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Cyclic plasticity and damage of a metal matrix composite by a gradient-enhanced CDM model

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Abstract: Cyclic plasticity and damage of a metal matrix composite have been studied in the framework of continuum damage mechanics. The material was considered as macroscopically homogeneous and a model incorporating damage gradient was applied. Strain-controlled fully reversed low-cycle fatigue uniaxial tests were performed to identify material parameters related to yield stress, isotropic and kinematic hardening, fatigue life and damage diffusion. From previous studies it has been found that in the most general case the parameters of the model are constant or depend exponentially on total strain so that only two or three tests are needed for the characterisation. The results obtained from the application of the model in terms of hysteresis loop shape and fatigue life were compared to the experimental results and to the Manson-Coffin curve. Very good agreement was found, even if the experimental results were much spread, mostly due to the effect of brittle reinforcement clustering.

keyword: Metal matrix composites, Low cycle fatigue, Continuum damage mechanics, Fatigue testing.

1 Nomenclature.

a, b	=	Kinematic hardening parameters
$\underline{\underline{c}}$	=	Tensor associated to kinematic hardening
$\underline{d}$	=	Energy dissipation rate
D	=	Damage
e	=	Specific internal energy
E	=	Elastic modulus
$F, \widehat{F}$	=	Dissipative potentials
g	=	Dissipative term
G	=	State Potential
$\underline{h}$	=	Body force vector
$\underline{H}$	=	Microscopic stress vector associated with $\nabla\beta$
k	=	Damage diffusivity constant
J	=	Von Mises equivalent stress
l	=	equilibrated inertia
M	=	Microscopic internal force
m	=	Microscopic external force
$\underline{n}$	=	Unit vector normal to the surface
n	=	Generic real number
p	=	Accumulated plastic strain
$\underline{q}$	=	Heat flux vector
R	=	Region occupied by the body
s	=	specific entropy
S_0	=	Fatigue life parameter
S_y	=	Yield Stress
$\underline{u}$	=	Displacement vector
v_1, v_2	=	Isotropic hardening parameters
$\underline{\underline{x}}$	=	Kinematic hardening tensor
y	=	Isotropic Hardening
z	=	Spatial coordinate
β	=	Damage associate variable
Γ	=	Boundary of region occupied by the body
$\underline{\underline{\varepsilon}}_e$	=	Elastic strain tensor
$\underline{\underline{\varepsilon}}_p$	=	Plastic strain tensor
θ	=	Temperature
$\lambda, \widehat{\lambda}$	=	Lagrange multipliers
ν	=	Poisson coefficient
ρ	=	Density
$\underline{\underline{\sigma}}$	=	Stress tensor
ψ_e, ψ_p	=	State potentials
ψ_c	=	State potentials
ψ	=	Specific free energy
Ω	=	Body domain

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2 Introduction.

The continuous increase in the performance requirements of materials for aerospace and automotive applications has led to the development of several structural composite materials. Among these, metal matrix composites (MMCs) emerged because of their high specific elastic modulus, strength-to-weight ratios, fatigue strength, temperature stability and wear resistance [Lloyd (1994, Torralba et al. (2003), Deuis et al. (1997)]. A major potential problem of particle-reinforced composites is non-uniform microstructure, often resulting from the manufacturing process. This can lead to the presence of clusters of particles, or regions without reinforcement [Hashim et al. (2002), Prangnell et al. (1996), Yotte et al. (2001)]. This intrinsic material inhomogeneity can give a wide scatter in strength and ductility [Doel and Bowen, (1996)], wear resistance [Deuis et al. (1997), Bindumadhavan et al. (2001)], fracture toughness [Hong et al. (2003)] and also in the fatigue behaviour [Llorca (2002)].

In this paper the cyclic plasticity and damage of an Aluminium matrix reinforced by alumina particles (AA6061/20vol.%Al2O3p) made by Compocasting ®has been studied in the framework of continuum damage mechanics (CDM). This theory uses a phenomenological approach to model the effect of microscopic geometric discontinuities induced by the deformation process (micro-cracks, micro-voids, etc.) on the macroscopic behavior of a structure. In continuum damage models, an internal variable related to the growth and coalescence of microscopic defects before the macroscopic crack initiation (whose definition and physical interpretation may vary from one model to the other) is introduced and the problem becomes to establish the constitutive relations for the damage variable as a function of the other state variables.

