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Conjugate Heat Transfer of Forced Convection with Viscous Dissipation for Visco-Elastic Fluid Past a Flat Plate Fin

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Summary

A conjugate forced convection with viscous dissipation heat transfer problem of a second-grade visco-elastic fluid past a flat plate fin has been studied. Governing equations include heat conduction equation of the fin, and continuity equation, momentum equation and energy equation of the fluid, were analyzed by a combination of a series expansion method, the similarity transformation and a second-order accurate finite-difference method. Solutions of a stagnation flow ($\beta = 1.0$) at the fin tip and a flat plate shape (wedge flow $\beta = 0.0$) on the fin surface were obtained by a generalized Falkner-Skan flow derivation. These solutions were used to iterate with the heat conduction equation of the fin to obtain distributions of the local convective heat transfer coefficient and the fin temperature. Ranges of dimensionless parameters, the Prandtl number (Pr), the elastic number (E), the viscous dissipation parameter (Ec) and the conduction-convection coefficient (Ncc) are from 0.1 to 100, 0.001 to 0.01, 0 to 0.1 and 0.05 to 2.0, respectively. Results indicated that Results indicated that elastic effect in the flow can increase the local heat transfer coefficient and enhance the heat transfer of a flat plate fin. Also, same as results from Newtonian fluid flow and conduction analysis of a flat plate fin, a better heat transfer is obtained with a larger Ncc, E, Ec and Pr.

keywords: conjugate heat transfer, second-grade fluid, flat plate fin, forced convection, viscous dissipation.

Introduction

The forced-driven and temperature-dependent nature of the interaction on interfaces causes the flow and the temperature fields to be specific according to the temperature distribution along surfaces of a fin. Fins are appendages intimately connected to the primary surface for the augmentation of heat transfer. With some special cases, combined conduction-convection effects, the most frequent application is one in which an extended surface is used specifically to enhance the heat transfer rate between a solid and an adjoining fluid. Such an extended surface is termed a fin. There are several fin applications. Consider the arrangement for cooling engine heads on motorcycles, automobiles, lawnmowers, air-conditioner, refrigerators, and for electric power transformers. The working fluids include air,

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refrigerants oil. Since the flow and temperature fields have a strong influence on the convective heat transfer coefficient that, in turn, strongly affects the fin temperature distribution, the tightness of the coupling is apparent. The analysis of a fin must deal with the energy conservation equation of the fin, the equations of mass, momentum and energy conservation in the surrounding fluid. Even in the coupling, apparently variations in the local thermophysical properties and the fluid flow temperature distributions still can be calculated by analyzing the coupled equations of the flow and the fin. This means that the flow and temperature fields in the fluid and the temperature distribution along surfaces of a fin must be solved simultaneously in a heat transfer problem of a fin/fluid system. Conjugate heat transfer analysis of a fin in a second-grade visco-elastic fluid flow is the major concern of the present investigation. The problem considered a fin that transfers heat to or from a surrounding second-grade fluid flow by the forced convection. The analysis of the flow field in a boundary layer adjacent to the fin is very important in the present problem, and is also an essential part in the area of the fluid dynamics and heat transfer. Especially, understanding boundary layer flows and heat transfer of non-Newtonian fluids has become important in recent year [1]. Srivatsava [2], and Rajeswari and Rathna [3] studied the non-Newtonian fluid flow near a stagnation point. Mishra and Panda [4] analyzed the behavior of second-grade visco-elastic fluids under the influence of a side-wall injection in an entrance region of a pipe flow. Rajagopal et al. [5] studied a Falkner-Skan flow field of a second-grade visco-elastic fluid. Massoudi and Ramezan [6] studied a wedge flow with suction and injection along walls of a wedge by the similarity method and finite-difference calculations. Hsu et al. [7] also studied the flow and heat transfer phenomena of an incompressible second-grade visco-elastic fluid past a wedge with suction or injection. An excellent review of boundary layers in non-linear fluids was recently written by Rajagopal [8]. These are related studies to the present investigation about second-grade visco-elastic fluids. However, conventional studies of conjugate problems have not included non-Newtonian fluids as the working fluid. The system to be analyzed in the present study is a flat plate fin in a second-grade visco-elastic fluid flow. Due to the coupling nature between the fin and the fluid, the present analysis is different from previous researches concerning forced convection about a flat plate fin. Those studies have dealt primarily with a plate having prescribed convective heat transfer coefficient that yield similar or non-similar solutions [9,10]. There are some related conjugate problems concerning a fin in a Newtonian flow, for instance, a complete model study about the forced convection on a rectangular fin has been investigated by Sparrow and Chyu [11]; the effect of the Prandtl number on the heat transfer from a rectangular fin has been studied by Sunden [12]. Also, Luikov and his co-workers solved the conjugate forced convective problem along a flat-plate both numerically [13] and analytically [14-16]. Lately, relative researches in connection with conjugate heat transfer almost all were working for Newtonian fluid [17-19]. On the other hand, researchers in connection with visco-elastic fluid or second grade non-Newtonian fluids [20-21], but there are not the conjugate heat transfer problems, therefore the plan proceed especially toward this ways.

