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Size effect studies on a notched plain concrete beam using initial stiffness method

B.K. Raghu Prasad[1], T.V.R.L. Rao[1], A.R. Gopalakrishnan[1]

Summary

A simple numerical method namely Initial Stiffness Method using finite element method has been employed to study the size effect which is prominent in concrete structures. Numerous experimental investigations performed on notched plain concrete beams subjected to three point or four-point bending have revealed the fracture process to be dependent on size of the structural member. It was found that, the nominal stress at maximum load decreases as the size of the structure increases. The nominal stress at failure on the characteristic dimension of structure is termed as size effect. This has also been explained in energy concepts as, the fracture energy G_f decreases with increase in structure size. This size effect is explained with regard to three parameters, namely, nominal stress at failure, post-peak slope of the load-displacement diagram and softening slope parameter α. The results obtained from the analysis of three point bend specimen of varying sizes using initial stiffness method, also confirms the size effect of concrete structures.

keywords: Initial stiffness method; Size effect; Concrete fracture energy; Notched plain concrete beam.

Introduction

The size effect is probably the most compelling reason for using the fracture mechanics in analyzing the cracking of concrete structures. The size effect can be explained clearly through a comparison of geometrically similar specimens but of different sizes. The nominal strength σ_N, at failure is calculated by dividing the maximum load with the uncracked ligament area. According to the classical theories, the above calculated apparent strength σ_N, should be same for all the specimens. But it is observed that this apparent strength σ_N, varies with the structure size. This dependence of σ_N on the specimen size is termed as size effect. The strength σ_N is found to decrease with increase in structure size. In the early stages, when fracture mechanics was introduced to analyze the concrete structures, principles of linear elastic fracture mechanics (LEFM) were used to explain this size effect. According to LEFM, the size effect is described by an inclined line of slope $1/2$ as illustrated in Fig. 1. In reality, the failures of concrete structures are not governed by LEFM principles. It is observed that the failure of concrete structures show a transitional behavior as shown by the curve in Fig. 1.

Hence, it can be said that neither strength criterion nor the principles of LEFM exactly describe the size effect of concrete structures. Even though, a similar size

[1]Department of Civil Engineering, Indian Institute of Science, Bangalore, India.

effect is observed in other materials, it is of paramount importance for concrete structures because the concrete structures are so large that real scale testing is impossible. Usually, small scale testing [RILEM Report 1,2] is done in laboratories and the results are extrapolated to real sizes. Hence, it is imperative for the design method to account for the dependence of the apparent strength of the structure on the characteristic dimension of the structure. This fact leads to the extensive amount of research to understand the size effect more precisely. An International workshop on the size effect in concrete structures [JCI Proc.,3] has been conducted in Japan. There has been a great emphasis on the inclusion of this size effect into the design codes.

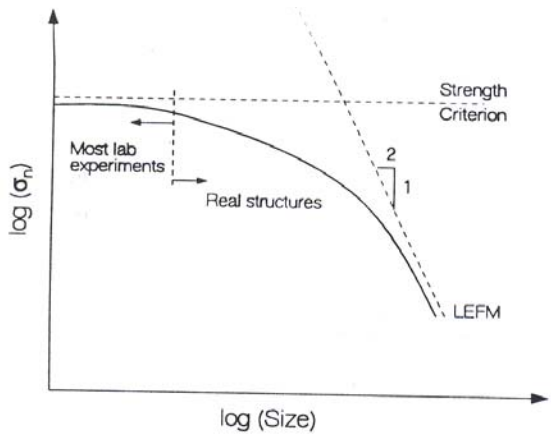

Figure 1: Size effect in concrete structures

Developments in size effect of concrete structures

Bazant and Cedolin [4] through numerical approach have predicted that, with the increase in structural size a transition from a strength collapse to a brittle collapse occurs. Similar effect was also reported by Petersson [5] and Carpinteri [6]. Gopalaratnam and Shaw [7] has reported softening response of plain concrete in direct tension. Reinforced concrete structures also show a similar size effect. This was reported by Bazant and Cedolin, Bazant and Kazemi [8-10], Mihashi and Nomura [11], and Michael D Kotsovos et al. [12]. Hence, it is generally accepted that smaller specimens tend to fail by plastic collapse, while the larger specimens fail by brittle failure. Plastic collapse can be analyzed by using the principles of limit analysis with a suitable strength criterion. LEFM principles are adequate to analyze brittle fracture behavior. Failures in the transition range in which, neither strength criterion nor LEFM principles are applicable are obviously more difficult to analyze. This seems to be the main problem in the failure analysis of concrete structures. Size effect is observed even in concrete specimens with dissimilar initial cracks. Weibull [13] attributed the size effect observed in the apparent strength of concrete structures to the randomness of the material strength. The observed size effect has been explained due to the variation in the amount of energy release by Bazant [14]. It has been shown that the rate of energy release from a larger structure is larger than from a smaller one. Apart from the size effects due to energy release and strength randomness, further size effects are introduced by diffusion phenomena such as drying of concrete or dissipation of hydration of heat, and boundary layer effects, caused by differences in composition and mechanical behaviour between the surface layer and the concrete in the core Sabnis [15]. Bazant [14] using the principles of dimensional analysis has shown that the structural size effect for geometrically similar specimens can be described by a simple relation of the following form,

$$\sigma_N = B\sigma_t' \left(1 + \frac{d}{\lambda_o} d_a\right)^{-1/2} \tag{1}$$