The mathematical model used considers a material macroscopically homogeneous and derives from the theories of continua with microstructures that have been proposed as an alternative approach to many complex physical problems since the pioneer work of Cosserat and Cosserat (1909) [Mindlin (1964), Costa Mattos et al. (1992), Costa Mattos and Sampaio (1995), Fleck and Hutchinson (1993), Chimisso and Costa Mattos (1994), Fremond and Nedjar (1996)]. In all these theories, in order to account for the microstructure, a reformulation of the kinematics (to include the possible microscopic motions) and of some basic governing principles of the classical Continuum Mechanics is necessary.

In the present paper the possibility of low cycle fatigue prediction for a MMC is shown through a continuum damage theory in which the material is considered to possess a substructure or microstructure. This kind of approach was adopted in the last years by a few different groups. Very promising results were obtained up to now in the case of homogeneous materials [Bonora (1997), Bonora and Newaz(1998), Nedjar (2001), Pirondi and Bonora (2003), Steglich et al. (2005)].

A damage scalar variable is related with the links between material points. An additional balance equation is included to account for the microscopic forces related to this variable. Besides, the free energy is supposed to be a function not only of this variable, but also of its gradient. The theory enables a convenient macroscopic (phenomenological) description of the degradation induced by the deformation process accounting for the strain-softening and localization behaviors. The present theory assumes that the damage is related with micro-cracks and not with micro-voids, and hence the damaged material is not considered a porous medium and the damage variable is not directly related with a volume change. The main features of such kind of approach are discussed performing simulations of low cycle fatigue tests on MMC bars. The results of the simulations and of the experimental tests are compared in terms of hysteresis loop shape and fatigue life.

3 Matematical Model

3.1 Preliminary definitions and summary of the basic balance equations

In this section we postulate appropriate conservation laws that govern the evolution of a continuous damageable body which is defined as a set of material points which occupies a region Ω of the Euclidean space at the reference configuration. For sake of simplicity the hypothesis of small deformation will be assumed throughout this work. The density $\rho$is assumed to be constant in time and the principle of conservation of mass is automatically satisfied.

In this theory, besides the classical variables that characterize the kinematics of a continuum medium (displacements, velocities and accelerations of material points), an additional variable $\beta \in[0,1]$ is introduced. A point, in such continuum theory, is representative of a given "volume element" of the real material and it is endowed with a microstructure that accounts for the kinetic energy and internal power of the microscopic motions associated to the microscopic geometric discontinuities (density of micro-cracks or cavities). This variable is related with the microscopic motions and can be interpreted as a measure of the damage state of the "volume element". If β=1, all the links between material points are preserved and the initial material properties are also preserved. If β=0, a local rupture is considered. When $\beta = 1$, the kinetic energy associated with the microscopic motions and also the power of the microscopic forces are equal to zero in the "volume element". Since the degradation is an irreversible phenomenon, the variation rate $\dot{\beta}$ must be negative or equal to zero.

A conservation law for the microscopic forces associated to β is postulated here. The proposed principles was briefly presented in Costa Mattos and Sampaio (1995) and may be regarded as a special case of the theories of continuum with microstructure [Mindlin (1964), Toupin, (1964), Goodman and Cowin (1972)]. In particular, these governing principles are very close to those proposed in the theory of elastic materials with voids [Cowin and Nunziato (1983)]. Nevertheless, the definition and the physical interpretation of the additional kinematic variable and also the proposed constitutive equations make both theories very different. In the theory of elastic materials with voids the additional variable is related with the change in solid volume fraction. The present theory assumes that the damage is related with micro-cracks and not with micro-voids. Hence, the damaged material is not considered a porous medium and the damage variable is not directly related with a volume change. The difference between these two types of microscopic geometric discontinuities are more evident when large deformations, heat transfer or wave propagation are considered.

