Copyright © 2015 Tech Science Press MCB, vol.12, no.1, pp.1-16, 2015

Mathematical Model for Skeletal Muscle to Simulate the Concentric and Eccentric Contraction

Chetan Kuthe[*], **R. V. Uddanwadiker**[†], **P. M. Padole**[‡] **and A. A. Ramteke**[§]

Abstract: Skeletal muscles are responsible for the relative motion of the bones at the joints and provide the required strength. They exhibit highly nonlinear mechanical behaviour and are described by nonlinear hyperelastic constitutive relations. It is distinct from other biological soft tissue. Its hyperelastic or viscoelastic behaviour is modelled by using CE, SEE, and PEE. Contractile element simulates the behaviour of skeletal muscle when it is subjected to eccentric and concentric contraction. This research aims to estimate the stress induced in skeletal muscle in eccentric and concentric contraction with respect to the predefined strain. With the use of mathematical model for contraction of skeletal muscle for eccentric and concentric contraction, the stress induced in the skeletal muscle is estimated in this research. Mathematical model is developed for the muscle using EMG signals and Force-velocity relationship calculated. With the use of force-velocity of contraction of muscle, mathematical model is developed. This can be useful to understand the mechanical behaviour of skeletal muscles in eccentric and concentric contraction with clinical relevance.
Authors are further working to develop the mathematical model with torsion force with proper activation function of muscle and experimentation for extraction of the anisotropic mechanical properties of skeletal muscle.

Keywords: CE (Contractile Element), SEE (Series Elastic Element), PE (Parallel Elastic Element), Anisotropy, hyperelastic, eccentric contraction, concentric contraction.

[*] Research Scholar. Department of Mechanical Engineering, Visvesvaraya National Institute of Technology, India. E-mail: 786Chetankuthe@gmail.com
[†] Assistant professor. Department of Mechanical Engineering, Visvesvaraya National Institute of Technology, India.
[‡] Professor. Department of Mechanical Engineering, Visvesvaraya National Institute of Technology, India.
[§] Knee & Hip Reconstruction & Replacement surgeon. Arthritis & Joint Replacement Clinic, Nagpur.

1 Introduction

Skeletal muscles are highly non linear and anisotropic material. It is mainly composed of cluster of muscle cells. Muscle shows the different characteristics to the isometric and isotonic contraction. Muscle fiber generates tension during the action of actin and myosin cross-bridge cycling. The term contraction implies shortening, when referring to the muscular system; it means muscle fibers generating tension with the help of neurons. Its electrical activity is measured using electromyography (EMG) or s-EMG.

In earlier era, the research was focused on physiological, biomechanical and human models based on rigid skeletons [1-6]. The physically unrealistic muscle model was used to present elastic deformation of soft tissues [7]. Several efforts were carried out to model realistic muscle to represent the muscle shape and deformable behaviour. The medical imaging technique has improved the visual quality and realism of muscle geometry and was used to developed models [8]. The realistic muscle geometry has been developed to describe the muscle deformable characteristics during its contraction. Numerous methods and approaches have been proposed to identify the mechanical properties of skeletal muscle like Mathematical formulation and experimentation under tension-compression test.

In the recent past, numerous bio mathematical models of skeletal muscle have been proposed. Existing models were focused on narrow (and at times, isolated) aspects of the excitation–contraction coupling process. Those were like (1) phenomenological energy-conversion models, which consider only one or two steps in the biochemical sequence of events [9, 10]. (2) Generalized viscoelastic Voigt, Maxwell, and Kelvin models, which applies to any one of many biological materials [11]. (3) Models that are applicable under certain conditions, such as Hill's model that applies to tetanised whole muscle; and (4) microscopic models, such as the H.E. Huxley cross-bridge model. First mathematical model proposed by V. Comincioli et al, describes the cross bridge mechanics for active muscle. It also described the dynamics of muscle contraction at macromolecular level [12-14]. Mathematical models based on electrical simulation were also used to predict non linear properties of skeletal muscle, which was used in physiological based design of control strategies of neuro-prosthesis [15-17].

More numbers of mathematical models were focused on force-length, force–velocity relation of active and passive contraction of the muscle. These models were based on differential equation with static and dynamic condition with fiber recruitment rate and fiber type within the muscle [18-22]. Several numbers of numerical models were formulated to simulate the mechanical properties of skeletal muscle with the assumption of isotropic and homogeneous material behaviour of the muscle [7, 23-30]. EMG-driven models measured muscle activity to estimate muscle force, these models implicitly account for a subject's individual activation patterns without the need to satisfy any constraints imposed by an objective function [31, 32]. Later on mathematical models were refined further and formulated based on strain energy density function with the consideration of hyperelastic, viscoelastic and anisotropic skeletal muscle [33-36]. These models were well examined for the active and passive response of the skeletal muscle with the fiber orientation within the muscle mass. Skeletal muscles drive the movement of human body so with this keen perception, various researchers have formulated the numerical tool to predict the functioning and mechanical properties of skeletal muscle.

