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Geometry related treatments for three-dimensional meshless method

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Summary

The meshless method has a distinct advantage that it needs only nodes without an element mesh which usually induces time-consuming work and inaccuracy when the elements are distorted during the analysis process. However, the element mesh provides the geometric information for numerical simulation without the need to judge if the nodes or integration points are inside the analysis domain as in the meshless method, such as the boundary of the analysis domain which is defined by the element's edges or faces and that the integration points are intrinsically inside the elements. Because the analysis model with only nodes in the meshless method lacks these types of geometry related information, some extra complicated treatments are usually required during the numerical simulation, especially, in the cases with three-dimensional irregular analysis domain. Therefore, two types of boundary and domain schemes, say, triangulated surface boundary scheme and constructive solid geometry scheme, are employed in this work. Several demonstrative cases prove the effectiveness of the proposed schemes.

Keywords: meshless method, geometry treatment, triangulated surface

Introduction

One of the main disadvantages of the finite element method (FEM) is it requires a mesh, including elements and nodes, which is usually tedious and time-consuming to build. Moreover, the FEM usually gets into troubles with the distortion of the elements in dealing with large deformation problems. The meshless method has an inherent advantage that it doesn't require any mesh but nodes only, and has become one of the most promising numerical methods. Although there were many pioneering successes in the early research works, most of the cases were two-dimensional problems. Till recent years, three-dimensional problems have then been successfully tackled [Chen and Guo (2001); Han and Atluri (2003); Li, Shen, Han and Atluri (2003); Han and Atluri (2004); Chen and Chen (2005); Chen and Lee (2005); Chen, Chi, and Lee (2009); Lee and Chen (2009)]. But most of those cases are

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dealing with regular geometry; part of the reason is the difficulty to handle three-dimensional irregular domain. So far, irregular geometry issues in the meshless method have not been elaborated yet.

Although the meshless method has the distinct advantage that it needs only nodes without an element mesh and can avoid the disadvantages of the element mesh. However, the element mesh provides some necessary information needed by the numerical simulation, such as, the boundary of the analysis domain is represented by the element's edges or faces, the integration points of all elements are intrinsically located inside the analysis domain without the need to judge if those are inside the analysis domain as in the meshless method. Because the analysis model in the meshless method lacks these types of geometry related information, some extra treatments are needed for the meshless method during the numerical simulation especially in the cases of three-dimensional irregular analysis domain for which the treatments will be much more difficult. Therefore, a triangulated surface geometry is employed to represent the boundary surfaces of the analysis domain. In this way, the boundary surfaces of the three-dimensional domain are defined with triangular facets. Triangulated surface geometry is widely used by most CAD/CAM/CAE fields and can be easily created whenever the geometry data are available.

In addition to the triangulated surface geometry, another simpler representation, constructive solid geometry (CSG), is also employed. Although it has limitation to represent irregular geometry models, it inherits some simple and efficient advantages due to a much smaller amount of the faces involved. This type of model can be utilized for some simulation fields in which the analysis models are regular and fixed.

On top of the information of boundary and domain, there still are some types of geometry treatments needed in the meshless method during the simulation processes, such as, determining if certain integration points are located inside the analysis domain for numerical integration purposes [Belytschko, Lu, and Gu (1994)], determining if the node inside the influence radius is blocked by any boundary and cannot be included into the influence domain. Here, a checking mechanism is proposed to handle these works

Treatments of analysis boundary and domain

Unlike the finite element method, i.e. the element model itself provides enough information of domain boundary either for two-dimensional or three-dimensional domain, the analysis model in the meshless method provides inadequate information about the boundary and analysis domain. Therefore, it is needed to employ some mechanisms to handle the boundary and domain issues which are actually geometrical issues. In two-dimensional cases, the boundary issue can be easily handled just by connecting the boundary nodes with straight lines. It's also easy to determine something inside or outside the domain [Belytschko, Lu, and Gu (1994)]. Nevertheless, in three-dimensional cases, the analysis domain and boundary are much more complicated and difficult to define, especially for irregular sculptural faces. Here, two types of boundary and domain representation, i.e. constructive solid geometry and triangulated surface geometry, are proposed to employ. Based on them, some checking schemes can then be further used to handle those geometry related issues for the meshless method.