The necessary thermal and mechanics field variables are introduced as primitive quantities: a density $\rho : \Omega \to \mathrm{R}$, a stress tensor $\underline{\underline{\sigma}} : \Omega \to \mathrm{R}^3$, a body force $h : \Omega \to \mathrm{R}^3$, a specific internal energy $e : \Omega \to \mathrm{R}$, a heat flux vector $\underline{q} : \Omega \to \mathrm{R}^3$, a specific entropy $s : \Omega \to \mathrm{R}$ and a temperature $\theta : \Omega \to \mathrm{R}$. In addition, we introduce a microscopic stress vector $\underline{H} : \Omega \to \mathrm{R}^3$, a microscopic internal force $M : \Omega \to \mathrm{R}$ and a microscopic distant force $m : \Omega \to \mathrm{R}$. In this work an arbitrary part of the body that occupies a region $R \subset \Omega$ at the reference configuration is taken as a mechanical system. By definition, the boundary of the region R will be called Γ. Besides the classical balance relations for mass, linear momentum and angular momentum the evolution of the damageable body was governed by the following balance relations:

Balance of microscopic forces:

$$\frac{d}{dt}\int_R \left(\rho l \dot{\beta}\right) = \int_\Gamma (\underline{H}\cdot\underline{n}) + \int_R (m - M); \quad \forall R \subset \Omega \tag{1}$$

The term l in equation (1) is called the equilibrated inertia. It can be shown (Costa Mattos and Sampaio, 1995) that l is such that $\dot{l} = 0$. In some basic work concerned with microstructure theories (Goodman and Cowin, 1972, for instance), the microscopic inertia term ρl is considered in the balance of microscopic forces. The role of l in the present theory is controversial due to the particular definition of β. Since the hypothesis of quasi-static evolution is adopted in this paper the role of the microscopic inertia is not discussed in the analysis.

The microscopic external force m must be introduced in the theory in order to take into account the non mechanical actions (chemic or electromagnetic) that affect the damage state of the material even if there is no mechanical deformation. The role of such kind of external microscopic force is discussed in Chimisso (1994). In the present paper it is assumed that $h = 0$.

Balance of energy:

$$\frac{d}{dt}\int_R \left[\rho\left(e + \frac{1}{2}\underline{\dot{u}}\cdot\underline{\dot{u}} + \frac{1}{2}l\dot{\beta}^2\right)\right] = \int_\Gamma \left[\left(\underline{\underline{\sigma}}\underline{\dot{u}} + \underline{H}\dot{\beta} - \underline{q}\right)\cdot\underline{n}\right] + \int_R \left(\underline{h}\cdot\underline{\dot{u}} + m\dot{\beta}\right); \quad \forall R \subset \Omega \tag{2}$$

Second law of thermodynamics:

$$\frac{d}{dt}\int_R (\rho s) \geq -\int_\Gamma \frac{\underline{q}\cdot\underline{n}}{\theta}; \quad \forall R \subset \Omega \tag{3}$$

The variable $\dot{u}$ is the time derivative of the displacement u. After some manipulation, the expressions(1), (2) and (3) lead, respectively to the local forms:

$$-M + div\underline{H} + m = \rho l \ddot{\beta} \tag{4}$$

$$\rho\dot{e} = -div\underline{q} + \underline{\underline{\sigma}} : \nabla\underline{\dot{u}} + \underline{H}\cdot\nabla\dot{\beta} + M\dot{\beta} \tag{5}$$

$$\rho\theta\dot{s} = div\underline{q} - \frac{1}{\theta}\underline{q}\cdot\nabla\theta \geq 0 \tag{6}$$

An alternative local form for the second law of thermodynamics (which will be useful later) can be obtained by the substitution of expression (5) in (6) and the introduction of the free energy $\psi = e - \theta s$ giving rise to the inequality:

$$d = \underline{\underline{\sigma}} : \dot{\varepsilon} + \underline{H} \cdot \nabla\dot{\beta} + M\dot{\beta} - \rho\left(\dot{\psi} + s\dot{\theta}\right) - \frac{1}{\theta}\underline{q} \cdot \nabla\theta \geq 0 \tag{7}$$

where $\varepsilon = \frac{1}{2}\left(\nabla\underline{u} + (\nabla\underline{u})^T\right)$ is the local strain. The expression (7) defines the rate of energy dissipation d and can be interpreted as the Clausius-Duhem inequality for this kind of continuum under the assumption of small deformations (which takes into account the power of the microscopic internal forces $\left(\underline{H} \cdot \nabla\dot{\beta} + M\dot{\beta}\right)$.

The fundamental principles presented in the previous section are valid for any kind of damageable body. In this section, the investigation is limited to elasto-plastic behavior and isothermal processes. The theory can be summarized in the four steps below.

