The Theory of Critical Distances: a History and a New Definition

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Abstract: Current theories of fracture recognize the importance of material length scales, i.e. parameters having the dimensions of length which are included, either explicitly or implicitly, in many methods of fracture prediction. This paper is a review of the development of one particular approach, which we have called the Theory of Critical Distances (TCD). The history of this approach – which is presented here for the first time - is a story of parallel developments in the areas of fatigue and brittle fracture and in different material fields: metals, polymers, ceramics and composites. A particular milestone in the development of the TCD was the incorporation of fracture mechanics concepts which allowed the critical distance parameter, L, to be calculated as a function of other mechanical properties. Over the last decade the theory has been rediscovered and extended by several workers, precipitating another phase of rapid development. This review concludes by proposing a precise definition for the TCD, which includes four related methods of analysis, and by suggesting some directions for future research.

keyword: Critical distance, Fracture, Fatigue, Stress concentration

Nomenclature

a	crack length
a_o	ElHaddad's constant
D	notch depth
F	geometry factor in the equation for K
FFM	Finite Fracture Mechanics
ICM	Imaginary Crack Method
K	stress intensity
K_c	fracture toughness
K_f	notch fatigue strength reduction factor
K_t	notch stress concentration factor
L	critical distance
LEFM	Linear Elastic Fracture Mechanics
LM	Line Method
PM	Point Method
r	distance measured from maximum stress point
r_c	Peterson's critical distance
ε	Neuber's critical distance
ΔK	range of stress intensity
ΔK_{th}	fatigue crack propagation threshold
$\Delta\sigma$	range of applied stress
$\Delta\sigma_o$	fatigue limit
$\Delta\sigma(r)$	local stress range as a function of r
ρ	root radius of notch
ρ'	a constant related to ε
σ	stress
$\sigma(r)$	local stress as a function of r
σ_o	material characteristic strength
σ_u	tensile strength

1 Introduction

This paper is concerned with the prediction of failure under nominally linear-elastic loading, i.e. when any plasticity, damage or other non-linear straining is confined to highly localised regions. Examples are the failure of brittle and quasi-brittle materials and those fracture modes which involve the initiation and growth of cracks from pre-existing notches. This includes high-cycle fatigue and brittle fracture in metallic materials.

Traditionally, failure in these circumstances was predicted in one of two ways, using either: (i) a critical stress or (ii) a critical stress intensity. In the first approach failure is assumed to occur when the maximum stress (or strain) in the body reaches some particular value, usually the material's tensile strength σ_u or, in the case of fatigue, the fatigue limit stress range $\Delta\sigma_o$. In the second approach, which is Linear Elastic Fracture Mechanics (LEFM), the failure of a pre-cracked body occurs when the stress intensity reaches a critical value K_c in fast fracture or ΔK_{th} in fatigue.

It has long been known that these approaches are insufficient in many cases: the first approach is suitable only for situations in which the gradient of stress is very low, such as plain tensile test specimens or notches which are

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large and blunt. LEFM is valid only for sharp cracks, not for notches having a non-zero root radius: major errors also arise when trying to apply LEFM to cracks which are physically short. Many different attempts have been made, over the last fifty years, to solve these problems and create a general method of failure prediction.

For convenience, we can consider a typical problem in this field as involving a notch, of depth D and root radius ρ, as shown in fig.1. In fact, the real aim is to predict the effect of any kind of stress-concentrating feature which might arise in an engineering component, including not-only geometric discontinuities such as holes and corners but also stress concentrations arising from other means, such as contact between bodies. Common features are the existence of a stress field described by a decrease in stress with distance from a 'hot spot' (a location of local maximum stress), and failure modes involving crack growth from the most highly stressed region, as shown in the figure.

