**ARTICLE**

# Optimizing Hybrid Fibre-Reinforced Polymer Bars Design: A Machine Learning Approach

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**ABSTRACT**

Fiber-reinforced polymer (FRP) bars are gaining popularity as an alternative to steel reinforcement due to their advantages such as corrosion resistance and high strength-to-weight ratio. However, FRP has a lower modulus of elasticity compared to steel. Therefore, special attention is required in structural design to address deflection related issues and ensure ductile failure. This research explores the use of machine learning algorithms such as gene expression programming (GEP) to develop a simple and effective equation for predicting the elastic modulus of hybrid fiber-reinforced polymer (HFPR) bars. A comprehensive database of 125 experimental results of HFPR bars was used for training and validation. Statistical parameters such as $R^2$, MAE, RRSE, and RMSE are used to judge the accuracy of the developed model. Also, parametric analysis and SHAP analysis have been carried out to reveal the most influential factors in the predictive model. Finally, the proposed model was compared to the available equations for elastic modulus. The results demonstrate that the developed GEP model performance is better than that of the traditional formula. Statistical parameters and K-fold cross-validation ensured the accuracy and reliability of the predictive model. Finally, the study recommends the optimal GEP model for predicting the elastic modulus of HFRP bars and improving the structural design of HFRP.

**KEYWORDS**

Optimization; fiber-reinforced polymer; corrosion resistance; machine learning; hybrid fiber-reinforced polymer; SHAP analysis

## List of Nomenclature

| | |
|---|---|
| FRP | Fiber Reinforced Polymer |
| HFRP | Hybrid Fiber Reinforced Polymer |
| GEP | Gene Expression Programming |
| RMSE | Root Mean Squared Error |
| MAE | Mean Absolute Error |
| RRSE | Root Relative Squared Error |
| $R^2$ | Coefficient of Determination |

| | |
|---|---|
| CFRP | Carbon Fiber Reinforced Polymer |
| GFRP | Glass Fiber Reinforced Polymer |
| BFRP | Basalt Fiber Reinforced Polymer |
| AFRP | Aramid Fiber Reinforced Polymer |
| ML | Machine Learning |
| ROM | Rule of Mixture |
| RoHM | Rule of Hybrid Mixtures |
| GP | Genetic Programming |
| GF % | Percentage of Glass Fiber |
| E-GF | Elastic Modulus of Glass Fiber |
| CF % | Percentage of Carbon Fiber |
| E-CF | Elastic Modulus of Carbon Fiber |
| BF % | Percentage of Basalt Fiber |
| E-BF | Elastic Modulus of Basalt Fiber |
| MoE | Modulus of Elasticity |
| ET | Expression Tree |
| cos | Cosine |
| sin | Sine |
| ln | Natural Logarithm |
| + | Addition |
| − | Subtraction |
| ∗ | Multiplication |
| / | Division |
| % | Percentage |
| avg | Average |
| kN.m | KiloNewtons.Metre |
| mm | Millimeter |
| MPa | Megapascal |
| GPa | Gigapascal |
| $b_j$ | Bias |
| Ec | FRP Bars Modulus of Elasticity |
| fr | FRP Bars Tensile Strength |
| K-FCV | K-Fold Cross Validation |
| f'c | Concrete Compressive strength |
| $E_{Hybrid}$ | Modulus of Hybrid Material |
| $E_{11,fi}$ | Young's Modulus of Fibers |
| $V_{fi}$ | Volume Fraction of Fibers |
| $E_m$ | Young's Modulus of Matrix |
| $V_m$ | Volume Fraction of Matrix |
| $CO_2$ | Carbon Dioxide |
| °C | Degree Celsius |
| Pi | Predictive Value |
| Ti | Target Value |
| ∞ | Infinity |
| CV | Cross Validation |

## 1 Introduction

Concrete has high brittleness and low tensile capacity [1]. The introduction of waste materials in concrete provides better durability and strength [2] but still needs to improve tensile reinforcement. Steel reinforced are used to improve the tensile capacity of concrete and ductility. However, reinforced concrete structures have faced various challenges, including steel corrosion, freezing damage in cold climates, and exposure to physical and chemical factors in corrosive conditions [3–6]. In humid environments, air pollutants penetrate through the concrete cover and cause steel reinforcement corrosion. The corrosion of steel within the reinforced concrete results in the expansion of steel, leading to internal cracking, surface cracking, spalling, and failure as shown in Fig. 1 [7].
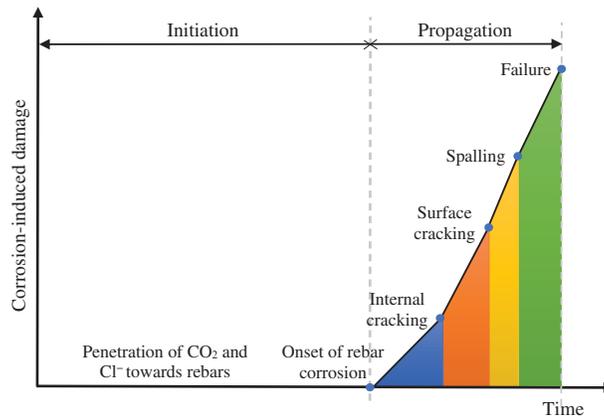


**Figure 1:** Conceptual model of rebar corrosion process [7]

The reinforcements become exposed to direct environmental attack which accelerates corrosion. The corrosion decreased the overall load-bearing capacity of the structures due to decreased bonds between the steel and concrete. The deterioration not only compromises the structural performance but also demands costly repairs and maintenance [8]. A study also noted that the corrosion of steel strands significantly the serviceability of prestressed concrete structures [9]. Similarly, a study [10] noted that fiber rope has many advantages as compared to steel in marine engineering. Therefore, it is necessary to adopt alternative materials as a solution to overcome the corrosion-related problems of traditional reinforcement. The research proposed different ways to decrease the corrosion of steel. A practical solution involves substituting steel bars with non-corrosive reinforcement materials such as fiber-reinforced polymer (FRP) bars.

Fiber-reinforced polymer (FRP) composites are creative and innovative ideas for addressing the growing problem of infrastructure [11]. FRP bars are promising alternative bars to steel reinforcement due to their high corrosion resistance and high strength properties [12]. Furthermore, FRP (carbon fiber-reinforced polymer) also improved seismic performance [13]. Studies show the anti-degradation performance of FRP bars in corrosive environments. Guo et al. [14] studied the degradation of carbon fiber-reinforced polymer (CFRP), glass fiber-reinforced polymer (GFRP), and basalt fiber-reinforced polymer (BFRP) in sea sand concrete. Results showed CFRP has the highest durability, followed by GFRP, BFRP, and steel. Li [15] explored CFRP and GFRP bars as replacements for steel reinforcement in sea sand concrete. The test findings indicated that the addition of CFRP greatly improves the shear capacity of beams without shear reinforcement, exceeding its effect on beams with appropriate shear reinforcement [16]. The study revealed differing corrosion resistance due to resin matrix dissolution in FRP bars. However, FRP bars demonstrate superior corrosion resistance to steel bars. Nevertheless, FRP bars have a lower elastic modulus than steel bars, and their brittleness is typically caused by their linear stress-strain behavior. Abdalla [17] developed simple methods to

estimate deflection in FRP-reinforced concrete members under flexural stress, highlighting the brittle behavior of CFRP-reinforced beams. Ferreira et al. [18] analyzed simply supported concrete beams reinforced with composite rebars, comparing the effects of reinforcement with composite and steel rebars on concrete. A study [19] reported that hybrid fiber-reinforced concrete (HFRC) shows better performance in terms of tensile and flexural capacity. Recently, GFRP bars have been utilized in various structural elements to investigate their behavior. Chaallal et al. [20] investigated the glass-fiber plastic rods as rebars in concrete structures, focusing on rod characterization, bond performance, and flexural behavior in concrete beams. Results indicate satisfactory performance of concrete beams with glass-fiber rebars compared to steel rebars. However, more cracking under moderate to high loading was observed. Numerous investigations on concrete beams reinforced with FRP bars have been carried out [21–24], and it has been found that using FRP bars enhanced the durability of FRP-reinforced structures [25–27]. However, FRP-reinforced structures often exhibit lower serviceability due to the lower modulus of elasticity and reduced ductility of FRP compared to those reinforced with steel bars [28,29].

The types of FRP bars that are commercially available are CFRP, GFRP, BFRP, and aramid (AFRP). Each FRP bar possesses distinct tensile strength, Young's modulus, ductility, and other characteristics depending on the type of fiber used [30]. Certain types of bars are relatively expensive to produce, and other varieties do not meet the strength requirements. The development of hybrid fiber-reinforced polymer (HFRP) bars presents an optimal solution. HFRP bars have shown significant attention due to their potential for enhanced mechanical properties.

Previous researches Hayashi [31], Belarbi et al. [32], and Ali et al. [33] have highlighted the importance of achieving specific characteristics in HFRP bars, particularly ductility under tensile load and variation of the modulus of elasticity through hybrid fiber configurations. However, experimental studies Bakis et al. [34], Cui et al. [35], You et al. [36], and Harris et al. [37] suggest that more efficient and cost-effective methods are required to optimize the design of HFPR bars. Table 1 [38] illustrates a comparison between machine learning (ML) models and traditional methods, focusing on different aspects such as computational cost, working mechanism, and model applicability. Studies [39–41] reported that ML is a cheaper, more efficient, and alternative approach to the traditional approach.

