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A new method for the dynamical symulation of mechanical systems using Matlab-Simulink

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Summary

The paper presents a new method for solving with precision of the differential equations of mechanical systems with two, three or more degrees of freedom. Always, the mechanical systems have coupled unknown quantities of differential equations. The method uncouples the unknown quantities using a graphic program: MATLAB- simulink.

Keywords: mechanical vibration, dynamical modeling, simulation

Introduction

A system with "n" degrees-of-freedom is represented in Fig.1 (where: k-suspension stiffness, c-suspension damping)

For this system the differential equations are:

$$[m]\{\ddot{x}.\} + [c]\{\dot{x}\} + [x]\{x\} = \{f(t)\} \tag{1}$$

Where:

[m] – the diagonal matrix of masses (nxn);

[c] – the matrix of dampings (nxn);

[k] – the matrix of stiffness –springs(nxn);

{f(t)} – the matrix of forces of disturbance (nx1);

$\{\ddot{x}\}$, $\{\dot{x}\}$, {x} – the matrixes of accelerations, speeds and **straight displacements**(nx1)

Always the matrix [c] is not a diagonal matrix and the resolution is possibly with classic approximate methods.

The principle of the new method

Considering the system with two degrees of freedom-Fig.2,the motion equations of the masses

m_1 and m_2 are:

$$m_1\ddot{x}_1 + c_1\dot{x}_1 + k_1x_1 + c_2(\dot{x}_1 - \dot{x}_2) + k_2(x_1 - x_2) = f_1(t)$$

$$m_2\ddot{x}_2 + c_3\dot{x}_2 + k_3x_2 - c_2(\dot{x}_1 - \dot{x}_2) - k_2(x_1 - x_2) = f_2(t) \tag{2}$$

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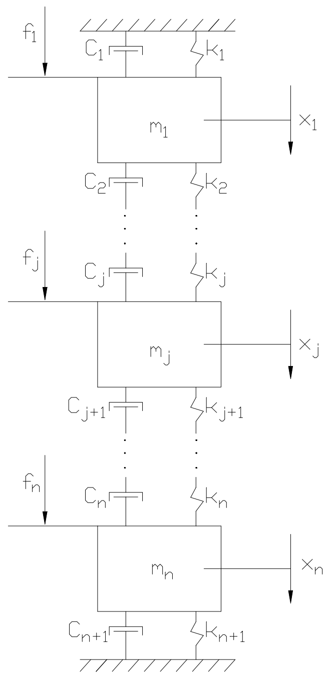

Figure 1: System with "n" degrees-of-freedom with straight displacement x

or:

$$m_1\ddot{x}_1 + (c_1+c_2)\dot{x}_1 + (k_1+k_2)x_1 - c_2\dot{x}_2 - k_2x_2 = f_1(t)$$
$$m_2\ddot{x}_2 + (c_2+c_3)\dot{x}_2 + (k_2+k_3)x_2 - c_2\dot{x}_1 - k_2x_1 = f_2(t) \tag{3}$$

The equations (3) can be written:

I/Method with "Derivative" blocks:

$$x_1 = \frac{1}{\mathrm{k}_1+\mathrm{k}_2}[f_1(t) - m_1\ddot{x}_1 - (c_1+c_2)\dot{x}_1 + c_2\dot{x}_2 + k_2x_2]$$
$$x_2 = \frac{1}{\mathrm{k}_1+\mathrm{k}_2}[f_2(t) - m_2\ddot{x}_2 - (c_2+c_3)\dot{x}_2 + c_2\dot{x}_1 + k_2x_1] \tag{4}$$

II/Method with "Integrator" blocks:

$$\ddot{x}_1 = \frac{1}{\mathrm{m}_1}[f_1(t) - (c_1+c_2)\dot{x}_1 - (k_1+k_2)x_1 + c_2\dot{x}_2 + k_2x_2]$$
$$\ddot{x}_2 = \frac{1}{\mathrm{m}_2}[f_2(t) - (c_2+c_3)\dot{x}_2 - (k_2+k_3)x_2 + c_2\dot{x}_1 + k_2x_1] \tag{5}$$

