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Fracture behavior of plain concrete beams – experimental verification of one parameter model

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Summary

Several different models have been proposed to characterize mode-I crack propagation in concrete. The fictitious crack model proposed by Hillerborg et al. and the blunt crack band theory developed by Bazant & Oh are particularly well suited for a finite element analysis. The two-parameter fracture model proposed by Jenq & Shah is found to be applicable only for beams with s/w=4, where s=span & w=depth of the beam. The general applicability of the model for other testing configurations is not published. In the present study an experimental verification of a one-parameter model based on fundamental equation of equilibrium developed by Ananthan, Raghu Prasad, and Sundara Raja Iyengar to explain the mode - I fracture behavior of notched and un-notched plain concrete beams subjected to three or four-point bending, also called Softening Beam Model are reported and discussed in this paper. The influence of structural size in altering the fracture mode from perfect brittle fracture to plastic collapse is explained through the stress distribution obtained across the un-cracked ligament. The key factor affecting the stress distribution is found to be the strain softening modulus and is considered to dependent on structural size. Based on large number of experimental results available in the literature pertaining to the testing of plain concrete beams in either three-point or four-point bending, an empirical relationship for the determination of process zone length (L_p) has been developed. With the knowledge of L_p, the maximum load P_{max} can be obtained. It is demonstrated that the model can predict P_{max} quite accurately. Here the objective is to experimentally verify the predicted P_{max}. The comparison is valid within a range of errors, reasonable in the cementetious materials like concrete. In this model only an inelastic fracture mechanics parameter 'L_p' has been used. The parameter L_p has been obtained as a function of the size of the beam and also softening modulus in order to consider the effect of softening on the tensile stress-strain behavior of concrete. Further, the P_{max} predicted by the one parameter model as obtained for mode-I failure has been experimentally verified to see how far it is valid for mixed mode type of failure. Since a very few experimental investigations have been so far carried out to study the fracture behavior of concrete under mixed mode condition, the present experimental program is carried out, so as to obtain a wide range of test results pertaining the mixed mode fracture of concrete.

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Keywords: Concrete softening; Fracture energy; Fracture Process zone; Mode-I fracture of concrete; Size effect.

Introduction

It is observed that structures are susceptible to failure due to stress concentration caused by presence of a crack which may be inherent either in the basic material or introduced during fabrication or service. The use of concrete as a primary structural material in complex structures such as tall buildings, submerged structures, bridges, dams, liquid and gas containment structure has increased in the recent past. Proper understanding of the structural behavior of plain and reinforced concrete is absolutely necessary in designing complex concrete structures. Engineering fracture mechanics can deliver the methodology to compensate the inadequacies of conventional design concepts. The presence of micro-cracks and other inherent flaws in concrete act as potential sources of crack propagation and fracture under external loadings. On the other hand, due to the application of repeated loads or due to a combination of loads and environmental attack these cracks will grow with time. Due to presence of the cracks the strength of the structure is decreased; it is lower than the original strength of the structure it was designed for. The residual strength of the structure decrease progressively with increasing crack size. A wide variety of techniques have been proposed to evaluate experimentally, the fracture mechanics parameters.

Brief literature review on fracture mechanics of plain concrete

There are two basic fracture mechanics approaches to describe how material fails.

1. A material fails when the intensity of stress concentration at micro-flaw (say, a crack tip) exceeds the intrinsic cohesive strength of the material (stress intensity approach).

2. A material fails when the energy stored in it during loading exceed the energy required for creating fresh macro-flaws (say, crack surfaces), the energy balance approach.

Hillerborg et al [1] have analyzed Single Edged Notched (SEN) beams using the fictitious crack model also known as Damage zone model. The tensile stress is assumed not to fall to zero immediately after the attainment of limiting value but to decrease slowly with increasing crack widths. Modulus of elasticity E, uniaxial tensile strength σ_t and fracture energy $G_{F,}$ defined as the area under post-peak stress vs. COD diagram are the material properties required to describe the tensile fracture behavior of concrete. Bazant & Oh [2] introduced the concept of crack band theory for fracture of concrete. The fracture front is modeled as a blunt smeared crack band. The material fracture parameters are characterized by three parameters G_F, σ_t and the width of crack band W_C (fracture process zone). G_F is however defined as the product of W_C and the area under the tensile stress-strain curve. Using this model the maximum load carrying capacity of several beams are predicted. G_F is found to depend on the specimen size. The values of G_F so obtained are used to obtain an empirical relationship to predict G_F from the knowledge of material properties. These model irrespective of the approach adopted requires a complete stress-crack opening relation. They are particularly well suited for numerical techniques like the finite element method. Some models which do not require the finite element technique are also proposed. Wecharatana & Shah [3] based on some simple and approximate extensions of the concepts of LEFM, have predicted the extent of the non-linear fracture process zone in concrete. Critical COD equal to 0.025mm and a constant closing pressure to exist along the length of the fracture process zone are assumed. Fracture loads of a large number of notched beams are reported to have been estimated with a reasonable degree of accuracy. Jenq & Shah [4] have proposed two parameter fracture model. The two parameters are critical stress intensity factor calculated at the tip of the effective crack and critical COD. Based on their test results, the two parameters are found to be size dependent. Alberto Carpinteri [5] presented a critical review of works dealing with concrete fracture. He concludes that heterogeneity is only a matter of scale and notch sensitivity is necessary but not sufficient condition for the applicability of the linear elastic fracture mechanics. Nallathambi & Karihaloo [6] have performed tests on cement mortar and concrete beams in two stages, with a view to study the influence of several variables upon the fracture behavior of concrete. On the basis of the results from the first stage of test in which a single water/cement ratio and type of coarse aggregate were used, a simple formula was established to estimate the fracture toughness of concrete in terms of specimen dimensions, maximum aggregate size and notch depth together with the mix compressive strength and modulus of elasticity (determined from separate standard cylinder test). It was found to predict with sufficient accuracy, the results from the second stage of test in which, besides variation of the type of coarse aggregate and water/cement ratio, some of the specimen sizes were outside the range used in the first series. Peterson [7] determined fracture energy G_{IC} using load-deflection curve. His test results indicate that $G_{IC}$is independent of both notch depth and beam depth. A simple numerical method called Initial stiffness method and Modified lattice model were proposed by Raghu Prasad et al.[8-9] to analyze fracture behavior of plain concrete beam (strain softening material) in mode-I using finite element method. A new parameter namely, strain softening parameter α has been introduced. The method is validated by analyzing a significant number of beams tested and reported by various researchers.