**Table 1:** Comparison between ML and traditional models [38]

| Type | Machine learning | Traditional methods |
|---|---|---|
| Computation cost | Medium | Low |
| Model structure | Complex | Simple |
| Working mechanism | Complex | Simple |
| Model accuracy | High | Low |
| Limitation | Low | High |
| Feasibility | High | Low |

To optimize cost and time efficiency in the construction industry, ML techniques are being increasingly used to precisely predict various properties [42]. A study [43] concluded that the random forest model shows better accuracy than the back propagation neural network (BPNN) ML model. A study [44] applied machine learning approaches, such as adaptive neuro-fuzzy inference systems (ANFIS), artificial neural networks (ANN), and gene expression programming (GEP), to forecast the compressive strength (CS) of steel fiber-reinforced concrete (SFRC) under high temperatures. The findings indicate that the GEP model has high accuracy and the lowest error in comparison to the ANN and ANFIS models. However, a study [45] predicted the split tensile strength of fiber-reinforced recycled aggregate concrete. Five prediction models namely two deep neural network models (DNN1 and DNN2), one optimizable Gaussian process regression (OGPR) model, and two genetic programming-based models (GEP1 and GEP2) are apply for analysis. The results indicate that all the models demonstrated high accuracy in predicting split tensile strength. The DNN2 model shows highest $R^2$ value 0.94, while the GEP1 model shows lowest $R^2$ value 0.76. The $R^2$ value of the DNN2 model was 3.3% and 13.5% greater than the $R^2$ values of the OGPR and GEP2 models, respectively. Therefore, a more detailed study is required on GEP to improve its accuracy.

Traditional regression models show lower computational costs due to their dependence on individual parameters for accuracy. However, several factors like hyperparameter tuning and model structure impact the predictive performance of ML models. These factors increase the computational cost and complexity of work than traditional methods. However, traditional models primarily rely on individual sample fittings, resulting in significantly lower prediction accuracy than ML models. Therefore, this study focuses on utilizing GEP, an advanced machine learning technique, to develop HFRP bars that can fill the gap by providing a systematic approach to design optimization, potentially offering improvements in mechanical performance, cost-effectiveness, and time efficiency compared to traditional experimental methods. The proposed approach reduces the limitations of conventional methods and offers an advanced methodology for predicting mechanical properties which bridge the gap between theory and practice. Integrating machine learning techniques into materials science holds significant potential for future advancements in structural design and engineering optimization.

## 2  Research Methodology

The methodology of this research is described in this section and the steps can be visualized in Fig. 2. Initially, the data was collected from previous published research articles and influential variables were defined. Afterward, the collected data underwent screening to process in which dependent and independent variables statistical analysis is conducted. For the utilization of ML approaches, the data was split into training and testing datasets. Subsequently, GEP algorithms were then employed to develop a highly accurate predictive model, and the best models were selected based on statistical parameters. Validation of the model was carried out. Furthermore, the proposed model was compared with the traditional method to assess robustness and accuracy. The conclusion recommends the most accurate model based on the comprehensive evaluation.
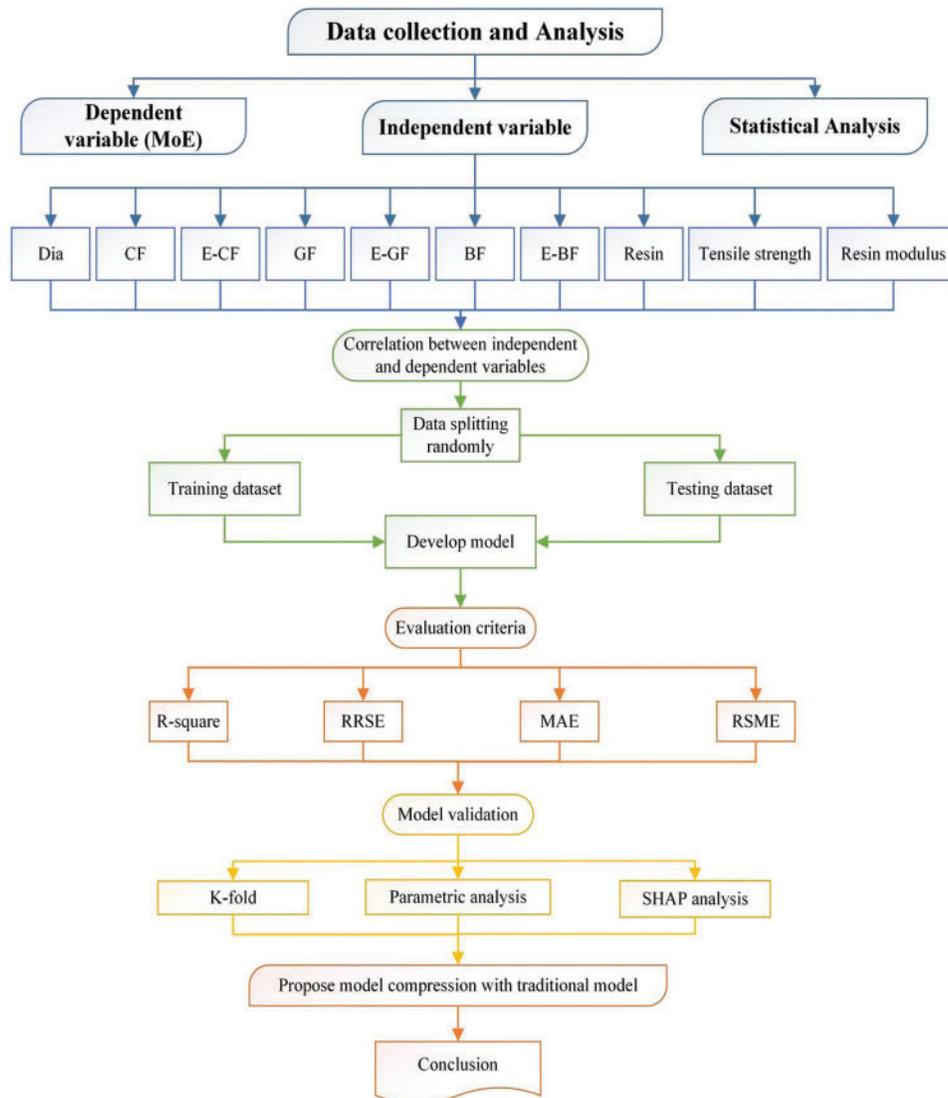
**Figure 2:** Research methodology

## 3  Existing Analytical Approach

There are various micromechanical models available in literature with differing accuracy and complexity including Halpin Tsai and rule of mixture (ROM) models. The ROM models are more often utilized as it is relatively simple and yield acceptable prediction values compared to experimental values. Andersons et al. [46] investigated the strength and stiffness of flax fiber composite under uniaxial tension using thermoset and thermoplastic polymer matrices. The study assesses the suitability of orientational averaging-based models and ROM models, originally developed for short fiber composites, to flax-reinforced polymer. Affdl et al. [47] concluded that the Halpin-Tsai equation is preferred for predicting the accuracy of short random fiber composites. Sandier et al. [48] studied the Cai-Pagano equation, which determines longitudinal and transverse modulus to assess composite stiffness, concluding that mixing rules for equations of state enables accurate predictions. Tham et al. [49] have conducted a detailed study on the ROM models, concluding that they assume homogeneity and neglect variations, interfacial effects, and non-linear behavior, thereby indicating

limitations in accurately predicting composite properties. Eq. (1) presents the mathematical model of ROM used to estimate the mechanical properties of composite materials based on their individual components' properties.

$$E_{Hybrid} = \sum_i E_{11,fi} V_{fi} + E_m V_m \tag{1}$$

where $E_{11}$ is Young's modulus of fibers along the longitudinal axis; $V_{fi}$ is the volume fraction of fibers; $E_m$ is Young's modulus of the matrix and $V_m$ is the volume fraction of the matrix. Table 2 reviews mixing approaches for composite materials, revealing inconsistent predictions with experimental values [50–61]. It emphasizes the need for improved mixing equations and alternative approaches in composite materials research.

## 4  Gene Expression Programming (GEP)

Genetic Algorithms (GA) are optimization and search algorithms inspired by genetics [62–65] Genetic Programming (GP) was developed by Cramer in 1985 and enhanced by Koza [66,67]. In GP, an initial population of programs with terminals and functions is provided. Functions play a crucial role in program excitation and defining terminals [68,69]. GEP introduced by Candida Ferreria mimics living organisms and involves generating complex tree-like structures/models. GEP considers input variable shapes, compositions, and sizes to search for optimal solutions. It leverages computational intelligence and genetic programming principles rooted in Darwin's and Mendel's theories [70,71].

In GEP, a linear chromosome encodes multiple computer programs, offering advantages like accurate predictive models and robust mathematical expressions [72]. In civil engineering, GEP models are increasingly popular for predictive equations, utilizing generated mathematical formulas. GEP has been successfully applied in structural engineering, predicting concrete parameters [73–77] strength characteristics of steel slag [78], prediction of compression strength of geopolymer concrete [79], determining the nominal shear capacity of steel fiber beam [80], etc. Additionally, GEP modeling is widely used for predicting functions, data mining pattern detection, and equation solving [81].

