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On the collapse condition for a thin-plate subjected to axial compression

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Summary

In the present study, the collapse behavior of a thin-plate subjected to axial compression is investigated parametrically using the finite element method. It is revealed that the axial collapse of the plate, which has various hardening characteristics, is induced by two dominant factors: the stress limitation of the material and the limitation of in-plane deformation at the side edges of the plate. Then, a simplified collapse condition, which corresponds to two modes, is derived based on the plastic buckling theory and the effective width concept, and the validity of the simplified collapse condition is then verified by the comparison of numerical results obtained using various material and geometric properties.

Introduction

Thin-walled members, such as those constructed of high tensile strength steel sheets (HTSSS) and Aluminum alloy, are used in a steadily widening range of fields because newer designs require high-stiffness and lightweight structures. Thus, it is becoming ever more important to develop methods for evaluating the axial collapse load of plate, which is a basic component of thin-walled members, in the conceptual designing phase. In the present study, we examine the axial collapse mode of plates having various hardening characteristics and dimensions using finite element analysis. The simplified collapse conditions are derived based on the combination of the plastic buckling theory [1] and Karman's effective width concept [2].

Numerical Analysis Method

In the present study, the commercial FEM analysis package MSC.Marc [3] was used to simulate the elastoplastic deformation of a plate under axial compression, as shown in Fig. 1. The plate was discretized using 3,750 four-node quadrilateral thickness shell elements (Element type 75). The top and bottom of the plate were completely constrained, and the axial compression was applied from the top by forced displacements. The boundary condition of side edges (y= 0, b) was set to the simply supported condition. The width of the plate was set to b= 50 mm, and its length and thickness were set to $6b$ and t, respectively. The updated Lagrange method was used to formulate the geometric nonlinear behavior.

The plate used in the analysis was assumed to be constructed of a homogeneous and isotropic elastoplastic material and was assumed to conform to the von Mises

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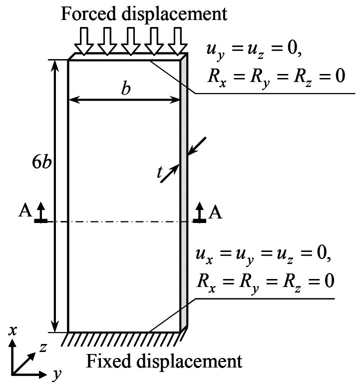


Figure 1: Analyzed model

yield criterion and to be well approximated by the bilinear (or trilinear) stress-strain relations and the Ludwik's hardening laws described by the following stress-strain relations:

$$\sigma_y = \sigma_0 + \frac{EE_h}{E - E_h}\bar{\varepsilon}^p, \tag{1}$$

$$\sigma_y = \sigma_0 + a(\bar{\varepsilon}^p)^n, \tag{2}$$

where σ_y is the yield stress, E is Young's modulus, E_h is the work-hardening coefficient, $\bar{\varepsilon}^p$ is the equivalent plastic strain, and a and n are material constants describing the work-hardening behavior. In the present study, we assume the Poisson's ratio to be ν = 0.3, Young's modulus to be E= 72.4 GPa, and the initial yield stress to be $\sigma_0 = 72.4$ MPa. The influence of the material properties on the axial collapse of a plate was investigated in terms of E_h, a and n.

Effective Width Concept and Collapse Modes

As has been shown by many researchers, plates can carry loads considerably in excess of the buckling loads because plates have a stable postbuckling behavior by side edge conditions. A method based on the concept of effective width proposed by Karman [2] is the most concise technique that can explain the collapse behavior of plates in postbuckling. In order to simplify design calculations, Karman ignored the center of the buckled plate and considered two fictitious strips of width b_{eff}(the effective width), which carry a uniform stress. In addition, he assumed that the elastic-perfectly plastic plate having a width of b_{eff}collapses when the buckling stress is equal to the yield stress σ_0. Thus, he derived a theoretical formula for the effective width and the collapse stress σ_{col}, as follows:

$$\frac{b_{eff}}{b}\sqrt{\frac{\sigma_{buc}^e}{\sigma_0}} = \sqrt{\frac{k\pi^2 E}{12(1-\nu^2)\sigma_0}}\left(\frac{t}{b}\right) \tag{3}$$

$$\sigma_{col} = \frac{b_{eff}}{b}\sigma_0. \tag{4}$$

However, based on the above-mentioned assumption, in order to evaluate the effective width at collapse, the stress limitation σ_0 must be known. That is, the conventional effective width concept cannot be applied to materials having a work-hardening property and a large t/b dimension, in which plastic buckling occurs.

In addition, Hopperstad et al. [4] proposed a sophisticated method of coupling the plastic buckling theory [1] and the effective width concept [2]. In their method, the plastic strain increment on the edge of both sides (the effective width) is assumed to be an input value, and the corresponding axial stress is calculated from the constitutive relation. By assuming the uniaxial stress state, the collapse stress is given as follows:

$$\sigma_{col} = \frac{b_{eff}(\varepsilon_{edge})}{b}\sigma_{edge}(\varepsilon_{edge}) \tag{5}$$

$$\sigma_{edge} = E_s\varepsilon_{edge}, \quad d\sigma_{edge} = C^{ep}d\varepsilon_{edge} \tag{6}$$

These equations show that the effective width b_{eff} and the stress at both sides of each step are functions of plastic strain ε_{edge}. Although evaluation of the effective width of each step requires knowledge of the stress limitation, they assumed the following plastic buckling stress for determining the stress limitation:

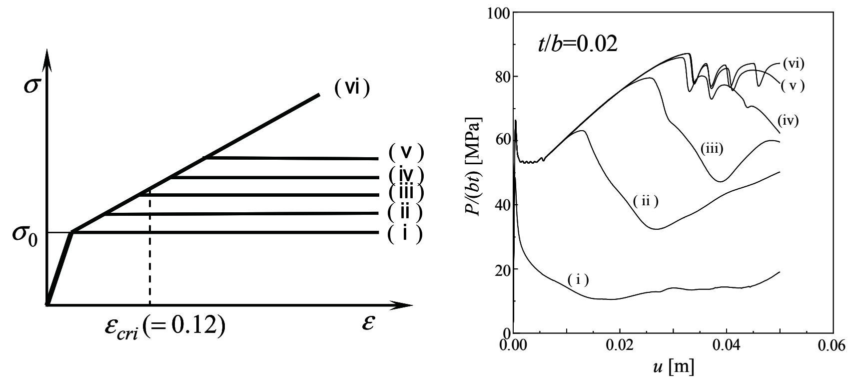
$$\sigma^p_{buc} = \eta\sigma^e_{buc} = \eta\frac{kE\pi^2}{12(1-\nu^2)}\left(\frac{t}{b_{eff}}\right)^2 = \sigma_{edge}, \tag{7}$$

where η is the function of the secant modulus E_s and the tangential stiffness C^{ep}. Calculating Eqs. (5)-(7) repeatedly reveals that the collapse stress occurs as a result of the balance of the reduction of the effective width and the stress increases.

As is evident from the above-mentioned discussion, the stress limitation is needed in order to employ the effective width concept. However, the collapse stress occurs after buckling, even if there is no stress limitation (e.g., bilinear hardening law). We then consider another factor of plate collapse using Fig. 2. Figure 2(b) shows the relationships between the compressive load P and the displacement u with the geometric property of t/b = 0.02 and with the material properties shown in Fig. 2(a). Figure 2(a) shows the trilinear stress-strain relations with same material properties of σ_0 = 72.4 MPa and E_h/E = 0.05, but with different tensile strengths σ_u(i)-(v), where the corresponding equivalent plastic strains for the five values of σ_u are set to be 0.0, 0.05, 0.1, 0.15, and 0.2, respectively. Figure 2 also shows the results for the case of bilinear hardening law (vi), i.e., $\sigma_u = \infty$, which indicate that larger collapse stresses σ_{col} appear with increasing tensile strength σ_u for stress-strain relations (i)-(iii). Thus, the dominant factor of plate collapse for stress-strain relations (i)-(iii) is the stress limitation of the material. In contrast, the collapse stresses for stress-strain relations (iv) and (v) do not depend on σ_u and are in approximate agreement with the value of the collapse stress for relation (vi). In addition, the in-plane deformations of folds at collapse are the same for these relations. Therefore, the dominant factor of plate collapse for stress-strain relations (iv)-(vi) is thought to be the limitation in-plane deformation. Furthermore, the parametric study reveals that the collapse mode due to the limitation of in-plane deformation at the side edge of the plate can be judged by the critical strain ε_{cri}, which is given as follows:

$$\varepsilon_{cri} = \gamma \frac{t}{b}, \tag{8}$$

where the coefficient γ prescribes the effect of the boundary conditions. In the case of condition S1, $\gamma = 6$.



(a) Constitutive relations (b) Compressive load – displacement relation

Figure 2: Collapse behavior of plate for several levels of tensile strength

Simplified Collapse Condition

In this section, we first examine the collapse conditions based on the plastic buckling theory [1] and the effective width concept [2], and we then attempt to derive a simplified prediction method for the collapse stress of a plate subjected to axial compression.

Let the collapse stress correspond to the maximum value of Eq. (5) because the compressive stress of the plate is a function of plastic strain, i.e.,

$$\left. \frac{d\sigma}{d\varepsilon_{edge}} \right|_{\sigma=\sigma_{col}} = \sigma_{edge} \frac{db_{eff}}{d\varepsilon_{edge}} + b_{eff} \frac{d\sigma_{edge}}{d\varepsilon_{edge}} = 0, \tag{9}$$

where the first term contributes to the reduction of the effective width due to plastic strain, and the second term contributes to the increase in axial stress on the edges due to plastic strain. Considering Eq. (7), the relationship between the effective width and the strain at the side edges after plastic buckling is given approximately as

$$b_{eff} \cong t\sqrt{A/\varepsilon_{edge}}. \tag{10}$$

Using Eqs. (5)-(10), the following expression is derived:

$$b\frac{d\sigma}{d\varepsilon_{edge}} = t\left(\frac{A}{\varepsilon_{edge}}\right)^{\frac{1}{2}}\left[C^{ep} - \frac{1}{2}E_s\right]. \tag{11}$$

Thus, the condition of collapse due to the stress limitation of the material is given by

$$f = C^{ep} - E_s/2 = 0. \tag{12}$$

In addition to the collapse condition (12), we examine a strain condition to judge the collapse stress caused by the stress limitation and the in-plane deformation. Next, we consider the condition whereby the effective width must satisfy $b_{eff} \leq b$. We can then obtain the condition of lower strain ε_{low} by Eqs. (6) and (7) as follows:

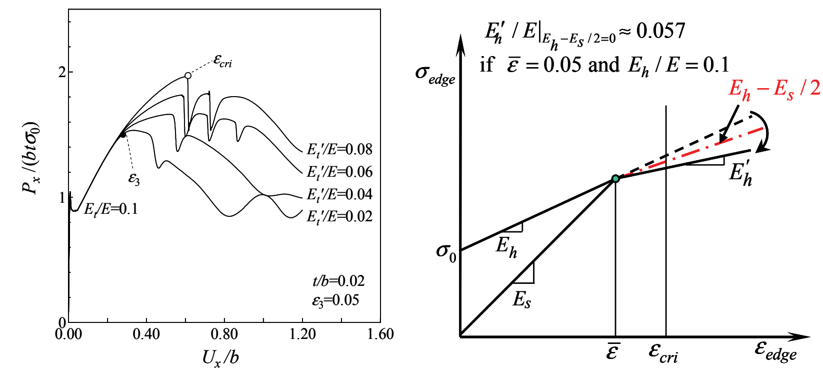
$$\frac{t}{b}\sqrt{\frac{A}{\varepsilon_{low}}} \leq 1. \tag{13}$$