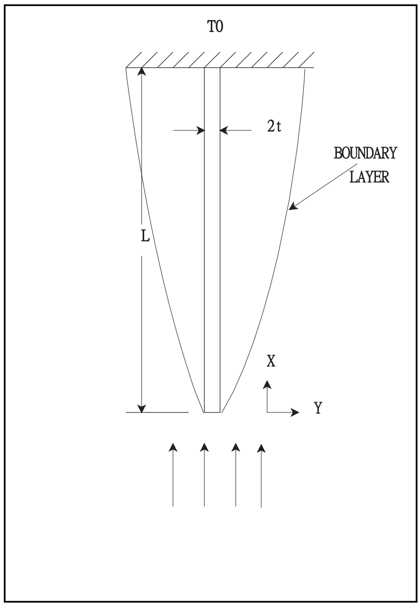

Figure 1: A sketch of the physical model for conjugate heat transfer of forced convection with viscous dissipation for visco-elastic fluid past a flat plate fin

The objective of the present analysis is to study the heat transfer of a flat plate fin cooled or heated by a high or low Prandtl-number, second-grade visco-elastic fluid with various conduction-convection parameters. An extension of previous works is then performed to investigate the conjugate heat transfer of a second-grade visco-elastic fluid past a flat plate fin. A schematic diagram of the flat plate fin is shown in Figure 1 to illustrate the physical situation and symbols of parameters needed for the analysis. Two types of flow fields, a stagnation flow and a pin shape flow respectively, are included. The Rivlin-Ericksen model for grade-two fluids is used in the momentum equations. Effects of dimensionless parameters, the Prandtl number (Pr), the elastic number (E) and the conduction-convection coefficient (N_{cc}) are main interests of the study. Flow and temperature fields of the stagnation flow and the wedge flow are analyzed by utilizing the boundary layer concept to obtain a set of coupled momentum equations and energy equations. A similarity transformation with wedge-type parameters and a series expansion method are then used to convert the nonlinear, coupled partial differential equations to a set of non-linear, decoupled ordinary differential equations. In the present conjugate problem, these decoupled equations and the conduction equation of the fin is then solved iteratively to obtain the temperature distribution and the local convective heat transfer coefficient along the fin by a second-order finite difference method. While the difference form of the fin conduction equation has previously been solved by either a relaxation procedure [23,24] or a direct matrix-inverse method [25,26] and the Runge-Kutta integration method [27,28], a simple and stable direct Gauss elimination method [29] is used in the present study. The authors Hsu et al. [35] had been studied the similar topic, but not considered the viscous dissipation. Present study is an extension to it, the energy equation increase the viscous dissipation item, through a similarity transformation and perturbation method to solve the conjugate heat transfer problem. To the author's knowledge, the influence of viscous dissipation on conjugate heat transfer of forced convection for visco-elastic fluid past a flat plate fin has not yet discussed in the literature. The detail formulas, data and figures have obtained different results as shown in this study.