The GA and GEP work mechanism shown in Figs. 3 and 4 involve generating random generation of chromosomes using the Karva language. A typical chromosome or gene in gene expression programming (GEP) consists of two parts: a head and a tail. The head contains function or terminal symbols, while the tail includes only parameters. The number of genes determines the number of sub ETs in the model. Chromosomes have fixed lengths and can be simply transformed into an algebraic expression [82]. Each GEP gene is composed of a fixed length list of terms, with each term derived from a set of functions. These functions can include arithmetic operations (e.g., +, −, x, /), Boolean logic functions (e.g., AND, OR, NOT, etc.), mathematical functions (e.g., cos, sin, natural log), conditional functions (e.g., IF, THEN, ELSE), or others [83].

**Table 2:** Composite material mixing approaches and outcomes in various research studies

| Ref. | Study focus | Research center | Purpose/Application | Composite material | Mixing approach | Outcomes |
|------|-------------|-----------------|---------------------|--------------------|-----------------|----------|
| [50] | Mechanical properties of pineapple leaf fiber-reinforced polypropylene composites | Malaysia Institute for Nuclear Technology Research (MINT). | Enhancing mechanical properties | Pineapple leaf fiber, Polypropylene (2.7, 5.4, 10.8, and 16.2%) | 1. Rule of Mixtures (ROM) <br> 2. $(E_c = E_f V_f + V_m E_m)$ | The experimental data revealed that the measured values were consistently lower than the theoretical predictions. |
| [51] | Interfacial and mechanical properties of environment-friendly "green" composites made from pineapple fibers and poly (hydroxybutyrate-co-valerate) resin | Fibre science program, Cornell University, Ithaca, USA. | Development of environment-friendly composites | Pineapple fiber, Poly (hydroxybutyrate-co-valerate) resin (20 and 30%) | 1. Rule of Mixtures (ROM) <br> 2. $(E_c = E_f V_f + V_m E_m)$ | The results indicate that the calculated theoretical values tend to be higher than the values observed in the experiments. |
| [52] | Effect of temperature on tensile properties of injection molded short glass fiber and glass bead-filled ABS hybrids | London Metropolitan Polymer Centre, London Metropolitan University, UK. | Effect of temperature on tensile properties of injection molded short glass fiber and glass bead-filled ABS hybrids | Chopped E-glass fibers/Spherical glass bead/ABS (5:5:90, 5:15:80, 10:10:80,15:15:70, 20:20:60) | 1. Rule of Mixtures (ROM) <br> 2. $(E_c = E_f V_f + V_m E_m)$ | It was found that the rule-of-mixtures for hybrids provided an excellent description of the experimental results. |
| [53] | Stiffness prediction of hybrid kenaf/glass fiber reinforced polypropylene composites using the rule of mixtures (ROM) and rule of hybrid mixtures (RoHM) | Mechanical and Manufacturing Engineering, University of Putra, Malaysia. | Prediction of the stiffness properties (specifically for hybrid composites) based on the combination of kenaf and glass fibers | Hybrid kenaf/glass fiber reinforced polypropylene composites | 1. Rule of Mixtures (ROM) <br> 2. Rule of Hybrid Mixtures (RoHM) <br> 3. Tsai-Pagano equations | The ROM and Tsai-Pagano equations aligned well with theoretical claims. The RoHM equation results showed a negative hybrid effect on stiffness. |
| [54] | Prediction of tensile properties of hybrid-natural fiber composites | Mechanical Engineering, Anna University, Chennai, India. | Predicting tensile properties of hybrid composites | Banana/sisal fibers (40:0, 30:10, 20:20, 10:30, 0:40) | 1. Rule of Hybrid Mixtures (RoHM) <br> $(E_c = E_f V_f + V_m E_m)$ | RoHM were closer to the experimental values but tended to slightly overestimate the properties of the composite. |

**Table 2 (continued)**

| Ref. | Study focus | Research center | Purpose/Application | Composite material | Mixing approach | Outcomes |
|---|---|---|---|---|---|---|
| [55] | Effects of fiber length on tensile strength of carbon/glass fiber hybrid composites | Engineering Faculty, Gifu University, Yanagido, Japan. | Evaluate the tensile strength of hybrid composites | Carbon fiber/Glass fiber (1.0:0, 0.5:0.5, and 0:1.0) | 1. Rule of Hybrid Mixtures (RoHM) | The experimental results closely match the calculated values, confirming the effectiveness of the hybrid mixtures rule in predicting tensile strength. |
| [56] | Mechanical properties of polypropylene matrix composites reinforced with natural fibers | Material Engineering Centre, University of Perugia, Italy. | Enhancing mechanical properties of polypropylene composites | Linum usitatissimum fiber/Polypropylene matrix | 1. Halpin-Tsai <br> 2. Tsai Pagano <br> 3. Rule of Mixtures (ROM) | The theoretical modal values obtained using these equations are slightly greater than the experimental values measured. |
| [57] | Tensile properties of wood flour/kenaf fiber polypropylene hybrid composites | The Research Forest of the Natural, Resources Faculty, University of Tehran, Iran. | Evaluating tensile properties of the hybrid composites | Wood flour/kenaf fiber polypropylene hybrid composites (40:0, 30:10, 20:20, 10:30, 0:40) | 1. Rule of Hybrid Mixtures (RoHM) <br> 2. Halpin-Tsai | The RoHM equation shows a linear trend for its predicted values, and all predicted values are higher than the experimental ones. However, the Halpin-Tsai equation exhibits a different nonlinear trend. |
| [58] | On the elastic modulus of hybrid particle/short fiber/polymer composites | MEEM Department, University of Hong Kong, China. | Study the elastic modulus behavior | Glass fiber/Acrylonitrile butadiene styrene | 1. Laminate Analogy Approach (LAA) <br> 2. Rule of Hybrid Mixtures (RoHM) | Based on the analysis of previous experimental results, the predicted results of the RoHM equation for the elastic modulus consistently underestimated the values, while the predictions of LAA aligned well with the experimental results. |

(Continued)

**Table 2 (continued)**

| Ref. | Study focus | Research center | Purpose/Application | Composite material | Mixing approach | Outcomes |
|------|-------------|-----------------|---------------------|--------------------|-----------------|----------|
| [61] | Predictions of Young's modulus of short inorganic fiber-reinforced polymer composites | Research Division of Green Function Material and Equipment, University of Technology, Guangzhou, China. | To provide valuable insights and predictive tools | Short Glass Fibre/ Polypropylene (2.5%, 5%, 10% and 15%) | 1. Liang's equation | The study indicates that the equation is a reliable tool for estimating the mechanical properties of short inorganic fiber-reinforced polymer composites. |
| [60] | Analysis of the tensile modulus of polypropylene composites reinforced with stone groundwood fibers | Chemical Engineering, University of Girona, Spain (Polypropylene: Eastman Chemical Products, Spain, and Stone groundwood fiber: Vetrotex, Chambery Cedex, France). | To analyze the effect of fiber content on tensile modulus | Stone groundwood fibers/ Polypropylene (20%, 30%, 40% and 50%) | 1. Cox-Krenchel model 2. Halpin-Tsai model 3. Tsai-Pagano equation | The Tsai-Pagano equation provides stiffness predictions that are more closely aligned with the experimental values compared to the Cox-Krenchel model and Halpin-Tsai model. |
| [59] | Predicting the elastic modulus of natural fiber-reinforced thermoplastics | Department of Chemical Engineering and Applied Chemistry, University of Toronto, Canada. | To predict the elastic modulus of the composite material | Hemp fibers, Hardwood fibers, rice hulls, and E-glass fibers/ High-density polyethylene (10%, 20%, 30%, 40%, 50%, and 60%) | 1. Rule of Mixtures 2. Halpin-Tsai Model 3. Mori-Tanaka Model 4. Shear-lag mode | ROM and IROM equations: These equations offer reasonable stiffness prediction bounds. Halpin-Tsai equation: Effective for composites reinforced with glass fibers, but not suitable for composites containing natural fibers. Shear-lag equations by Nairn and Mendel: Consistently overestimate tensile modulus and inadequately represent composites with staggered fiber distribution in a matrix. |
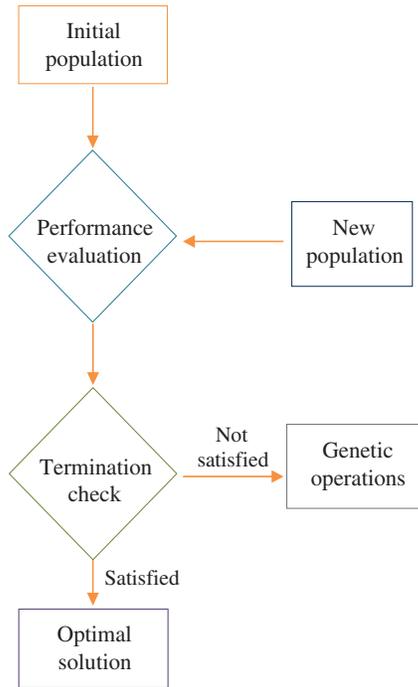
**Figure 3:** GA flowchart
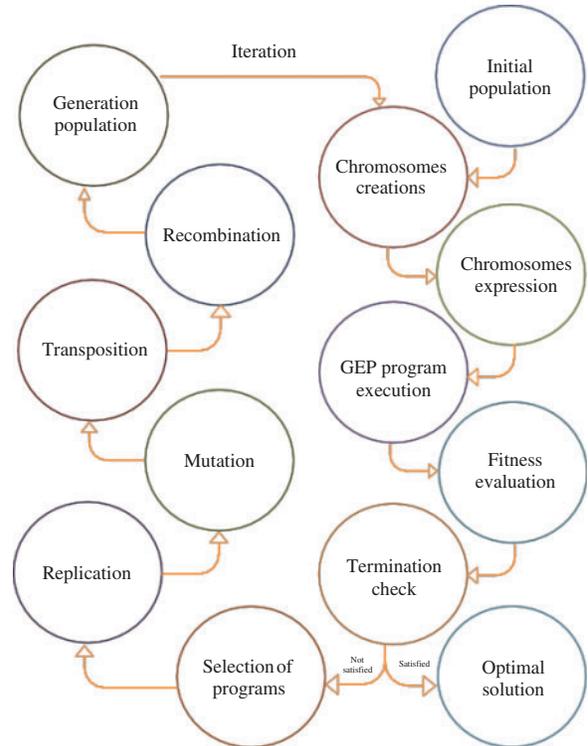