2 Method

28 male subjects (age 22 $\pm$ 3 years; BMI 18.5-22.5 kg-m^{-2}) volunteered for the study after a detailed explaination of the procedures and possible risk involved. They were asked to perform isometric exercises (hold) at an elbow angle maintained at 100^0 to record their maximum voluntary contraction using s-EMG. Each contraction was to be held for 60 seconds. The experimental protocol of the study was ethically approved by local ethics committee, India. Isometric contraction is performed to get the skeletal mscles maximum force T_0 at resting length of the muscle L_{p0}. Also Isotonic contraction [eccentric and concentric contraction on the same subjects are performed for the period of 5 sec in order to get the muscle activation in each contraction. The Mathematical model is used to compute the stress in contractile element of the biceps brachii with using the muscle dynamics parameters from the EMG data.

2.1 EMG signal recording

The signals are recorded from the biceps brachii of each subject's right arm. After the skin preparation a wireless bipolar surface electrode was attached to the skin over the belly of the biceps at the area where maximum bulk of biceps was found. All EMG data recordings are carried out using a Power Lab 2/26 (AD Instruments, Australia). EMG signals were sampled at a rate of 1 kHz with a 24-bit analog-to-digital resolution.

2.2 EMG Signal acquisition and analysis

Recorded EMG signal from the subject is the raw EMG data. To remove the artefacts from the raw EMG data; band pass filter of 20-450 Hz is used. Low end cutoff removes electrical noise associated with wire sway and biological artifacts while high end cutoff eliminates tissue noise at the electrode site. Filtered EMG data is then further processed using full wave rectification to get absolute value of each data point see figure 1. The force generated by the muscle during the Isometric contraction is given by equation (1).

$$F_t = gE(t) \tag{1}$$

Where, F_t is the muscle force at instant t during static contraction and E(t) be the rectified EMG at that instant of contraction.

g – gain factor of S-EMG

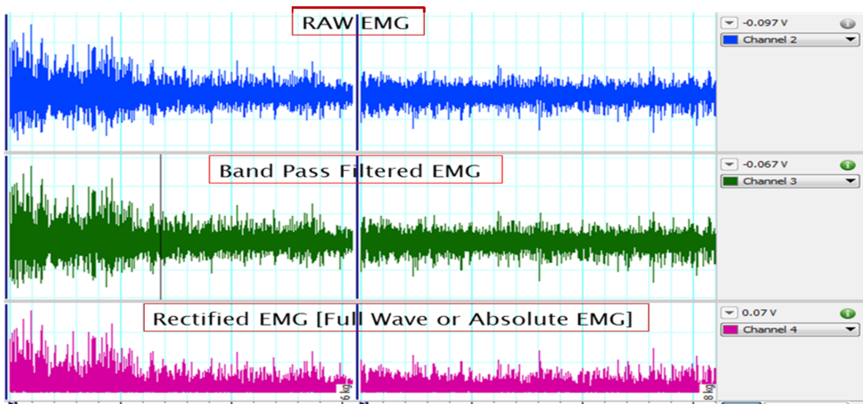


Figure 1: EMG signal in isometric contraction [Raw emg to Filtered EMG to Rectified EMG].

2.3 Mathematical Model

In the present study, mathematical model is used for simulating behaviour of muscle for eccentric and concentric type of contraction. Hyperelastic or viscoelastic behaviour is modelled by using Contractile Element, Series Elastic Element, and Parallel Elastic Element refer figure 2. This model simulates the contractile element of the muscle. Stress stretch relations are estimated using current mathematical model and recorded EMG signals of muscle electrical activity.

In eccentric contraction, the muscle elongates while under tension. Contractions that permit the muscle to shorten are referred as concentric contractions.

Hills tree element basic muscle model consist of contractile element in series with series elastic element and those in parallel with parallel elastic element (figure 2). Most important equation of the muscle mechanics to refer skeletal muscle property by Hills is given by

$$(v+b)(T+a) = b(T_0+a) \tag{2}$$

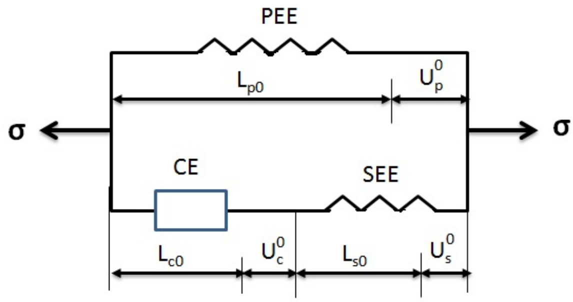


Figure 2: Hills Three Element model.

