Quantum Multi-User Detection Based on Coherent State Signals

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Abstract: Multi-user detection is one of the important technical problems for modern communications. In the field of quantum communication, the multi-access channel on which we apply the technology of quantum information processing is still an open question. In this work, we investigate the multi-user detection problem based on the binary coherent-state signals whose communication way is supposed to be seen as a quantum channel. A binary phase shift keying model of this multi-access channel is studied and a novel method of quantum detection proposed according to the conclusion of the quantum measurement theory. As a result, the average interference between deferent users is presented and the average error probability of the quantum detection is derived theoretically. Finally, we show the maximum channel capacity of this effective detection for a two-access quantum channel.

Keywords: Multi-user detection, multi-access channels, quantum communication, quantum information processing.

1 Introduction

For optical communication, some quantum resources, such as entangled states and squeezed states, cannot maintain long-distance quantum properties due to large-scale diffraction loss in space. However, the coherent state of light field produced by conventional laser emitters can maintain coherence under the loss of diffraction. If it is used as a signal carrier, it will be a better choice [Dixon (1984); Flikkema (1997); Simon, Omura and Scholtz (1994); Vilnrotter (2012); Vilnrotter and Lau (2001)]. The classical optical receivers for deep space communications are currently under consideration, which use photon counting or coherent detection to detect coherent state signals, and even extract

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information from single photon on average. Dolinar receiver is introduced in reference [Helstrom (1976)]. Dolinar receiver is the first classical structured receiver. It uses real-time optical feedback and physical measurements. Error probability can theoretically reach Helstrom bound. However, it is pointed out in literature [Lau, Vilnrotter, Dolinar et al. (2006); Cook, Martin and Geremia (2007)] that precise local laser intensity control is very challenging, and it is difficult to achieve the desired results when the data rate is high. Literature [Sasaki and Hirota (1996)] describes a different method proposed by Sasaki and Hiroshima, which does not use optical feedback but achieves quantum optimum bounds by unitary transformation and photon counting. However, the practical implementation of Sasaki-Hiroshita receiver requires multi-photon non-linear optical processing, which also leads to complex receiver structure. According to the recent results of quantum mechanics, the optimal quantum measurement of binary signals can be achieved by partitioning the signals into disjoint parts and then performing the optimal measurement of each segment.

On the other hand, with the development of large-capacity multi-access communication such as CDMA wireless communication, multi-user detection technology has become an effective way to combat multi-access interference (inter-user interference). Verdu took the lead in putting forward multi-user detection model and optimal multi-user detection algorithm [Verdu (1998)]. Because most of the multi-user detection algorithms are NP-hard to solve, the application of multi-user detection technology is facing great difficulties. Subsequently, a series of novel quantum methods have been proposed [Liu, Wang, Yuan et al. (2016); Liu, Gao, Yu et al. (2018); Liu, Xu, Yang et al. (2018); Liu, Chen, Liu et al. (2018); Qu, Wu, Wang et al. (2017); Qu, Zhu, Wang et al. (2018); Qu, Li, Xu et al. (2019); Qu, Cheng and Wang (2019); Wang, Yang and Mousoli (2018)].

Firstly, the detection problem in classical multi-user communication can be solved more quickly by using quantum parallelism. In 2002, S. Imre et al. proposed a method to obtain the optimal solution of quantum multiuser detection in classical multiple access channels through quantum registers and quantum search algorithm [Imre and Bahlzs (2002); Imre and Bahlzs (2002)].

The second is to solve the detection problem of multi-user quantum information in quantum channel. Concha et al. [Concha and Poor (2000)] proposed the analysis method of multi-user detection in quantum channel. Concha et al. [Concha and Poor (2004)] then proposed the modeling method of multi-access quantum channel, and gave the result that the bit error rate of the system is close to the single user boundary. The optical field model of multi-input and multi-output quantum optical systems is presented in Concha et al. [Concha (2001); Concha and Poor (2002)]. Allahverdyan et al. [Allahverdyan and Saakian (1997)] propose a multi-access quantum channel model. The results show that quantum multiuser detection algorithm is superior to classical optimal multiuser detection algorithm under the same conditions.

