^2}\right)^{\frac{1}{2}}\int_c^1\left(\frac{1-y^2}{y^2-c^2}\right)^{\frac{1}{2}}\frac{ydy}{y^2-x^2} \frac{C_1}{\sqrt{(x^2-c^2)(1-x^2)}},\ c<x<1, \tag{46}$$

C_1 being an arbitrary constant determined by the condition (43). The analytic solution of equation (42) is not easily possible for $h_2 << 1$. If we assume $h_2 >> 1$ and $h_2 \geq h_1$ or $h_1 \geq h_2$ then K_1 may be expanded in the inverse powers of h_2 in the form:

$$K_1(y,t) = \left[\frac{I_0 t}{h_2^2}+\frac{I_1 t}{h_2^4}(t^2+3y^2)+O(h_2^{-6})\right], \tag{47}$$

where

$$I_i = \frac{(-1)^i}{R_0\underline{|1+2i}}\int_0^\infty u^{2i+1}\left[\tanh(\varepsilon u) \frac{c_{44(1)}(1+a_{22}L_1(\varepsilon u))-a_{21}e_{15(1)}L_1(\varepsilon u)}{[(1+a_{11}L_1(\varepsilon u))(1+a_{22}L_1(\varepsilon u))-a_{12}a_{21}L_1^2(\varepsilon u)]} - R_0\right]du, \tag{48}$$

$$\varepsilon = \frac{h_1}{h_2},\ L_1(\varepsilon u) = \frac{\tanh(\varepsilon u)}{\tanh(u)},\ i=0,1. \tag{49}$$

And we find that

$$K(x^2,t) = \frac{2}{\pi}\left(\frac{x^2-c^2}{1-x^2}\right)^{\frac{1}{2}} \left[\frac{I_0 t}{h_2^2}+\frac{I_1 t}{h_2^4}\left(t^2+3x^2-\frac{3}{2}k^2\right)+O(h_2^{-6})\right], \tag{50}$$

where

$$k^2 = 1-c^2. \tag{51}$$

Integrating both sides of equation (42) with respect to c and using equation (43) we find that

$$C_1 = \frac{2}{\pi F}\int_c^1 w(x^2)dx+\frac{2}{\pi F}\int_c^1 h(t^2)\left[\frac{I_0 t}{h_2^2}(E-C^2F) + \frac{I_1 t}{h_2^4}\left\{\left(t^2-\frac{3}{2}k^2\right)(E-c^2F)-c^2(E+F) + 2E\right\}\right]dt+O(h_2^{-6}), \tag{52}$$

where

$$E = E(\frac{\pi}{2},k),\ F=F(\frac{\pi}{2},k),\ k=(1-c^2)^{\frac{1}{2}}, \tag{53}$$

$$w(x^2) = \tau_0\left(\frac{x^2-c^2}{1-x^2}\right)^{\frac{1}{2}}\int_c^1\left(\frac{1-y^2}{y^2-c^2}\right)^{\frac{1}{2}}\frac{ydy}{y^2-x^2}. \tag{54}$$

E and F are elliptic integrals of the second and first kind respectively as defined in the book of Gradshteyn and Ryzhik ((5), pp. 904-905). Now $h(x^2)$ must satisfy the integral equation

$$h(x^2)+\int_c^1 h(t^2)M(x^2,t)dt = S(x^2),\ c<x<1, \tag{55}$$

where

$$M(x^2,t) = \frac{2t}{\pi\sqrt{(x^2-c^2)(1-x^2)}}\left[\frac{I_0}{h_2^2}\left(x^2-\frac{E}{F}\right) + \frac{I_1}{h_2^4}\left\{\left(t^2+\frac{3}{2}k^2\right)\left(x^2-\frac{E}{F}\right) +3x^2(x^2-1)+\frac{E}{F}+c^2-\frac{2c^2E}{F}\right\}\right]+O(h_2^{-6}), \tag{56}$$