2 History

2.1 Early Work: Neuber and Peterson

The story begins with the work of Heinz Neuber in Germany and Ralph Earl Peterson in the USA. Their ideas, originally found in papers in the 1930s, are clearly expressed in two seminal publications (Neuber, 1958; Peterson, 1959). Neuber proposed that the fatigue limit of a specimen containing a notch could be predicted using the average stress along a line drawn from the notch root (i.e. the average of the stress shown in fig.1). The length of the line was assumed to be a material constant: Neuber's original symbol for this distance was ε as shown in fig.2. Thus, if the stress range as a function of distance r is written $\Delta\sigma(r)$ then Neuber's approach can be expressed as:

$$\frac{1}{\varepsilon}\int_0^\varepsilon \Delta\sigma(r)\,dr = \Delta\sigma_o \qquad (1)$$

We call this method the Line Method (LM). For Neuber it was originally not a method of fatigue prediction but rather a fundamental tenet of stress analysis. Neuber argued that, since a material is not a homogeneous continuum, then methods for calculating stresses should not use the standard calculus of infinitesimal intervals (dx, dy, dz) but rather should use finite differences, the magnitude of which should reflect the internal structure of the
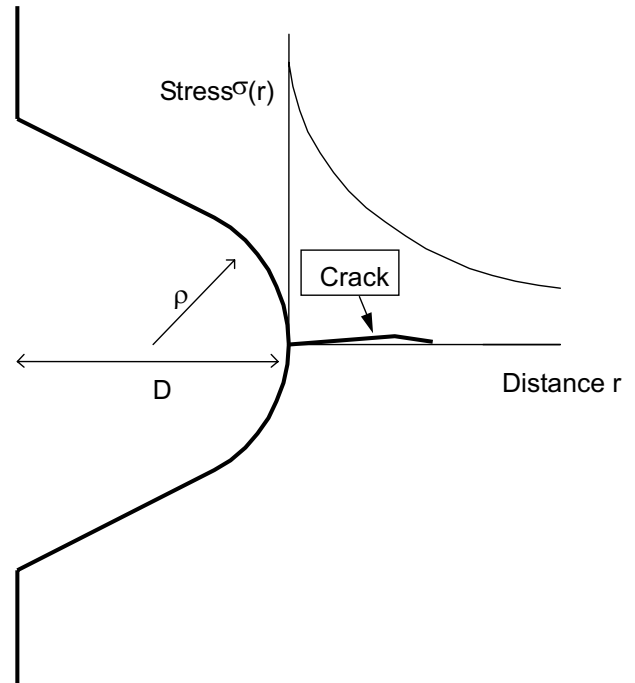

Figure 1 : A typical notch, with depth D and root radius ρ. TCD methods use either the stress/distance curve or else compute the stress intensity factor for a notch-root crack.

material. Neuber said little about real microstructures – he imaged the material as composed of 'finite structural particles' whose size corresponded to his parameter ε. For him the effect of these particles was to smooth out any stress gradients that occurred over the length of the particle. Peterson had followed this work closely, and suggested a modified, and even simpler, approach, which can be written as follows:

$$\Delta\sigma(r_c) = \Delta\sigma_o \qquad (2)$$

Thus, according to Peterson, the fatigue limit of the notched specimen occurs when the stress at a point, located a distance r_c from the notch root, is equal to the plain-specimen fatigue limit. We call this the Point Method (PM). The critical length constant here, r_c, will of course be different from Neuber's ε. Both of these approaches have been used extensively in fatigue prediction ever since, but these days they are rarely used in the explicit forms given in equations 1 and 2. The reason for this is that in the 1950s the stress-distance curve was difficult to obtain, finite element analysis not being a practical proposition for engineering components. So Neuber

and Peterson combined their criteria with approximate methods of stress analysis; Neuber's equation was:

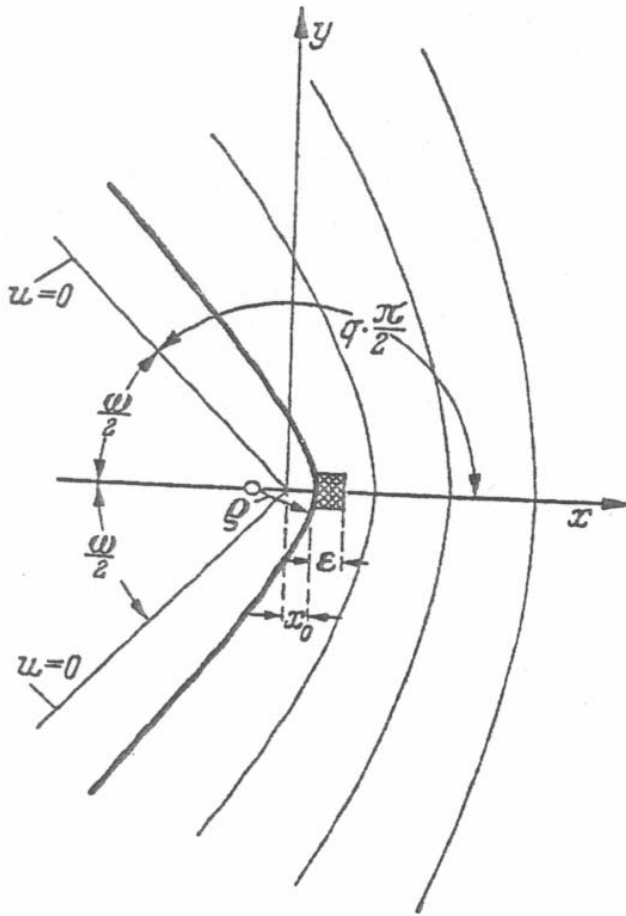$$K_f = 1 + \frac{K_t - 1}{1 + \sqrt{\rho'/\rho}} \qquad (3)$$

Figure 2 : A figure from Neuber's 1958 book, showing a notch and the critical distance, labelled ε.

Here K_t is the elastic stress concentration factor of the notch and K_f is the predicted fatigue-strength reduction factor; ρ' is a material constant, related to ε. This equation relies on the fact that the stress field near a notch can be approximately described knowing K_t and ρ. Various equations of this general type became common in fatigue analysis and are still used to this day by engineering designers. These methods have limited applicability and poor accuracy owing to their underlying approximations and to the fact that K_f cannot be precisely defined in most component geometries. These days it would be

much better to forget these equations and use the LM and PM explicitly, though currently these methods are not incorporated into any of the generally available software for fatigue analysis.

Recently the explicit use of the TCD has been taken up again as a research topic, for the study of notch fatigue (Taylor, 1999; Lanning *et al.*, 2004; Lanning *et al.*, 2005). Some quite complex situations have been successfully analysed, including welded joints (Taylor *et al.*, 2002) and some engineering components (Taylor, 2005a).

2.2 Composites: the introduction of LEFM

The story of the TCD moves forward in time to the 1970s and in subject matter to the monotonic fracture of composites. Whitney and Nuismer described methods for predicting the effect of holes and notches on the static strength of fibre-reinforced polymers (Whitney and Nuismer, 1974). These methods are identical to the PM and LM: the only difference was that, instead of the fatigue limit, $\Delta\sigma_o$, they used the tensile strength of the material, σ_u, to predict the onset of unstable, brittle fracture in tensile tests. No reference was made to the work of Neuber or Peterson, leading us to conclude that these authors were unaware of the earlier work. However, in the intervening two decades an important development had occurred: LEFM, initially developed by Griffith for brittle materials, was now being applied to many fracture problems. So Whitney and Nuismer were able to make an important theoretical link between the PM and LM and LEFM, as follows. The stress distribution close to a crack tip can be described by:

$$\Delta\sigma(r) = \frac{K}{\sqrt{2\pi r}} \qquad (4)$$

Using this, it is a simple matter to show that the critical distances for the PM and LM can both be expressed in terms of a length parameter L, where:

$$L = \frac{1}{\pi}\left(\frac{K_c}{\sigma_u}\right)^2 \qquad (5)$$

Applying the PM to the case of a crack we find that the distance from the crack tip to the critical point is L/2; likewise applying the LM we find that the distance over which stresses should be averaged is 2L. This is a very useful result because it allows us to calculate the value of
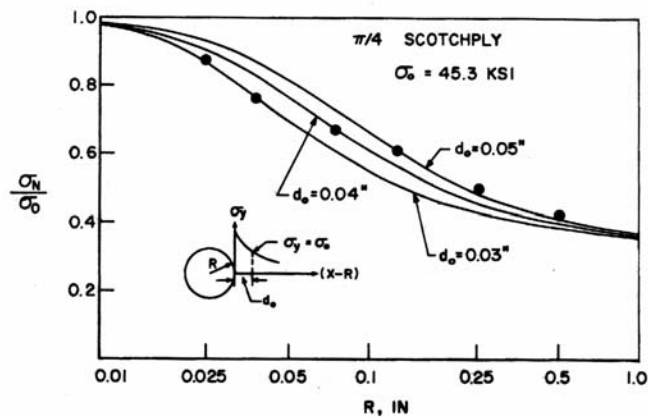
Figure 3 : A figure from Whitney and Nuismer 1974, showing how the PM can be used to predict the effect of hole radius (R) on fracture stress (normalised by the plain-specimen value). The lines are predictions at three different values of the critical distance d_o (which is equal to L/2).

Figure 4 : Data on the effect of root radius on measured fracture toughness, normalised by the toughness for a sharp crack, for a rubber-toughened epoxy material (Kinloch *et al.*, 1983).

the critical distance from these two well-known mechanical properties.

It should be noted, however, that this theoretical argument applies only to cracks. Whitney and Nuismer showed that the same value can be used to predict experimental data from specimens containing two other features: circular holes and sharp notches. Fig.3 shows an example, which illustrates an important property of critical distance methods: their ability to predict size effects. The fracture strength of the specimens is strongly affected by hole radius, even though the K_t factor is virtually constant: the hole only exerts the full effect of K_t if it is much larger than L, which in this case is 0.08 inches (about 2mm). Holes much smaller than L have a negligible effect; the strength of the specimen approaches the plain-specimen value. In fibre composites the value of L is quite large – of the order of millimetres – so this prediction is very useful for assessing holes of a size which commonly occur in components. The work of Whitney and Nuismer was widely adopted in the field of composite laminates, and is still extensively used today, both in research (e.g. (Cowley and Beaumont, 1997)) and in industrial design (e.g. (Zetterberg *et al.*, 2001)).

2.3 Polymers: non-damaging notches

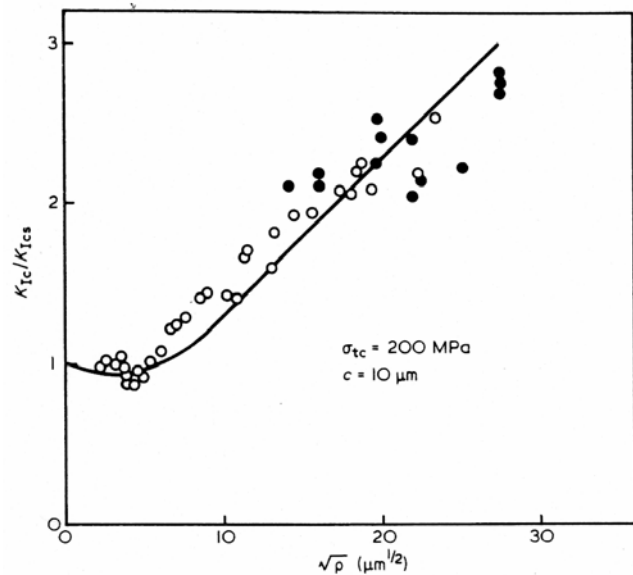The TCD can also be used to predict brittle fracture in polymers. It seems that this was first discovered by Kin-

loch, Williams and co-workers who wrote several papers on the subject in the 1980s (e.g. (Kinloch and Williams, 1980; Kinloch *et al.*, 1983)). Their aim was rather different – to investigate the effect of crack-tip blunting on fracture toughness – but the method they describe is essentially the PM. Again, these workers do not seem to have been aware of the earlier publications discussed above, in the fields of metal fatigue and composites. A typical result from this work was a prediction of the effect of notch root radius on measured fracture toughness, as shown in fig.4, taken from Kinloch and Williams' original work. In the case where the root radius is zero, the notch is equivalent to a crack, so a valid measurement of the toughness, K_c, is possible. As ρ is increased, producing a long, narrow slot rather than a crack, the measured value of K_c first stays fairly constant, and then gradually increases. The prediction of this behaviour, and especially of the critical value of ρ at which the upturn occurs, is of great practical interest.