This paper investigates the multi-user detection in the case of binary coherent-state signals. The channel model is given and the theoretical analysis and performance for the proposed quantum detection method shown.

2 Multi-user detection based on minimum error discrimination (MED) measurement

Theoretically the complex envelope α of a coherent state signal is able to be any value within the complex plane. Therefore here we consider following case: Let $\{\alpha_k | k = 0, 1 \cdots K-1\}$ be a set of 2K symmetric complex values which have different angels but share the same module $|\alpha|$. It can be seen in Fig. 1.

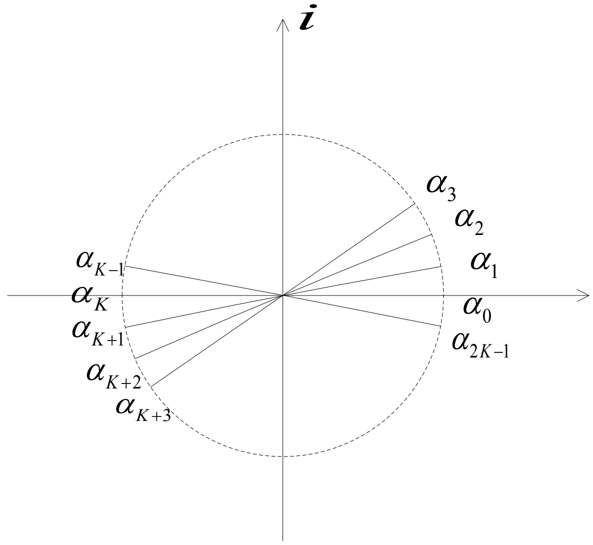

Figure 1: Symmetrical coherent states

In Fig. 1 $\alpha_j = |\alpha| e^{\frac{\pi j i}{K}}$, all the module values of complex numbers $\{\alpha_j\}$ equal $|\alpha|$.

$$\left|\alpha_j\right\rangle = e^{-\frac{1}{2}|\alpha|^2} \sum_{n=0}^{\infty} \frac{\alpha_j^{\ n}}{\sqrt{n!}} |n\rangle, \left(j \in \{0, 1, \cdots 2K-1\}\right) \tag{1}$$

It can be seen from Fig. 1 an arbitrary α has a negative argument other than its central symmetric one. There are totally K pairs of α.

According to the analysis above, we propose a multi address access channel based on the coherent signals. In order to facilitate the unification of formulas and indicators we assume the number of users is from 0 to $K-1$, obviously K users in total。If $2K$ coherent states are assigned to K users the j th user is supposed to have one pair of coherent states $\{|\alpha_j\rangle, |\alpha_{j+K}\rangle\}$. Here $j \in \{0, 1, \cdots K-1\}$ and $|\alpha_{j+K}\rangle = |-\alpha_j\rangle$. Every user deploys BPSK method to send information through coherent states $\{|\alpha_j\rangle, |\alpha_{j+K}\rangle\}$ like Fig. 2 shows.

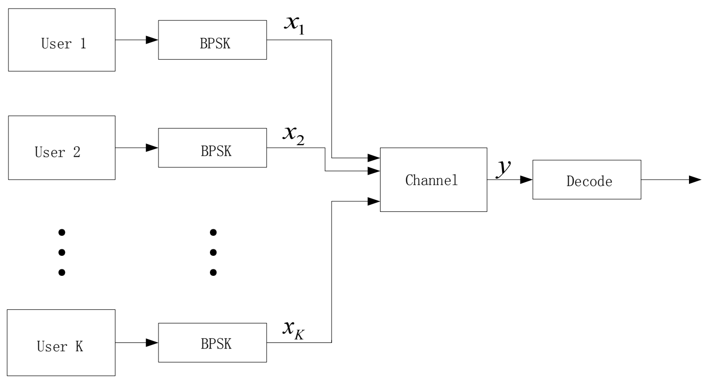

Figure 2: Multi-access channel based on BPSK

The user shown in Fig. 2 has the following priori assumptions for his signal

$$\text{User } j: \begin{cases} H_0^j & |\alpha_j\rangle \\ H_1^j & |\alpha_{j+K}\rangle \end{cases}, \ (j = 0, 1, \cdots, K-1) \tag{2}$$