3.2 State Variables

Under the hypothesis of small deformations and isothermal processes, the local state of a elasto-plastic material is supposed to be a function of the total strain ε, of the plastic strain ε^p, of the damage variable β, of its gradient $\nabla\beta$ and also of a scalar variable$p$associated to the isotropic hardening and of a second order tensor variable $\underline{\underline{c}}$ associated with kinematic hardening.

3.3 State Laws

Following the classical assumption of Thermodynamic of Irreversible Processes, the free energy is supposed to be a function of the state variables. Thus, the following expression is proposed for the free energy:

$$\psi(\underline{\underline{\varepsilon}}, \underline{\underline{\varepsilon}}^p, \underline{\underline{c}}, p, \beta, \nabla\beta) = \beta\left[\psi_e(\underline{\underline{\varepsilon}} - \underline{\underline{\varepsilon}}^p) + \psi_p(p) + \psi_c(\underline{\underline{c}})\right] + \frac{1}{2}k(\nabla\beta \cdot \nabla\beta) \tag{8}$$

with

$$\psi_e = \frac{E}{2(1+\nu)} \times \left\{\frac{\nu}{1-2\nu}\left[tr(\underline{\underline{\varepsilon}} - \underline{\underline{\varepsilon}}^p)\right] + (\underline{\underline{\varepsilon}} - \underline{\underline{\varepsilon}}^p) \cdot (\underline{\underline{\varepsilon}} - \underline{\underline{\varepsilon}}^p)\right\} \tag{9}$$

$$\psi_p = v_1\left[p + \exp(-v_2 p)\right] + p\sigma_y \tag{10}$$

$$\psi_c = \frac{1}{2}a\left(\underline{\underline{c}} \cdot \underline{\underline{c}}\right) \tag{11}$$

where $\underline{\underline{\varepsilon}}^e = (\underline{\underline{\varepsilon}} - \underline{\underline{\varepsilon}}^p)$ is the elastic strain tensor. The term $\frac{1}{2}k(\nabla\beta \cdot \nabla\beta)$ is considered so as to give to βa diffusive behavior, thus smoothing the field β in Ω.

The thermodynamic forces$\left(\underline{\underline{\sigma}}, \underline{\underline{x}}, y, G, \underline{H}\right)$, related to the state variables$\left(\underline{\underline{\varepsilon}}, \underline{\underline{c}}, p, \beta, \nabla\beta\right)$, are obtained from the free energy by the state equyations:

$$\underline{\underline{\sigma}} = \frac{\partial\psi}{\partial\underline{\underline{\varepsilon}}^p} = \frac{\beta E}{1+\nu} \times \left[\frac{\nu}{1-2\nu}tr(\underline{\underline{\varepsilon}} - \underline{\underline{\varepsilon}}^p)^2 + (\underline{\underline{\varepsilon}} - \underline{\underline{\varepsilon}}^p)^2 : (\underline{\underline{\varepsilon}} - \underline{\underline{\varepsilon}}^p)^2\right] \tag{12}$$

$$\underline{\underline{x}} = \frac{\partial\psi}{\partial\underline{\underline{c}}} = \beta(a\underline{\underline{c}}) \tag{13}$$

$$y = \frac{\partial\psi}{\partial p} = \beta\left[v_1(1 - \exp(-v_2 p) + \sigma_y\right] \tag{14}$$

$$G = \frac{\partial\psi}{\partial\beta} = \psi_e + \psi_p + \psi_c \tag{15}$$

$$\underline{H} = \frac{\partial\psi}{\partial\nabla\beta} = k\nabla\beta \tag{16}$$

To complete the constitutive equations, additional information about the dissipative behavior must be given. This information can be obtained from a plastic potential F and are called the evolution laws.