Where, σ_N = nominal stress at failure; P = maximum load; b = width thickness; σ_t' = direct tensile strength; B, λ_o = empirical constants to be determined by fitting test results for geometrically similar specimens of various sizes; d_a = maximum aggregate size.

The above size effect law was confirmed by various experimental results by Bazant and Pfeiffer, analytical models by Bazant and Lin and Bazant and Kazemi, [16,17,10]. Karihaloo [18] has presented analytical model for the size effect in notched three-point bend fracture specimen of concrete and other quasi-brittle materials based on the cohesive crack model. Prado and Van Mier [19] have reported mode-I fracture of concrete in a series of numerical analyses with a simple beam lattice model on the effect of material structure both on the pre-peak and softening regimes of the stress-deformation diagram in uniaxial tension. Other models called the softening beam model Ananthan et al. [20], Rao,TVRL and Raghu Prasad [21-23] and the Lattice model Raghu Prasad et al. [24-26] were attempted to study the fracture behavior of plain concrete beam.

Attempts to characterize the effects of structure size on the fracture behaviour by some non-dimensional numbers also known as brittleness numbers are also evident in the literature. Hillerborg [27] has used the non-dimensional ratio d/ℓ_{ch} and Carpinteri [28] has an another non-dimensional ratio $S = K_{1c}/\sigma_y b^{1/2}$. Only when the values for these numbers are known, quantitative predictions can be made. Bazant and Pfeiffer [16] have pointed that these numbers are objective only for comparison of different sizes of structure of the same geometry. They have proposed a brittleness number, $\beta = \frac{d}{\delta_o d_a}$. It is to be noted that β can be calculated only after determining δ_o either experimentally or by finite element analysis. β is also found to be independent of the shape and size of the structure. They have also concluded that for $\beta < 0.1$, the behaviour is close to plastic limit analysis and for $\beta > 10$, it is closer to LEFM. For $0.1 < \beta < 10$, nonlinear fracture analysis must be used. Thus, the brittleness number can only serve as basic qualitative indicator of the type of fracture of a given structure of some shape and size. The extensive amount of results in size effect emphasizes the need for the revision of building design codes. With this in view, the size effect in concrete structures using numerical model based on initial stiffness method [21-22] are studied for notched plain concrete beam subjected to three point bending. In addition to the size effect on nominal stress at failure two new parameters, namely post peak slope of load -deflection curve and softening slope parameter α have been studied to confirm the size effect in concrete structures.

Initial Stiffness Method of Analysis

Conventional linear elastic fracture mechanics principle cannot be directly applied to concrete due to the existence of slow crack growth, formation of nonlinear fracture process zone ahead of crack tip and softening of concrete. Hence, new fracture models have been developed to get a better description of the actual fracture behaviour of concrete. Keeping this in view, a simple numerical model called Initial Stiffness Method was developed for Mode-I fracture analysis in plain concrete beam [21-22]. The method leading to the present model was obtained from the Initial Stress Method by Zienkiewicz et al. [29]. The softening property of concrete enables the material to bear stress even after crossing the tensile strength as shown in Fig. 2. In the present method, stiffness matrices of the elements are generated only once based on the initial Young's modulus and is used for the rest of the analysis, even in the softening portion of concrete. Hence, the method is named as Initial stiffness method. Fracture is assumed to start as soon as the maximum principal stress reaches the limiting tensile stress. Thereafter, the stress carrying capacity of the softened concrete (decrease of stress with increasing deformation) is taken care of, without any modification in the stiffness of the element.

Ultimately, when the principal strain reaches the ultimate value, the element is removed by making the elements of stiffness matrix extremely small. Thus, although fracture is assumed to start as soon as the maximum principal stress reaches tensile strength σ_t, final fracture occurs when ultimate strain ε_u shown in Fig. 2 is reached. This ultimate strain ε_u, is a function of the softening slope parameter α as shown in Fig. 2. This parameter is a size dependent parameter and is based on the fracture mechanics concepts. The area under the stress-strain curve shown in Fig. 2 is a measure of the fracture energy G_f, which is the energy consumed in the formation and opening of all micro-cracks per unit area of crack plane. For a given value of E and σ_t as the value of the α increases, the area under the stress-strain curve increases and hence the fracture energy, G_f increases.

