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Free vibrations of magnetoelectric bimorph beam devices by third order shear deformation theory

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Summary

The axial and flexural natural frequencies of magneto-electro-elastic bimorph beam devices are analyzed in the framework of the third-order shear deformation theory (TSDT). Although the assumption of parabolic transverse shear strain distribution along the thickness leads to higher order stress resultants the use of the TSDT allows to avoid the need for shear correction factor. Moreover, since the electric and magnetic potentials strictly depend on the shear strains, a more accurate modeling of the magneto-electric coupling can be achieved by expanding the kinematical model up to the cubic term. The natural frequencies for different mechanical boundary conditions are computed by varying the magnetoelectric bimorph configuration. The results are compared to those obtained by a first-order shear deformation theory (FSTD).

Keywords: magneto-electro-elastic, third-order shear deformation theory, bimorph, natural frequencies.

Introduction

A new class of material, the so called magnetoelectric composites (MEE), are receiving an increasing attention for the design of smart devices, Fiebig (2005). In fact, these materials can be profitably used in the field of Smart Structure technology due to the capability of coupling different fields such as elastic, electric and magnetic ones. In particular, due to the coexistence of piezoelectric and piezomagnetic phases which provide the composite with both the electro-mechanical and the magneto-mechanical coupling, a magneto-electro-elastic medium is able to couple the electric and magnetic fields through the elastic one, Nan (1994), Eerenstein, Mathur and Scott (2006). Due to this unique feature, called magneto-electric effect, the MEE materials can be used for magnetic field sensors, Fetisov, Bush, Kamentsev, Ostashchenko and Srinivasan (2006), Dong, Li and Viehland (2004), as well as generators, transformers and wireless power supply for microelectronics devices, Bayrashev, Robbins and Ziaie (2004). The increasing interest towards the magnetoelectric composite gives rise to the development of analytical and numerical models to investigate both the static and the dynamic responses of this new class of materials. In the field of modal analysis, the finite element method has been used by Annigeri, Ganesan and Swarnamani (2007) to analyze the natural frequencies of multiphase and layered magnetoelectric beams while the propagation matrix method

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has been employed by Pan and Heyliger (2002) to study the free vibrations of simply supported multilayered magneto-electro-elastic plates. Analytical solutions for the free and forced vibration problems of magneto-electric beams, based on the classical Bernoulli beam theory and on a first-order Timoshenko-like beam theory (FSTD), have been derived in the works of Milazzo, Orlando, and Alaimo (2009), where the shear influence on the magneto-electric fields has been pointed out. In the present paper the free vibration problem of magneto-electric bimorph beam has been modeled by using the Third Order Shear Deformation Theory (TSDT), which assumes parabolic distributions of the shear strains with respect to the thickness coordinate, Reddy (2004). By using the TSDT the shear factor correction is not needed overcoming the drawback of its dependence on the magnetic and electric fields in the FSTD model for magneto-electro-elastic beams. Computation of the natural frequencies of homogeneous and bimorph magneto-electro-elastic beams are presented.

Model Derivation and Solution

Let us consider a magneto-electro-elastic composite bimorph beam of length L and thickness h whose laminae are considered perfectly bonded from the mechanical, electric and magnetic point of view. Under the assumptions of zero electric density charge and current and quasi-static electric and magnetic fields, the electric and magnetic state of the beam are described in terms of the electric and magnetic potential function ϕ and ψ, respectively. Moreover, the electric polarization and magnetization directions are supposed to be directed along the y-axis, namely the thickness direction, while the components of the electric and magnetic fields along the x-axis, namely the beam length direction, are assumed to be negligible. The presented bimorph beam model relies upon the third order shear deformation beam theory, Reddy (2004), which is here extended by considering the magneto-electro-mechanical constitutive relationships, written assuming a monoaxial stress state, Milazzo, Orlando and Alaimo (2009). Accordingly, the following kinematical model is assumed for the beam

$$\begin{aligned} u(x,y,t) &= u_0(x,t) - y\vartheta(x,t) + \alpha y^3\left[\vartheta(x,t) - \frac{\partial v(x,t)}{\partial x}\right] \\ v(x,y,t) &= v_0(x,t) \end{aligned} \tag{1}$$