Theory and Analysis

The Rivlin-Ericksen model [30] for a homogeneous, non-Newtonian, second-grade visco-elastic fluid is used in the present wedge flow. The model equation is expressed as follows:

$$\mathbf{T} = -P\mathbf{I} + \mu\mathbf{A}_1 + \alpha_1\mathbf{A}_2 + \alpha_2\mathbf{A}_1^2 \tag{1}$$

Where P is pressure, μ is dynamic viscosity, α_1 and α_2 are first and second normal stress coefficients which are related to the material modulus. The kinematic tensors $\mathbf{A}_1$ and $\mathbf{A}_2$ are defined as

$$\mathbf{A}_1 = grad\mathbf{V} + (grad\mathbf{V})^T \tag{2}$$

$$\mathbf{A}_2 = \frac{d}{dt}\mathbf{A}_1 + \mathbf{A}_1 \cdot (grad\mathbf{V}) + (grad\mathbf{V})^T \cdot \mathbf{A}_1 \tag{3}$$

Where $\mathbf{V}$ is velocities and d/dt is the material time derivative. As done by Rajagopal [25], the present researchers substituted equation (1) into momentum equations

$$\rho\frac{d}{dt}\mathbf{V} = div\mathbf{T} + \rho\mathbf{b} \tag{4}$$

And assumed that the fluid is incompressible and the flow is in isochoric motion to obtain

$$div\mathbf{V} = 0 \tag{5}$$

For the steady, two-dimensional laminar flow under conservative body force $\mathbf{b}$, the following were defined:

$$P^* = P - (2\alpha_1 + \frac{\alpha_2}{2})(\frac{\partial u}{\partial y})^2 + \rho\Phi \tag{6}$$

$$\mathbf{b} = \nabla\Phi \tag{7}$$

From Bernoulli's principle and the substitution of the edge velocity u_e, the following was obtained:

$$u_e\frac{\partial u_e}{\partial\chi} = -\frac{1}{\rho}\frac{\partial P^*}{\partial\chi} \tag{8}$$

Consequently, one can eliminate the pressure term in the momentum equation and obtain the dimensionless boundary-layer equations

$$\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0 \tag{9}$$

$$U\frac{\partial U}{\partial X} + V\frac{\partial U}{\partial Y} = U_e\frac{dU_e}{dX} + \frac{\partial^2 U}{\partial Y^2} + E\left[\frac{\partial}{\partial X}\left(U\frac{\partial^2 U}{\partial Y^2}\right) + \frac{\partial U}{\partial Y}\frac{\partial^2 V}{\partial Y^2} + V\frac{\partial^3 U}{\partial Y^3}\right] \tag{10}$$

Where $E = \alpha_1 Re_L/\rho L^2$, Re_L is the Reynolds number, L is the characteristic length. The corresponding dimensionless parameters are

$$X = \frac{x}{L} \quad Y = \frac{y}{L}\sqrt{Re_L} \quad U = \frac{u}{U^*} \quad V = \frac{\upsilon}{U^*}\sqrt{Re_L} \quad U_e = \frac{u_e}{U^*} \tag{11}$$

The dimensionless boundary conditions are

$$Y = 0 \quad U = 0 \quad V = 0 \quad Y \to \infty \quad U \to U_e(X) \tag{12}$$

By using the stream function ψ one can define

$$U = \frac{\partial\psi}{\partial Y} \quad V = -\frac{\partial\psi}{\partial X} \tag{13}$$

And substitute into equation (10) to get

$$\begin{aligned}\frac{\partial\psi}{\partial Y}\frac{\partial^2\psi}{\partial X\partial Y} - \frac{\partial\psi}{\partial X}\frac{\partial^2\psi}{\partial Y^2} &= U_e\frac{dU_e}{dX} + \frac{\partial^3\psi}{\partial Y^3}\\ &+ E\left[\frac{\partial}{\partial X}\left(\frac{\partial\psi}{\partial Y}\frac{\partial^3\psi}{\partial Y^3}\right) - \frac{\partial^2\psi}{\partial Y^2}\frac{\partial^3\psi}{\partial Y^2\partial X} - \frac{\partial\psi}{\partial X}\frac{\partial^4\psi}{\partial Y^4}\right]\end{aligned} \tag{14}$$

Where $E = \frac{\alpha_1 Re_L}{\rho L^2}$ is the visco-elastic parameter. Where L is characteristic length or fin length and $T_w = T_\infty + A = constant$. The boundary conditions are written as

$$Y = 0 \quad \frac{\partial\psi}{\partial Y} = 0 \quad \frac{\partial\psi}{\partial X} = 0 \quad Y \to \infty \quad \frac{\partial\psi}{\partial Y} \to U_e(X) \tag{15}$$