The details of the softening slope (either ultimate strain ε_u or the parameter, α)

are not mentioned by the researchers for the beams tested by them. Hence, in the present model in order to introduce the size dependent nature of the parameter, α it is varied and adjusted such that the value of the maximum load obtained from the analysis is close to the experimental one, according to the size of the beam by trial and error. Such a value of α is considered as the softening slope of the material used for that particular size. If the obtained maximum load from the analysis is less than the experimental value, the softening slope parameter α is increased and vice-versa.

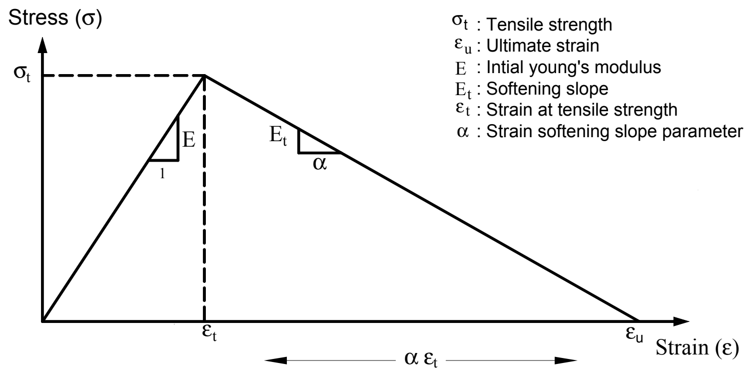

Figure 2: The stress strain curve of softening material like concrete

The strain softening property of concrete is considered in finite element analysis in a novel and simple way. Here, the principal stress in the region ahead of crack tip is assumed to be mainly in the direction perpendicular to the crack [22]. The analysis is based on displacement control. Suppose at some prescribed deflection, the maximum principal stress in some elements exceeds the tensile strength σ_t, the stress in excess of the tensile strength is converted into equivalent nodal loads of the element and applied in a reverse direction and the structure is analyzed again. The load and the stresses in all the elements obtained in this analysis are added to the values obtained during the prescribed displacement. Another check is applied to see whether the resultant maximum principal stresses in all the elements are according to the true stress-strain curve of the material. Thus, it is possible to trace the softening slope for all the elements. At any stage, if it is found that the strain in any element has reached its ultimate strain ε_u, the element is said to have completely failed and the stiffness matrix of the element is made in effective by making the components of the matrix negligibly small. The analysis is continued until the load bearing capacity of the structure reduces to zero. The model has been validated with many of the published experimental results on three point bend specimens.

Discussion of Results

The size effect as explained in the previous section has been observed in the results obtained from the present model. It is found that the nominal stress σ_N decreases with increase in the characteristic dimension of the structure. This has also been explained in energy concepts as the fracture energy G_f decreases with increase in structure size. The results obtained from the analysis of three point bend specimens of varying sizes using Initial stiffness method, also confirm the size effect. The size effect observed in the results obtaining from Initial stiffness method is explained in three ways. They are,

- Variation of nominal stress at failure with structure size.
- Variation of post peak slope of load-deflection diagram with structure size.
- Variation of softening slope parameter 'α' (of stress-strain curve) with structure size.

It is observed that the results tend to simulate the experimental size effect satisfactorily. The variation of nominal stress at failure with structure size is well known. The later two, that is, the variation of post peak slope of load-deflection diagram and softening slope parameter α with structure size are new in literature. The variations of these two parameters also confirm the existing size effects in concrete structures.

Variation of Nominal Stress at Failure with Structure size

The nominal stress at failure for the various beams analyzed using Initial stiffness method is calculated. The nominal stress at failure is calculated by dividing the maximum load P_{max} with the uncracked ligament area, Fig. 3. Although, it is known that load at failure P_{fail} is lower than P_{max}, P_{max} itself has been considered as P_{fail} for, in most of the results, P_{fail} is not mentioned. Initially, the nominal stress is calculated based on the maximum load obtained in the experiments. Table 1 gives the nominal stress at the maximum load, obtained for various beams. It can be seen from the Table 1 that in every set of beams performed by some experimentalist, the nominal stress at failure decreases as the structure size increases. The results of beams tested by Bazant et al. [16], indicate that the nominal stress at failure decreases from 1.56 N/mm^2 to 0.94 N/mm^2 as the structure size (uncracked ligament, d-a) increases from 31.6 mm to 126.6 mm. Similarly, it can be seen that σ_N of beams tested by Petersson et al. [30] decreases from 0.177 N/mm^2 to 0.137 N/mm^2 as the structure size (d-a) increased from 25mm to 100mm.