Where T represents tensional force in muscle, v the velocity of the muscle contraction, and a; b and T_0 are constants. The constant T_0 is the maximum force developed in the muscle under isometric condition at resting length of the muscle (L_{p0}) . The maximum Isometric force is computed by using equation (1) from EMG signal see figure 3. A dimensionless form of Hill's equation is

$$\frac{T}{T_0} = \frac{1 - v/v_0}{1 + c\,(v/v_0)} \tag{3}$$

$$c = T_0/a \tag{4}$$

From the figure 2, following relation is for the muscle without activation or undeformed muscle

$$L_{p0} + U_p^0 = L_{c0} + U_c^0 + L_{s0} + U_s^0 \tag{5}$$

Where L_{c0} and L_{s0} are initial lengths of contractile and series elastic elements. U_p^0 , U_c^0 and U_s^0 =0 are elongations (displacements) of the corresponding elements at Isometric contraction.

We further suppose that we have the relations between the lengths given by

For undeformed skeletal muscle, elongations in CE, SEE, and PEE are zero.

Hence, equation (5) can be rewrite as

$$L_{p0} = L_{c0} + L_{s0} \tag{6}$$

Relations between lengths is given by equation (7)

$$L_{s0} = kL_{m0} \tag{7}$$

Where k is constant.

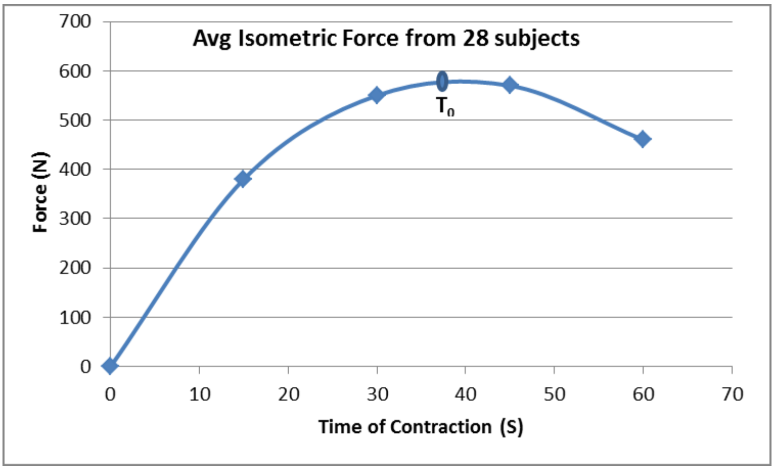


Figure 3: Average Isometric Force for Biceps Brachii of 28 subjects.

Dividing equation (5) by $L_{c0,}$

$$\frac{L_{p0}}{L_{c0}}+\frac{U_p^0}{L_{c0}}=1+\frac{U_c^0}{L_{c0}}+\frac{L_{s0}}{L_{c0}}+\frac{U_s^0}{L_{c0}} \tag{8}$$

From equation (6), (7) and (8)

$$\delta_c^0=(1+k)\,\delta_p^0-k \tag{9}$$

$\delta_p^0=1$, it shows the initial state corresponds to the undeformed state of the skeletal muscle.

Hence $\delta_c^0=1$.

At time instant t, equation for lengths given by equation (10)

$$L_p^t=L_{c0}+U_c^0+\int_{t_a}^{t} v_c dt+L_{s0}+U_c^t=L_{p0}+U^t \tag{10}$$

Where v_c is the rate of change of the muscle length, and t_a is activation time. Dividing this equation by Lm0, obtaining equation (11)

$$(1\text{+}k)\,\delta_p^t=\delta_c^0+\int_{t_a}^{t}\frac{v_c}{L_{c0}}dt+k\delta_s^t \tag{11}$$

Rewriting the above equation for the end of the time step with increments of stretches as equation (12)

$$(1+k)\,\delta_p^{t+\Delta t}=\delta_c^t+\Delta\delta_c+k\delta_s^t+\Delta\delta_s \tag{12}$$

The relation between tension in the skeletal muscle and its velocity of shortening is further modified in an incremental form as follows

For concentric contraction

The stress produced by contractile element of the Muscle at time $t+\Delta t$ is

$$\sigma_c^{t+\Delta t} = \sigma_0^t f_{ce}^{t+\Delta t} \tag{13}$$

Where σ_0^t: stress corresponding to ${}^t\delta$

$$\sigma_0^t = \alpha_a^{t+\Delta t} * \sigma_0$$

$$\sigma_0 = \frac{T_0}{A}$$

Where A is the physical cross section area of the muscle.