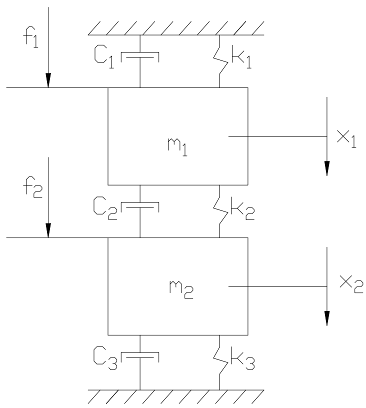

Figure 2: The system with two degrees of freedom with straight displacement

Where:

m_1, m_2 – masses;

k_1, k_2, k_3 – stiffness;

c_1, c_2, c_3 – vibration damping.

For exemplification, the system with differential equation (5) can be solved using a software in MATLAB-simulink. The block diagram in "simulink", using the system(5) is shown in Fig.3.

The programme-Fig.3, contains two subsystems(Ports & Subsystems):

-**subsystem 1**,for the first equation;

-**subsystem 2**,for the second equation.

The subsystems contain virtual blocks from:

*Math operations(Math function-pow, Product, Sum);
*Continuous (Integration, Derivative)

The inputs are:

*Constant(Sources):

m_1=20kg; m_2=40 kg; k_1=1000N/m; k_2=1500N/m; k_3=1200N/m; c_1=40Ns/m; c_2=60Ns/m; c_3=50Ns/m;

*Signal Generator(Sources), with Wave Form: $f = F \sin \omega t$ (f_1=100 N-5Hz, f_2=200 N-20Hz)

The outputs"Scope"represent the displacements,speeds and accelerations of the masses

$m_1$and m_2-Fig.4

Also, this method can be used for mechanical systems with **torsional vibra-**

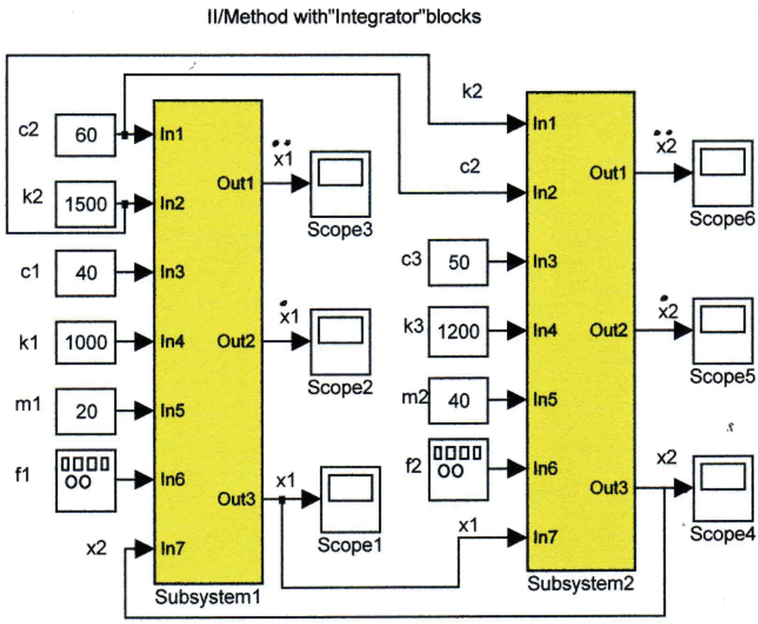

Figure 3: Block diagram for dynamic simulation in MATLAB-simulink

tions, where:
-mass “m” becomes mass moment of inertia “J”;
-force “F” becomes moment “M”;
-straight displacement “x” becomes angular displacement “φ”.

Applications of the method

This method has useful domain in the every **dynamic problems**.