where u and v are the displacement components along the x and y axis, respectively, u_0 and v_0 are the displacement components at the beam mean-line, ϑ is the cross section rotation and t denotes the time variable. The coefficient $\alpha = 4/3h^2$ is determined by ensuring that the shear stress vanishes at the beam top and bottom surfaces. Under the aforementioned assumptions, following the procedure proposed in Milazzo, Orlando and Alaimo (2009), the Gauss' laws for electrostatic and magnetostatic are firstly satisfied in terms of the kinematical variables derivatives obtaining

$$\begin{aligned}\phi^{\pm} &= \alpha A_{\phi}^{\pm}\left(\frac{\partial \vartheta}{\partial x}-\frac{\partial^2 v}{\partial x^2}\right)\frac{y^4}{4}+\left(A_{\phi}^{\pm}\frac{\partial \vartheta}{\partial x}+B_{\phi}^{\pm}\frac{\partial^2 v}{\partial x^2}\right)\frac{y^2}{2}+a_1^{\pm}y+a_2^{\pm}\\ \psi^{\pm} &= \alpha A_{\psi}^{\pm}\left(\frac{\partial \vartheta}{\partial x}-\frac{\partial^2 v}{\partial x^2}\right)\frac{y^4}{4}+\left(A_{\psi}^{\pm}\frac{\partial \vartheta}{\partial x}+B_{\psi}^{\pm}\frac{\partial^2 v}{\partial x^2}\right)\frac{y^2}{2}+a_3^{\pm}y+a_4^{\pm}\end{aligned} \tag{2}$$

where the superscript $\pm$ is used to denote variables pertaining to the upper or lower layers of the bimorph. The A and B coefficients definitions and the computation of the integration constants a_i are omitted for the sake of brevity; the interested reader can find a full description of the employed procedure in Milazzo, Orlando and Alaimo (2009). It is just worth noting that once the magneto-electric boundary conditions are imposed the electric and magnetic potentials through-the-thickness distribution results known; only their $x-$dependence stands unknown and is related to the beam kinematical variables. Thus, in order to determine the electric and magnetic fields variables, the beam equations of motion need to be derived and solved. Following Reddy (2004), the Hamilton's principle for a linear elastic body is first considered. It points out that the primary variables of the problem are u_0, v_0, ϑ and $\partial v_0/\partial x$, while the corresponding secondary variables are N_x, $\bar{T}_x+\alpha\left(\partial P_x/\partial x - J_4\partial^2\vartheta/\partial t^2+\alpha I_6\partial^3 v_0/\partial x\partial t^2\right)$, $\bar{M}_x$, $-\alpha P_x$. The beam equilibrium equations are obtained as follows

$$\begin{aligned}&\frac{\partial N_x}{\partial x} = I_0\frac{\partial^2 u}{\partial t^2}\\ &\frac{\partial \bar{T}_x}{\partial x}-\alpha\frac{\partial^2 P_x}{\partial x^2} = I_0\frac{\partial^2 v}{\partial t^2}-\alpha\left(J_4\frac{\partial^3\vartheta}{\partial x\partial t^2}+\alpha I_6\frac{\partial^4 v}{\partial x^2\partial t^2}\right)\\ &\frac{\partial \bar{M}_x}{\partial x}+\bar{T}_x = K_2\frac{\partial^2\vartheta}{\partial t^2}+\alpha J_4\frac{\partial^3 v}{\partial x\partial t^2}\end{aligned} \tag{3}$$

where, denoted by ρ the mass density and set $I_i=\int_{-\frac{h}{2}}^{\frac{h}{2}}\rho y^i dy$, one has $J_4=I_4-\alpha I_6$ and $K_2=I_2-2\alpha I_4+\alpha^2 I_6$. The new mechanical variables used in writing the equilibrium equations, *i.e.* Eq. 3, are defined in terms of stress resultants according to

$$\begin{aligned}\bar{M}_x &= M_x-\alpha P_x\\ \bar{T}_x &= T_x-3\alpha R_x\end{aligned} \tag{4}$$