Similar size effect is observed in beams of same dimensions with varying initial crack length. This has been experimentally shown by Kim, et al. [31]. It is found that as the size of the initial crack increases, the nominal stress σ_N at failure decreases. The values in Table 1 indicate that the nominal stress σ_N at failure decreases from 0.356 N/mm^2 to 0.254 N/mm^2 as the initial crack length increased from 15.2 mm to 30.4 mm for the beams tested by Nallathambi et al. [32]. If the

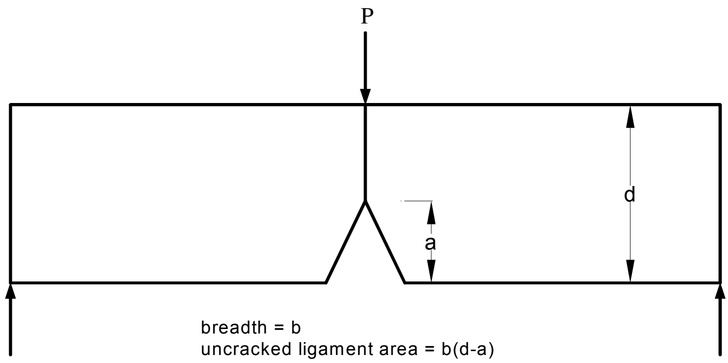

Figure 3: Calculation of nominal stress at maximum load

initial crack length 'a' increases while the characteristic dimension d is held constant, which means that uncracked ligament size (d-a) decreases, σ_N decreases. It can be inferred that, if the overall size 'd' is held constant and as the crack length 'a' is increased, the nominal stress σ_N decreases, and if the overall size 'd' increases, along with crack length 'a', holding d/a constant, the nominal stress σ_N decreases.

Table 1: Comparison of nominal stresses at maximum load (P_{max}) for various beams (both experimental and ISM results)

Names of Investigators	$S \times W \times B$ mm	a mm	(d-a) mm	P_{EXP} kN	$\sigma_{N_{EXP}}$ N/mm^2	P_{ISM} kN	$\sigma_{N_{ISM}}$ N/mm^2
Bazant *et al.* [1987]	95 × 38 × 38	6.4	31.6	1.87	1.56	1.881	1.566
	191 × 76 × 38	12.7	63.3	3.17	1.317	2.90	1.205
	381 × 152 × 38	25.4	126.6	4.74	0.98	4.543	0.94
Petersson [1980]	600 × 50 × 50	25.0	25.0	0.22	0.176	0.222	0.177
	2000 × 200 × 50	100.0	100.0	0.69	0.138	0.686	0.137
Jenq & Shah [1984]	305 × 76 × 26	25.4	50.6	0.82	0.623	0.822	0.624
	914 × 229 × 86	76.0	153.0	6.20	0.471	6.133	0.466
Hilsdorf *et al.* [1984]	500 × 100 × 100	50.0	50.0	1.92	0.38	1.88	0.376
	4000 × 800 × 400	400.0	400.0	42.91	0.268	42.17	0.263
Nallathambi *et al.* [1984]	600 × 76 × 80	15.2	60.8	1.70	0.349	1.731	0.356
	600 × 76 × 80	22.8	53.2	1.07	0.251	1.029	0.241
	600 × 76 × 80	30.4	45.6	0.90	0.247	0.929	0.254
Kaplan [1961]	305 × 76 × 101	12.7	63.3	4.40	0.688	4.543	0.710
	305 × 76 × 101	25.4	50.6	2.74	0.536	2.86	0.559
	305 × 76 × 101	38.0	38.0	1.58	0.411	1.55	0.404
Carpinteri *et al.* [1986]	800 × 100 × 100	50.0	50.0	1.13	0.226	1.01	0.202
	1260 × 250 × 100	125.0	125.0	3.63	0.29	3.51	0.281
	800 × 100 × 100	50.0	50.0	1.04	0.208	1.08	0.209
	1260 × 250 × 100	125.0	125.0	3.75	0.30	3.92	0.313

Suffix 'EXP' denotes experimental value.
Suffix 'ISM' denotes value obtained from Initial Stiffness method.

The nominal stress σ_N at failure are calculated using the maximum load obtained from the analysis using Initial stiffness method (P_{ISM}). The nominal stresses thus obtained are given in the Table 1 along with the experimental one. It can be observed that, the results from the initial stiffness method show a similar size effect as seen in experiments. The nominal stress σ_N decreases from 1.566 N/mm^2 to 0.94 N/mm^2 as the structure size increases from 31.6 mm to 126.6 mm for the beams tested by Bazant et al.[16]. From the Table 1, it can be seen that similar trend is observed in other cases also. The nominal stress σ_N at failure from experiment, P_{EXP} for various sizes of beams of the same experiment are plotted against the characteristic dimension of the structure (uncracked ligament, d-a) on a logarithmic scale. Fig. 4 shows such a curve for the beams tested by Bazant et al.[16]. The figure also shows the logarithmic plot of the nominal stress σ_N at failure based on maximum load obtained from the analysis using Initial stiffness method (P_{ISM})against the characteristic dimension of the structure uncracked ligament (d-a). The results from initial stiffness method are found to satisfactorily predict the experimentally observed size effect.