Objective & scope of present study

Several different models have been proposed to characterize mode-I crack propagation in concrete. The fictitious crack model proposed by Hillerborg et al [1] and the blunt crack band theory developed by Bazant & Oh [2] are particularly well suited for a finite element analysis. The two-parameter fracture model proposed by Jenq & Shah [4] is found to be applicable only for beams with s/w=4, where s=span & w=depth of the beam. In the present study an experimental verification of a one-parameter model based on fundamental equation of equilibrium developed by Ananthan et al.[10-11] to explain the mode-I fracture behavior of notched and un-notched plain concrete beams subjected to three or four-point bending also called as softening beam model are presented. The influence of structural size in altering the fracture mode from perfect brittle fracture to plastic collapse is explained through the stress distribution obtained across the un-cracked ligament. The key factor affecting the stress distribution is found to be the strain softening modulus and is considered to dependent on structural size. Based on large number of experimental results available in the literature pertaining to the testing of plain concrete beams in either three-point or four-point bending, an empirical relationship for the determination of process zone length (L_p) has been developed [10]. With the knowledge of L_p the maximum P_{max} can be obtained. It is demonstrated that the model can predict P_{max} quite accurately.

Proposed model

Since concrete is generally known to exhibit strain softening in tension, brittle fracture or perfect plastic collapse seem to be only idealizations. Hence by modeling concrete according to the strain softening behavior and by varying the strain softening slope between infinity to zero a transition between the two extreme idealizations can be achieved. Since M_{max}/M_n can characterize the type of fracture of a beam of given dimensions, it becomes necessary to determine the stress block across the un-cracked ligament. The stress–strain diagram for concrete in tension can be approximated to be bilinear as shown in Fig. 1(a). The stress σ in the strain softening range can be expressed as, for details see [10]

$$\sigma = \sigma_t - E_T(\varepsilon - \varepsilon_t) \tag{1}$$

in which σ_t = limiting uni-axial tensile strength, E_T = strain softening modulus, ε , ε_t = tensile strain corresponding to σ and σ_t respectively. With the additional assumptions regarding compressive stresses to vary linearly Fig. 1(b) with increasing

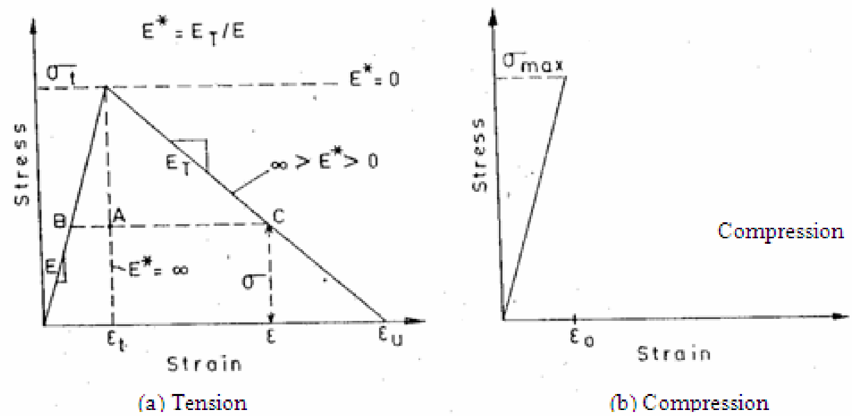

Figure 1: Stress-Strain for concrete

strains and the strain distribution to possess linear variation across the un-cracked ligament, ε can be expressed as

$$\varepsilon = \frac{1-\alpha}{\beta}\varepsilon_t \tag{2}$$

Substituting for ε as given by (2) in (1), one obtains

$$\sigma = \sigma_t - \frac{E_T\varepsilon_t(1-\alpha-\beta)}{\beta} \tag{3}$$

Replacing ε_t by σ_t/E and denoting E_T/E as E*, (3) can be written as

$$\sigma = \sigma_t(1 - E^*\frac{(1-\alpha-\beta)}{\beta}) \tag{4}$$

The maximum compressive stress σ_c can be expressed as

$$\sigma_c = \frac{\alpha}{\beta}\,\sigma_t \tag{5}$$

Thus with a knowledge of α , β and E_T, stress block across the un-cracked ligament can be completely characterized through (4) and (5). Since the cracked section is in equilibrium, fundamental equations of equilibrium have to be satisfied up to the onset of fracture. As the cracked section is not subjected to any externally applied horizontal force, the total compressive force shall be equal to the total tensile force. Further, the moment of resistance developed by the net section shall be equal to the external bending moment. Invoking the first condition of equilibrium

$$\frac{1}{2}\sigma_c\,\alpha\,Db\;=\;\frac{1}{2}\sigma_t\,\beta\,Db\,+\,\frac{1}{2}(\sigma_t+\,\sigma)(1-\alpha-\beta)Db \tag{6}$$

is obtained.