Taking into account the constitutive relationships for the stress components σ_x and σ_{xy} and the expressions of the electric and magnetic potential functions given by Eq. 2, the stress resultants can be written as

$$N_x = \int_{-\frac{h}{2}}^{\frac{h}{2}} \sigma_x dy = K_u^N \frac{\partial u_0}{\partial x} + K_\vartheta^N \frac{\partial \vartheta}{\partial x} + K_v^N \frac{\partial^2 v_0}{\partial x^2} + N_{EM} \tag{5}$$

$$M_x = \int_{-\frac{h}{2}}^{\frac{h}{2}} \sigma_x y dy = K_u^M \frac{\partial u_0}{\partial x} + K_\vartheta^M \frac{\partial \vartheta}{\partial x} + K_v^M \frac{\partial^2 v_0}{\partial x^2} + M_{EM} \tag{6}$$

$$P_x = \int_{-\frac{h}{2}}^{\frac{h}{2}} \sigma_x y^3 dy = K_u^P \frac{\partial u_0}{\partial x} + K_\vartheta^P \frac{\partial \vartheta}{\partial x} + K_v^P \frac{\partial^2 v_0}{\partial x^2} + P_{EM} \tag{7}$$

$$T_x = \int_{-\frac{h}{2}}^{\frac{h}{2}} \sigma_{xy} dy = \left(c_{44}^+ + c_{44}^-\right) \frac{h}{3} \left(\vartheta - \frac{\partial v_0}{\partial x}\right) \tag{8}$$

$$R_x = \int_{-\frac{h}{2}}^{\frac{h}{2}} \sigma_{xy} y^2 dy = \left(c_{44}^+ + c_{44}^-\right) \frac{h^3}{60} \left(\vartheta - \frac{\partial v_0}{\partial x}\right) \tag{9}$$

where c_{44} is the shear stiffness. The coefficients ${K_u}^i$, ${K_\vartheta}^i$, ${K_v}^i$ (i=N, M, P) are beam equivalent extensional and flexural stiffness depending on both the elastic and magnetoelectric material constants, whereas the terms $N_{EM,}, M_{EM,}, P_{EM}$ account for the applied magneto-electric loads. The expression of these coefficients is not given for the sake of brevity. By substituting Eq.5-9 into Eq.3 the equations of motion are obtained. The free vibration problem is solved by using the method of separation of variables based on modal expansion of the kinetic variables. According to the standard procedure, one assumes

$$u_0 = U_n(x) e^{i\omega_n t},\ v_0 = V_n(x) e^{i\omega_n t},\ \vartheta = \Theta_n(x) e^{i\omega_n t} \tag{10}$$

where U_n, V_n and Θ_n are the mode shapes and ω_n is the natural circular frequency, determined imposing the beam boundary conditions. In the present work the clamped-clamped (CC), cantilever (CF) and simply supported boundary conditions are considered. They are summarized in the following

$$S-S: \begin{cases} U(0) = V(0) = \bar{M}_x(0) = \alpha P_x(0) = 0 \\ U(L) = V(L) = \bar{M}_x(L) = \alpha P_x(L) = 0 \end{cases} \tag{11}$$

$$C-C: \begin{cases} U(0) = V(0) = \Theta(0) = V'(0) = 0 \\ U(L) = V(L) = \Theta(L) = V'(L) = 0 \end{cases} \tag{12}$$

$$C-F:\begin{cases} U(0)=V(0)=\Theta(0)=V'(0)=0 \\ N_x(L)=\bar{T}_x^*(L)=\bar{M}_x(L)=\alpha P_x(L)=0 \end{cases} \tag{13}$$

where $\bar{T}_x^*=\bar{T}_x+\alpha\left(P_x'+\omega_n^2 J_4\Theta-\alpha I_6\omega_n^2 V'\right)$. It is worth nothing that in order to obtain homogeneous governing equations the equivalent electromagnetic stress resultants $N_{EM},, M_{EM},, P_{EM},$are set to zero by enforcing proper electromagnetic boundary conditions.