An important modification was introduced by these workers: they noticed that, in order to predict the static strength for notched components of brittle polymers such as epoxy, the critical stress parameter to be used is not the plain-specimen tensile strength σ_u but a higher value, which we can call σ_o. We have discussed this matter in
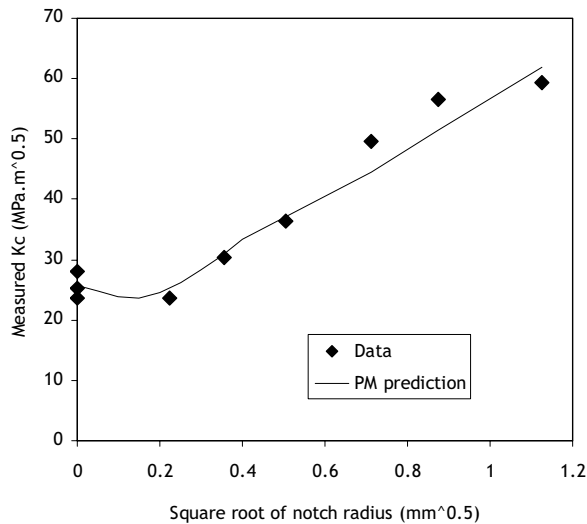
Figure 5 : Data and predictions similar to fig.4, but showing measured K_c for cleavage fractures in steel at low temperature (Taylor 2005b).
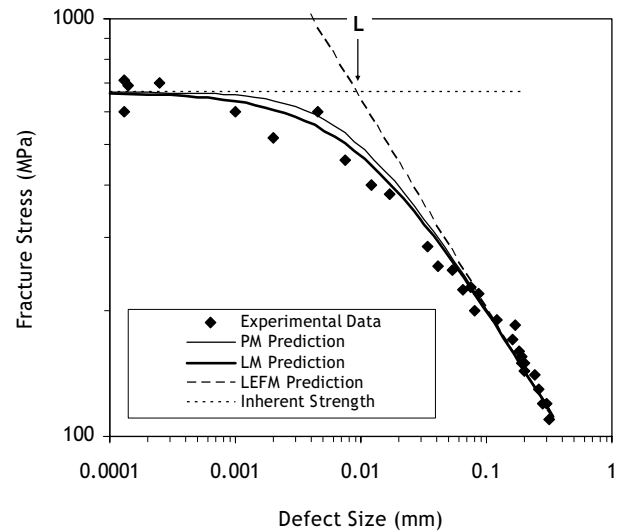

Figure 6 : Fracture stress as a function of defect size for crack-like defects in silicon carbide (Taylor 2004). Data deviate from the LEFM prediction at sizes of the order of L. The PM and LM both give reasonable predictions.

detail in a recent publication (Taylor *et al.*, 2004): an important consequence of this modification is that there will be some notches – those for which $K_t < \sigma_o/\sigma_u$ – which are predicted to have no effect on strength (beyond any effect they may have in reducing the cross section). These non-damaging notches do indeed exist in practice.

The work of Kinloch and Williams, unlike that of Whitney and Nuisner, was not widely adopted. Indeed there seems to have been almost no further work using the TCD in the field of polymers until some recent papers by ourselves (Taylor *et al.*, 2004), and by some researchers who considered the particular problem of V-shaped notches of zero radius (e.g. (Seweryn, 1998)). This is a great pity, because the PM and LM could certainly be of value in the design of engineering components made from polymeric materials.

Recently we have also investigated the application of the PM and LM to other materials, showing that it can be used to predict brittle fracture in both ceramics (Taylor, 2004) and metals (Taylor, 2005b). In the case of ceramics, the critical stress was found to be equal to the plain-specimen tensile strength σ_u, whilst metals displayed the same effect as found above in polymers, with σ_o being greater than σ_u, sometimes by factors as large as ten. Figs 5 and 6 show examples of the prediction of notch effects in metals and short crack effects in ceramics.