$$S(x^2) = \frac{(x^2-E/F)\tau_0}{\sqrt{(x^2-c^2)(1-x^2)}}. \tag{57}$$

Solution of equation (55) may be written in the form

$$h(x^2) = g_0(x^2)+\frac{g_1(x^2)}{h_2^2}+\frac{g_2(x^2)}{h_4^2}+O(h_2^{-6}),\ c<x<1, \tag{58}$$

where

$$\{g_0(x^2), g_1(x^2)\} = \frac{(x^2-E/F)\tau_0}{[(x^2-c^2)(1-x^2)]^{\frac{1}{2}}}\left\{1, -\frac{1}{2}I_0C_0\right\} \quad (59)$$

$$g_2(x^2) = \frac{C_0}{4[(x^2-c^2)(1-x^2)]^{\frac{1}{2}}}[I_0^2C_0(x^2-E/F) - 2I_1(3x^4+C_1x^2+C_2)], \quad (60)$$

with

$$C_0 = 1+c^2-\frac{2E}{F}, \quad (61)$$

$$C_1 = \frac{k^4}{4C_0}-(1+c^2), \quad (62)$$

$$C_2 = c^2+\frac{E}{F}\left[C_1-\frac{k^4}{2C_0}\right]. \quad (63)$$

Using equations (34), (37) and (41) we find that

$$\sigma_{zy(1)}(x,0) = -\frac{2}{\pi}\int_c^1 \frac{th(t^2)dt}{t^2-x^2} - \frac{2}{\pi}\int_c^1 h(t^2)K_1(x,t)dt + \tau_0, \quad 0 \le x < c, \ x > 1. \quad (64)$$

From equations (47) and (58) - (60) we find that

$$\int_c^1 h(t^2)K_1(x,t)dt = \frac{\tau_0\pi}{8}\left[\frac{2I_0C_0}{h_2^2} - \frac{I_0^2C_0^2}{h_2^4} + \frac{2I_0C_0}{h_2^4}\left(3x^2+C_1+\frac{3}{2(1+c^2)}\right)\right] + O(h_2^{-6}). \quad (65)$$

For $0 < x < c$, we have

$$-\int_c^1 \frac{th(t^2)dt}{t^2-x^2} = \tau_0\frac{\pi}{2}\left[\left(\frac{E/F-x^2}{X_1}-1\right)\left(1-\frac{I_0C_0}{2h_2^2}+\frac{I_0^2C_0^2}{4h_2^4}\right) + \frac{I_1C_0}{2h_2^4}\left\{\frac{3x^4+C_1x^2+C_2}{X_1}+3\left(\frac{1+c^2}{2}+x^2\right) + C_1\right\}\right] + O(h_2^{-6}), \quad (66)$$

and for $x > 1$

$$\int_c^1 \frac{th(t^2)dt}{x^2-t^2} = \tau_0\frac{\pi}{2}\left[\left(\frac{x^2-E/F}{X_2}-1\right)\left(1-\frac{I_0C_0}{2h_2^2}+\frac{I_0^2C_0^2}{4h_2^4}\right) - \frac{I_1C_0}{2h_2^4}\left\{\frac{3x^4+C_1x^2+C_2}{X_2}-3\left(\frac{1+c^2}{2}+x^2\right) - C_1\right\}\right] + O(h_2^{-6}), \quad (67)$$

where

$$X_1 = \sqrt{(c^2-x^2)(1-x^2)}, \quad (68)$$

$$X_2 = \sqrt{(x^2-c^2)(x^2-1)}. \quad (69)$$

We find that the stress intensity factors which are defined by

$$K_c^\tau = \lim_{x\to c^-}[2(c-x)]^{\frac{1}{2}}[\sigma_{yz}(x,0)_{0<x<c}], \quad (70)$$

$$K_1^\tau = \lim_{x\to 1^+}[2(x-1)]^{\frac{1}{2}}[\sigma_{yz}(x,0)_{x>1}] \quad (71)$$

are given by

$$K_c^\tau = \frac{\tau_0}{\sqrt{(1-c^2)c}}\left[(E/F-c^2)\left(1-\frac{I_0C_0}{2h_2^2}+\frac{I_0^2C_0^2}{4h_2^4}\right) + \frac{I_1C_0}{2h_2^4}\{3c^4+C_1c^2+C_2\}+O(h_2^{-6})\right], \quad (72)$$

$$K_1^\tau = \frac{\tau_0}{\sqrt{(1-c^2)}}\left[(1-E/F)\left(1-\frac{I_0C_0}{2h_2^2}+\frac{I_0^2C_0^2}{4h_2^4}\right) - \frac{I_1C_0}{2h_2^4}(3+C_1+C_2)+O(h_2^{-6})\right]. \quad (73)$$

The equations (72) and (73) give the stress intensity factors at the edges of the crack.