**Figure 4:** Algorithm of GEP

Afterward, the chromosomes are represented by expression trees (ETs) that come in various shapes and sizes. Then, the main genetic operators such as crossover, mutation, transposition and recombination (including 1-point, 2-point, and gene recombination) are executed on the chromosomes based on their ratios [84]. The ETs are then expressed using karva notation or K-expression [85]. The entire process concludes once the stopping condition (highest number of generations or suitable solution) is met.

## 5 Data Acquisition and Model Development

### 5.1 Data Acquisition

In this research, a dataset of 125 values is considered for the GEP and ROM prediction model for HFRP elastic modulus. The data section 1–11 is acquired from [70], data section 12–23 [86], data section 24–25 [87], data section 26–38 [88], data section 39–46 [89], data section 47 [90], data section 48–65 [91], data section 66–106 [92], data section 107–112 [93], data section 113–118 [94], and data section 119–125 [95]. In the current study, 100 data points for training and 25 data points for testing were considered for the generation of the GEP model. Moreover, ten input variables and one dependent variable were taken, i.e., the Diameter of the bar (Dia of the bar), percentage of glass fiber (GF), the elastic modulus of glass fiber (E-GF), percentage of carbon fiber (CF), the elastic modulus of carbon fiber (E-CF), percentage of basalt fiber (BF), elastic modulus of basalt fiber (E-BF), percentage of resin (Resin), tensile strength of resin elastic and modulus of resin (Resin Modulus) against HFRP elastic modulus. The statistical analysis and frequency distribution of the database are shown in Table 3 and Fig. 5, respectively. These are highly dependent variables for governing the elastic modulus of HFRP bars which are collected from previous studies.

**Table 3:** Statistical analysis of the database

| Parameters | Dia | GF | E-GF | CF | E-CF | BF | E-BF | Resin | Tensile strength | Resin modulus | MoE |
|---|---|---|---|---|---|---|---|---|---|---|---|
| **Compiled database** | | | | | | | | | | | |
| Unit | (mm) | (%) | (GPa) | (%) | (GPa) | (%) | (GPa) | (%) | (MPa) | (GPa) | (GPa) |
| Mean | 10.45 | 40.99 | 43.04 | 8.55 | 103.11 | 15.84 | 20.08 | 34.62 | 75.04 | 3.27 | 62.87 |
| Standard error | 0.28 | 2.6 | 2.9 | 1.57 | 13.29 | 2.67 | 3.36 | 0.85 | 0.59 | 0.02 | 2.5 |
| Median | 9.5 | 54.01 | 45 | 0 | 0 | 0 | 0 | 35 | 75 | 3.35 | 49.6 |
| Mode | 8 | 0 | 45 | 0 | 0 | 0 | 0 | 40 | 75 | 3.35 | 43.87 |
| Standard deviation | 3.08 | 29.06 | 32.38 | 17.6 | 148.64 | 29.87 | 37.53 | 9.49 | 6.64 | 0.2 | 27.97 |
| Kurtosis | −0.6 | −1.4 | −1.45 | 5.21 | −0.92 | 0.07 | −0.2 | −0.47 | 1.15 | −1.02 | 3.48 |
| Skewness | 0.67 | −0.54 | −0.24 | 2.48 | 0.9 | 1.4 | 1.34 | 0.07 | 1.06 | −0.11 | 1.99 |
| Range | 12 | 75 | 81 | 65 | 377 | 80 | 92 | 41 | 28 | 0.7 | 110.98 |
| Minimum | 6 | 0 | 0 | 0 | 0 | 0 | 0 | 20 | 62 | 3 | 39.42 |
| Maximum | 18 | 75 | 81 | 65 | 377 | 80 | 92 | 61 | 90 | 3.7 | 150.4 |
| Count | 125 | 125 | 125 | 125 | 125 | 125 | 125 | 125 | 125 | 125 | 125 |
| **Training database** | | | | | | | | | | | |
| Mean | 10.54 | 41.14 | 43.66 | 8.48 | 101.87 | 15.58 | 19.71 | 34.8 | 75.28 | 3.27 | 62.77 |
| Standard error | 0.31 | 2.91 | 3.26 | 1.76 | 14.83 | 2.98 | 3.73 | 0.97 | 0.69 | 0.02 | 2.79 |
| Median | 9.5 | 53.67 | 45 | 0 | 0 | 0 | 0 | 35 | 75 | 3.35 | 49.57 |
| Mode | 8 | 0 | 0 | 0 | 0 | 0 | 0 | 40 | 75 | 3.35 | 66.5 |
| Standard deviation | 3.14 | 29.1 | 32.6 | 17.56 | 148.28 | 29.76 | 37.31 | 9.69 | 6.85 | 0.21 | 27.89 |
| Kurtosis | −0.66 | −1.4 | −1.46 | 5.23 | −0.88 | 0.17 | −0.11 | −0.39 | 0.83 | −0.97 | 3.51 |
| Skewness | 0.61 | −0.55 | −0.28 | 2.47 | 0.92 | 1.44 | 1.37 | 0.13 | 1 | −0.07 | 1.99 |
| Range | 12 | 75 | 81 | 65 | 377 | 80 | 92 | 41 | 28 | 0.7 | 110.98 |
| Minimum | 6 | 0 | 0 | 0 | 0 | 0 | 0 | 20 | 62 | 3 | 39.42 |
| Maximum | 18 | 75 | 81 | 65 | 377 | 80 | 92 | 61 | 90 | 3.7 | 150.4 |
| Count | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 |
| **Testing database** | | | | | | | | | | | |
| Mean | 10.09 | 40.41 | 40.56 | 8.8 | 108.08 | 16.88 | 21.56 | 33.92 | 74.08 | 3.25 | 63.26 |
| Standard error | 0.57 | 5.91 | 6.4 | 3.62 | 30.61 | 6.18 | 7.83 | 1.76 | 1.15 | 0.04 | 5.76 |
| Median | 9.5 | 54.01 | 45 | 0 | 0 | 0 | 0 | 35 | 75 | 3.35 | 49.63 |
| Mode | 8 | 0 | 45 | 0 | 0 | 0 | 0 | 35 | 75 | 3.35 | N/A |
| Standard deviation | 2.85 | 29.53 | 31.99 | 18.11 | 153.03 | 30.91 | 39.16 | 8.8 | 5.77 | 0.19 | 28.82 |
| Kurtosis | −0.11 | −1.48 | −1.39 | 6.59 | −1.01 | −0.08 | −0.35 | −1.2 | 3.95 | −1.58 | 4.39 |
| Skewness | 0.95 | −0.56 | −0.12 | 2.65 | 0.87 | 1.36 | 1.3 | −0.33 | 1.36 | −0.38 | 2.11 |
| Range | 9.9 | 75 | 81 | 65 | 377 | 80 | 92 | 29 | 28 | 0.5 | 107.33 |
| Minimum | 6 | 0 | 0 | 0 | 0 | 0 | 0 | 20 | 62 | 3 | 40.69 |
| Maximum | 15.9 | 75 | 81 | 65 | 377 | 80 | 92 | 49 | 90 | 3.5 | 148.02 |
| Count | 25 | 25 | 25 | 25 | 25 | 25 | 25 | 25 | 25 | 25 | 25 |

**Figure 5:** (Continued)

**Figure 5:** Frequency distribution

Moreover, the study explores the relationships between various variables, as shown in Fig. 6. The results indicate that "GF%" has a positive correlation with "Bar Dia" and "E-GF" (correlation coefficients of 0.301 and 0.311, respectively). This suggests that an increase in glass fiber percentage or effective glass fiber is connected to a larger bar diameter. Additionally, the correlation coefficients

reveal connections among resin properties. Specifically, "Resin%" exhibits a positive correlation with "Resin Tensile Strength" (correlation coefficient: 0.0327) and "Resin Modulus" (correlation coefficient: 0.0688). These correlations indicate that an increase in resin percentage is linked to higher tensile strength and modulus.