Substituting for σ and σ_c according to (4) and (5), (6) after simplification can be written as

$$\alpha^2\,(1+E^*)+2\alpha(\beta+E^*\beta-E^*)+[(\beta^2(1+E^*)-2\beta(1+E^*)+E^*]=0 \tag{7}$$

The roots of the above quadratic equation are given by

$$\alpha=\frac{-(\beta+E^*\beta-E^*)\pm\sqrt{2\beta E^*+2\beta-E^*}}{(1+E^*)} \tag{8}$$

According to moment equilibrium condition, if M is the external bending moment acting on the cross-section, then moment of the tensile forces about the centre of compression can be written as

$$M=\frac{bD^2}{6}[2\sigma_t\beta(\alpha+\beta)+\sigma(1-\alpha-\beta)(\alpha+3\beta+3) \\ +(\sigma_t-\sigma)(1-\alpha-\beta)(\alpha+2\beta+1)] \tag{9}$$

Substituting for σ according to (4), (9) after simplification can be obtained in the following non-dimensional form

$$\frac{M}{M_n}=3-2\alpha+E^*\frac{(\beta+2)}{\beta}(2\beta+2\alpha-1)-(\alpha+\beta)^2(1+\frac{E^*(\beta+2)}{\beta}) \tag{10}$$

It can be easily verified from (10), that for E* = ∞, β=α= 0.5, M/M_n equal to unity is obtained. M_{max}/M_n values obtained for various values of E^* in the range 0-1 is shown in Fig. 2. Likewise for E* = 0, α=β= 0, M/M_n equal to three results.

Solution procedure

Only two equations (8) and (10) are available to determine three unknown parameters α, β and E*, required to completely characterize the stress block across the un-cracked ligament. Thus, the solution in a straight forward manner seems to be impossible. However, by assuming values for E* and β, α can be obtained according to (8) and hence M/M_n according to (10). Since arbitrary values are obtained

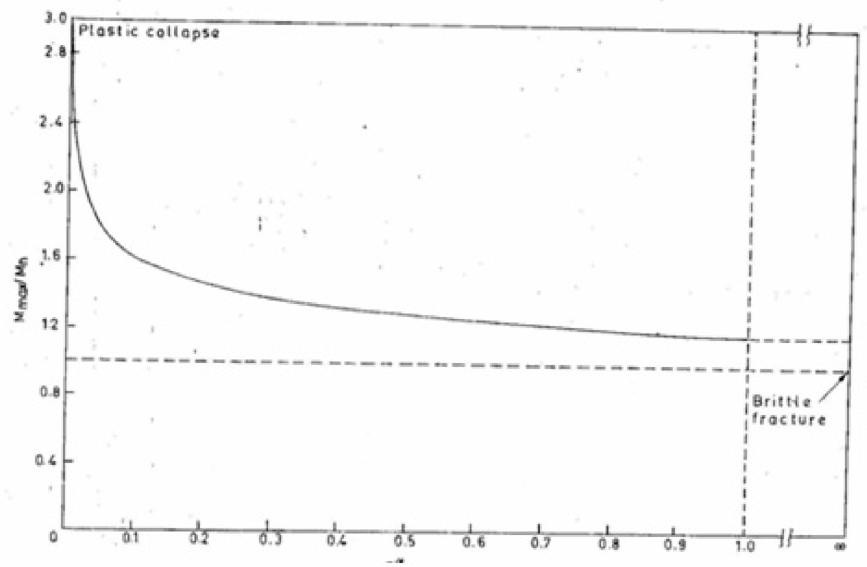

Figure 2: M_{max}/M_n for all values of E*

with this approach, it becomes necessary to check for their physical admissibility. With this inview, the following additional conditions are developed.

Condition 1

For any given value of E*, the minimum value of β shall be such that the quantity inside the radical sign in (8), that is $\sqrt{2\,\beta\,E^* + 2\beta - E^*}$should be equal to zero for α to be positive and real.

Condition 2

Since two distinct values for α are obtained according to (8) for various values β and E*, the physically admissible value for α, is the one which is not only positive but also yields M/M_n obtained from (10) as a positive quantity.

Condition 3

The value of α and β shall be such that $(\alpha + \beta)$ cannot be greater than unity.

Condition 4

Experimental studies [12–15] reveal that corresponding to any stress level, pre-peak strain is less than post-peak strain as measured from the peak. i.e., AB<AC in fig.1(a). With this assumption, maximum value of E* is fixed to be unity, with the minimum value of E* as equal to zero.

With the conditions 1 to 4, operating on the values of α, β and E*, the following solution procedure is proposed.

1. For any assumed value of E* in the range fixed by condition 4, the minimum value of β is obtained according to condition 1.The corresponding value of

α is evaluated from (8).

2. This value of α is checked for its admissibility according to conditions 2 and 3.

   The corresponding value of M/M_n is obtained from (10).

3. The minimum value of β is then incremented by a small amount and the whole procedure is repeated till a limiting stage according to condition 3 is reached.

