

Determination of Interior Point Solutions for 3D Generally Anisotropic Elastic Solids by the Boundary Element Method

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Summary

In the boundary element method (BEM), interior point solutions for the displacements and the stresses at an interior point of an elastic body are obtained through the numerical evaluation of the Somigliana's identities. It is carried out as a secondary exercise in the BEM analysis, after the boundary integral equation (BIE) has been solved for all the unknown displacements and tractions on the surface of the domain. In the integrals of these identities, the integrands contain terms with up to second order derivatives of the Green's function for the displacements of the elastic problem.

The Green's function, or fundamental solution, for displacements and that for tractions are necessary items for the direct formulation of the BEM for elastic stress analysis. For 3D general anisotropic solids, the Green's function for displacements has been obtained by Lifschitz and Rozentsweig (1947) many years ago. In the development of the BEM to treat such bodies, the numerical evaluation of these fundamental solutions has remained a subject of investigation over the past few decades; see, e.g. Wilson and Cruse (1978), Sales and Gray (1998), Phan et al (2004), Tonon et al (2001), Wang and Denda (2007), Tan et al (2009). This is because of their mathematical complexity. In the BEM formulation presented by the present authors very recently, Tan et al (2009), the fundamental solutions employed in the BIE are expressed in algebraic, real-variable explicit forms, unlike those used by the other authors previously. They were derived by Ting and Lee (1997) for displacements, and Lee (2003) for their first derivatives which are then utilized for the derivation of the traction solution, respectively. These Green's functions were used for the first time in a BEM formulation. Because of their algebraic forms, they can be numerically evaluated in a fairly straightforward manner. Their implementation into an existing BEM code which had been developed for 3D isotropic elastostatics was also carried out without any difficulty. It was, however, discovered that a significant proportion of the computational effort is spent on evaluating high-order tensor terms which appear in Lee's (2003) solution. Lee (2009) re-examined her solution and showed how a simpler analytical form for the first derivatives of the displacement Green's function could be obtained without the high-order tensors. This can be achieved by carrying out the partial differentiation in a spherical coordinate system as an intermediate step; the explicit expressions are, however, pre-

sented only for the special case of transverse isotropy. Following this development, the present authors derived the corresponding fully explicit forms of the solution for the displacement first derivatives in general anisotropy. Their validity and superior efficiency of using these alternative fully explicit forms of the fundamental solutions in the BIE is demonstrated very recently in Shiah et al (2010).

Of significance to note too is that Lee's (2009) revised approach also lends itself readily to obtaining higher order derivatives of the Green's function for the displacements without the need to introduce high-order tensor quantities. The present authors have further derived the expressions, in fully explicit algebraic forms, of the second derivatives of the displacement fundamental solution. This enables the implementation of the BEM to obtain the displacements and stresses at an interior point of a 3D generally anisotropic solid as well; it is the focus of the present paper. To the authors knowledge, this development has never been reported previously in the literature. Some examples are presented in which the numerical solutions obtained are compared with those obtained using the FEM or by a finite difference approach to demonstrate their validity.

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