2.4 Imaginary Cracks and Finite Crack Extensions

If we return to the 1970s and 1980s, we find the development and use of another theory for predicting fracture and fatigue which at first sight seems rather different from the PM and LM described above. The main idea in this approach – which we will call the Imaginary Crack Method (ICM) – is to assume the existence of a small crack, located at the root of the notch. This concept is illustrated on fig.1: the crack length, a_o, is assumed to be constant. If we assume that the behaviour of this notch obeys LEFM then we can predict brittle fracture or fatigue based on the calculated stress intensity of the notch-plus-crack system. The same concept can also be applied to a crack – the real crack now being imagined to increase its length by a certain amount. This turns out to be very useful in predicting the non-LEFM behaviour of short cracks.

Like the PM and LM, this is an idea which has been invented and re-invented many times. In fatigue we can find its use in the late 1970s for notches (Lukas and Klesnil, 1978) and for short cracks (El Haddad *et al.*, 1979), but the same approach had already been advocated almost a decade earlier (Waddoups *et al.*, 1971) to predict brittle fracture in composite materials. Some workers have suggested that these notch-root defects have a
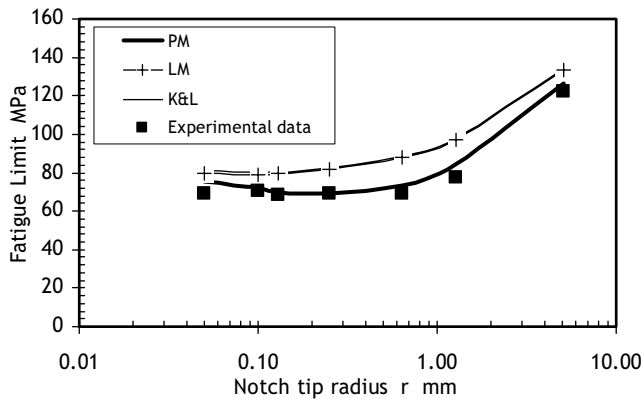
Figure 7 : Experimental data on the effect of notch tip radius on fatigue limit, with three theoretical predictions (Taylor and Wang 2000). K&L refers to the method of Klesnil and Lukas (Klesnil and Lukas, 1980), which is based on the ICM. Typically for this kind of data, the PM provides a very accurate prediction whilst the LM and ICM, which are almost identical, are slightly non-conservative.

real physical existence (e.g. (Usami *et al.*, 1986; Ostash and Panasyuk, 2001), but this raises a theoretical problem, because a real crack of this size will be a short crack by definition, and so cannot be analysed using LEFM. This leads to a circular argument which, in the author's opinion, can only be resolved by accepting that the crack is imaginary: there is no real crack or, if there is, it is not the size assumed. Waddoups, for example, argued that the function of this crack was to act as a simple representation of the complex damage zone that develops in a composite material prior to fracture.

We can calculate the length of this imaginary crack from first principles, simply by noting that, for the case of a plain specimen, failure will occur at an applied stress of σ_u and at a stress intensity (applied to the imaginary crack) of K_c. We use the standard LEFM equation for K as a function of crack length a and geometry factor F:

$$K = F\sigma\sqrt{\pi a} \qquad (6)$$

The modification necessary to apply the ICM for the case of a pre-existing crack is:

$$K = F\sigma\sqrt{\pi(a+a_o)} \qquad (7)$$

Letting $K = K_c$ and $\sigma = \sigma_u$ when $a = 0$ gives:

$$a_o = \frac{1}{\pi}\left(\frac{K_c}{F\sigma_u}\right)^2 \qquad (8)$$
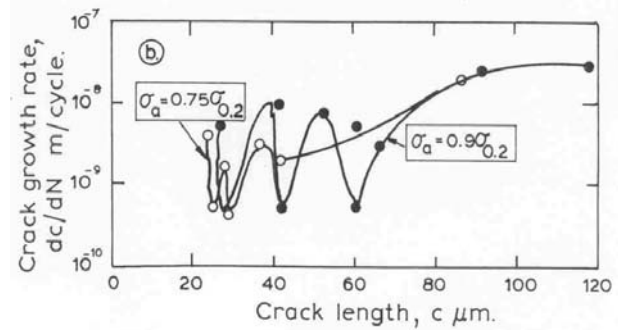

Figure 8 : Variation of crack growth rate with length for a slow-growing fatigue crack (Blom *et al.*, 1986).