Here the objective is to experimentally verify the predicted P_{max}. The comparison is valid within a range of errors, reasonable in the cementitious materials like concrete. In this model only an inelastic fracture mechanics parameter 'L_p' has been used. The parameter L_p has been obtained as a function of the size of the beam and also softening modulus in order to consider the effect of softening on the tensile stress-strain behavior of concrete. Jenq and Shah has extended the concepts of two parameter fracture model to characterize the mixed mode fracture behavior of concrete[15]. The procedure developed by them to predict P_{max}, again requires the use of finite element analyses.

Further, the P_{max} predicted by the one parameter model as obtained for mode-I failure has been experimentally verified to see how far it is valid for mixed mode type of failure. Since a very few experimental investigations have been so far carried out to study the fracture behavior of concrete under mixed mode condition, the present experimental program is carried out, so as to obtain a wide range of test results pertaining to the mixed mode fracture of concrete.

Experimental set-up and other details

Materials used

Cement: Ordinary Portland cement conforming to Indian standard specification (IS:269-1976) was used.

Aggregate: Sand obtained from local river beds was used as fine aggregate. Broken granite stone of maximum size 20mm was used as coarse aggregate.

Water Ordinary potable water was used.

Mix Adopted

A concrete mix of 1:1.5:3 by weight with a water cement ratio of 0.45 was used. Generally this proportion of mix gives M20 grade concrete.

Casting and curing of specimens

The notch is made by a plastic plate of 3mm thickness with V-shape at tip. The notch plate is fixed to the mould with nails. Concrete was poured in three layers and vibrated. Six cubes and six cylinders were also cast along with each batch of casting of the beams to take the measurement of cube-crushing and tensile strength of concrete. After 24 hours of the casting of the beam the notch making plate was carefully removed and after that mould also removed. Specimens were cured under damped gunny bags for 28 days. Only two beams of the first set $B1_{-4/8}$, $B1_{-2/8}$) were tested on 14^{th} day. The beams were dried and kept for 24 hours in the laboratory at room temperature and humidity before testing.

Test Specimens

A total of 15 beams were cast for testing, but one beam failed due to handling before testing.

The 15 beams were in three different sizes.

1^{st} set 1400mm * 300mm * 150mm

2^{nd} set 1580mm * 230mm * 115mm

3^{rd} set 1800mm * 200mm * 100mm

The spans of the beam were 1200mm, 1380mm and 1600mm for the 1^{st} set, 2^{nd} set and 3^{rd} set respectively. The experimental set up for beam testing ,I^{st} , 2^{nd}, and 3^{rd} set of beams are shown in Plates 1-4 respectively.

In each set of the beams the notch position was varied from centre to $1/8^{th}$ span from support.

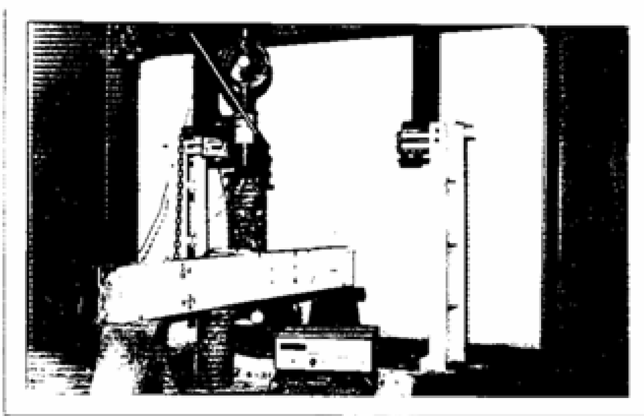

Plate 1. Experimental Setup for Beam Testing

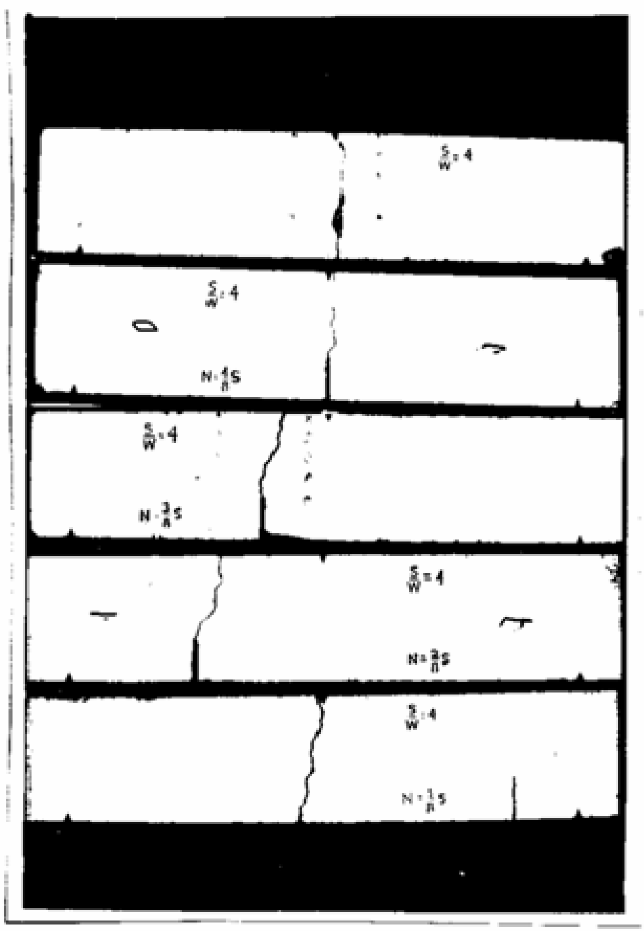

Plate 2. 1st Set of Beams

$B_{I,II,III}$ = Beams of I, II and III sets without notch.