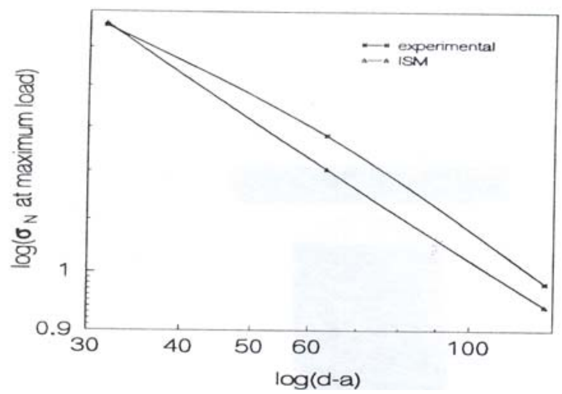

Figure 4: Variation of nominal stress at maximum load, P_{max} with increasing size, for the beams tested by Bazant and Pfeiffer [1987] (Size effect)

Variation of Post-Peak Slope of the Load-Deflection Curve with Structure Size

The experimental results confirm the fact that is already known viz., that as the structure size increases, the failure transforms from ductile nature to brittle nature. It can be observed that in the load-deflection diagrams, the post peak slope becomes steeper as the structure size increases. If the post peak slope is steeper Fig. 5(a), it indicates a brittle failure. The shallow post peak slope indicates a ductile failure Fig. 5(b). However, the above view of the size effect although implied has not been explicitly expressed in the literature available till now. In reality, the post peak slope of load-displacement diagram is not linear and hence, a definite slope cannot be determined for the same. Here an average notional slope is established for post peak curve of the load-displacement diagram. Two extreme points on the post peak slope of the load-displacement diagram (leaving the horizontal tail portion) are selected for this purpose. The slope (N/mm) denoting the rate of decrease of load

(N) per unit increase in displacement (mm) is calculated by dividing the difference in loads by the difference in displacement at those two points.

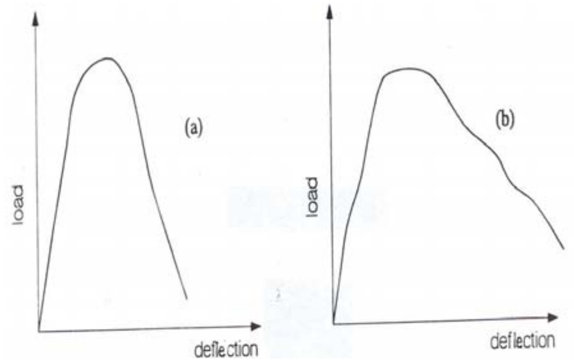

Figure 5: (a) Typical load-displacement diagram for a large size beam, (b) Typical load-displacement diagram for a smaller beam

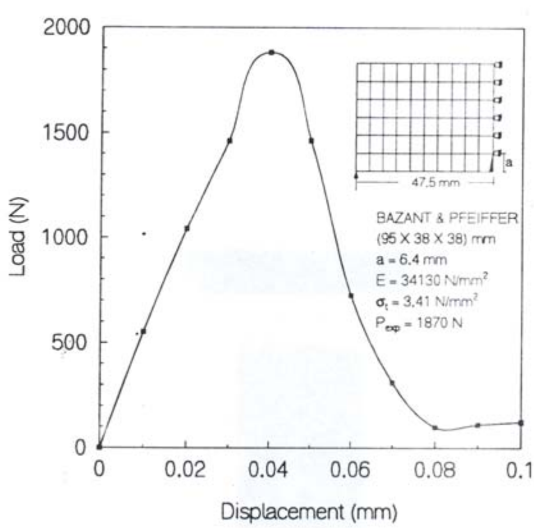

Figure 6: Predicted load vs displacement diagram using Initial Stiffness method of a beam tested by Bazant and Pfeiffer [1987] (using uniform mesh)

A higher value of this slope represents a steep post-peak load-displacement diagram thus showing brittle nature. This is usually seen in large structures, which fail in brittle manner. The shallow post peak load-displacement diagram is represented by a low value of this slope thus showing a ductile nature. This is generally observed in smaller structures, which show plastic collapse. Fig. 6 show the load-displacement diagram obtained from the initial stiffness method for the beam tested by Bazant et al.[16] using uniform mesh. Fig. 7 show the load-displacement diagram obtained from the initial stiffness method for the beam tested by Hilsdorf et al.[33] using finer mesh near the crack tip. It can be seen that the post peak slope becomes steeper as the structure size increases.