3.4 Evolution laws

The potential F is presumed to have the form:

$$F = J\left(\underline{\underline{\sigma}} - \underline{\underline{x}}\right) - y + g\left(\underline{\underline{x}}, G; \underline{\underline{\varepsilon}}, \underline{\underline{\varepsilon}}^p, p, \underline{\underline{c}}, \beta\right) \leq 0 \tag{17}$$

with $J\left(\underline{\underline{\sigma}} - \underline{\underline{x}}\right)$ being von Mises equivalent stress:

$$J(\underline{\underline{\sigma}} - \underline{\underline{x}}) = \left[\frac{3}{2}(\underline{\underline{\sigma}} - \underline{\underline{x}})_{dev} : (\underline{\underline{\sigma}} - \underline{\underline{x}})_{dev}\right]^{1/2} \tag{18}$$

and

$$g = \frac{b}{2a}\left(\underline{\underline{x}} : \underline{\underline{x}}\right) - \frac{ab}{2}\left(\beta^2\underline{\underline{c}} : \underline{\underline{c}}\right) + \frac{G^2}{2S_0} - \frac{1}{2S_0}\left(\frac{\psi_e + \psi_p + \psi_c}{\beta}\right)^2 \tag{19}$$

a dissipative term.

Together with the plastic potential F, another potential, $\hat{F}(\dot{\beta}) = \dot{\beta}$, is utilized to account for the condition $\dot{\beta} \leq 0$.

The following evolution relations are postulated:

$$\underline{\underline{\dot{\varepsilon}}}^p = \lambda \frac{\partial F}{\partial \underline{\underline{\sigma}}} = \lambda \frac{\frac{3}{2}\left(\underline{\underline{\sigma}} - \underline{\underline{x}}\right)_{dev}}{J\left(\underline{\underline{\sigma}} - \underline{\underline{x}}\right)} \tag{20}$$

$$\underline{\underline{\dot{c}}} = -\lambda \frac{\partial F}{\partial \underline{\underline{x}}} = \underline{\underline{\dot{\varepsilon}}}^p - \frac{b}{a}\underline{\underline{x}}\lambda \tag{21}$$

$$\dot{p} = -\lambda \frac{\partial F}{\partial y} = \lambda \tag{22}$$

$$\dot{\beta} = M - \lambda \frac{\partial \mathrm{F}}{\partial \mathrm{G}} - \hat{\lambda} \frac{\partial \hat{F}}{\partial \dot{\alpha}} = M - \frac{\lambda(\psi_e + \psi_p + \psi_c)}{S_0} - \lambda \tag{23}$$

The expressions from (20) to (23) are the set of evolution equations of the dissipative process, where : $\lambda \geq 0$, $F \leq 0$, $\lambda F = 0$ and $\hat{\lambda} \geq 0$, $\hat{F} \leq 0$, $\hat{\lambda}\hat{F} = 0$.

M is the microscopic internal force associated with β, λ is the Lagrange multiplier associated with the condition $F \leq 0$, and $\hat{\lambda}$ is the Lagrange multiplier associated with the condition $\hat{F} \leq 0$.

It can be proved [Chimisso (1994)] that, together with the evolution equations, the state equations define a complete set of constitutive equations thermodynamically admissible.

Introducing equation (16) and (23) in (4), the following balance equation is obtained

$$\dot{\beta} = k\Delta\beta - \frac{\lambda(\psi_e + \psi_p + \psi_c)}{S_0} - \hat{\lambda} \tag{24}$$

Introducing the damage variable: D=1-β, the following expression can be derived :

$$\begin{aligned} \dot{D} &= k\Delta D + \frac{\lambda(\psi_e + \psi_p + \psi_c)}{S_0} + \hat{\lambda} \\ &= \left\langle k\Delta D + \frac{\lambda(\psi_e + \psi_p + \psi_c)}{S_0} \right\rangle \end{aligned} \tag{25}$$

where $< n >= max\{0, n\}$. From now on we will use Dinstead of βbecause the definition this variable is closer to the one usually adopted in the traditional works of continuum damage mechanics.

Equations (12)-(16) and (20)-(23) form a complete set of constitutive equation. These constitutive equations, in their evolutionary form, simplified and reduced for the one-dimensional case, are presented below.