|  | Diameter of bar | GF (%) | E GF | CF (%) | E CF | BF (%) | E BF | Resin (%) | Tensile strength | Resin Modulus | Modulus (GPa) |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Diameter of bar | 1.00 | 0.30 | 0.31 | -0.18 | -0.31 | -0.20 | -0.20 | 0.03 | -0.02 | 0.37 | -0.28 |
| GF (%) | 0.30 | 1.00 | 0.93 | -0.48 | -0.15 | -0.75 | -0.75 | 0.20 | -0.18 | -0.25 | -0.35 |
| E GF | 0.31 | 0.93 | 1.00 | -0.43 | 0.02 | -0.79 | -0.78 | 0.42 | -0.20 | -0.19 | -0.30 |
| CF (%) | -0.18 | -0.48 | -0.43 | 1.00 | 0.50 | -0.15 | -0.12 | 0.08 | 0.50 | 0.34 | 0.93 |
| E CF | -0.31 | -0.15 | 0.02 | 0.50 | 1.00 | -0.21 | -0.17 | 0.18 | -0.06 | -0.23 | 0.57 |
| BF (%) | -0.20 | -0.75 | -0.79 | -0.15 | -0.21 | 1.00 | 0.97 | -0.57 | -0.13 | 0.02 | -0.26 |
| E BF | -0.20 | -0.75 | -0.78 | -0.12 | -0.17 | 0.97 | 1.00 | -0.53 | -0.09 | 0.10 | -0.23 |
| Resin (%) | 0.03 | 0.20 | 0.42 | 0.08 | 0.18 | -0.57 | -0.53 | 1.00 | 0.03 | 0.07 | 0.14 |
| Tensile strength | -0.02 | -0.18 | -0.20 | 0.50 | -0.06 | -0.13 | -0.09 | 0.03 | 1.00 | 0.69 | 0.51 |
| Resin Modulus | 0.37 | -0.25 | -0.19 | 0.34 | -0.23 | 0.02 | 0.10 | 0.07 | 0.69 | 1.00 | 0.23 |
| Modulus (GPa) | -0.28 | -0.35 | -0.30 | 0.93 | 0.57 | -0.26 | -0.23 | 0.14 | 0.51 | 0.23 | 1.00 |

**Figure 6:** Pearson correlation matrix of the dataset

### 5.2 Model Development

GeneXproTool 5.0, a flexible GEP data modeling software, was used to predict the HFRP modulus of elasticity. This tool allows for the adjustment of various parameters, facilitating the creation of diverse GEP models with different characteristics. Table 4 presents a comprehensive GEP prediction model for HFRP elastic modulus, each configuration corresponds to a unique GEP setup identified by a code. For instance, "C25-G1-Addition" shows 25 chromosomes, 1 gene per chromosome, and addition as the linking function. The chromosome count remains consistent at 25, 50, 75, and 100 in each configuration. Additionally, all configurations have 8 genes per chromosome, influencing the overall model behavior. The linking function determines gene combinations within chromosomes, denoted by symbols such as "+", "−", "∗", "/", or "Average," representing addition, subtraction, multiplication, division, or averaging of input variables.

**Table 4:** GEP model development details

| GEP model | Model details | | | |
|---|---|---|---|---|
|  | Chromosomes | Head size | Genes | Linking function |
| C25-G1-Addition | 25 | 8 | 1 | + |
| C25-G1-Average | 25 | 8 | 1 | Average |

(Continued)

**Table 4 (continued)**

| GEP model | Model details | | | |
|---|---|---|---|---|
| | Chromosomes | Head size | Genes | Linking function |
| C25-G1-Division | 25 | 8 | 1 | / |
| C25-G1-Multiplication | 25 | 8 | 1 | ∗ |
| C25-G1-Subtraction | 25 | 8 | 1 | − |
| C50-G1-Addition | 50 | 8 | 1 | + |
| C50-G1-Average | 50 | 8 | 1 | Average |
| C50-G1-Division | 50 | 8 | 1 | / |
| C50-G1-Multiplication | 50 | 8 | 1 | ∗ |
| C50-G1-Subtraction | 50 | 8 | 1 | − |
| C75-G1-Addition | 75 | 8 | 1 | + |
| C75-G1-Average | 75 | 8 | 1 | Average |
| C75-G1-Division | 75 | 8 | 1 | / |
| C75-G1-Multiplication | 75 | 8 | 1 | ∗ |
| C75-G1-Subtraction | 75 | 8 | 1 | − |
| C100-G1-Addition | 100 | 8 | 1 | + |
| C100-G1-Average | 100 | 8 | 1 | Average |
| C100-G1-Division | 100 | 8 | 1 | / |
| C100-G1-Multiplication | 100 | 8 | 1 | ∗ |
| C100-G1-Subtraction | 100 | 8 | 1 | − |

## 6 Model Evaluation Criteria

### 6.1 Statistical Metrics

The statistical performance of the proposed GEP models to predict MOE was measured using four analytical standard measures including the coefficient of determination ($R^2$), root relative squared error ($RRSE$), Root mean squared error ($RMSE$) and mean absolute error ($MAE$) [96–98] The equation of each fitness measure is shown in Table 5. The best-fitted predictive model is usually based on the parameters. $R^2$ represents the proportion of the variance in the dependent variable (predicted from the independent variables). Therefore, the fitness of a regression model measures how well the model fits the observed data points [99,100]. RRSE measures the standard deviation of the errors in a prediction model, relative to the range of the target variable. Therefore, the average difference between the predicted and actual values is normalized by the range of the target variable [96]. RMSE measures the average magnitude of the errors in a prediction model. It represents the squared differences between the predicted and actual values [101–103]. MAE measures the average magnitude of the errors in a prediction model. It represents the absolute differences between the predicted and actual values [104].

**Table 5:** Models evaluation

| Equation | Preferred fitness | Range |
|---|---|---|
| $RRSE = \sqrt{\dfrac{\sum_{i=1}^{m}(P_i - T_i)^2}{\sum_{i=1}^{m}(P_i - \bar{T})^2}}$ | A lower value indicates better accuracy | 0–∞ |

(Continued)

**Table 5 (continued)**

| Equation | Preferred fitness | Range |
|---|---|---|
| $RMSE = \sqrt{\dfrac{\sum_{i=1}^{m}(P_i - T_i)}{m}}$ | A lower value indicates better accuracy | 0–∞ |
| $MAE = \dfrac{1}{m}\sum_{i=1}^{m}|P_i - T_i|$ | A lower value indicates better accuracy | 0–∞ |
| $R^2 = 1 - \dfrac{\sum_{i=1}^{n}(P_i - T_i)^2}{\sum_{i=1}^{n}(P_i - T_i)}$ | A higher value indicates a better fit | 0–1 |

Note: $P_i$ is the predictive value and $T_i$ is the targeting value.

### 6.2 K-Fold Cross-Validation

To assess the machine learning model's flexibility and performance, the Cross-Validation (CV) technique is used. Various CV techniques, such as the Monte Carlo test, Jack Knife test, bootstrapping, three-way split cross-validation, and disjoint sets test, are utilized [105]. However, among these techniques, K-fold cross-validation (CV) is the most unbiased approach to address sampling and overfitting concerns.

The dataset is divided into "K" equal subsets to assess the model's accuracy. The K-fold CV methodology creates K-1 subsets for training and one subset for testing to validate the training dataset [106]. This process is repeated K times with different data samples for training and testing. The best model is chosen based on approximation statistics for other errors from all the datasets. K-fold CV ensures complete dataset validation to obtain the most optimal model. The following steps are followed in K-fold validation: The dataset is divided into "K" equal subsets, where K-1 subsets are used for training, and one subset is reserved for testing and validation. The best model is developed from the K-1 subsets and validated on the remaining subset, considering all training subsets. The complete flow diagram of the K-fold CV algorithm is shown in Fig. 7.
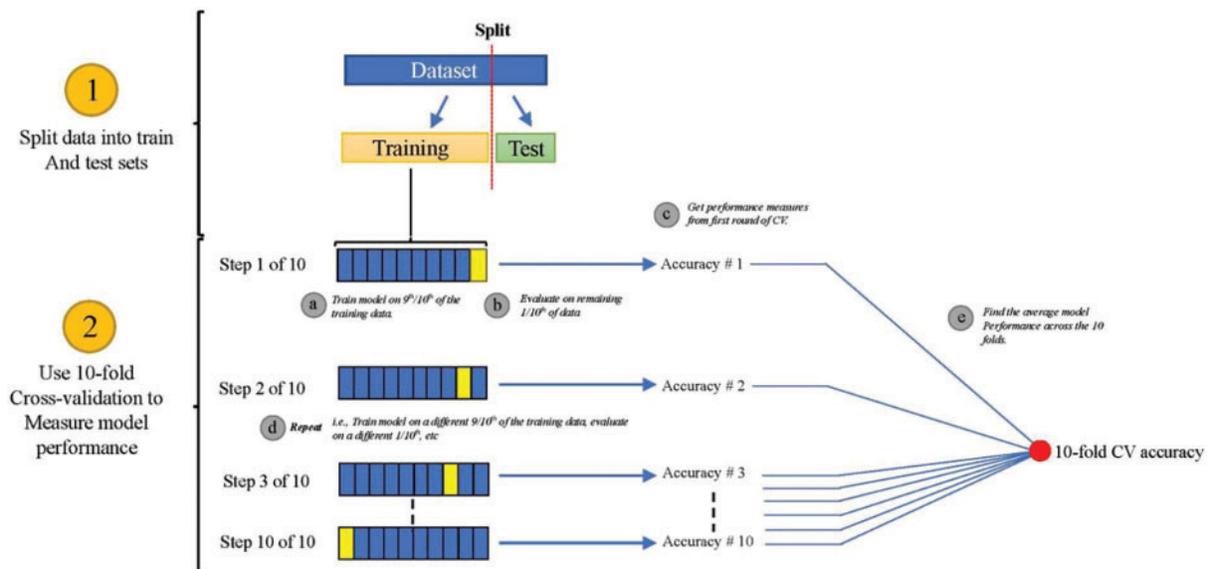


**Figure 7:** K-fold cross-validation flow diagram

## 7  Results and Discussion

Table 6 shows the different GEP models using various evaluation metrics for training and testing. The $R^2$, RMSE, MAE, and RRSE values for each model are used for accuracy. The "C100-G1-Addition" model demonstrates better performance than models with an $R^2$ value of 0.96 in training and 0.96 in testing. Therefore a strong correlation exists between predicted and actual values. The model shows the lowest RMSE of 5.59, indicating accurate predictions with minimal error. The MAE value of 3.87 also supports the models accuracy (average magnitude of errors). Finally, the RRSE value of 0.19 shows that the models performance is better than others, reflecting the normalized error relative to the target variables.