$B_{I,II,III}$ = Beams of I, II and III sets with notch at $4/8^{th}$ span from support (N=4/8 S)

$B_{I,II,III}$ = Beams of I, II and III sets with notch $3/8^{th}$ span from support (N=3/8 S)

$B_{I,II,III}$ = Beams of I, II and III sets with notch at $2/8^{th}$ span from support (N=2/8 S)

$B_{I,II,III}$ = Beams of I, II and III sets with notch at $1/8^{th}$ span from support (N=1/8 S)

2^{nd} beam of 3^{rd} set ($B_{III-4/8}$) failed due to handling before testing.

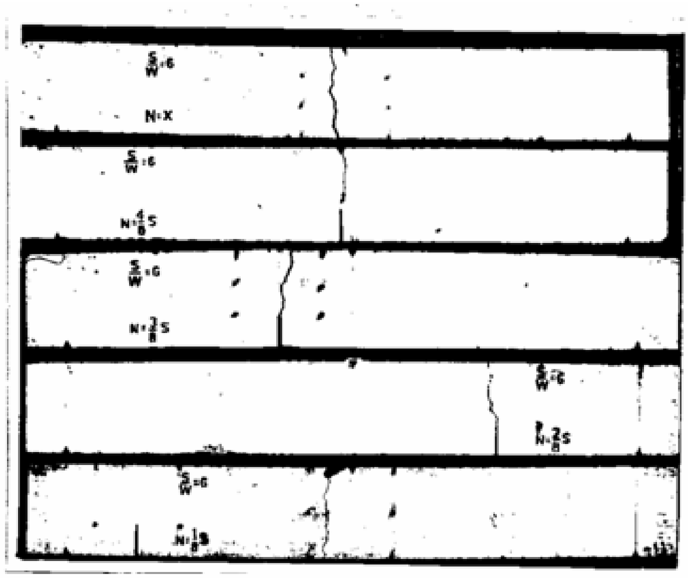

Plate 3. 2nd Set of Beams

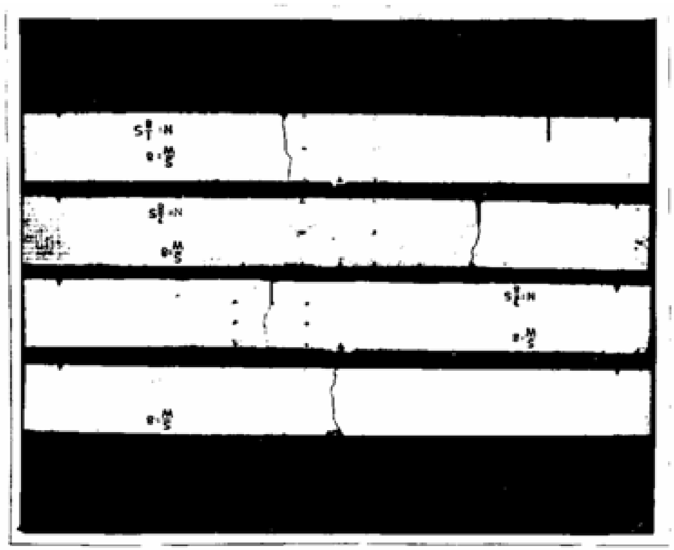

Plate 4. 3rd Set of Beams

Details of Experimental Set-up

The general view of experimental set-up for three-point loading is shown in Plate 1.The specimen was supported on steel rollers which were again supported by steel blocks.

Loading for all the beams was done using hydraulic jack. The jack was manually operated for loading and unloading. The applied load was measured by means of pre-calibrated proving ring of capacity 10 tons. Using the mechanical dial gauge with 0.01 mm least count the vertical deflection of the beam at center was measured. The crack-mouth opening displacement (CMOD) was measured by linear variable digital transducer (LDVT). The strain at different layers of the beam was measured by Demec gauge with a least count of 0.00254mm.

Test Procedure

Load was applied in the downward direction at center of the beam by a hydraulic jack which was supported on a mild steel roller on the beam. At every load increment, three observations viz., CMOD, deflection and strain at different levels of the beam were noted, using which load Vs CMOD, load Vs deflection curves were plotted. In initial stages load increments were 952.5 N, while nearer to the peak load, they were 635 N.

To measure CMOD, two aluminum angles were fixed at the bottom of the beam on either side of the notch. Transducer was fixed to these angles, which is again connected to digital meter. Initial reading of digital meter before applying load was noted. Soon after applying the load there was movement of crack mouth and it was again shown by digital meter as final reading. The deflection between the final and initial readings of the digital meter gives CMOD. For measuring strain at different levels of the beam, demec points were used. For 1^{st} set beams the strains at five layers and 2^{nd} and 3^{rd} set beams the strains at three layers, were noted. The layers were at top level, crack tip level and in between mid level. Vertical deflection at center of the beam for each increment of load was noted.

Experimental methods to obtain softening portion

When the load was nearing the maximum, the pointer in the deflectometer of the proving ring showed a sudden jerk to the backward direction. But the deflection suddenly increased. From that point the load becomes less as deflection increases. At some point, the system becomes stable. Again on slight increase of load, the system again starts changing as like as before. In this way, with decreasing load, higher deflection is recorded before complete fracture of the beam. The negative inclination of the curve load Vs. deflection is called softening portion of the curve.