For a round bar submitted to a one-dimensional loading such that a push-pull low cycle fatigue test, the state law equations and the evolution law equations are reduced in the following evolutive form:

$$\dot{\sigma} = (1-D)E(\dot{\varepsilon} - \dot{\varepsilon}^p) - \dot{D}E(\varepsilon - \varepsilon^p) \tag{26}$$

$$\dot{x} = (1-D)(a\dot{\varepsilon}^p - b\lambda x) - \dot{D}ac \tag{27}$$

$$\dot{y} = (1-D)v_1 v_2 e^{-v_2 p}\dot{p} - \dot{D}\left[v_1(1-e^{-v_2 p}) + \sigma_y\right] \tag{28}$$

$$\dot{\varepsilon}^p = \lambda \frac{\sigma - x}{|\sigma - x|} \tag{29}$$

$$\dot{p} = \lambda \tag{30}$$

$$\dot{c} = \dot{\varepsilon}^p - \frac{b}{a}\lambda x \tag{31}$$

$$\begin{aligned} \dot{D} = \Bigg\langle & \frac{\lambda}{2S_0}\Big\{E(\varepsilon - \varepsilon^p)^2 + ac^2 \\ & + 2\left[v_1\left(p + \frac{e^{-v_2 p}}{v_2}\right) + p\sigma_y\right]\Big\} + k\frac{\partial^2 D}{\partial z^2} \Bigg\rangle \end{aligned} \tag{32}$$

where an initially undeformed bar, clamped and with prescribed axial displacements at the extremities is considered.

The boundary conditions are:

u(z = 0, t) = 0 , u(z = L, t) = 0 ; D(z = 0, t) = D(z = L, t) = 0

The initial conditions are:

ε^p(z, t = 0) = p(z, t = 0) = c(z, t = 0) =0 and

D(z, t = 0) = D_0(z)

The choice of D_0(z) will depend on possible presence of a flaw. For a bar without flaws we have: D_0(z)=0.

In the general three dimensional case, the model parameters could be function of the maximum strain amplitude defined as:

$$Max\left(J\left(\underline{\underline{\varepsilon}}(t)\right) = \left[\frac{3}{2}\left(\underline{\underline{\varepsilon}}(t)\right)_{dev} : \left(\underline{\underline{\varepsilon}}(t)\right)_{dev}\right]^{\frac{1}{2}}\right) \tag{33}$$

4 Material and experimental methods

The composite selected for investigation was based the 6061 aluminium alloy reinforced with 20vol.% of Al2O3 particles (W6A20A).

Table 1 : Chemical compositions (in wt.%) of the matrix

	Si	Fe	Cu	Mn
W6A20A	0.65	0.15	0.18	0.10
Mg	Zn	Ti	Cr	Zr
0.97	0.009	0.02	0.19	-

The nominal chemical compositions (in wt.%) of the matrix alloys are given in Table 1.

This composite was produced by DURALCAN (USA), using a proprietary molten metal process, based on the Compocasting method [Huda at al. (1995)]. The material was then extruded and heat-treated to the T6 condition (details in Table 2).

Table 2 : T6 heat treatment conditions

Material	Solution Treatment	Quenching	Ageing
W6A20A	560 $^\circ$C - 2 h	H_2O at 25 $^\circ$C	177 $^\circ$C - 10 h

Low-cycle fatigue tests were carried out on a servo-hydraulic test machine (INSTRON 8032), equipped with a 100 KN load cell. Strain was measured with a clip-on extensometer, attached directly to the gauge length.

Flat specimens, machined with diamond cutting tools, with the tensile axis perpendicular to the extrusion direction, were prepared. Strain controlled low-cycle fatigue tests were performed at room temperature, according to ASTM E606, using specimens with 12.5 mm gauge length, 4 mm gauge width and 4 mm thickness. A triangular waveform was used, the strain amplitude range was from 0.6 % to 2%. The tests were carried out at constant cyclic frequency of 0.1 Hz and the load ratio was R=-1. Data were acquired by means of a National Instruments board in order to be processed nearly in real time. This was possible being low the frequency of the test.

The testing conditions were chosen on the basis of literature data [Ding et al. (2003)], since specific guidelines for fatigue tests on particle reinforced metal matrix composites are not available.

5 Results

One small subset of specimens was used to determine the material parameters of the model, while the whole set of tests was considered to compare the results.

5.1 Material parameters determination

The hysteresis loop was analyzed by means of LabView virtual instrument that could determine the parameters for each cycle, not just for the stabilized ones. Stabilization occurred after few cycles in every test.