f_{ce} : function of contractile element

$$f_{ce} = \alpha_a^{t+\Delta t} * \frac{1+\Delta\delta_c/\Delta\delta_{c0}}{1-c\delta_c/\Delta\delta_{c0}} \tag{14}$$

Where $\Delta\delta_m$: Stretch increment in CE

α_a : Activation function and it is given by the equation(15)

$$\alpha_a^{t+\Delta t} = F^{t+\Delta t}/F_c \tag{15}$$

$\mathrm{F}^{t+\Delta t}$ is the instantaneous force generated by skeletal muscle at time $t+\Delta t$ and F_c is the maximum tetanised force generated by skeletal muscle. These forces are computes from the s-EMG signal of the muscle. To compute the forces from s-EMG signal, first raw signal is rectified using full wave rectifier and then convert it into force from the amplitude of rectified signal see figure 4.

$$\Delta\delta_{c0} = \Delta t\,\dot{\delta}_{c0} \tag{16}$$

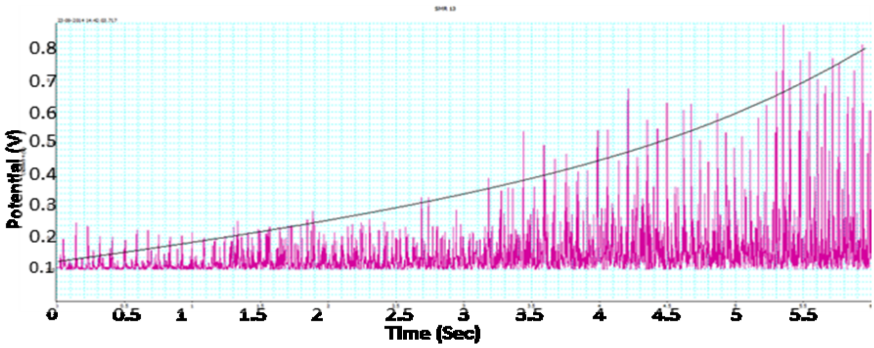
Where $\dot{\delta}_{c0}$: stretch rate of the CE.

For eccentric Contraction

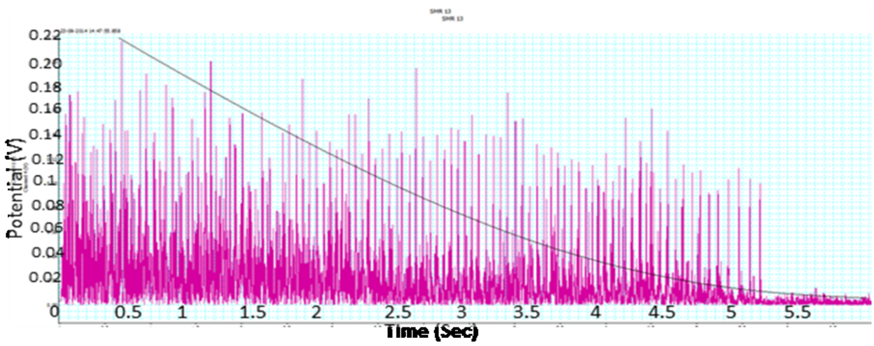
The tension-velocity relation during muscle lengthening was then characterized in the form of an equation by Otten, (1987) and later adopted by Van Leeuwen, (1991) [37, 38]. This tension –velocity general relation during eccentric contraction of skeletal muscle is given by equation (17)

$$\frac{T}{T_0} = \left[d-(d-1)\left(\frac{1-v/v_0}{1+c\,(v/v_0)}\right)\right] \tag{17}$$

Where d is a parameter quantifying the force offset with respect to the isometric case due to the eccentric movement.



(a)



(b)

Figure 4: EMG signal recorded for concentric and eccentric contraction of biceps brachii (a) Rectified EMG in concentric contraction. (c) Rectified EMG in eccentric contraction.

This relation is modified into an incremental form to obtain stress-stretch relation by the contractile element for describing the eccentric contraction. Hence the stress produced by contractile element of the skeletal Muscle at time $t+\Delta t$ is given by equation (18).

$$\sigma_c^{t+\Delta t} = \sigma_0^t f_{ce}^{t+\Delta t} \tag{18}$$

$$f_{ce} = \alpha_a^{t+\Delta t}\left[d-(d-1)\frac{1+\Delta\delta_c/\Delta\delta_{c0}}{1-cS_p\Delta\delta_c/\Delta\delta_{c0}}\right] \tag{19}$$

Sp: the shape parameters of force-velocity curve of the CE From equation (12),

$$\Delta\delta_c = a_1 - k\Delta\delta_s \tag{20}$$

where

$$a_1 = (1+k)\,\delta_p^{t+\Delta t} - \delta_c^t - k\delta_s^t \tag{21}$$

Stresses σ_c and σ_s in the contractile element and the series elastic element are considered to be equal at any moment. Hence

$$\sigma_c^{t+\Delta t} = \sigma_s^{t+\Delta t} \tag{22}$$

Force stretch relation for non linear elastic element (SEE) is given by

$$S = (S^* + \beta)\, e^{a(\delta - \delta^*)} - \beta \tag{23}$$

Where α and β are constants, and S^* is the force corresponding to stretch δ^*.