**Table 6:** GEP model results

| GEP Model | Training | | | | Testing | | | |
|---|---|---|---|---|---|---|---|---|
| | $R^2$ | RMSE | MAE | RRSE | $R^2$ | RMSE | MAE | RRSE |
| C25-G1-Addition | 0.8978 | 8.872 | 6.304 | 0.3197 | 0.9604 | 5.684 | 4.142 | 0.2013 |
| C25-G1-Average | 0.8778 | 9.796 | 6.581 | 0.3529 | 0.9332 | 7.455 | 4.898 | 0.264 |
| C25-G1-Division | 0.8837 | 9.466 | 6.647 | 0.3411 | 0.9445 | 6.731 | 4.791 | 0.2384 |
| C25-G1-Multiplication | 0.8999 | 8.783 | 6.256 | 0.3164 | 0.9548 | 6.039 | 4.403 | 0.2138 |
| C25-G1-Subtraction | 0.8921 | 9.117 | 6.826 | 0.3285 | 0.9404 | 7.101 | 5.267 | 0.2514 |
| C50-G1-Addition | 0.8977 | 8.877 | 6.346 | 0.3198 | 0.9665 | 5.315 | 3.988 | 0.1882 |
| C50-G1-Average | 0.9003 | 8.764 | 6.309 | 0.3158 | 0.9585 | 5.846 | 4.373 | 0.2071 |
| C50-G1-Division | 0.8782 | 9.801 | 6.478 | 0.3531 | 0.937 | 7.273 | 4.731 | 0.2575 |
| C50-G1-Multiplication | 0.8884 | 9.272 | 6.982 | 0.3341 | 0.9456 | 6.655 | 5.423 | 0.2356 |
| C50-G1-Subtraction | 0.8899 | 9.211 | 6.443 | 0.3319 | 0.9527 | 6.193 | 4.336 | 0.2193 |
| C75-G1-Addition | 0.9053 | 8.539 | 6.016 | 0.3077 | 0.9605 | 5.722 | 4.206 | 0.2026 |
| C75-G1-Average | 0.8922 | 9.113 | 6.487 | 0.3283 | 0.9683 | 5.176 | 3.989 | 0.1833 |
| C75-G1-Division | 0.8954 | 8.977 | 6.403 | 0.3234 | 0.9511 | 6.282 | 4.866 | 0.2225 |
| C75-G1-Multiplication | 0.8803 | 9.601 | 6.899 | 0.3459 | 0.9371 | 7.217 | 5.618 | 0.2556 |
| C75-G1-Subtraction | 0.8977 | 8.879 | 6.299 | 0.3199 | 0.9443 | 6.725 | 4.451 | 0.2405 |
| C100-G1-Addition | 0.9627 | 5.625 | 3.874 | 0.1934 | 0.9683 | 5.126 | 4.017 | 0.1961 |
| C100-G1-Average | 0.8726 | 9.906 | 6.786 | 0.3569 | 0.9428 | 6.868 | 4.801 | 0.2432 |
| C100-G1-Division | 0.8849 | 9.454 | 6.247 | 0.3405 | 0.9339 | 7.414 | 4.853 | 0.2625 |
| C100-G1-Multiplication | 0.8934 | 9.062 | 6.221 | 0.3265 | 0.9627 | 5.537 | 3.855 | 0.1815 |
| C100-G1-Subtraction | 0.8864 | 9.362 | 6.636 | 0.3373 | 0.9371 | 7.284 | 4.837 | 0.2579 |

### 7.1  Proposed GEP Model

Numerous models were generated by adjusting various parameters in GeneXprotool such as the number of chromosomes, genes, and functions. Statistical parameters such as $R^2$, MAE, RMSE, and RRSE were employed to evaluate the performance of the developed GEP models. The optimal fitness model, "C100-G1-Addition" shows better performance with $R^2$ values of 0.96 during training and 0.96 during testing. Additionally, the GEP regression plot and error distribution diagram are illustrated in Figs. 8 and 9, respectively. The solid lines represent the predicted value, while the markers (e.g., circles and squares) indicate the observed data points. The different colors distinguish between training and testing datasets. In Fig. 9a, the comparison of experimental and predicted values of the modulus of elasticity is illustrated. The blue markers represent the experimental data, while the orange markers

denote the predicted values from the GEP model. Each data point corresponds to a specific dataset, and the lines connecting the markers help visualize the trend and correlation between the experimental and predicted values. Fig. 9b represents the error value (difference between experimental and predicted values) for each dataset.
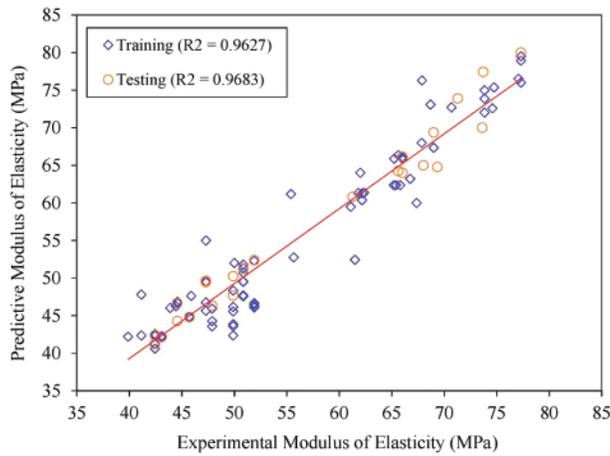


**Figure 8:** Regression plot
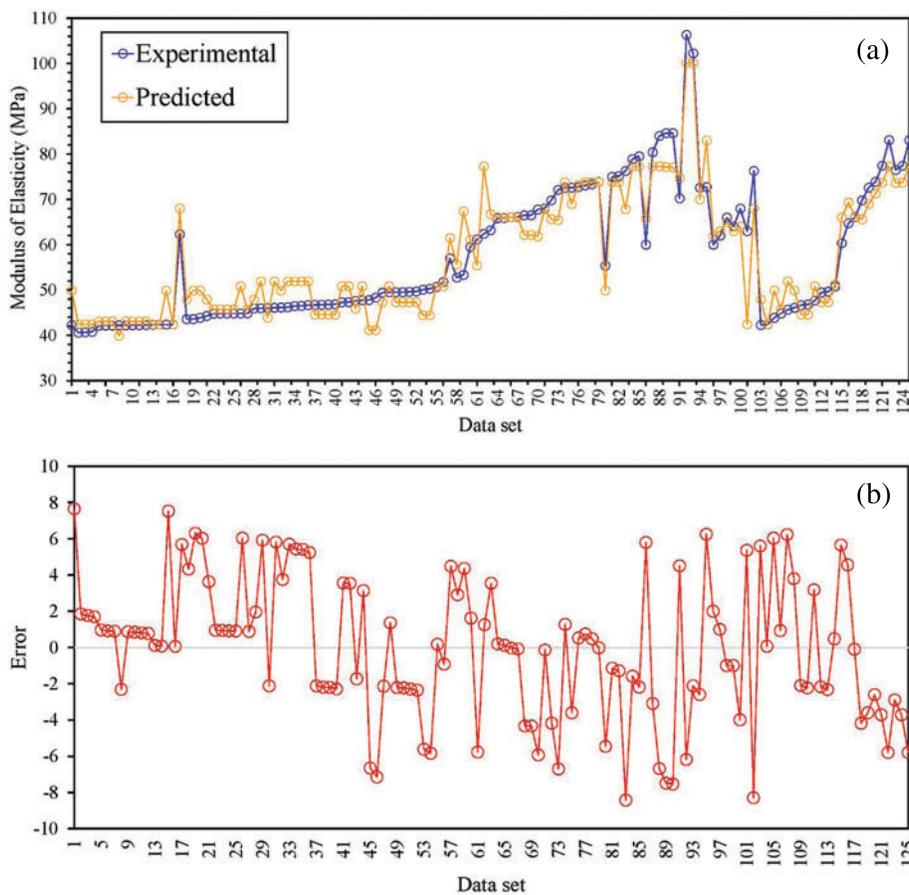


**Figure 9:** (a) Experimental *vs*. Predicted values (b) Error

The expression tree generated by GEP is from the best model "C100-G1-Addition" utilized to formulate the mathematical equation for the predictive model as shown in Fig. 10 and Eq. (2). This equation is for the utilization of a model generated by machine-learning techniques in the real world. By decoding the expression tree, a concise mathematical relationship between various input variables and mathematical operations (e.g., addition, subtraction, multiplication, division) is established. This simplified equation enables the calculation of the modulus of elasticity for HFRP bars.