Constructive solid geometry (CSG) and set operations for regular three-dimensional analysis domains

The CSG is one of the popular schemes to represent geometry models in CAD field. The geometry models are constructed by combining primitives, such as, block, sphere, cylinder, etc., with some set operations, such as union, intersection, subtraction. This scheme for domain definition is easy and simple to manipulate for the meshless method. Although it has some limitations, such as difficult to create irregular geometry and unable to be deformed during solution processes, it still can be used for many simulation cases with the simplicity advantage. For example, in linear structural analysis and Eulerian-formulation simulation, such as electrostatic and flow analyses, in which the analysis domains are fixed in the space without any change.

The typical primitives used by this scheme are block, sphere, and cylinder, as shown in Fig. 1. It is easy to define the territories for those primitives, just by a set of coordinate ranges or a simple analytical equation. For example [Zeid (1991)],

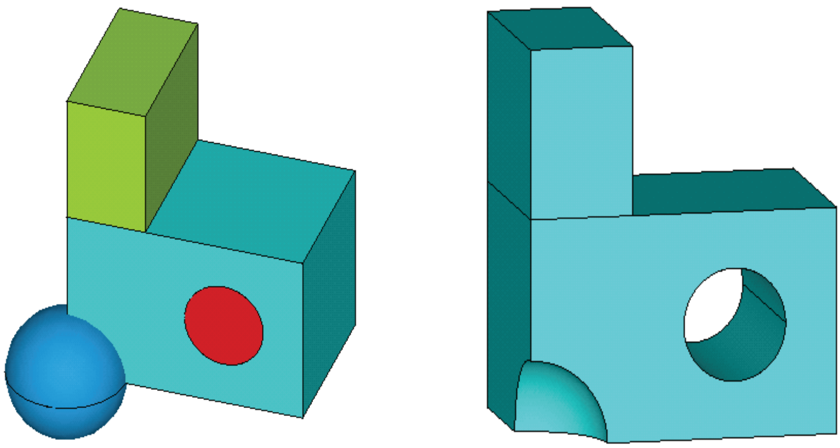
$$\text{Block}(L \times W \times H): \quad \{(x,y,z): \quad 0 \le x \le L,\ 0 \le y \le W,\ 0 \le z \le H\} \tag{0.1}$$

$$\text{Sphere}(r): \quad \{(x,y,z): \quad x^2+y^2+z^2 \le r^2\} \tag{0.2}$$

$$\text{Cylinder}(r \times H): \quad \{(x,y,z): \quad x^2+y^2 \le r^2,\ 0 \le z \le H\} \tag{0.3}$$

These basic shapes can be combined to represent some more complicated analysis domains by using set operations. For example, as shown in Fig. 1, the analysis domain can be defined with two different blocks, a cylinder and a sphere by using set operations, i.e. union the two blocks first and then subtract the cylinder and sphere from it. These operations can be easily programmed into the meshless solvers without consuming much computing time.

For linear structural analysis and Eulerian-formulation analyses, this scheme provides an easy and efficient way to handle the definition of analysis domain for the meshless method. On judging if a point is inside the analysis domain, it can be simply done by checking whether that point is in the model set or not.



(a) combination (b) final domain after set operations

Figure 1: Regular three-dimensional analysis domain by set operation

This type of representation can be used to solve those problems with regular three-dimensional analysis domains and those simulations of which the analysis domains are fixed during the simulation processes.

Triangulated surface geometry

For a three-dimensional irregular domain, a triangulated surface geometry is employed to represent the boundary surfaces of the analysis domain, i.e. the surfaces of the three-dimensional domain are represented with triangular facets. This employment is also adopted by some other research [Han and Atluri (2004)]. The triangulated surface geometry can be generated by most CAD packages and pre-processors and widely used for rapid prototyping, CAM, and computer graphics. The triangular facets are not triangular elements in the FEM and are used only for geometry purpose. Fig. 3 shows an example. In addition to representing three-dimensional irregular domain, the triangular facet which is simply defined by and deflected with the three vertices, i.e. nodes, is so easy to be manipulated that the complicated geometry represented by the triangular facets can be deformed and updated with the moved nodes in accordance with the analysis results during the simulation processes.