The constitutive equation for the stress in SEE from equation (23) is given by

$$\sigma_s^{t+\Delta t} = \beta \left[e^{\alpha(\delta_s^t - 1 + \Delta\delta_s)} - 1 \right] \tag{24}$$

The governing equation for solving the stress increament of SEE from equations (20), (22) and (24), $\Delta\delta_s$ is given by

$$f(\Delta\delta_s) = (a_2 + a_3\Delta\delta_s)\, e^{a\Delta\delta_s} - a_4\Delta\delta_s - a_5 = 0 \tag{25}$$

Where for concentric contraction

$$a_2 = \left(\sigma_s^t + \beta\right)\left(1 - \frac{ca_1}{\Delta\delta_{c0}}\right) \tag{26}$$

$$a_3 = \left(\sigma_s^t + \beta\right)\left(\frac{kc}{\Delta\delta_{c0}}\right) \tag{27}$$

$$a_4 = \left(\frac{\beta c - \sigma_0^t \alpha_a^{t+\Delta t}}{\Delta\delta_{c0}}\right) k \tag{28}$$

$$a_5 = \left(\sigma_0^t \alpha_a^{t+\Delta t} + \beta + \frac{\sigma_0^t \alpha_a^{t+\Delta t} - \beta c}{\Delta\delta_{c0}} a_1\right) \tag{29}$$

and for eccentric contraction

$$a_2 = \left(\sigma_s^t + \beta\right)\left(1 + \frac{cs_p a_1}{\Delta\delta_{c0}}\right) \tag{30}$$

$$a_3 = -\left(\sigma_s^t + \beta\right)\left(\frac{ks_p c}{\Delta\delta_{c0}}\right) \tag{31}$$

$$a_4 = \left(\frac{\beta s_p c - \sigma_0^t \alpha_a^{t+\Delta t}\,(ds_p c + d - 1)}{\Delta\delta_{c0}}\right) k \tag{32}$$

$$a_5 = \left(\sigma_0^t \alpha_a^{t+\Delta t} + \beta - \frac{\sigma_0^t \alpha_a^{t+\Delta t}\,(1 - d - dcs_p) - \beta s_p c}{\Delta\delta_{c0}} a_1\right) \tag{33}$$

Equation (25) can be solved numerically by standard Newton's method. Once the value of $\Delta\delta_s$ is known, the values of σ_c and σ_s at time $t + \Delta t$ can be obtained using equations (13), (18) and (24). All the constants are used from the study of Kojic et al see table 1.

Table 1: Material Parameters for simulating skeletal muscle [41].

b	c (N/m^2)	α	β (N/m^2)	d	s_p	$\dot{\delta}_{c0}$ (s^{-1})	k	a (N/m^2)
2.1	8210	100	0.1	1.65	3.14	2	0.3	70x10^3

2.4 Statistical data

In this study, the SIGMAPLOT software package was used for data analysis. To assess the variations in the stress in the contractile element of the muscle associated with the effects of muscle activation on muscular contractions. Data were represented as mean ± standard deviation (mean ± SD). Statistical testing between males test model and existing Martins et al. model is examined by student t-test. Significance test for the alpha value was set at 0.05.

3 Results and Discussion

Force and velocity of shortening of skeletal muscle relationship curve is extracted from recorded EMG signals. Also activation functions of skeletal muscle during the concentric and eccentric contraction are recorded from EMG. In this article, activation function is considered in the form of ratio of amount of instantaneous force generated to that of maximum force generated by the muscle during the isotonic contraction. In order to simplify the computation, activation functions are considered as 0%, 25%, 50%, 75% and 100% in isotonic contraction of skeletal muscle.Numerous mathematical equations were described in the literature used to explain the eccentric and concentric contraction of skeletal muscle [38-40].The present model compute eccentric and concentric behaviour of skeletal muscle. The stress in the contractile element during concentric contraction is computed using equation (13) see table 2 and during eccentric contraction is computed by equation (18) see table 3.

The stress in the contractile element in eccentric contraction is calculated using equation (18) see table 3.