The expressions (71) and (83) represent the expressions for stress intensity factors only for $h_2 >> 1$ and not for general values of h_2.

If we assume that $h_2 \to \infty$ then the integrals I_0 and I_1 are divergent and results for the stress intensity factors from equations (72) and (73) are not easily possible. If we assume that $h_1 = h_2$ then the integrals I_0 and I_1 are convergent.

If $h_1 = h_2, h_2 \to \infty$ in equations (72) and (72) we get the results for two collinear cracks in an infinite homogeneous medium under shear load as shown in reference (13). If we assume the the thickness of the layers tend to infinite in references (11) and (12) we get the result for stress intensity factor for one crack lying at the interface of two infinite materials, which are independent of the materials. In the same way in this case the two collinear cracks lying at the interface of two infinite piezoelectric materials, their stress intensity factors are not dependent on the piezoelectric materials

In the same way intensity factors for the electric displacement at the crack tips can be obtained.

Using equations (19) and (28) we can write equation (18) in the following form:

$$D_{y(1)}(x,0^+) = D_0 - \int_0^\infty \left[\frac{a_{21}\varepsilon_{11(1)}J_{12}(\xi) + e_{15(1)}(1+a_{22}J_{12}(\xi))}{1+a_{22}J_{12}(\xi)}\right] \times \xi \tanh(h_1\xi)\cos(x\xi)A_1(\xi)d\xi. \quad (74)$$

Now substituting for A_1 from equation (37), the above equation may be written as

$$D_{y(1)}(x,0^+) = D_0 - \frac{2}{\pi}\int_0^\infty \xi\,[R_1 + R_2(\xi,h_1,h_2)]\cos(x\xi)M(\xi)d\xi, \quad (75)$$

where

$$R_1 = \frac{a_{21}\varepsilon_{11(1)} + e_{15(1)}(1+a_{22})}{(1+a_{11})(1+a_{22}) - a_{12}a_{21}}, \quad (76)$$

$$R_2(\xi,h_1,h_2) = \frac{a_{21}\varepsilon_{11(1)}J_{12}(\xi) + e_{15(1)}[1+a_{22}J_{12}(\xi)]}{[1+a_{11}J_{12}(\xi)][1+a_{22}J_{12}(\xi)] - a_{12}a_{21}J_{12}^2(\xi)} \times \tanh(h_1\xi) - R_1. \quad (77)$$

Substituting for M from equation (41) we may rewrite equation (75) in the form

$$D_{y(1)}(x,0^+) = D_0 - \frac{2R_1}{\pi R_0}\int_c^1 \frac{th(t^2)dt}{t^2-x^2} - \frac{2}{\pi R_0}\int_c^1 h(t^2)K_2(x,t)dt, \quad (78)$$

where

$$K_2(x,t) = \int_0^\infty R_2(\xi,h_1,h_2)\cos(x\xi)\sin(t\xi)d\xi. \quad (79)$$

The electric intensity factors at the edge of the crack are defined by

$$K_c^D = \lim_{x\to c^-}[2(c-x)]^{\frac{1}{2}}[D_{y(1)}(x,0^+)_{0<x<c}], \quad (80)$$

$$K_1^D = \lim_{x\to 1^+}[2(x-1)]^{\frac{1}{2}}[D_{y(1)}(x,0^+)_{x>1}]. \quad (81)$$

Now from equations (78), (80) and (81) we find that

$$\left(K_c^D, K_1^D\right) = \frac{R_1}{R_0}\left(K_c^\tau, K_1^\tau\right) \quad (82)$$

where K_c^τ and K_1^τ are given by equations (72) and (73) respectively.