The visco-elastic model is applicable for diluted polymer fluids under retarded-motion expansion. So one can assume E«1 and expand the stream function ψ with respect to E as

$$\psi = \psi_0 (X,Y) + E\psi_1 (X,Y) + \cdots + E^n \psi_n (X,Y) \tag{16}$$

Substituting equation (16) into equations (14) and (15), and introducing the similar transformation parameters

$$\eta = \left(\frac{m+1}{2}\right)^{1/2} X^{(m+1)/2} Y \tag{17}$$

$$f_0 (\eta) = \left(\frac{m+1}{2}\right)^{1/2} \psi_0 X^{-(m+1)/2} \tag{18}$$

One can obtain a set of nonlinear ordinary differential equations from the concepts of perturbation technique and power series expansion. The equation of the zeroth-order term, f_0 is of the form

$$f_0''' + f_0 f_0'' + \beta \left[1 - (f_0')^2\right] = 0 \tag{19}$$

Where $\beta = 2m/(m+1)$ is the shape factor of the wedge. Also from the potential flow theory, the edge velocity U_e is expressed as

$$U_e = X^m \tag{20}$$

The boundary conditions are then written as

$$f_0(0) = 0 \quad f_0'(0) = 0 \quad f_0'(\infty) \to 1 \tag{21}$$

Similarly, by assuming

$$f_1 (\eta) = \left(\frac{2}{m+1}\right)^{1/2} \psi_1 X^{(1-3m)/2} \tag{22}$$

And performing the similarity transformation, one can also obtain a nonlinear ordinary differential equation

$$\frac{m+1}{2} f_1''' + \frac{m+1}{2} f_0 f_1'' - (3m-1) f_0' f_1' + \frac{3m-1}{2} f_0'' f_1 + (3m-1) f_0' f_0''' - \frac{3m-1}{2} \left(f_0''\right)^2 - \frac{m+1}{2} f_0 f_0'''' = 0 \tag{23}$$

For the first-order term, f_1 the corresponding boundary conditions of the equation are

$$f_1(0) = f_1'(0) = 0, \quad f_1'(\eta \to \infty) \to 0. \tag{24}$$

In the present study, we simply set $\beta = 0\,(m = 0)$ and $\beta = 1.0\,(m = 1.0)$ respectively, to represent a wedge flow and a stagnation flow. Consequently, the velocity distribution can be obtained by solving equations (19-21) and (23-24) with numerical methods. By introducing the non-dimensional temperature,

$$\theta = \frac{T - T_e}{T_f - T_e} \tag{25}$$

The non-dimensional energy equation (including the viscous dissipation) in the boundary layer is written as

$$U\frac{\partial \theta}{\partial X} + V\frac{\partial \theta}{\partial Y} = \frac{1}{\Pr}\frac{\partial^2 \theta}{\partial Y^2} + \quad \frac{\upsilon}{C_p}(\frac{\partial U}{\partial Y})^2 \tag{26}$$

With the boundary conditions

$$\theta\,(X,0) = 1, \quad \theta\,(X,\infty) = 0. \tag{27}$$

To utilize the concept of the local similarity transformation, one can define

$$\xi = X^{m-1} \tag{28}$$

And assume that the non-dimensional energy equation can be expanded according to E as

$$\theta = g_0\,(\xi,\eta) + E g_1\,(\xi,\eta) + \cdots + E^n g_n\,(\xi,\eta)\,. \tag{29}$$

Finally, by substituting equation (29) into equation (26), the zero-order equation with respect to E and corresponding boundary conditions are

$$\frac{\partial^2 g_0}{\partial \eta^2} + \Pr.f_0\frac{\partial g_0}{\partial \eta} = \Pr\frac{2\,(m-1)}{m+1}.\xi.f_0'.\frac{\partial g_0}{\partial \xi} - \Pr\cdot Ec\cdot \xi^{\frac{2m}{m-1}}\cdot f''^2_0 \tag{30}$$