Discussion of results

In a total of 15 beams, one beam could not give any useful forms of graphs and tables. The results are presented in the form of load Vs. deflection, load Vs. CMOD and load Vs. offset ratio diagrams.

Load Vs. deflection

The Figs. 3 and 4 show the load vs. deflection for the beams of the second set and third set beams. As the experiment progressed, the recording of the data improved as also beam testing was done under improved set up conditions. Therefore second and third set results are completely reliable.

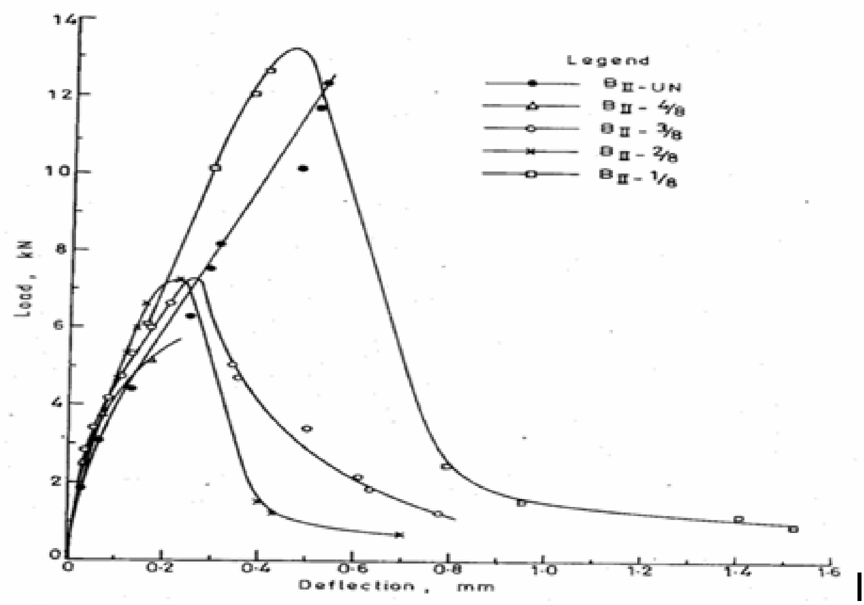

Figure 3: Load vs. deflection curves for 2^{nd} beams

It may be observed that the curve is mostly linear in the initial stage but in the pre-peak load region, it is highly non-linear. This confirms the assumption namely, post-peak softening modulus E_T is less than the peak elastic modulus, made in the derivation of expression for L_P [10]. The peak loads are shown in Table 1.

It is also observed that at peak load, for the same offset ratio for the 3^{rd} set beams, the deflections (at centre) are more than that of the second set beams. This is due to the fact that the third set beams span / depth ratio is more than that of second set beams.

Load Vs. CMOD

The Figs. 5-6, shows the load vs. CMOD for the second and third set beams respectively. As earlier observed by Jenq & Shah[15], the diagrams could be seen to be easily classified into 3 parts, i.e., linear, stable crack growth and unstable crack growth. It can be seen that the initial stage of the load – CMOD curve is

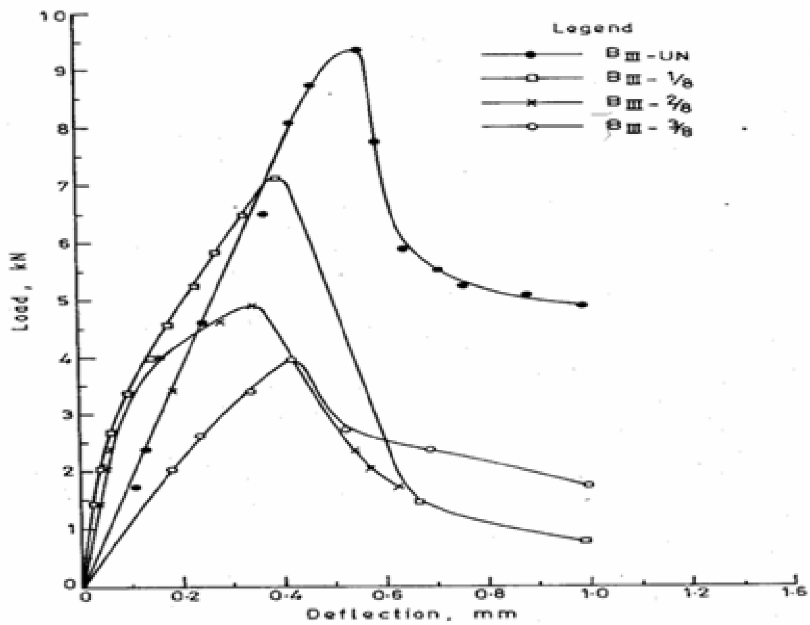

Figure 4: Load vs. Deflection for 3rd set beams

linear since there is no (or negligible) crack growth. In this stage, the whole system behaves linearly. As the applied load increases, the specimen starts to respond non-linearly. This nonlinear effect is mainly due to the generation and coalescence of micro-cracks (also termed as stable crack growth). As the micro-cracks coalesce into a macro-crack, further propagation of these macro-cracks may be arrested by aggregates and air voids unless the applied load is increased. As a result of this arresting mechanism, the crack may deviate from its original propagation plane and additional load and energy are required for the crack to propagate further. When the applied load reaches the peak load, the crack will begin to propagate in an unstable manner under load testing environment.