5 Numerical results and discussion

For numerical results we assume the upper layer of material PZT-4 and lower layer of material PZT-5H. The material properties of PZT-4 are:

$$c_{44(1)} = 2.56\times 10^{10} N/m^2, \quad (83)$$

$$e_{15(1)} = 12.7 C/m^2, \quad (84)$$

$$\varepsilon_{11(1)} = 64.6\times 10^{(-10)} C/Vm \quad (85)$$

and the material properties of PZT-5H are:

$$c_{44(2)} = 2.3\times 10^{10} N/m^2, \quad (86)$$

$$e_{15(2)} = 17.0 C/m^2, \quad (87)$$

$$\varepsilon_{11(2)} = 150.4\times 10^{(-10)} C/Vm \quad (88)$$

By using the above values of the material constants the numerical values of the stress intensity factors K_c^τ and K_1^τ have been calculated from equations (72) and (73) respectively.

For the numerical values we assume that $h_1 = h_2 = h$(constant). The numerical values of $\frac{K_c^\tau}{\tau_0\sqrt{c}}$ and $\frac{K_1^\tau}{\tau_0}$ have been displayed in Figs. 2-5 against the layer thickness h which are shown by thick lines. In Figs. 2-5 dotted lines shows the curves of the stress intensity factors for two collinear cracks in the homogeneous elastic layer under torsion

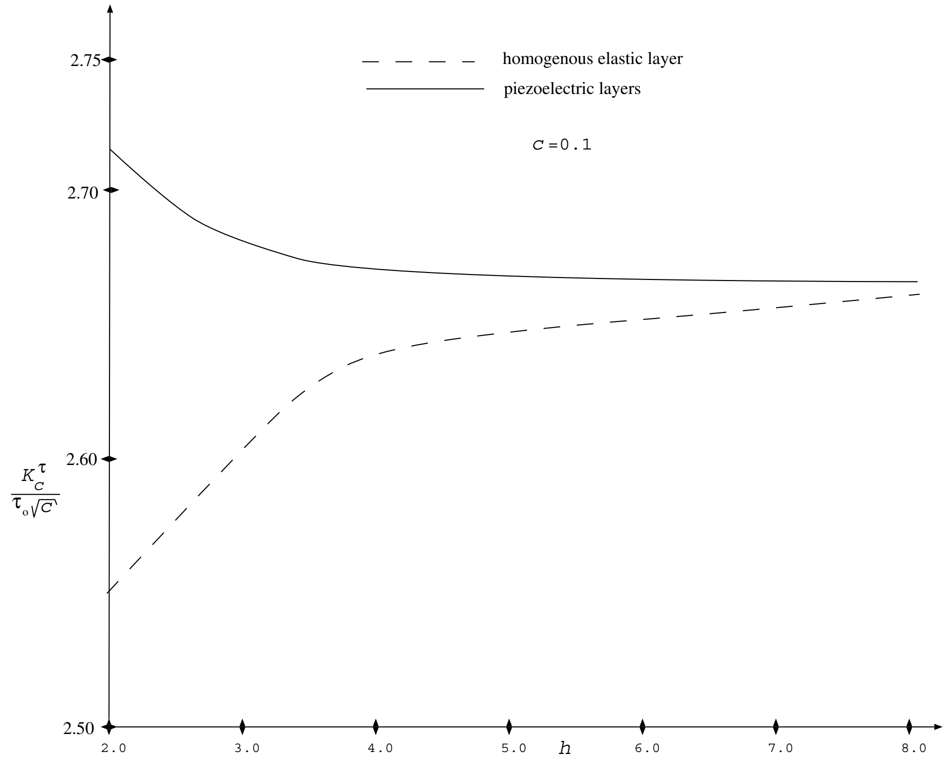

Figure 2: The variation of stress intensity factors $\frac{K_c^\tau}{\tau_0\sqrt{c}}$ with h for $h_1 = h_2 = h, c = 0.1$.