$$g_0\,(\xi,\eta = 0) = 1 \quad g_0\,(\xi,\eta \to \infty) = 0. \tag{31}$$

The first-order equation and boundary conditions are

$$\frac{\partial^2 g_1}{\partial \eta^2} + \Pr\left(f_0\frac{\partial g_1}{\partial \eta} + \frac{3m-1}{2}.\xi.f_1.\frac{\partial g_0}{\partial \eta}\right)$$
$$= \Pr\left(\frac{2\,(m-1)}{m+1}.f_0'.\xi.\frac{\partial g_1}{\partial \xi} + (m-1)\,.f_1'.\xi^2.\frac{\partial g_0}{\partial \xi}\right) + Ec\cdot(\frac{m+1}{2})^2\cdot f_1''^2\cdot \xi^{\frac{4m-2}{m-1}} \tag{32}$$

$$g_1(\xi,\eta=0)=0, \quad g_1(\xi,\eta\to\infty)=0 \tag{33}$$

One can solve equations (30-33) by neglecting terms with ξ-derivatives (a local similar concept) to obtain temperature distributions. The heat flux on the surface of the fin is

$$q_w=-k\left.\frac{\partial T}{\partial y}\right|_{y=0}=h\left(T_f-T_e\right) \tag{34}$$

And with some manipulations the local Nusselt number can be expressed as

$$Nu_x=hx/k=-\left(\frac{m+1}{2}\right)^{1/2}Re_x^{1/2}\left.\frac{\partial\theta}{\partial\eta}\right|_{\eta=0} \tag{35}$$

The corresponding local heat convective heat transfer coefficient can be written as

$$h=-\left(k/x\right)\left(\frac{m+1}{2}\right)^{1/2}Re_x^{1/2}\left.\frac{\partial\theta}{\partial\eta}\right|_{\eta=0} \tag{36}$$

The constant, related to the wedge angle and the temperature gradient in equations (35) and (36), may be expressed as

$$\theta'(0)=-\left(\frac{m+1}{2}\right)^{1/2}\left.\frac{\partial\theta}{\partial\eta}\right|_{\eta=0} \tag{37}$$

Or expanded according to the order of E as

$$\theta_0'(0)=-\left(\frac{m+1}{2}\right)^{1/2}\left.\frac{\partial g_0}{\partial\eta}\right|_{\eta=0} \tag{38}$$

And

$$\theta_1'(0)=-\left(\frac{m+1}{2}\right)^{1/2}\left.\frac{\partial g_1}{\partial\eta}\right|_{\eta=0} \tag{39}$$

The formulation of the first analysis principle for forced convection along a fin involves the energy conservation for the fin and the boundary layer equations for the flow. For a slender fin, ample evidence based on finite difference solutions shows that a one-dimensional model is adequate [30]. The fin temperature at any x location serves as the wall temperature for the adjacent fluid and is denoted as $T_f(x)$. The energy equation for the fin may be written in two different forms, depending on how the coupled-fin/boundary-layer problem is solved. The method used here involves a succession of consecutive iteration solutions for the fin and the boundary-layer flow, with the sequence continued until there is no change (within a preset tolerance) between the nth iteration and the (n - 1) iteration. Within each iteration information must be transferred from the boundary-layer solution which is current for that period and be used as input to update the fin solution. This information may be either in the form of the local heat flux $g(x)$ or the local forced convective heat transfer coefficient $h(x)$. Both $q(x)$ and $h(x)$ are available from the current wedge-type boundary layer solution. The flat plate fin energy equation can be expressed as

$$\frac{d^2T_f}{dx^2} = \frac{q}{k_f t} \tag{40}$$

or

$$\frac{d^2T_f}{dx^2} = \left(\frac{q}{h_f t}\right)(T_f - T_e) \tag{41}$$

Where k_f and t are the thermal conductivity and the half thickness of the fin, respectively. For the solutions of either equations (40) or (41) at a given cycle of the iterative procedures, h and q can be regarded as known quantities. At first glance, it appears advantageous to use equation (40) rather than equation (41) because it is easier to solve; however, equation (41) is employed in the solution scheme. This choice made was based on experience, which has shown that at any stage of an iterative cycle h is closer to the final converged result than q. Thus, equation (41) is chosen to obtain rapid convergence of the iterative procedure, whereby this objective is satisfactorily fulfilled, as will be documented shortly. Equation (41) is recast in a dimensionless form by the substitutions

$$X = x/L, \quad Y = y/L, \quad \theta_f = (T_f - T_e)/(T_0 - T_e) \tag{42}$$

Where T_0 is the base temperature of the fin, so that

$$\frac{d^2\theta_f}{dx^2} + \frac{d^2\theta_f}{dy^2} = \hat{h}N_{cc}\theta_f \tag{43}$$

