 ICCES, vol.14, no.4, pp.99-101

Markov approach to analysis of mechanical interaction of surfaces during friction

Yu.A. Goritskiy[1], **D.G. Tigetov**[1]

Summary

It is well-known that process of friction depends immensely on the surface roughness. Also, surfaces change while in friction. Roughness change is influenced by many factors such as shape of asperities, physical characteristics of materials, load, sliding velocity, lubricant and others. It was shown experimentally that roughness reaches a steady form (known as "equilibrium roughness") while running-in process. It is "equilibrium roughness" that determines a stationary friction mode.

Firstly, it is proposed to analyze the process of friction by means of the discontinuous model where surfaces are represented by asperities of random height. Load applied, and some pairs of surface asperities contact. Pairs of contacting asperities are changed when the shift occurs. Asperities in contact deform and destruct each other, that is why heights of contacting asperities may change. Total height change of all surface asperities leads to roughness transformations and variation of friction characteristics (real contact area, friction coefficient, wear).

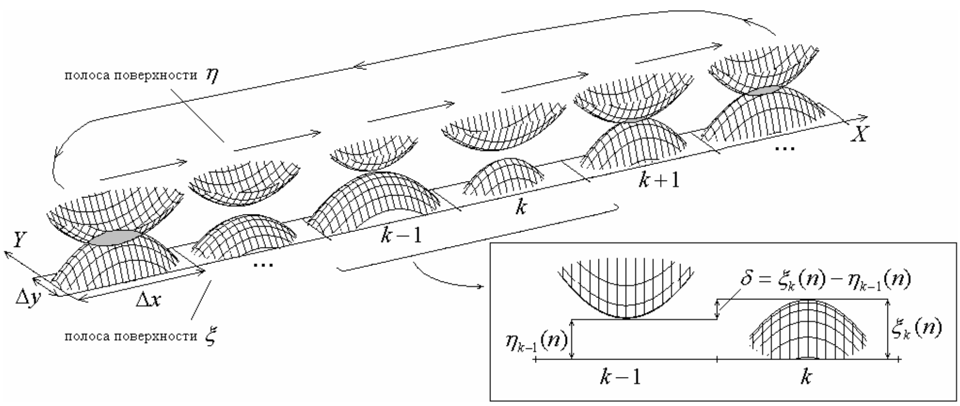

Figure 1: Model surfaces and sketch of cyclical shifts

Let's consider K pairs of asperities positioned in one raw. Time is discontinuous and is measured in shift counts. After n shifts heights of asperities of one surface are represented by vector $\xi_k(n)$ $(k = 1,...,K)$, and heights of asperities of the other surface are represented by vector $\eta_k(n)$. The asperity height change of the kth pair in one shift is described by general equations:

$$\xi_k(n+1) = \Psi(\xi_k(n), \eta_{k-1}(n)),$$

[1]Moscow Power Engineering Institute, Moscow, Russia

$$\eta_k(n+1) = \Phi(\xi_k(n), \eta_{k-1}(n)), \quad k = \overline{1,K}, \quad \eta_0(n) = \eta_K(n),$$

where Ψ and Φ determine stochastic mechanism of asperity interaction as a sphere interaction, [1], with random radiuses. Presented model of asperity interaction takes into account elastic-plastic deformation and wear.

Model is realized on a computer.

After N shifts are performed, asperity height histogram is depicted, real contact area; friction force and wear are measured. Presented results agree to well known observations, such as independence of "equilibrium roughness" from initial surface roughness.

Secondly, it is proposed to describe the trajectory of height changes of single asperity by means of Markov series, [2]. The height ξ of single asperity changes when it interacts with asperities of other surface, having heights $\eta(t)$ at the moment t. Time step Δt is selected in such a way, that values $\eta(t+\Delta t)\eta(t)$ are close to uncorrelated. It is supposed, that asperity with the height ξ interacts with the series of asperities of the other surfaces with heights $\eta_1, \ldots, \eta_n, \ldots$ distributed with density $q_\eta(y)$. Thus series of heights $\xi_1, \ldots, \xi_n, \ldots$ is obtained, height value ξ_{n+1} depends on ξ_n and η_{n+1}:

$$\xi_{n+1} = \Psi(\xi_n, \eta_{n+1}),$$

where Ψ determines accepted mechanism of asperity contact interaction. Series $\xi_1, \ldots, \xi_n, \ldots$ is Markov series. Moreover, heights $\xi_1, \ldots, \xi_n, \ldots$ are discontinuous, series $\xi_1, \ldots, \xi_n, \ldots$ becomes Markov chain. Similarly, ξ is distributed with density $p_\xi(x)$, and next equation is obtained:

$$\eta_{n+1} = \Phi(\xi_n, \eta_{n+1}).$$

Once two functions Ψ and Φ are elaborated the system of equations for stationary distributions p^* and q^* is obtained:

$$\begin{cases} p^* = p^* \cdot P_\xi(q^*, \Psi), \\ q^* = q^* \cdot P_\eta(p^*, \Phi), \end{cases}$$

where $P_\xi(q^*, \Psi)$ and $P_\eta(p^*, \Phi)$ are matrices of transition probabilities for series of ξ_n and η_n. Elements of matrices are determined by distributions p^* and q^* correspondingly and by two functions Ψ and Φ. For example, elements of matrix $P_\xi(q^*, \Psi)$:

$$\left\| P_\xi \right\|_{ij} = P\{\xi_{n+1} = j | \xi_n = i\} = \sum_{l:\Psi(i,l)=j} q^*(l).$$

Elements of $P_\eta(p^*, \Phi)$ are determined similarly.

Solution of system is presented; it is obtained by simple iteration method. Characteristics of friction are estimated with the help of obtained solution. Estimations agree with experimental observations.

References

1. **Johnson K. L**. Contact Mechanics, Cambridge University Press, 1987.
2. **Doob J.L.** Stohastic Processes. New York – Jonh Willey & Sons, 1953.