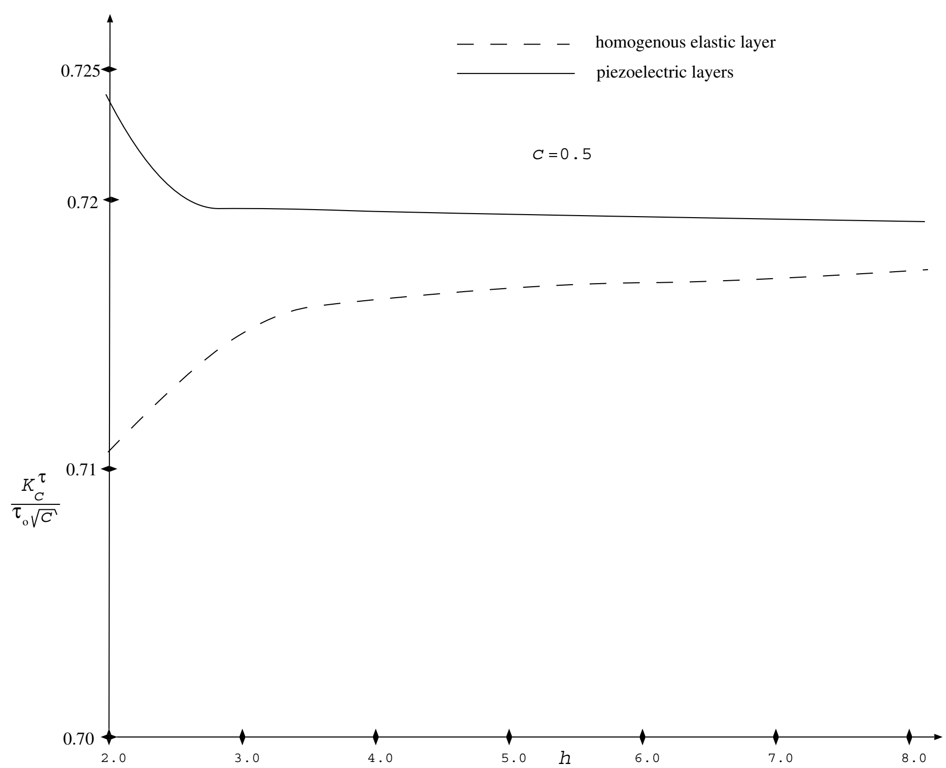

Figure 3: The variation of stress intensity factors $\frac{K_c^\tau}{\tau_0\sqrt{c}}$ with h for $h_1 = h_2 = h, c = 0.5$.