$$E_{HFRP} = \frac{3BF_\% BF_E + GF_\% GF_E}{(10.6 - d - BF_\% BF_E)(CF_\% CF_E + GF_\% GF_E - 34.77)} + 0.68CF_\% CF_E - d + \frac{0.5}{M_\% M_t M_E} + 57.81$$

(2)
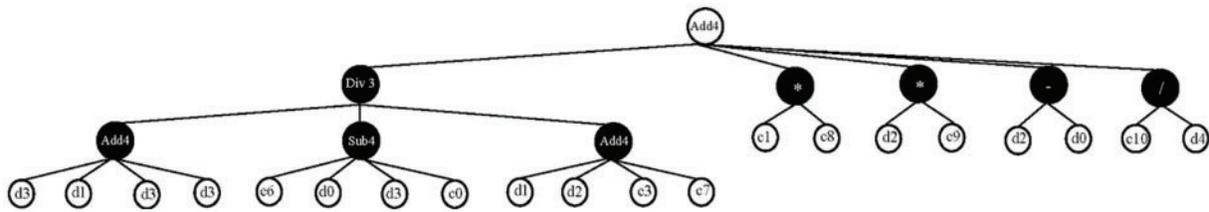


**Figure 10:** Expression tree of model "C100-G1-Addition"

### 7.2 K-Fold Cross-Validation

In this study, a 10-fold cross-validation technique was utilized to assess the performance of a predictive model. Cross-validation is a widely employed method in machine learning and statistical analysis to evaluate a model's generalization capability. Fig. 11 illustrates the performance of the predictive model across the 10-fold cross-validation iterations.
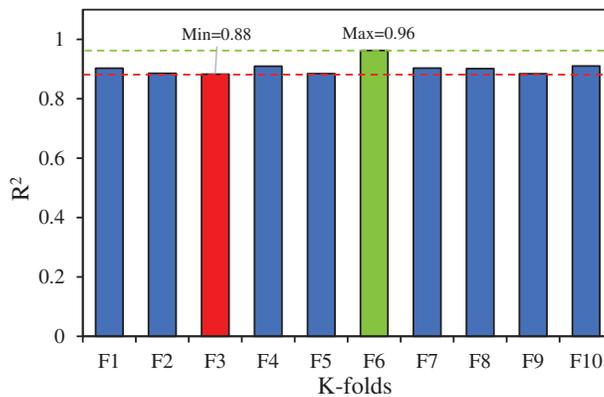


**Figure 11:** GEP model K-fold cross-validation results, $R^2$

## 8 Parametric Analysis

Fig. 12 illustrates the correlations between the elastic modulus of hybrid fiber-reinforced polymer (HFRP) bars and each variable such as bar diameter, carbon fiber percentages, the elastic modulus of carbon fibers, the elastic modulus of basalt fibers, basalt fibers percentages, the elastic modulus of glass fibers, glass fibers percentages, resin matrix percentages, resin modulus, and tensile strength. It can be observed that all the variables positively influence the elastic modulus of HFRP bars. The discovered beneficial impacts of each variable on the elastic modulus of HFRP bars indicate that

changes in each parameter lead to an increase in the elastic modulus of the HFRP bars. Furthermore, the tensile strength more effectively increased the elastic modulus of HFRP bars than the other variables. These findings indicate that increasing the tensile capacity of concrete is a very effective approach to increasing its elastic modulus. Therefore, the study recommends that special attention should be given to the tensile strength to ensure better elastic modulus of HFRP bars. Also, it can be observed that each variable (other than tensile strength) has a significant role in increasing the elastic modulus of HFRP bars. Therefore, optimizing all the considered variables leads to an increase in the elastic modulus of the HFRP bars. These results have implications for carrying out efficient parametric studies and improving the design and manufacturing processes of HFRP bars to fulfill precise structural criteria.
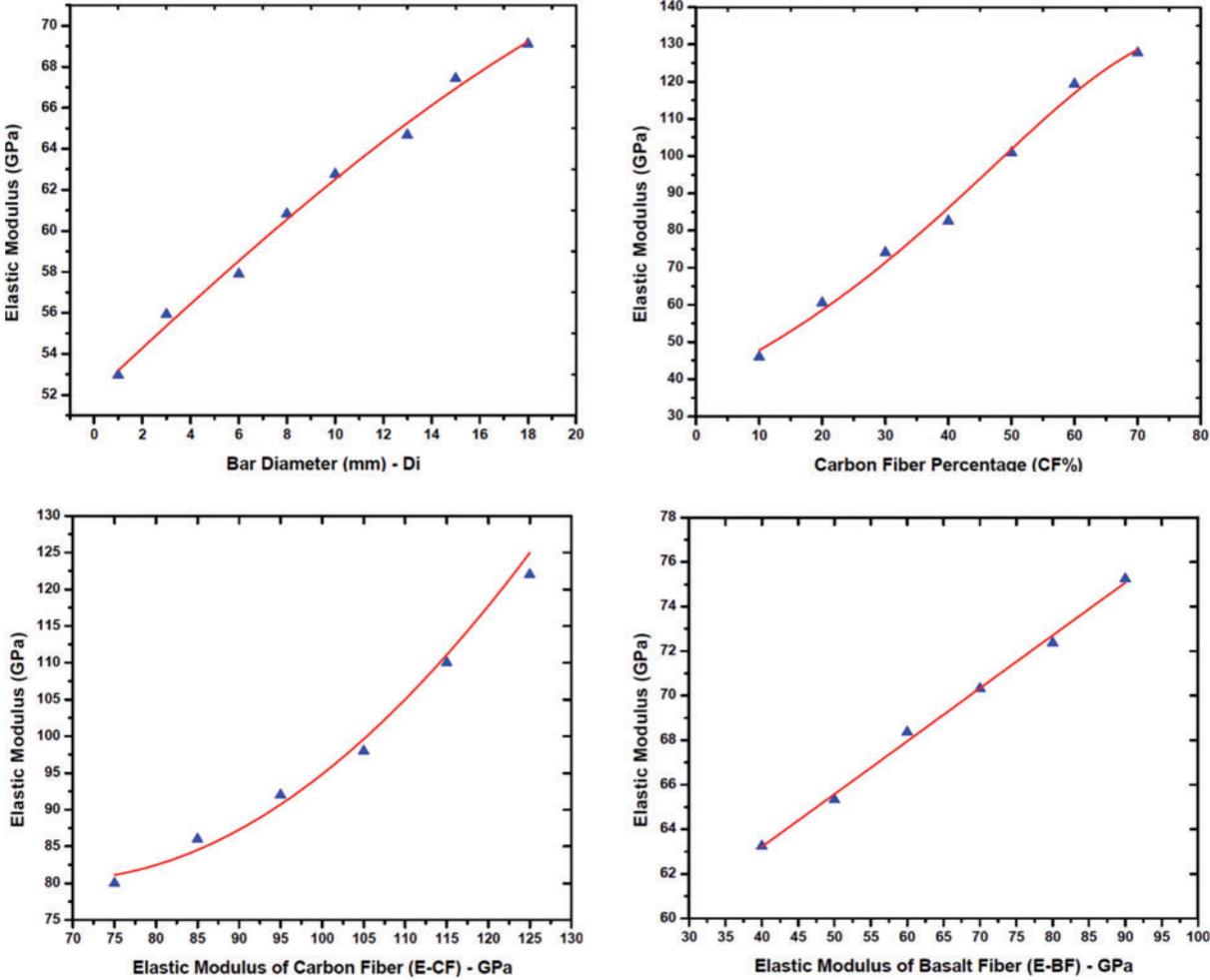