Inside/outside determination for analysis domain

In the meshless method, there are several occasions needing to determine if an object inside the analysis domain. One is to determine if certain integration points are located inside the analysis domain for numerical integration purpose [Belytschko, Lu, and Gu (1994)], because the integration cells and points are paved in the space independently of the analysis domain. Another is to determine if certain nodes are located inside the analysis domain when using the background grid scheme [Chen, Chi, and Lee (2009)], because the background nodes are also paved in the space independently of the analysis domain.

As mentioned above, when using the CSG scheme, the inside/outside determination can be simply done by checking if that point is in the model set.

When the object or analysis domain is alternatively represented by the triangulated surface geometry, the model still lacks enough information for determining whether a point is inside or outside the domain. Therefore extra judging operations are needed to work for that. Here, a checking mechanism is proposed to handle these works. At first, put a reference point inside the analysis domain. After the reference point is created properly, when we want to check if an integration point or a node inside the analysis domain, we connect the discussed point and the reference point with a connecting line as shown in Fig. 2 (b). Then, we check how many times the connecting line crosses the boundary facets. A criterion is then set: when the number of times that the connecting line crosses the boundary is odd, the point is outside the analysis domain; when it is even, the point is inside the analysis domain.

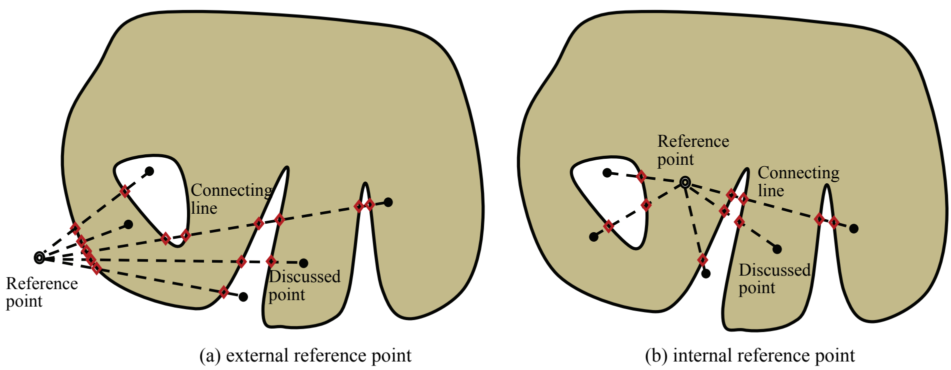


Figure 2: Types of reference points: (a) external (b) internal

Alternatively, we can employ a reference point outside the analysis domain. It's easier to pick a point assured to be outside the analysis domain. First, we decide the bounding box which is just big enough to cover the entire analysis domain by checking the minimum and maximum of the coordinates of all the nodes of the analysis model. The bounding box is constructed by the minimal and maximal values in x, y, z direction, respectively. Then, create a reference point outside the bounding box. After the reference point is created properly, we similarly connect the discussed point and the reference point with a connecting line. Then, we check how many times the connecting line crosses the boundary facets as shown in Fig. 2 (a). For this case, i.e. external reference point, the criterion is opposite to the internal-reference-point case. It is if the number of times is odd, the point is inside the analysis domain; if it is even, the point is outside the analysis domain.

To compare the two kinds of reference points mentioned above, it is obvious that the outer reference point is easier to choose without confusion. But for internal-reference-point case, we can check the shortest distance to the boundary in the beginning of the analysis process. The information is useful that we can check if the length of certain connecting lines is shorter than that shortest distance, if yes, we can directly pass those discussed points without further checking. This can save a lot of computing time.

Determination of the node set for the influence domain

After above checking operations, the nodes inside the analysis domain will have been determined. Another need for the meshless method is to determine if the node which is inside the influence radius range [Belytschko, Lu, and Gu (1994)] is blocked by any boundary; if yes, the node will not be included in the node set which defines the influence domain.