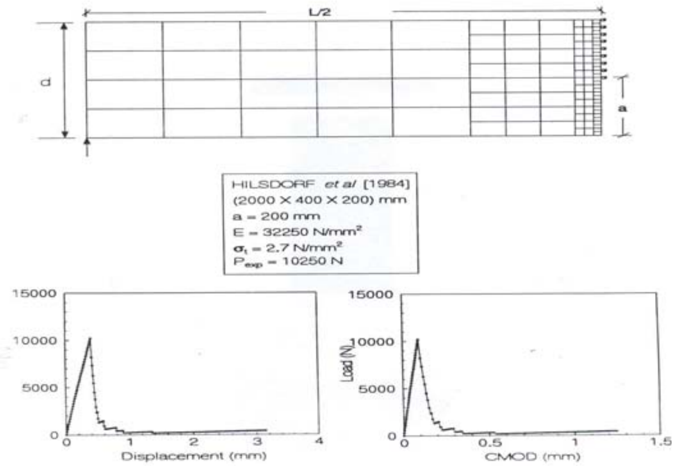

Figure 7: Predicted load vs displacement, load vs CMOD diagrams using Initial Stiffness method of a beam tested by Hilsdorf et al. using graded mesh (including discretization details)

A notional value of the post peak slope is evaluated to have a better understanding of the size effect. Table 2 gives the values of these slopes for some of the beams analyzed using the present model. It is seen that the value of this slope increases from 52700 N/mm to 93575 N/mm as the structure size (uncracked ligament, d-a) increases from 31.6 mm to 126.6 mm for the beams tested by Bazant et al.[16]. This clearly indicates the transition from ductile to brittle failure as the structure size increases. For the beams with varying crack length and constant overall dimensional the value of the post peak slope increases as the crack length decreases. This can be observed in the results of the beams tested by Nallathambi et al.[32], whereas the crack length is varied from 15.2 mm to 30.4 mm, the value of the post peak slope decreased from 15391 N/mm to 8800 N/mm. It can be inferred that, if the overall size 'd' is held constant, and as 'a' is increased; there is gradual transition from brittle failure to ductile failure.

Variation of Softening Slope Parameter α of Stress-Strain Curve with Structure Size

The results obtained by using initial stiffness method are analyzed with respect to the strain softening slope parameter α. Fig. 8 shows the stress-strain curve of a pure plastic material and a pure brittle material. If the value of α is infinity, it indicates pure plasticity (line BB'). If the value of α is zero, it indicates pure brittle

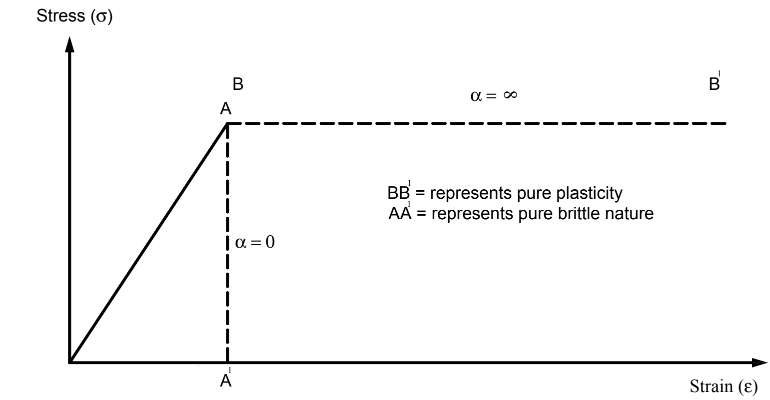

Figure 8: Variation of softening slope parameter α from plastic to brittle nature as 'a' is increased, there is gradual transition from brittle failure to ductile failure

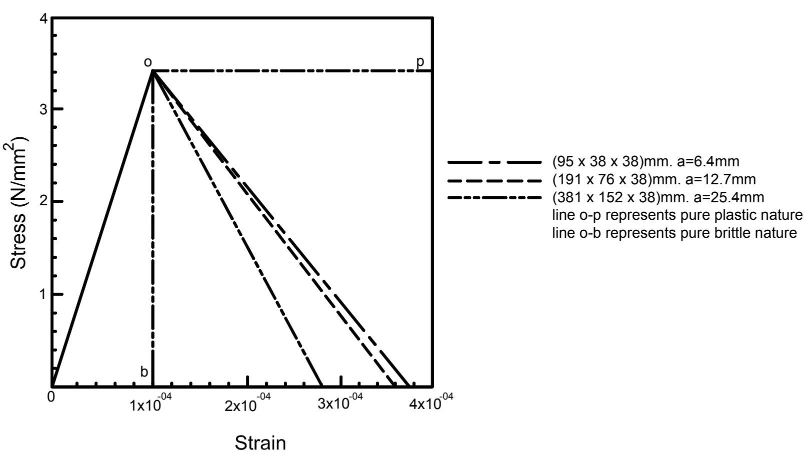

Figure 9: Variation of softening slopes for different sizes of beams tested by Bazant and Pfeiffer [1987]

nature (line AA').

