

The collocation multipole method for acoustic problems with elliptical boundaries

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Summary

A semi-analytical approach is presented to solve two-dimensional interior and exterior acoustic problems with elliptical boundaries. The eigenproblems for the interior domain and the scattering problems for the exterior domain are both to be investigated. To satisfy the Helmholtz equation in elliptical coordinates, the multipole expansion for the acoustic pressure is formulated in terms of angular and radial Mathieu functions. The boundary conditions are satisfied by uniformly collocating points on the boundaries. The acoustic pressure at each boundary point is directly calculated in each local elliptical coordinate system. For the multiply connected domain problems, the normal derivative of the acoustic pressure is calculated by using the appropriate directional derivative in different coordinate systems, an alternative to the addition theorem. By truncating the multipole expansion, a finite linear algebraic system is derived. For the interior eigenproblem, the direct searching approach is employed to determine the natural frequencies by using the singular value decomposition (SVD). Eigensolutions obtained by the present method are compared well with those of the analytical method and finite element method (ABAQUS). The spurious eigensolutions are not found in the proposed formulation. For the exterior acoustic scattering problem subject to an incident plane sound wave, the scattered field is constructed which satisfies the Helmholtz equation as well as the radiation condition at infinity. Both Dirichlet (sound-soft) and Neumann (sound-hard) conditions are considered. The proposed results are compared well with those of the analytical method and the boundary element method. The near-field acoustic pressure and the far-field scattering pattern are both investigated to consider the effect of the space between scatterers and the incident wave number on them. Excellent accuracy and fast rate of convergence are the main features of the present method thanks to the semi-analytical procedure.