A dynamic model of the motor vehicle [2],[5]with linear parameters: **suspension** and damper elements is an application of the program [4],that solves the systems with two degrees of freedom, having unknown coupled quantities. The program is usefully for the suspensions optimum designing.

In the domain of bioenginering, in the case of mobile vertebral prosthesis [1],[3], the method can be used for the dynamic simulation of **biomechanisms**. This biomechanisms can be assimilated with a mechanical system with more degrees--of –freedom-Fig.1.

Conclusions

The new method, in MATLAB-simulink, solves exactly the differential equations of Mechanical systems with more degrees-of-freedom, having unknown coupled

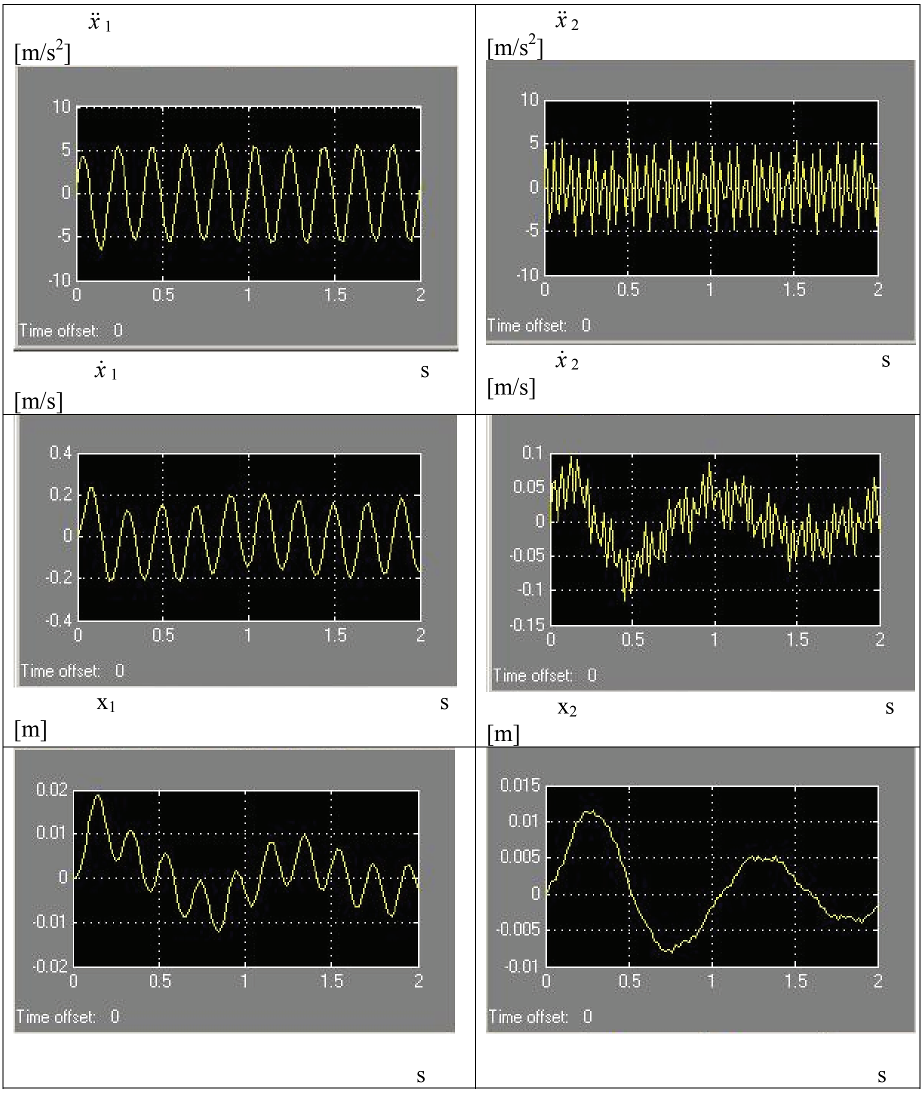

Figure 4: The displacements, speed and accelerations of masses m_1 and m_2

quantities [4]. The classic methods solves approximately this problem.

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