

# **Solving the Cauchy problem of nonlinear steady-state heat conduction equations by using the polynomial expansion method and the exponentially convergent scalar homotopy method (ECSHA)**

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## **Summary**

In this paper, the Cauchy problem of the nonlinear steady-state heat conduction is solved by using the polynomial expansion method and the exponentially convergent scalar homotopy method (ECSHA). The nonlinearity involves the thermal dependent conductivity and mixed boundary conditions having radiation term. Unlike the regular boundary conditions, Cauchy data are given on part of the boundary and a sub-boundary without any information exists in the formulation. We assume that the solution for a two-dimensional problem can be expanded by polynomials as: where  $T$  is the temperature distribution,  $n_p$  is the maximum order of polynomial expansion,  $x$  and  $y$  are Cartesian coordinates,  $R_x$  and  $R_y$  are characteristic lengths in  $x$ - and  $y$ - directions, and  $c_{ij}$  are undetermined coefficients. This expression is substituted into the interior point equations (satisfying governing equations) and boundary point equations (satisfying boundary conditions). The residual of each equation can construct a residual vector  $F$ . Then, the ECSHA which is originated from minimizing the residual norm for the all equations can derive an evolution form as: where  $F$  is the vector containing the equations required to be satisfied,  $B$  denotes as the Jacobian matrix, and are control parameters that make the convergence of ECSHA faster and  $t$  is the fictitious time. Several examples are given to show the validity of the current method. Numerical results show that the proposed method can deal with the ill-posed nature of the Cauchy problem successfully and it can have a very good noise resistance.