A different behavior can be seen in the load-CMOD curves when the offset ratio (2X/S, where X is the distance of the notch position from the centre of the beam and S is the span of the beam) is more than 0.75. The curves seem to have a "SNAP-BACK". A possible reason is that when the failure initiates at the centre notch, the CMOD at the off centre notch might decrease because of the crack opening at the centre.

Beam No.	Span (S) [mm]	Depth (W) [mm]	Breadth (B) [mm]	S/W	Notch Position from Support	a/W	Cube Strength [N/mm]	Days	Split Tensile strength [N/mm]	P_{max} [KN] Experimental	P_{max} [KN] Predictor	Prediction Pexp	Failed at
B_{I-UN}					_	0	48.89	28	3.348	30.57	29.6	0.968	Centre
$B_{I-4/8}$					4S/8=600		23.67	14	1.726	7.24	6.95	0.955	Notch
$B_{I-3/8}$	1200	300	150	4	3S/8=450	1/3	48.89	28	3.348	12.8	17.7	1.383	Notch
$B_{I-2/8}$					2S/8=300		23.67	14	1.726	13.27	13.83	1.042	Notch
$B_{I-1/8}$					S/8=150		48.89	28	3.348	30.57	29.6	0.968	Centre
B_{II-UN}					_	0	46.7		3.422	12.99	11.74	0.904	Centre
$B_{II-4/8}$					4S/8=690		46.7		3.422	5.7	5.598	0.982	Notch
$B_{II-3/8}$	1380	230	115	6	3S/8=517.5	1/3	46.7	28	3.422	7.27	7.464	1.027	Notch
$B_{II-2/8}$					2S/8=345		41.78		2.98	7.27	9.375	1.289	Notch
$B_{II-1/8}$					S/8=172.5		41.78		2.98	12.67	10.44	0.824	Centre
B_{III-UN}					_	0				9.38	6.41	0.683	Centre
$B_{III-4/8}$					4S/8=800					Failed at carrying & handling			
$B_{III-3/8}$	1600	200	100	8	3S/8=600	1/3	34.96	28	3.084	3.98	3.863	0.971	Notch
$B_{III-2/8}$					2S/8=400					4.93	5.80	1.176	Notch
$B_{III-1/8}$					S/8=200					7.15	6.41	0.896	

Table. 1 Comparison of P_{max} predicted by one parameter model Experimental values

Load Vs. offset ratio curve

Fig. 7 shows load Vs. offset ratio. This figure indicates that the final failure may not occur at the location of the notch always. In the off-central notches, there are two possible location of final failure, i.e., the location of notch and mid-span where the bending moment is maximum. It can be observed that the failure took place at mid-span for a very high offset ratio, while failure occurred at the notch position for low value of offset ratio. This is exactly similar to the behavior which has been observed by Jenq & Shah[15]. In fact, the predicted P_{max} from the one parameter model is very close to the experimental P_{max} even when the beams have failed at the off centre notches, which shows that the one parameter model obtained can also predict for mixed mode type of failure.

The Table 1 gives the P_{max} values which are predicted by one parameter model and comparison with the experimental values of the P_{max}. The agreement is very close in most of the cases. Out of 15 beams, in 9 cases, the results are within + 10% of experimental values. In the case of two beams ($B_{II-1/8}$, $B_{III-2/8}$) it is around +18%. But other three beams ($B_{I-3/8}$, $B_{II-2/8}$ & B_{III-UN}) give bad results. This is probably the erroneous readings shown by the deflectometer of the proving ring.

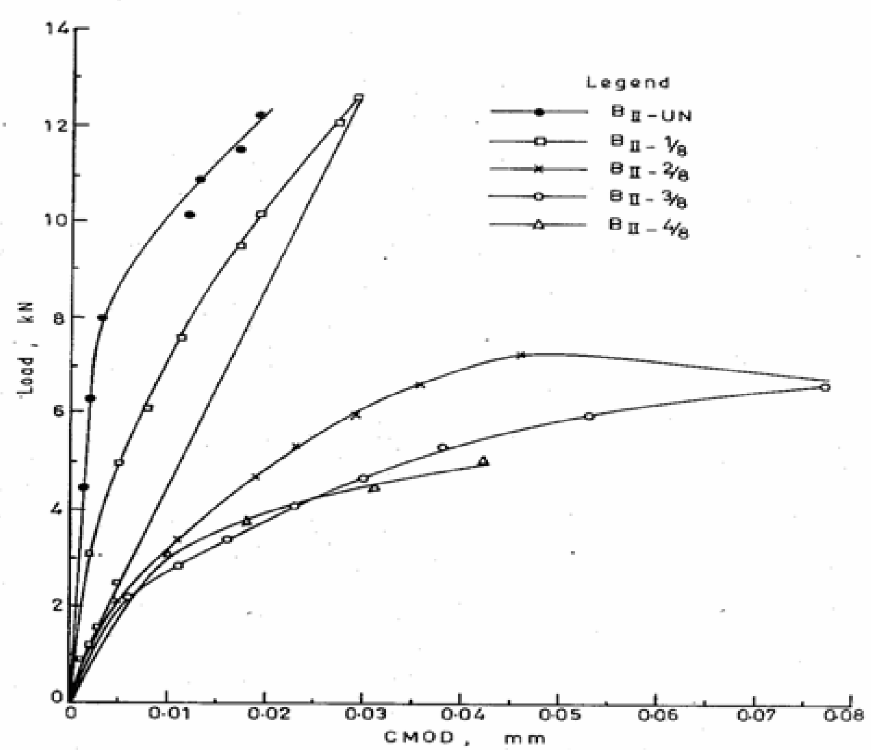

Figure 5: Load vs. cmod curves for 2^{nd} set beams

Conclusions

1. The one parameter model where, 'L_P',the process zone length is the only parameter dependent on the size, can very well predict, peak load of the beam which fails in mode-I type failure. It may be said that the errors are within permissible limit in cementetious materials like concrete.