The plastic arcs obtained were interpolated using the techniques described in [Chimisso et al. (2000)]. For the determination of isotropic hardening parameters the maximum stress was plotted as a function of the accumulated plastic strain. The coefficient $v_1$was obtained from the asymptotic part of the curve, while $v_2$was derived from the transient behaviour. In previous works [Ferrari et al. (2002), Minak et al. (2001)] we found that in general the parameters a and b and characteristic life S_0 depend exponentially from the total strain range. The minimum number of tests necessary to find the values of the coefficient and exponent is two, but it can be shown that much better approximation can be found using three specimens tested at different strain ranges

Considering specimens loaded at 0.4%-0.6%-1.0% the following trends were found:

$$a = 14680\Delta\varepsilon^{-1.0012} \tag{34a}$$

$$b = 132\Delta\varepsilon^{-1.4796} \tag{34b}$$

$$S_0 = 19.9\Delta\varepsilon^{-3.9123} \tag{34c}$$

The parameters a and S_0 are expressed in MPa.

The values of a and b determine the shape of the hysteresis loop in the plastic zone. In fact, the kinematic hardening in this model follow an exponential law (being a and b its parameters) as a function of the plastic strain. The parameter S_0 is a measure of the fatigue resistance of the material for every strain level. In figure 1 the stress range as a function of the number of cycles for different strain range levels is plotted. It is possible to see that the material did not show isotropic hardening [Ceschini et al. (2005)]. A slight continuous softening during the test characterized the whole specimen set. This softening is mainly due to damage evolution in the form of the mutual action of interfacial decohesion and particles cracking [Ceschini et al. (2005), Poza and LLorca (1995), Vyletel et al. (1993), Vyletel et al. (1995), Srivatsan (1995)].

Due to these considerations the parameters $v_1$and v_2 have been set to zero.

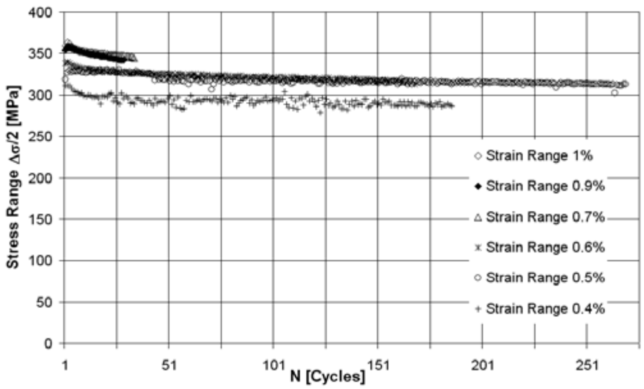

Figure 1 : Stress range as a function of life

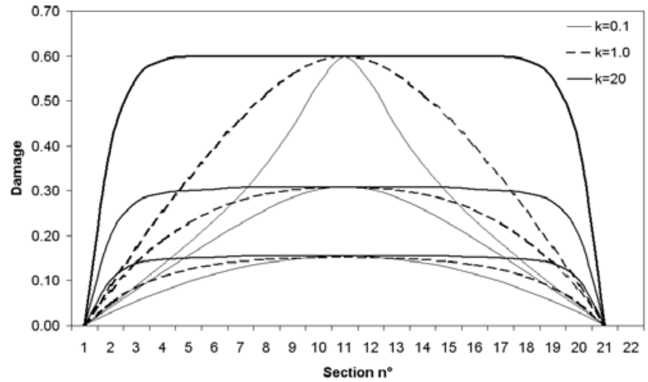

Figure 2 : Effect of the damage diffusion parameter k

In Ferrari et al. (2002) and Minak et al. (2001) different values were found for the paramenter k in different aluminium alloys In fact it for AA6351 k=0.01 mm^2/s, for AA2011 k=0.1 mm^2/s and for AA2030 k=1 mm^2/s. In all these cases the damage evolution is localized more or less in the middle of the specimen.

For the material under investigation the damage, that consists in particle breaking, particle debonding, microvoids nucleation and growth [Ceschini et al. (2005)], was weakly localized, i.e. the whole gauge length was characterized by similar levels of damage. Due to this consideration the damage diffusion parameter k, that measures the damage localization was set to 20 mm^2/s.

This choice is arbitrary, but the use of different values of k within the same order of magnitude slightly changes the results in terms of fatigue life.

The effect of the value of k on damage localization is shown in figure 2 at different stages of damage evolution in different sections of the calibrated length.

5.2 Comparison of experimental and numerical results

Two example of hysteresis loops located at half life are shown in figure 3.