The size effect based on the softening slope parameter α as shown in Fig. 2 is also new to the literature. Due to the absence of the value of this parameter α for the various beams reported in literature, α was varied keeping Young's modulus E, tensile strength σ_t constant until the maximum load obtained was closer to the experimentally observed one. The analysis of the softening slopes used for the various beams in the present model, indicate the size effect. It is observed that as the size of the beam increases, the softening slope parameter α required for the analysis of that beam decreases, thus showing a transition from plastic collapse to brittle failure. Beams of the same material properties are chosen for explaining the

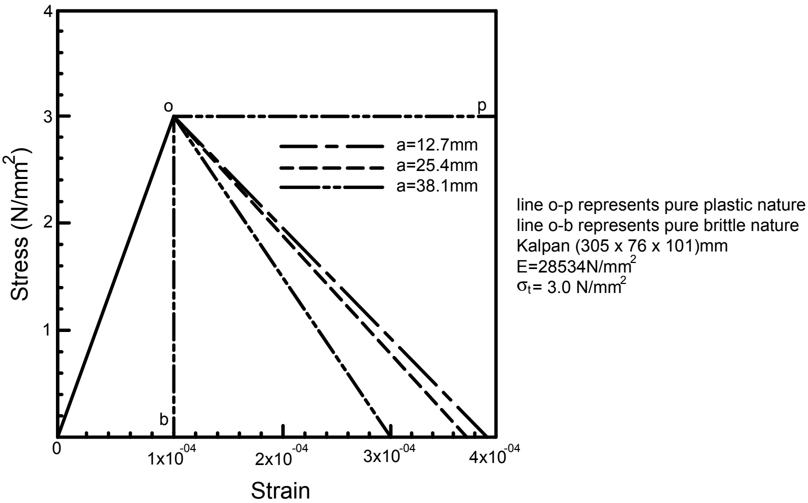

Figure 10: Variation of softening slopes for beams with varying initial crack length, tested by Kaplan [1961]

size effect. Table 3 indicates the values of the softening slopes used for the analysis of various beams using uniform mesh. Table 4 indicates the values of softening slopes used for the analysis of various beams using finer mesh near the crack tip. It can be concluded that, by and large, the softening slope becomes steeper as the size increases. Fig. 9 shows this variation clearly for the set of beams tested by Bazant et al.[16]. Similar variation of softening slope is seen in Fig. 10, for the set of beams tested by Kaplan [34]. It is observed from the Figures 9 and 10, that as the size of the specimen increases, the softening slope moves from pure plastic nature (line BB') to pure brittle nature line (AA').

Summary and Conclusions

The results from the initial stiffness method confirm the size effect, which is prominent in concrete structures. This size effect is explained with regard to three parameters, namely, nominal stress at failure, post-peak slope of the load-displacement diagram and softening slope parameter α. It is observed that as the structure size (uncracked ligament, d-a) is varied the nominal stress at the maximum load also varies. For the beams where a/d is held constant, the nominal stress σ_N is found to decrease with increasing uncracked ligament size. When the overall dimensions of the beams are kept constant and as the crack size is increased, the nominal stress σ_N is found to decrease.

Similar effect is observed in the analysis of the results with regard to post-peak slope. The value of the post-peak slope is found to increase for the beams with increasing uncracked ligament size provided a/d is held constant. For the

Table 2: Comparison of post peak slopes of load-displacement diagrams for various beams

Names of Investigators	$S \times W \times B$ mm	a mm	(d-a) mm	P_{EXP} kN	P_{ISM} kN	Postpeak slope N/mm
Bazant *et al.* [1987]	95 × 38 × 38	6.4	31.6	1.87	1.881	52700
	191 × 76 × 38	12.7	63.3	3.17	2.90	78333
	381 × 152 × 38	25.4	126.6	4.74	4.543	93575
Hilsdorf *et al.* [1984]	500 × 100 × 100	50.0	50.0	1.92	1.88	13292
	2000 × 400 × 200	200.0	200.0	10.25	10.22	81018
	4000 × 800 × 400	400.0	400.0	42.91	42.88	418813
Nallathambi *et al.* [1984]	600 × 76 × 80	15.2	60.8	1.70	1.73	15391
	600 × 76 × 80	22.8	53.2	1.07	1.078	9591
	600 × 76 × 80	30.4	45.6	0.90	0.903	8800
Petersson [1980]	600 × 50 × 50	25.0	25.0	0.22	0.211	1385
	2000 × 200 × 50	100.0	100.0	0.69	0.686	5218
Jenq & Shah [1984]	305 × 76 × 26	25.4	50.6	0.82	0.822	5450
Carpinteri *et al.* [1986]	800 × 100 × 100	50.0	50.0	1.13	1.17	8050
	1260 × 250 × 100	125.0	125.0	3.63	3.636	24928
	800 × 100 × 100	50.0	50.0	1.04	1.082	7321
	1260 × 250 × 100	125.0	125.0	3.75	4.09	27850

Suffix 'EXP' denotes experimental value.
Suffix 'ISM' denotes value obtained from Initial Stiffness method.