**Figure 12:** (Continued)
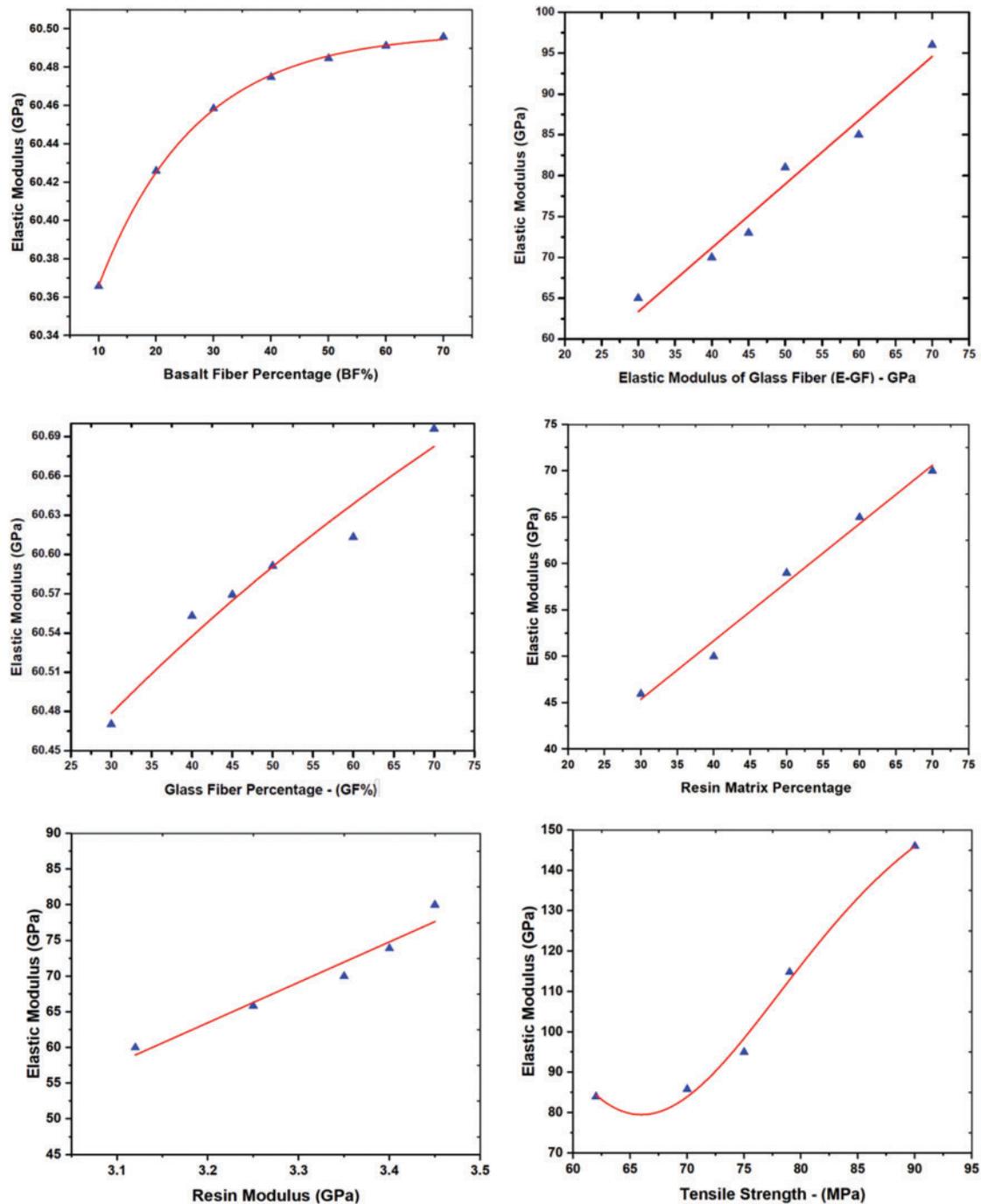
**Figure 12:** Parametric analysis

## 9  SHAP Analysis

The Shapley additive explanation (SHAP) framework provides internal and external explanations for investigating a critical variable. It is particularly suitable for collective machine learning techniques due to its robustness and ability to offer qualitative and quantitative insights, comparable to widely used feature significance metrics. In Fig. 13, prominent attributes significant contributions to the model's output, notably tensile strength, CF%, GF%, bar diameter, elastic modulus GF, resin%,

BF% and elastic modulus CF, resin modulus and elastic modulus BF influencing the prediction of elastic modulus. These influential parameters display distinct thresholds dictating their effects on the model's output. The intensity of each variable is judged through different colors. The red color indicates a highly sensitive variable that significantly impacts the elastic modulus. The SHAP analysis indicates that tensile strength is the most sensitive variable which significantly impacts the elastic modulus. Therefore, the study suggests that special attention should be given to the tensile strength while designing hybrid fiber-reinforced polymer (HFRP) bars.
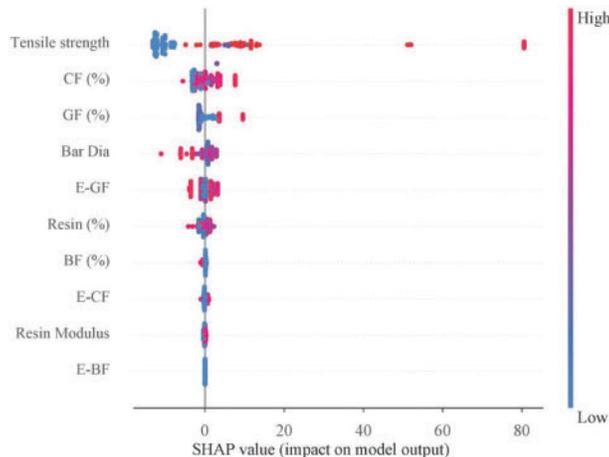


**Figure 13:** SHAP analysis

## 10  Comparative Analysis of GEP and Traditional Prediction Formula

Fig. 14 shows the comparison between the developed models of the GEP and the traditional prediction formula (ROM) available for elastic modulus. The alignment between the GEP model and experimental values is comparable. However considerable variation was noted between the experimental results and ROM value. Therefore, the findings indicate that the developed GEP model captures the patterns and relationships more accurately than the ROM model.
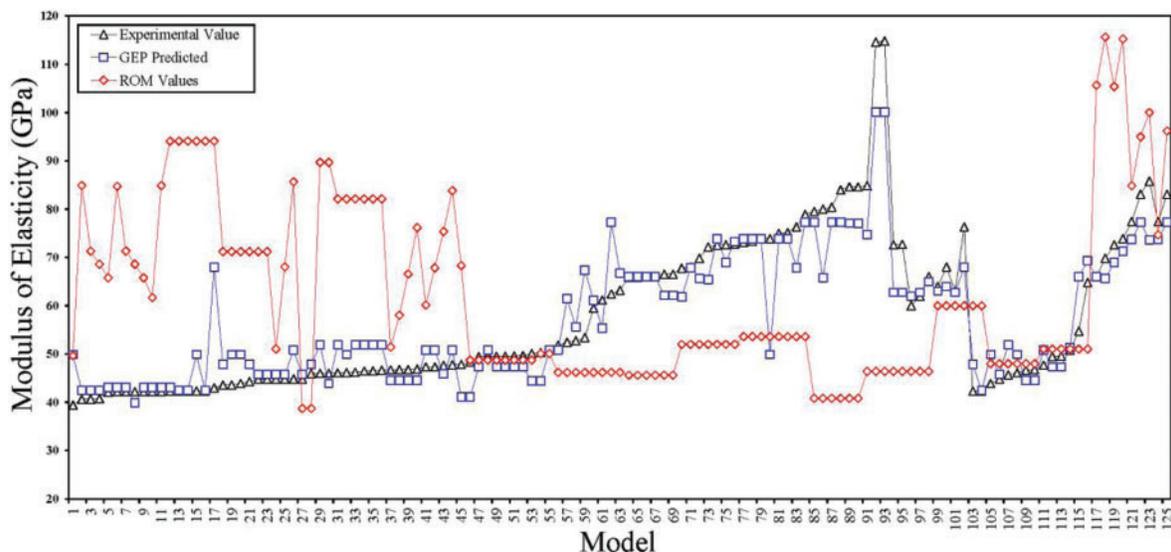


**Figure 14:** Comparison between results

## 11  Conclusion

The application of machine learning was applied to develop a predictive model for the elastic modulus of HFRP bars. A dataset containing 125 experimental results of HFRP bars was collected from the literature and used for model development and validation. The detailed conclusions are:

- The developed GEP model performance is better than the traditional available formula for predicting the elastic modulus of HFRP bars.
- A more accurate model (optimal predictive model) was identified with 100 chromosomes, 1 gene and an addition linking function.
- The statistical parameters such as $R^2$, RMSE, MAE, and RRSE values are 0.96, 5.62, 3.87, and 0.1934, respectively, for training, and 0.96, 5.12, 4.01, and 0.196 for testing, respectively.
- A simplified mathematical equation for predicting the elastic modulus of HFRP bars is developed which offers a more straightforward approach for practical applications.
- Parametric and SHAP analyses identify tensile strength, carbon fiber, and glass fiber were the most significant contributing factors.
- Validation was performed through K-fold cross-validation (CV), demonstrating satisfactory performance across all models and further confirming the accuracy and reliability.

## 12  Limitations and Recommendations

Although the study provides promising results. However, the study has certain limitations and recommends future work to improve performance and accuracy.

- The study focused primarily on the prediction of the elastic modulus of HFRP bars. However, it might be possible that HFRP other properties like tensile strength and flexural behaviour influence overall performance. The study recommends future studies to explore different properties.
- Gene expression programming (GEP) was employed for predictive modelling and accuracy could be increased with the use of advanced algorithms. Therefore, the study recommends some advanced algorithms such as random forest or XGBoost to improve accuracy.
- Although the dataset used in this study is compressive, the variation in testing conditions, material properties, and fabrication methods may still have limitations. Therefore, the study recommends detailed controlled and standardized studies.

**Author Contributions: Aneel Manan:** Software, Data curation, Visualization, Conceptualization, Methodology, Writing–original draft. **Pu Zhang:** Supervision, Methodology, Writing–review and editing. **Shoaib Ahmad:** Investigation, Conceptualization, Formal analysis, Writing–review and editing. **Jawad Ahmad:** Conceptualization, Methodology, Data curation, Visualization, Writing–review and editing. All authors reviewed the results and approved the final version of the manuscript.

**Availability of Data and Materials:** Data will be made available upon request.

**Ethics Approval:** Not applicable.

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