For an ideal quantum multiple access channel, when the signal states transmitted by different users are orthogonal, there is no inter-user interference. For coherent state multiple access channels, because the coherent states are always non-orthogonal, the inter-user interference will always exist. Therefore, the average inter-user interference can be reflected by the degree of overlap of coherent state signals of different users, i.e.,

$$r = \frac{1}{4K(K-1)} \sum_{\substack{j,j'=0, j'\neq j \\ j'\neq (j+K) \bmod 2K}}^{2K-1} \left|\langle \alpha_{j'} | \alpha_j \rangle\right|^2 = \frac{1}{2(K-1)} \sum_{\substack{j=1 \\ j\neq K}}^{2K-1} \exp\left(-\left| \alpha \left(1 - e^{\frac{\pi j i}{K}} \right) \right|^2 \right) \tag{3}$$

The value of r is from 0 to 1, the closer to 0, the smaller the interference, the closer to 1, the stronger the interference. Fig. 3 shows the curve of average user interference with uniform change of user number from K=2 to K=20.

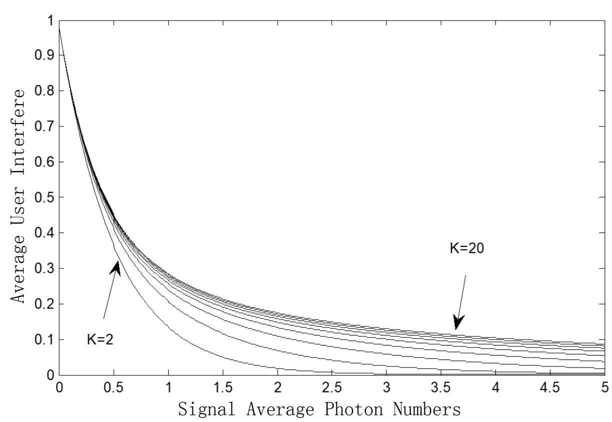

Figure 3: Interference between users

3 Two-user detection based on MED measurement

Equations and mathematical expressions must be inserted into the main text. Two different types of styles can be used for equations and mathematical expressions. They are: in-line style, and display style. Next, we take the two-access channel of two users as the research object. The coherent state two-access channel for two users is shown in Fig. 4.

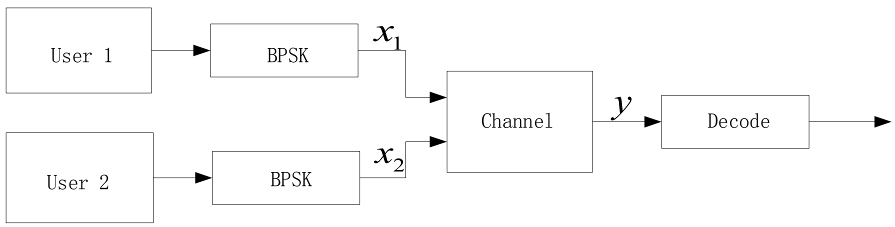

Figure 4: Two users' channel

Two users have a set of coherent state signals modulated by BPSK respectively.