The stress in contractile element of Hills three model is used to simulate the muscle condition during the contraction. The stress response is different in shortening and lengthening of the muscle see figure 5 and 6. In the shortening of the muscle the stress response is negative while in lengthening it is positive.. The stress in the contractile element is negative during the shortening of the muscle because the change in length of the muscle is shortened by 10 % of its resting length while positive stress in lengthening due to increase in muscle length by 10% in steps. The maximum stress in contractile element of biceps brachii muscle is 1644.74 KPa at

Table 2: Stress in the contractile element in concentric contraction expressed in mean±(SD).

Time (sec)	Activation	Stretch (λ_f)	Test Model Stress σ^m	Exisitng Model Stress σ^m
0	0	1	0	0
1	0.25	0.9	-0.24 (0.016)	-0.39
2	0.5	0.8	-1.17 (0.06)	-2.15
3	0.75	0.7	-3.6 (0.09)	-4.25
4	1	0.6	-11.7 (0.18)	-11.52
P<0.05				

Table 3: Stress in the contractile element in eccentric contraction expressed in mean±(SD).

Time (sec)	Activation	Stretch (λ_f)	Test Model Stress σ^m [KPa]	Exisitng Model Stress σ^m [KPa]
0	0	1	0	0
1	0.25	1.1	101.23 (7.66)	101.6
2	0.5	1.2	409.19 (20.73)	200.7
3	0.75	1.3	924.20 (23.84)	409.2
4	1	1.4	1644.74 (10.589)	780.9
P<0.05				

40 % elongation of its resting length see table 3. This shows that the muscle can generate more force during eccentric contraction than the concentric contraction. Results from the current numerical formulation shows the significance difference (p<0.05) from the results of Johansson et al and Martins et al [39, 40] and this discrepancy in the results is because of hyperelasticity or fiber orientation within the muscle.

The results of the present study shows that the eccentric contraction of the muscle plays a vital role in monitoring the muscle conditioning during the contraction. If the stress is increased further there are chances of damage to the muscle during the specific task. If the analyzed EMG data is available before the simulation of model, the solution converges in a very short time. This study has limitation that the the range of motion or joint angle is not considered during the simulation of eccentric and concentric contraction. The maximum yield stress or safe stress condition is

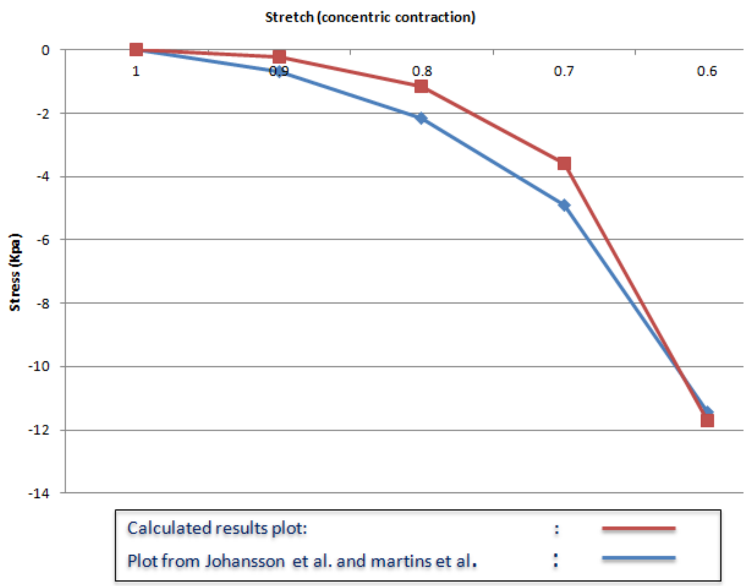


Figure 5: Stress vs. Stretch in Concentric contraction.

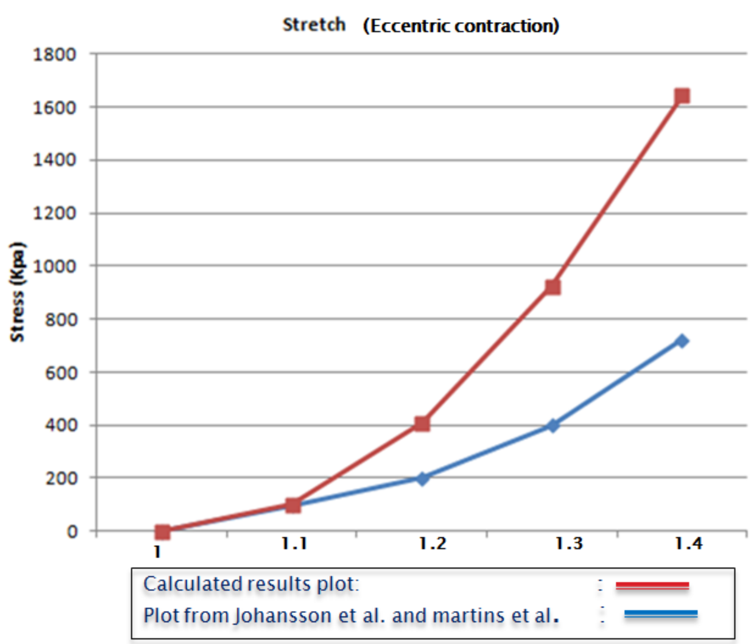


Figure 6: Stress vs. Stretch in eccentric contraction.

not computed in this study. Furthermore the limitations of the study needs to be investigated in future. Neverthless this study can be useful to compute the stress in the muscle at any instant during its contraction.