Based on the above considerations, the primary conditions for collapse require Eq. (12). Furthermore, in order for collapse due to stress limitation to occur, the following strain condition must hold:

$$\varepsilon_{low} < \varepsilon < \varepsilon_{cri}. \tag{14}$$

Next, we verify the validity of collapse conditions (12) and (14) by comparing the obtained results with the numerical results obtained by FEM. In the calculation, the trilinear hardening law is adopted for the plate with *t*/*b*= 0.02. Figure 3(a) shows that the relation between compressive load and displacement with the first hardening coefficient was set to $E_h/E = 0.1$ until $\bar{\varepsilon} = 0.05$, while the second hardening coefficient was varied as E'_h/E = 0.02, 0.04, 0.06, and 0.08 (see Fig. 3(b)). Since the critical strain for the plate with *t*/*b*= 0.02 is $\varepsilon_{cri} = 0.12$, $\bar{\varepsilon} = 0.05$ satisfies Eq. (14). The second hardening coefficient that satisfies Eq. (12) is $E'_h/E \approx 0.057$. Figure 3 indicates that the collapse due to the stress limitation appears in the case of $E'_h/E < 0.057$. In contrast, the collapse due to the in-plane deformation appears when the strain on the edges reaches ε_{cri}, in the case of $E'_h/E < 0.057$. As mentioned above, we can judge the collapse mode of the plate beforehand by using proposed conditions (12) and (14).

We now consider the prediction method for the collapse stress due to in-plane deformation. In the collapse mode of this case, nearly the total load is carried by



(a) Compressive load – displacement relation (b) Four types constitutive relation

Figure 3: Verification of collapse condition

two strips of width $b_{eff}/2$ directly adjacent to the edges of the plate, and thus the effective width concept can be applied. By considering Eqs. (5), (8) and (10), the effective width at collapse due to in-plane deformation is given by

$$\left.\frac{b_{eff}}{b}\right|_{In-plane} = \frac{t}{b}\left(\frac{A}{\varepsilon_{cri}}\right)^{1/2} = \sqrt{\frac{t}{b}}\sqrt{\frac{A}{\gamma}}. \tag{15}$$

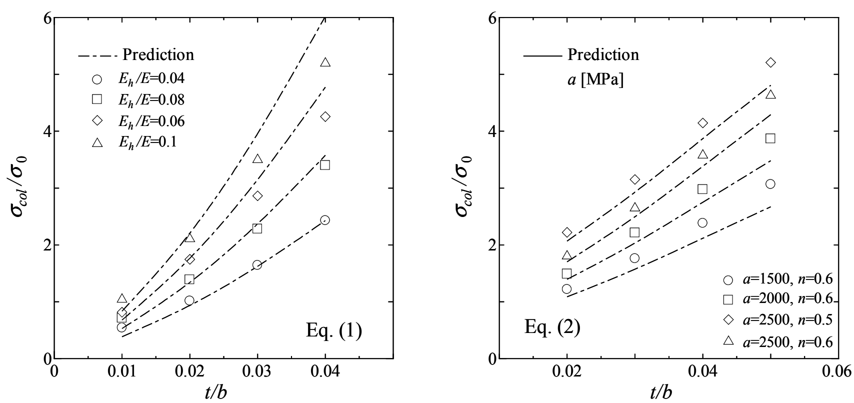


Figure 4: Comparison of collapse stress due to in-plate deformation between the proposed method and BEM

Figure 4 compares the values for σ_{col} obtained by the FEM with the values obtained using Eqs. (5) and (15). These data sets are in reasonably good agreement with each other.

Conclusions

The collapse mode of a plate subjected to axial compression was revealed. The simplified collapse condition based on the plastic buckling theory and the effective width concept was proposed, the validity of which was verified by comparing simplified collapse condition to the results obtained by FEM. The collapse stress due to stress limitation can be also predicted using Eqs.(5)-(11).

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