Numerical results

The natural frequencies for two different magnetoelectric beam configurations are presented. The first application deals with the free vibration analysis of a homogeneous beam made of BF60, a particulate magnetoelectric composite with a 60% volume fraction of $BaTiO_3$ and 40% volume fraction of $CoFe_2O_4$ whose material properties have been taken from Milazzo, Orlando and Alaimo (2009). The results have been obtained for the three different mechanical boundary conditions, that is both ends clamped (CC), one end clamped and the other free (CF) and both ends simply supported (SS). The magneto-electric boundary conditions are those characterizing the sensing capability of the magneto-electro-elastic device and are obtained by setting to zero the electric and magnetic potentials on the bottom beam surface and the normal component of the electric displacement and magnetic induction vectors on the top beam surface. The natural frequencies for these three different mechanical boundary conditions are listed in Table 1.

Table 1: Natural frequencies (Hz) for the BF60 beam (axial mode in italic style).

Mode	C-C			C-F			S-S		
	f_{TSDT}	f_{FSDT}	*% difference*	f_{TSDT}	f_{FSDT}	*% difference*	f_{TSDT}	f_{FSDT}	*% difference*
1	1052	1054	-0.19 %	170	170	0 %	475	475	0 %
2	2792	2800	-0.28 %	1042	1043	-0.09 %	1858	1861	-0.16 %
3	5230	5250	-0.38 %	2827	2832	-0.17 %	*3961*	*3961*	*0 %*
4	*7922*	*7922*	*0 %*	*3961*	*3959*	*0.05 %*	4037	4050	-0.32 %
5	8211	8250	-0.47 %	5307	5323	-0.3 %	6869	6901	-0.46 %
6	11613	11675	-0.53 %	8350	8386	-0.43 %	10208	10272	-0.62 %

It can be observed that the values of the natural frequencies obtained by using TSDT slightly differs to those obtained by the FSTD beam model: they decrease due to the less stiffness associated to the kinematical model of the TSDT. However, even if the difference between the natural frequencies obtained with TSDT and FSDT is very small, practically negligible, the higher order theory gives a meaningful improvement in the appraisal of the through-the-thickness distribution of electric and magnetic potentials and it does not require a preparatory assessment of the shear factor.

Table 2: Natural frequencies (Hz) for $BaTiO_3$/ $CoFe_2O_4$ beam (axial mode in italic style).

Mode	C-C			C-F			S-S		
	f_{TSDT}	f_{FSDT}	% *difference*	f_{TSDT}	f_{FSDT}	% *difference*	f_{TSDT}	f_{FSDT}	% *difference*
1	1095	1101	-0.54 %	177	178	-0.56 %	495	498	-0.60 %
2	2901	2919	-0.61 %	1086	1092	-0.55 %	1935	1948	-0.67 %
3	5421	5457	-0.66 %	2940	2959	-0.64 %	4126	4155	-0.71 %
4	*8309*	*8311*	*-0.02 %*	*4155*	*4155*	*-0 %*	*4224*	*4230*	*-0.14 %*
5	8492	8551	-0.69 %	5508	5549	-0.74 %	7126	7191	-0.90 %
6	11983	12069	-0.71 %	8647	8718	-0.81 %	10565	10678	-1.06 %

The second example investigates the free vibrations of a bimorph beam obtained by stacking a piezoelectric $BaTiO_3$ layer and a piezomagnetic $CoFe_2O_4$ layer. The beam has length L= 0.3 m and height h= 0.02 m and the mass density is 5550 kg/m^3. Also for this case the results are compared to those obtained by Milazzo, Orlando and Alaimo (2009) with a first order beam theory. The computed natural frequencies are listed in Table 2. As expected the same considerations as in the previous example can be pointed out.

Conclusion

In this paper an analytical model based on the third order shear deformation theory for magnetoelectric bimorph beam has been presented. The piezoelectric, piezomagnetic and electromagnetic coupling effect have been included in the beam equivalent flexural and axial stiffness coefficients. Free vibration analyses have been carried out by using the proposed model for three distinct boundary conditions. The results obtained show that the difference between the natural frequencies computed by TSDT and FSDT is practically negligible. Nonetheless, the higher order theory can give a meaningful improvement in the appraisal of the through-the-thickness distribution of electric and magnetic potentials and it does not require a preparatory assessment of the shear factor.

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