Clearly this is very similar to the value of L in equation 4 above, except for the factor of F^2. In the particular case of a through crack in an infinite plate in tension, when F=1, L and a_o will be identical. In many cases of practical interest F takes values close to unity, so the differences between the two distances will be small. Alternatively, one can think of the strength parameter as being $F\sigma_u$. It is not surprising, then, that attempts to compare this method with the PM and LM have shown that the predictions are almost identical, both for brittle fracture (Awerbuch and Madhukar, 1985) and fatigue (Taylor and Wang, 2000). Fig.7 shows an example of the comparison of these theories, for the case of notch fatigue limits. In fact, one can show that, for the case of a crack with F=1, the predictions of the LM and the ICM are mathematically identical (Taylor, 1999).

Recently we developed another method which is similar to the ICM, but with an important difference. Returning to the classic Griffith energy-balance approach which is the basis of LEFM, we rewrote the equations using a finite amount of crack extension, Δa, instead of the usual infinitesimal extension da. This approach mirrors the original idea of Neuber, to use finite quantities rather than the infinitesimal ones of continuum mechanics, but applied now to a thermodynamic, energy-based argument rather than a stress-based argument. This approach, which has been outlined in detail elsewhere (Taylor *et al.*, 2005) leads to the following result:

$$\int_{a}^{a+\Delta a} K^2 da = K_c^2 \Delta a \qquad (9)$$

This equation can be solved for any case in which the variation of K with a is known, such as a notch-plus-crack situation, many of which are available in stress in-

tensity factor handbooks (e.g. (Murakami, 1987)). The physical meaning of this approach is that a crack will not initiate and grow from the notch unless there is sufficient energy available to allow it to grow by an amount Δa: smaller amounts of crack growth are not permitted. This reflects the actual mechanism of crack propagation in many cases of brittle fracture and fatigue: crack growth is often not smooth and continuous but rather discontinuous: the crack jumps suddenly from one length to another. Fig.8 shows an example of this kind of behaviour in near-threshold fatigue crack growth, where it leads to large variations in the rate of crack growth da/dN (data from (Blom *et al.*, 1986)).

The most interesting aspect of this approach is its close relationship to the three preceding ones: if we solve equation 8 for the case of a sharp crack (allowing the crack to extend by an amount Δa) the result is exactly the same as that of the ICM (equation 6) and the value of Δa is exactly equal to $2a_o$, and therefore to 2L for the case of F=1. This approach is theoretically more sound than the ICM because it is not necessary to assume an imaginary crack: now the crack is a real crack, constrained to grow in a discontinuous way. We have given this approach the name Finite Fracture Mechanics (FFM). As far as we know, equation 8 was presented for the first time in our recent paper (Taylor *et al.*, 2005), though the idea of finite crack extension was also explored recently by Seweryn and co-workers (e.g. (Seweryn and Lukaszewicz, 2002)). Not surprisingly, given the above remarks, the FFM approach gives very similar predictions to the PM, LM and ICM when applied to problems in brittle fracture and fatigue (Taylor *et al.*, 2005).

3   A Definition of the TCD

The above review has charted the historical development of four different theories: two (the PM and LM) can be seen as modifications of the original stress-based criterion of material failure; the other two (ICM and FFM) are developments of the stress-intensity (LEFM) approach. We have seen that all four methods give similar predictions, comparing favourably with the experimental data for many practical problems. We have also seen that the critical distances used in the four methods are all quite similar, related to the parameter L as defined in equation 4. Another important similarity between these methods is that they are all continuum mechanics approaches and all assume linear elastic material behaviour.

At first sight this assumption would seem to invalidate the PM and LM because in most cases the actual stresses near the notch will be different from those predicted by a linear elastic analysis, owing to plasticity, damage and other sources of non-linear deformation. This is certainly a problem: the solution may lie in the similarity of the PM and LM to the ICM and FFM. In these latter methods, the use of linear, elastic assumptions can be justified on the same grounds normally used to justify LEFM: that the zone of non-linear deformation is contained within a surrounding elastic zone, which controls material behaviour.