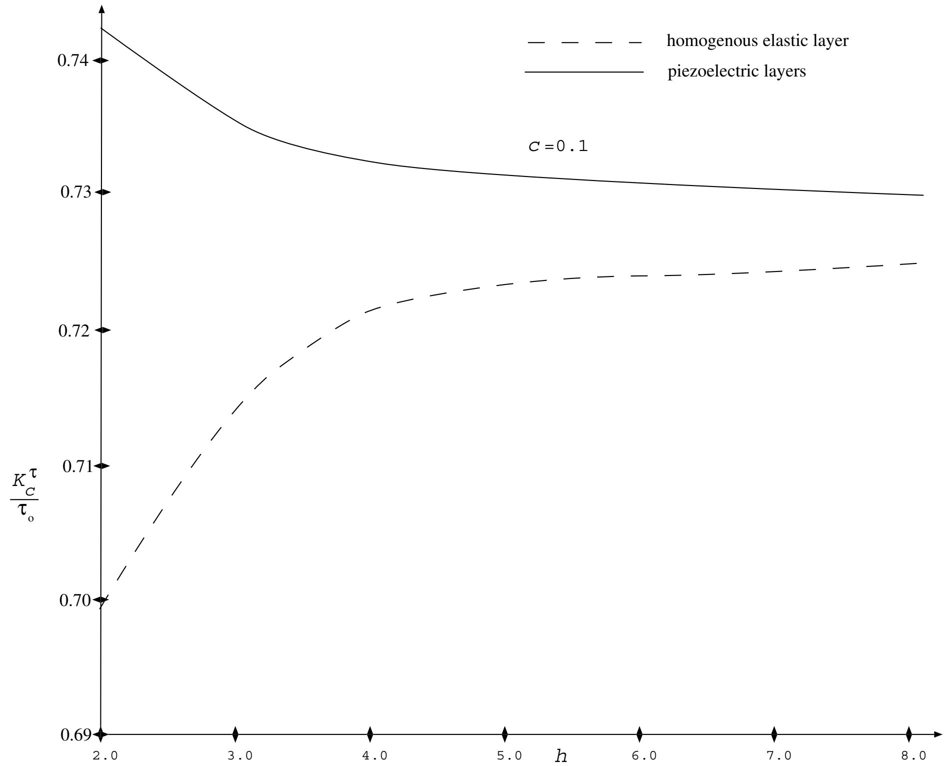

Figure 4: The variation of stress intensity factors $\frac{K_1^\tau}{\tau_0}$ with h for $h_1 = h_2 = h, c = 0.1$.

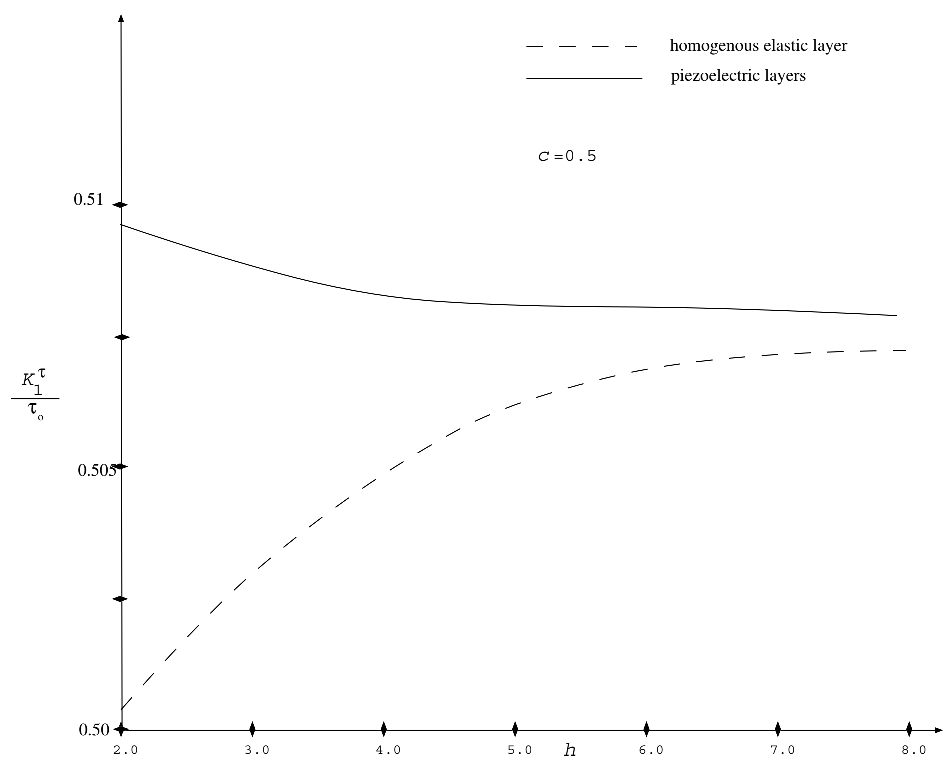

Figure 5: The variation of stress intensity factors $\frac{K_1^\tau}{\tau_0}$ with h for $h_1 = h_2 = h, c = 0.5$.

when the plane surfaces of the layers are clamped. The numerical results for two collinear cracks in a homogeneous layer are obtained from the expressions of stress intensity factors of the paper Dhaliwal, Singh and Chehil (3) and are plotted by dotted lines in Figs. 2-5. If we compare the curves for stress intensity factors of cracks in piezoelectric layers with stress intensity factors of cracks in a homogeneous elastic layer, then the results of stress intensity factors for two cracks in piezoelectric layers are reasonable and we find that the trends of the stress intensity factors change more for $h < 4$.

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