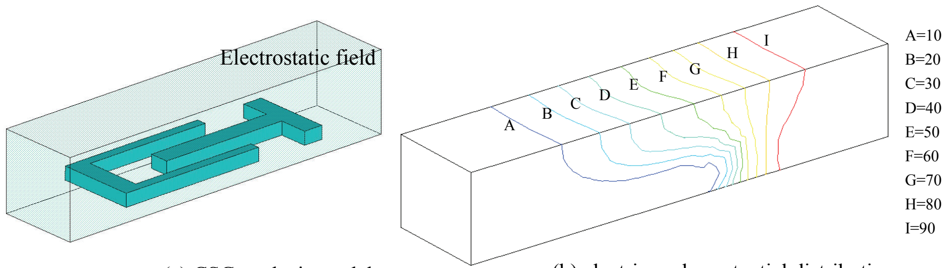
In the case of triangulated surface boundary scheme, to determine if certain node is blocked by any boundary, we use the similar way as mentioned above. We connect the center point, an integration point or a node, and the discussed node with a connecting line. In this case, if the connecting line crosses any triangular facet, it means the two points have some boundary in between and the discussed node should be excluded from the influence domain.

In the case of the CSG scheme, although there are more types of surfaces than triangular facet, we can use the same mechanism, i.e. to check if the connecting lines cross any boundary faces.

Numerical examples and results

Here, two analysis cases were conducted to demonstrate the effectiveness of the proposed schemes. For comparison purposes, the FEM will also be employed to solve the same problems.

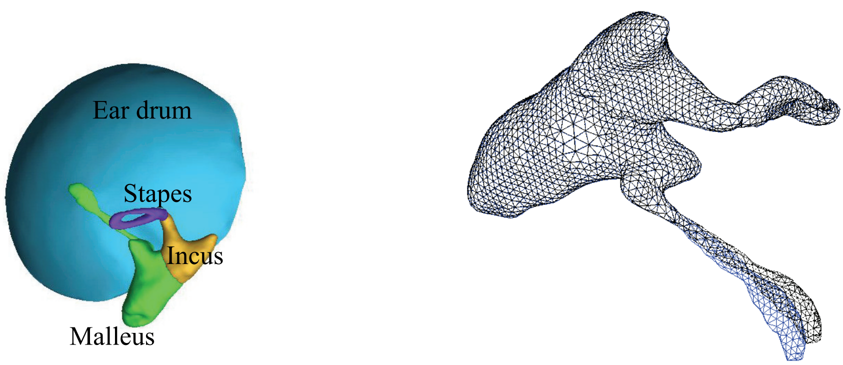
First case is an electrostatically actuated comb-drive component. Here, an electrostatic analysis of a pair of comb-drive fingers is carried out to obtain the electric scalar potential distribution of the electrostatic field. The shape of the analysis domain is a regular three-dimensional geometry. To solve this case, the CSG scheme was employed. The results are also shown in Fig. 3. The ANSYS results were also obtained for comparison purpose.



(a) CSG analysis model (b) electric scalar potential distribution

Figure 3: Electrostatic analysis of a comb drive case (one pair of fingers, unit: Volt)

The second case is about three ossicles, i.e. malleus, incus, and stapes, of mouse's middle ear [Sun, Gan, Chang, Dormer (2002)] as shown in Fig. 4(a). They play the role of passing outside sound pressure on ear drum to inner hearing nerves. Normally, the geometry data are obtained by computed tomography (CT) and very irregular. Here, the triangulated surface boundary scheme can easily be used to solve the stiffness of the bones. The Fig. 4(b) shows the deformation of the analysis model.



(a) CT scan geometry (b) triangularized geometry and deformation

Figure 4: Structural analysis of osiccles of middle ear

Conclusions

In this work, we have proposed to employ two types of geometry representation, the triangulated surface geometry and the CSG, to define three-dimensional analysis domains for the meshless method. Combined with the domain representation, a checking mechanism, the connecting line way, is also proposed to deal with some location addressing issues needed for the meshless method. The CSG or triangulated surface geometry either has different advantages. The CSG scheme is simpler and cheaper. Therefore it is good for regular-domain and Eulerian-formulation problems. On the other hand, the triangulated surface geometry scheme can handle any kind of irregular geometry. With these two schemes which both are comparatively simple and concise and have involved minimal geometrical calculation, the meshless method can thus handle any three-dimensional irregular analysis domain effectively. Above demonstrative cases prove the effectiveness of the proposed schemes.

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