2. The model can also be extended to predict peak load of the beam which fails in the mixed mode. A correction factor depending on the crack initiation angle can be applied in which case the error in predicted load could be further improved.

3. It may be required to repeat more number of experiments in order to obtain a consistent set of results.

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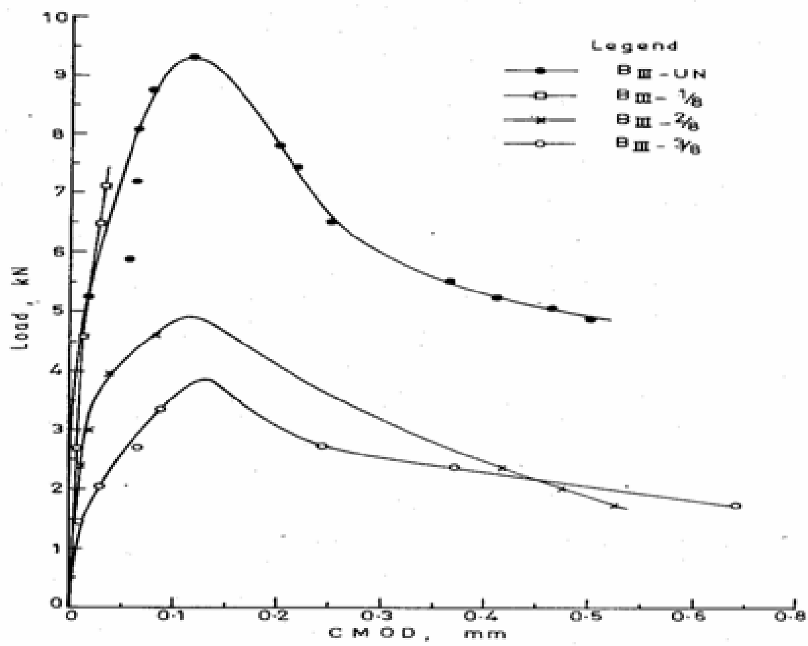

Figure 6: Load vs. cmod curves for 3rd set beams

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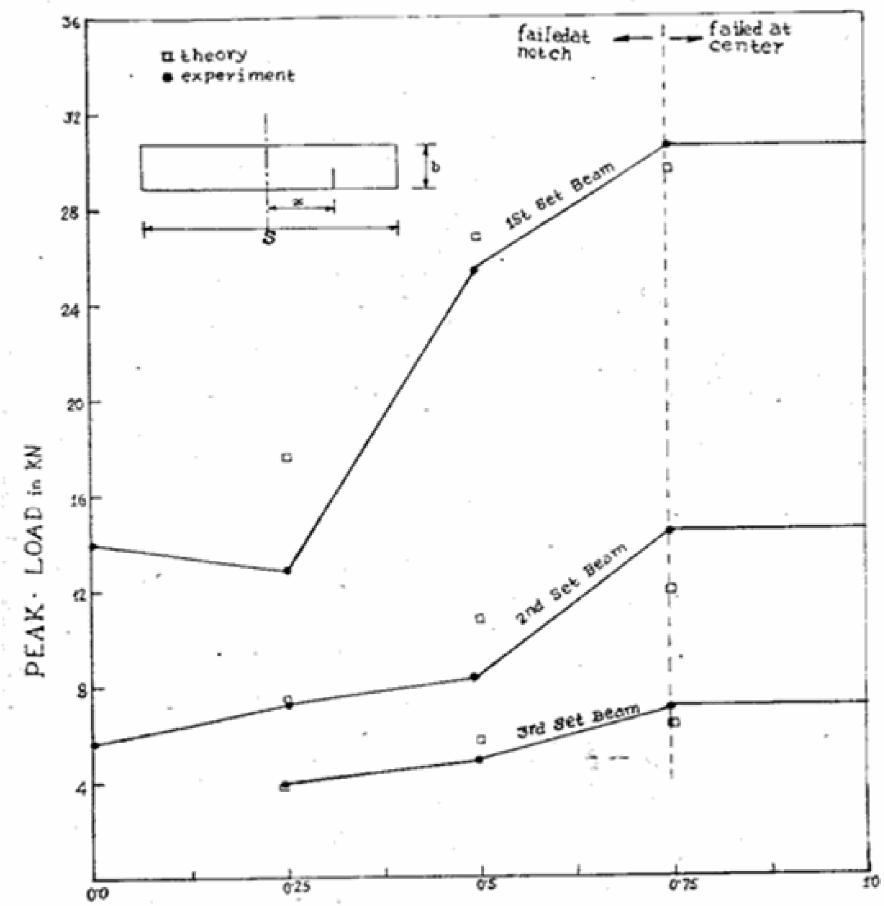

Figure 7: Comparison of experimental peak-load and Theoretical Predictions using One Parameter Model for different off-set ratios

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