In both cases the agreement is very good and the main difference was found in the elastic-plastic transition. In the experimental loops there was a slight difference in the elastic modulus in traction and in compression. This difference is probably due to the non perfect alignment of the load on the specimen and to the possible buckling onset, especially in the higher strain range case. We must note that the cycle at 0.7% strain rate was not one of

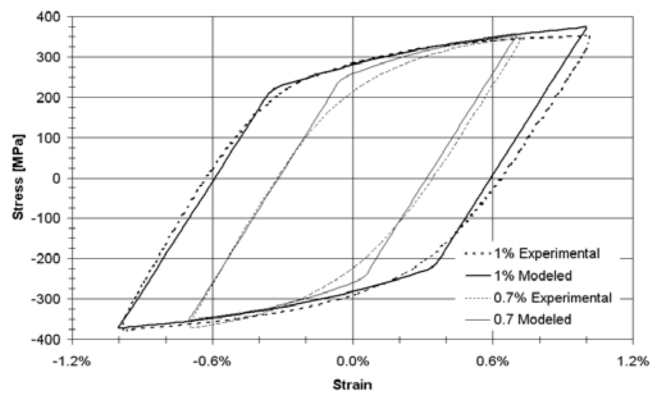

Figure 3 : Comparison of experimental and modeled hysteresis loops for two different strain ranges

those used for the parameter determination, nevertheless the agreement is very good.

The strain-life experimental results and the interpolation of the data according to the classical Manson-Coffin model and the results of the numerical computation are reported in figure 4. During the tests specimens were considered failed after a sharp variation of their stiffness, that correspond to a macroscopic crack initiation. In the numerical simulations a damage level of 0.6 was conventionally indicating the failure of the specimens. Anyhow, after a value of 0.4, the damage increase was very fast and the residual life was always negligible.

The experimental data showed a high dispersion, which can be interpreted on the basis of the material inhomogeneity. In all the tests fatigue fracture of the specimen occurred by unstable propagation of the crack between the extensometer blades.

It is interesting to note that the numerical model is in better agreement with the Manson-Coffin curve than the original experimental data from which it was calculated. Obviously this is due to the fact that material parameters (calculated based only three specimens) are function of the strain range only and they are not affected by the dispersion in the physical distribution of the particles. This effect, present in the experimental results is averaged in the numerical ones.

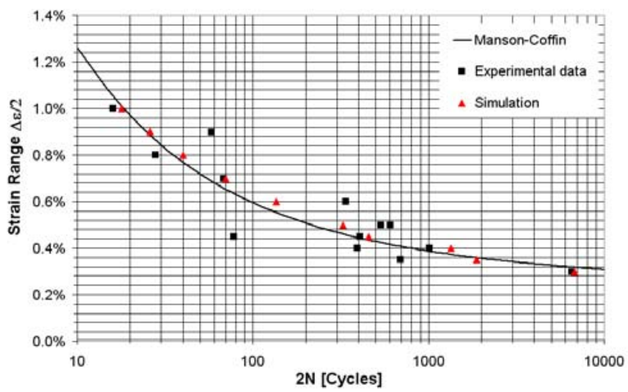

Figure 4 : Comparison of experimental life data, simulation results and Manson-Coffin curve

6 Discussion and Conclusions

Fracture in metal matrix composite is controlled by three main mechanisms: interfacial decohesion, fracture of reinforcing particles, void nucleation and growth.

The model proposed does not take into account the presence of microvoids (i.e. density is assumed to be constant) and considers only microcracks. Nevertheless the numerical results are in good agreement with the experimental ones.

The material parameters necessary for the CDM numerical model were dependent from the total strain and the determination of their trend has been done by means of three tests.

We found that damage was homogeneously distributed in the specimens gauge length so that an high value of the damage diffusion parameter has been used.

Some asymmetry was found in the experimental hysteresis loops but the overall agreement with the modelled loops was good. The fatigue life prevision made by the model is the same extimated by the Manson-Coffin curve. This is a good property of the model since in this simple case we expect to find results comparable to well know and commonly utilized methods.

In order to apply the model to the design of components made of MMCs it would be necessary to take into account the large spread of the experimental data.

The applicability of a CDM model for the simulation the cyclic plasticity and damage of a MMC, at that macroscopic scale, was demostrated.

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