4 Conclusion

This article is availing the numerical procedure for the stress calculation for a three-element Hill's model of muscle. The computational procedure of the current research is numerically efficient and reliable. The presented approach is applicable to real life muscle problems, with various muscle groups, arbitrarily oriented in space, with external loading and computed activation functions from s-EMG.

This research output will be helpful for further investigation of mechanical behaviour of muscle for various types of load. The results of such numerical methods can be used to enhance the Finite element analysis of the skeletal muscle. The model presented can be used to study numerical biomechanical experiments in different areas, i.e. muscle recruitment, muscle architecture, and stabilization. Future scope of the study is the anisotropic properties of skeletal muscle using in vivo method.

Acknowledgement: This project is funded by the Department of Science and Technology of India. The authors would like to thank Dr. Neha Deshpande for her support during the research.

Reference

1. Hanson, J. & Huxley, H. E. (1953) The structural basis of the cross-striations in muscle. Nature, 172, 530–532.

2. Huxley, A. F. (1969) Theories of muscular contraction. Proceedings of the Royal Institution of Great Britain, 43, 71-84.

3. Huxley, A. F. & Simmons, R. M. (1971) Proposed mechanism of force generation in striated muscle. Nature, 233, 533–538.

4. Badler, N. I. & Smoliar, S. W. (1979) ACM Computing Surveys. Digital representations of human movement, 11, 19–38.

5. Magnenat-Thalmann, N. (1985) Computer animation: theory and practice. Springer-Verlag New York, Inc., USA.

6. Paul, J., Vignos, J. R. & Lefkowitz, M. (1959) A biochemical study of certain skeletal muscle constituents in human progressive muscular dystrophy. J. Clin Invest, 38(6), 873–881.

7. Chadwick, J. E., Haumann, D. R. & Parent, R. E. (1989) Layered construction for deformable animated characters. SIGGRAPH Computer Graphics, 243–252.

8. Ng-Thow-Hing, V. & Fiume, E. (1999) B-spline solids as physical and geometric muscle models for musculoskeletal systems. In Proceedings of the VIIth International Symposium of Computer Simulation in Biomechanics, 68–71

9. Bornhorst, W. J. & Minardi, J. E. (1970) A phenomenological theory of muscle contraction. I. Rate equations at a given length based on irreversible thermodynamics. Biophys. J, 10 (2), 137–154.

10. Bornhorst, W. J. & Minardi, J. E. (1970) A phenomenological theory of muscle contraction II. Generalized length variation. Biophys. J, 10 (2), 155–171.

11. Fung, Y. C. (1993) Biomechanics: Mechanical Properties of Living Tissues, 2nd Edition, Springer, New York.

12. Comincioli, V., Torelli, A., Poggesi, C. & Reggiani, C. (1984) A four-state cross bridge model for muscle contraction. Mathematical study and validation. J. Math. Biol, 20, 277-304.

13. Comincioli, V. & Naldi, G. (1990) Mathematical models in muscle contraction: Parallelism in the numerical approach. Mathematical and Computer Modelling: An International Journal, 13(1), 109-115.

14. Hill, T. L., Eisenberg, E., Chen, Y. D. & Podolsky, R. J. (1975) Some self-consistent 2-state sliding filament models of muscle-contraction. Biophysical Journal, 15 (4), 335–372.

15. Frey Law, L. A. & Shields, R. K. (2006) Predicting human chronically paralyzed muscle force: a comparison of three mathematical models. Journal of Applied Physiology, 100, 1027–1036.

16. Frey Law, L. A. & Shields, R. K. (2005) Mathematical models use varying parameter strategies to represent paralyzed muscle force properties: a sensitivity analysis. Journal of Neuro-engineering and Rehabilitation, 2, 2-12.

17. Stephen J. Dorgan (1997) A non linear mathematical model of electrically simulated skeletal muscle. IEEE transactions on rehabilitation engineering, 5 (2), 179-194.

18. Wexler, A. S. & Ding, J. (1997) A mathematical model that predicts skeletal muscle force. IEEE Transactions on Biomedical Engineering, 44, 337–348.