These considerations lead to a definition of the Theory of Critical Distances, as follows. The TCD can be defined as a group of theories for predicting material failure in the presence of stress concentrations and stress gradients. Common features are the assumption of linear elastic material behaviour and the existence of a material parameter with the dimensions of length: the critical distance L. These four theories are:

1. The Point Method, in which failure occurs if the stress at a point a certain distance from the hot spot exceeds a characteristic strength σ_o. In some cases (e.g. high-cycle fatigue, fracture of composites) σ_o may coincide with the plain-specimen strength of the material, in other cases it may take a different, higher value.

2. The Line Method, in which failure occurs if the average stress along a line of a certain length, drawn starting at the hot spot, exceeds σ_o.

3. The Imaginary Crack Method, in which a crack of a certain length is imagined to be present at the root of the notch, whose propagation occurs at a stress intensity of K_c. LEFM is assumed in calculating K for this crack.

4. Finite Fracture Mechanics, in which failure occurs if there is sufficient energy to propagate a crack a certain distance from the notch root. LEFM is again assumed in making the calculation.

Viewing these four methods as essentially four manifestations of the same underlying approach is useful, for two reasons. Firstly, it allows the user to choose whichever method most suits a particular problem. For example, the PM and LM are convenient for use with finite element

analyses for problems of complex geometry and loading, such as are encountered in many engineering components and structures. The ICM and FFM, on the other hand, are useful in cases where a stress-intensity solution is available in closed form, allowing one to conduct parametric studies.

Secondly, this unified view helps us to understand these methods from a theoretical standpoint, suggesting avenues for future investigation. It is still not completely clear why the TCD works as well as it does: the author's own view is that the answer lies in FFM, which appears to capture an important aspect of the real mechanism of failure in materials. The other methods may work simply because they are approximations to the FFM.

4   Current and Future Work

After a period of relative dormancy in the late 1980s and most of the 1990s, the TCD is experiencing a period of renewed interest. In some cases this has taken the form of a re-invention of the basic methods by authors who were unaware of the earlier work: this is a forgivable omission because in some areas – indeed in all areas except composites – the subject has been largely untouched for two decades.

A number of workers (e.g. (Leguillon, 2002; Seweryn, 1998)) have used TCD methods in conjunction with rigorous analytical approaches applied to the prediction of brittle fracture, concentrating especially on sharp, V-shaped notches. Others have taken the TCD into novel areas such as fretting fatigue (Vallellano *et al.*, 2003), where the stress concentration is caused not by a geometrical feature but by localised contact between bodies. Work in our laboratories has consisted of validating the basic methods using large amounts of experimental data, of extending the use of the methods to other types of materials in which it has largely been unused (e.g. brittle fracture in metals and ceramics, fatigue in polymers), and in extending the theoretical basis of the method through the development of FFM and the use of some combined stress/energy methods (Taylor and Cornetti, 2005).

Some interesting future challenges on the practical side are the extension of the TCD to consider fatigue in the medium and low cycle range (Susmel and Taylor, 2005), and to incorporate multiaxial criteria into the predictions (Susmel and Taylor, 2003), both for fatigue and brittle fracture. On the theoretical side there is a need to link the methods of the TCD more closely to actual physical mechanisms of damage at the microstructural level. In some cases the value of L seems to be similar to that of microstructural features such as the grain size, but in other cases L is significantly larger and may correspond to the size of the damage zone at failure.

5   Conclusions

1. The Theory of Critical Distances has a long history but, with the exception of the field of composite materials, it is not currently being used to any great extent in industrial practice.

2. The TCD has demonstrated excellent accuracy in predicting fatigue and brittle fracture; it has been extensively validated for different types of materials and stress concentration features.

3. A formal definition of the TCD includes four different but related approaches: two are stress based (the PM and AM); the other two are energy (or stress-intensity) based (the ICM and FFM). Common features are the use of linear elastic analysis and a single, material constant length parameter.

4. The stress-based forms of the TCD have excellent potential for use in conjunction with FEA for the analysis of engineering components.

5. Many interesting topics for future research exist, both in extending the practical applications of the TCD and in developing a fuller understanding of the operation of the method and the meaning of the critical distance and critical stress parameters.

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