Table 3: Comparisonof softening slopes used for obtaining P_{max} using ISM (using uniform mesh)

Names of Investigators	$S \times W \times B$ mm	a mm	E N/mm^2	σ_t N/mm^2	P_{EXP} kN	P_{ISM} kN	α
Petersson [1980]	600 × 50 × 50	25.0	42500	4.2	0.22	0.222	1.0
	2000 × 200 × 50	100.0	30000	3.3	0.69	0.664	0.8
Jenq & Shah [1984]	305 × 76 × 26	25.4	34311	3.0	0.82	0.822	3.5
	914 × 229 × 86	76.0	34311	3.0	6.20	6.133	2.4
Hilsdorf *et al.* [1984]	500 × 100 × 100	50.0	32250	2.7	1.92	1.88	3.2
	4000 × 800 × 400	400.0	32250	2.7	42.91	42.17	1.9
Nallathambi *et al.* [1984]	600 × 76 × 80	15.2	33000	3.1	1.70	1.731	4.0
	600 × 76 × 80	30.4	33000	3.1	0.90	0.929	2.8
Bazant *et al.* [1987]	95 × 38 × 38	6.4	34130	3.41	1.87	1.88	2.7
	191 × 76 × 38	12.7	34130	3.41	3.17	2.88	2.1
	381 × 152 × 38	25.4	34130	3.41	4.74	4.95	1.8
Kaplan [1961]	305 × 76 × 101	12.7	28534	3.0	4.40	4.543	2.8
	305 × 76 × 101	25.4	28534	3.0	2.74	2.86	2.5
	305 × 76 × 101	38.0	28534	3.0	1.58	1.55	1.8
	305 × 76 × 101	25.4	38538	4.0	3.41	3.49	2.3
	305 × 76 × 101	38.0	38538	4.0	1.96	1.78	1.7
Carpinteri *et al.* [1986]	800 × 100 × 100	50.0	57400	3.58	1.13	1.01	1.3
	1260 × 250 × 100	125.0	57400	3.58	3.63	3.51	1.2
	800 × 100 × 100	50.0	60600	3.90	1.04	1.08	1.3
	1260 × 250 × 100	125.0	60600	3.90	3.75	3.92	1.2

Suffix 'EXP' denotes experimental value.
Suffix 'ISM' denotes value obtained from Initial Stiffness method

beams where the overall dimensions are held constant and as crack size is varied the

Table 4: Comparison of softening slopes used for obtaining P_{max} using ISM (using finer mesh)

Names of Investigators	S×W×B mm	a mm	E N/mm^2	σ_t N/mm^2	P_{EXP} kN	P_{ISM} kN	α
Petersson [1980]	600x50x50	25.0	42500	4.2	0.22	0.211	5.0
	2000x200x50	100.0	30000	3.3	0.69	0.686	4.6
Jenq & Shah [1984]	305x76x26	25.4	34311	3.0	0.82	0.78	6.0
Hilsdorf et al.[1984]	500x100x100	50.0	32250	2.7	1.92	1.84	8.5
	2000x400x200	200.0	32250	2.7	10.25	10.22	5.4
	4000x800x400	400.0	32250	2.7	42.91	42.89	5.6
Nallathambi et al. [1984]	600x76x80	15.2	33000	3.1	1.70	1.73	7.0
	600x76x80	22.8	33000	3.1	1.07	1.078	4.5
	600x76x80	30.4	33000	3.1	0.90	0.903	4.3
Carpinteri et al.[1986]	800x100x100	50.0	57400	3.58	1.13	1.17	6.0
	1260x250x100	125.0	57400	3.58	3.63	3.636	4.6
	800x100x100	50.0	60600	3.90	1.04	1.08	5.0
	1260x250x100	125.0	60600	3.90	3.75	4.08	4.5

Suffix 'EXP' denotes Experimental value.
Suffix 'ISM' denotes value obtained from Initial Stiffness Method.

value of the average post-peak slope is found to increase with increasing uncracked ligament size. The variation of the softening slope parameter α also confirms the size effect. The value of this parameter α is found to decrease as the structure size is increased thus showing a transition from plastic collapse to brittle failure. Thus, it can be seen that the results from the initial stiffness method confirms the size effect, which is prominent in concrete structures.

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