19. Rasmussen, J., Damsgaard, M. & Voigt, M. (2001) Muscle recruitment by min/max criterion-a comparative numerical study. Journal of biomechanics, 34, 409-415.

20. Neidhard-Doll, A. T., Phillips, C. A., Repperger, D. W. & Reynolds, D. B. (2004) Biomimetic model of skeletal muscle isometric contraction: II. A phenomenological model of the skeletal muscle excitation-contraction coupling process. Computers in Biology and Medicine, 34, 323–344.

21. Zakotnik, J. (2006) Biomechanics and neural control of targeted limb movements in an insect. PhD thesis, University of Bielefeld.

22. Chin, L., Yue, P., Feng, J. J. & Seow, C. Y. (2006) Mathematical simulation of muscle cross bridge cycle and force-velocity relationship. Biophysical Journal, 91, 3653-3663.

23. Colli, P. & Math, J. (1986):, A mathematical model of heterogeneous behavior of single muscle fibres. *Biology*, Vol. 24, pp. 103-118.

24. Colli, P., Comincioli, V., Naldi, G. & Reggiani, C. (1988) Mathematical modelling for contracting muscle. J biomathematics and related problems, 603-613.

25. Colli, P. (1989) On a nonlinear and nonlocal evolution equation related to muscle contraction. Nonlinear Anal, 13, 1149-1162.

26. Comincioli, V. & Torelli, A. (1983) Mathematical aspects of the cross-bridge mechanism in muscle contraction. Nonlinear Anal, 7, 661-683.

27. Comincioli, V. & Torelli, A. (1988) A mathematical model of contracting muscle with viscoelastic elements. SIAM J. Math. Anal., 19 (3), 593-612.

28. Gastaldi, L. & Tomarelli, F. (1987) A nonlinear and nonlocal evolution equation describing the muscle contraction. Nonlinear Anal, 11, 163-182.

29. Gastaldi, L. & Tomarelli, F. (1984) Numerical approximation of a nonlinear problem arising in physiology. Pubbl. I. A. N. (Zstituto di Analisi Numerica) del C. N. R. Pavia, 429.

30. Allain, G. & Colli, P. (1988) A mathematical study of a muscle contraction model in which the fibre is a continuum of elements. Advances in applied mathematics, 9, 104-126.

31. Buchanan, T. S. & Lloyd, D. G. (1995) Muscle activity is different for humans performing static tasks which require force control and position control. Neuroscience Letters, 194, 61–64.

32. Tax, A. A., Denier van der Gon, J. J. & Erkelens, C. J. (1990) Differences in coordination of elbow flexor muscles in force tasks and movement tasks. Experimental Brain Research, 81, 567–572.

33. Calvo, B., Ramírez, A., Alonso, A., Grasa, J., Soteras, F., Osta, R. & Muñoz, M. J. (2010) Passive nonlinear elastic behaviour of skeletal muscle: experimental results and model formulation. J. Biomech, 43, 318–325.

34. Grasa, J., Ramirez, A., Osta, R., Munoz, M.J., Soteras, F. & Calvo, B. (2011) A 3D active passive numerical skeletal muscle model incorporating initial tissue strains: validation with experimental results on rat tibialis anterior muscle. Biomech model mechanobiol, 10, 779-787.

35. Martins, J. A. C., Pires, E. B., Salvado, R. & Dinis, P. B. (1998) A numerical model of passive and active behavior of skeletal muscles. Computer Methods in Applied Mechanics and Engineering, 151, 419–433.

36. Odegard, G. M., Haut Donahue, T. L., Morrow, D. A. & Kaufman, K. R. (2008) Constitutive modeling of skeletal muscle tissue. Journal of Biomechanical Engineering, 130 (6), 610-617.

37. Otten, E. (1987) A myocybernetic model of the jaw system of the rat. Journal of neuroscience methods, 21, 287–302.

38. Van Leeuwen, J. L. & Kier, W. M. (1997) Functional design of tentacles in squid: linking sarcomere ultra-structure to gross morphological dynamics. Philosophical Transactions of the Royal Society B, 352, 551–571.

39. Martins, J. A. C., Pato, M. P. M. & Pires, E. B. (2006) A finite element model of skeletal muscles. *Virtual and Physical Prototyping,* 1, 159–170.

40. Johansson, T., Meier, P. & Blickhan, R. (2000) A finite-element model for the mechanical analysis of skeletal muscles. Journal of Theoretical Biology, 206, 131–149.

41. Kojic, M., Mijailovic, S., Zdravkovic, N. (1998) Modelling of muscle behaviour by the finite element method using Hill's three-element model. *International Journal for Numerical Methods Engineering*, 43, 941–953.