User 1： $$\begin{cases} H_0^0 & |\alpha_0\rangle = |\alpha\rangle \\ H_1^0 & |\alpha_2\rangle = |-\alpha\rangle \end{cases} \quad (4)$$

User 2： $$\begin{cases} H_0^1 & |\alpha_1\rangle = |i\alpha\rangle \\ H_1^1 & |\alpha_3\rangle = |-i\alpha\rangle \end{cases} \quad (5)$$

Based on the analysis of the previous section and the conclusion of MED measurement in reference [Eldar, Megretski and Verghese (2004); Elron and Eldar (2005)], the measurement probability of two-user MED measurement falling on each result can be calculated as follows:

$$P\left(k \mid H_0^0\right) = \frac{1}{4} \times \left(\sum_{m,n=0}^{3} \frac{\exp\left(\left(e^{\frac{\pi m}{2} i} - 1\right)|\alpha|^2 + \frac{\pi m\left(\left(m-k\right) \bmod 4\right)}{2} i\right)}{2\sqrt{1 + e^{-|\alpha|^2 + \left(|\alpha|^2 + \frac{\pi n}{2}\right) i} + e^{-2|\alpha|^2 + \pi n i} + e^{-|\alpha|^2 - \left(|\alpha|^2 - \frac{3\pi n}{2}\right) i}}} \right)^2, \left(k \in \{0,1,2,3\}\right) \quad (6)$$

The above formula represents the signal state $|\alpha\rangle$ sent by user 0 and the probability of the measurement result *k* at the receiving end. Based on the equality and symmetry of the transcendental probability of the transmitted signal, a more general expression of the measurement probability is deduced as follows.

$$P\left(k \mid H_l^j\right) = P\left(\left(k - j - 2l\right) \bmod 4 \mid H_0^0\right) \quad (7)$$

Among them $k \in \{0,1,2,3\}$, $j,l \in \{0,1\}$. Thus, from the upper formula, it can be seen that the detection error probability measured by two-user MED is as follows.

$$P_{\mathrm{e}}^{\mathrm{MED}}=\frac{1}{4}\sum_{j=0}^{1}\sum_{\substack{k=0,k\neq j\\ k\neq j+2}}^{3}\left[P\left(k\mid H_0^j\right)+P\left(k\mid H_1^j\right)\right]=P\left(1\mid H_0^0\right)+P\left(3\mid H_0^0\right) \tag{8}$$

As far as the channel capacity of two-user communication is concerned, the maximum channel capacity of coherent state two-access channel will be obtained theoretically by MED measurement. The maximum channel capacity obtained here is equivalent to the Von Neumann Entropy of coherent state signals. The mixed state of the transmitted signal is

$$\rho=\frac{1}{4}\sum_{j=0}^{3}\left|\alpha_j\right\rangle\left\langle\alpha_j\right| \tag{9}$$

According to Eq. (9), MED measurement can be used. The maximum channel capacity of coherent state two-access channel is

$$C^{\mathrm{MED}}=S\left(\rho\right)=S\left(\frac{1}{4}\sum_{j=0}^{3}\left|\alpha_j\right\rangle\left\langle\alpha_j\right|\right)=\frac{1}{4}\sum_{j=0}^{3}f_j\left(\alpha\right)\left(2-\log\left(f_j\left(\alpha\right)\right)\right). \tag{10}$$

Here

$$f_j\left(\alpha\right)=\left(1+e^{-|\alpha|^2+\left(|\alpha|^2+\frac{\pi j}{2}\right)i}+e^{-2|\alpha|^2+\pi ji}+e^{-|\alpha|^2-\left(|\alpha|^2-\frac{3\pi j}{2}\right)i}\right). \tag{11}$$

4 Conclusion

Figures and tables should be inserted in the text of the manuscript. According to the calculation results of Fig. 3, it depicts the curves of the interference between users whose average photon number of varies uniformly from 0 to 5. It is easy to know that the communication performance gets dramatically better as the average photon number or the users' amount gets larger because of the interference between users decreasing. In order to access the larger channel capacity derived by Eq. (10), the higher average photon number is supposed to be applicable for practical detection even if we just take two users of coherent-state model under consideration.

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