With boundary conditions

$$\theta_f = 1\ (X = 1), \quad k_f\frac{d\theta_f}{dX} + h\theta_f = 0\ (X = 0) \tag{44}$$

Where Ncc is the conduction-convection number and is defined as

$$Ncc = \left(\frac{hL}{k_f}\right)Re_L^{1/2} \tag{45}$$

The quantity $\hat{h}$ is a dimensionless form of the local convective heat transfer coefficient and can be written as

$$\hat{h} = \left(\frac{hL}{k}\right) Re_L^{1/2} \tag{46}$$

The Biot number is not an appropriate parameter in the present problem because the heat transfer coefficient varies with x and is also unknown prior at the beginning of the computations. It will be change the Ncc number to the Biot number for this conduction-convection problem. These conjugate ordinary differential equations are discretized by a second-order accurate central difference method, and a computer program has been developed to solve these equations. To avoid errors in discretization and calculation processing and to ensure the convergence of numerical solutions, some conventional numerical procedures have been applied in order to choose a suitable grid size, a suitable η range and ξ positions, etc, and a directly gauss elimination with Newton's method [33] is used in the computer program to obtain solutions of these difference equations. Calculation steps of the entire conjugate system are as follows:

1. Estimate the fin temperature distribution $T_f(x)$.
2. Solve flow fields [equations (19),(21), (23)-(24) and (30)-(33)] and the local convective heat-transfer coefficient (equation (46)) according to the local Prandtl number, elastic parameter, and the local fin temperature from the related equations.
3. Solve the heat-conduction equation of the fin (equation (43)) with the renewed local convective heat-transfer coefficient.
4. Compute thermodynamic fluid properties from the fin temperature and free-stream temperature.

The sequences from 2 to 4 are repeated until an acceptable convergence for fin temperature has been reached. The conditions of continuity in the heat flux and temperature at the fluid-solid interface are then satisfied and all relevant heat transfer characteristics can be calculated.

Results and Discussion

Many previous studies of conventional and conjugate problems didn't consider convective effects of the stagnation flow at the tip, but simply substituted the convective condition by an adiabatic boundary condition. However, from the fin-flow configuration shown in Figure 1, the heat transfer at the tip of the fin should not be ignored. It is important to include stagnation flow effects at the tip point of the fin in either conventional heat transfer problems or conjugate problems. Results also support that the heat transfer at the tip is significant, and will be discussed in the later part of the present study. A generalized Falkner-Skan flow derivation is used to analyze a stagnation flow (shape factor β = 1.0 at the fin tip and a pin shape flow β = 0.0) on the fin surface.

Fig 1. A sketch of the physical model for conjugate heat transfer of forced convection with viscous dissipation for visco-elastic fluid past a flat plate fin.

Table 1: $f_0''(0)$ and $f_1''(0)$ vs. β

	$f_0''(0)$	Present		f_1''	Present	
β	$Ref.[5]$	Solution	Errors	$Ref.[5]$	solution	Errors
0.05	0.5311	0.5312	0.0001	0.8214	0.8278	0.0064
0.10	0.5870	0.5871	0.0001	0.5296	0.5279	0.0017
0.20	0.6867	0.6869	0.0002	0.3009	0.2985	0.0024
0.30	0.7748	0.7751	0.0003	0.1418	0.1401	0.0017
0.40	0.8544	0.8548	0.0004	-0.0112	-0.0123	0.0011
0.50	0.9277	0.9282	0.0005	-0.1708	-0.1717	0.0035
0.60	0.9958	0.9965	0.0007	-0.3409	-0.3419	0.0010
0.80	1.1202	1.1211	0.0009	-0.7164	-0.7181	0.0017
1.00	1.2326	1.2337	0.0011	-1.1390	-1.1420	0.0030
1.20	1.3357	1.3371	0.0014	-1.6064	-1.6112	0.0048
1.60	1.5215	1.5234	0.0019	-2.6641	-2.6744	0.0003

Table 2: $\theta_0'(0)$ vs. Pr β=0.0

	$\theta_0'(0)$	$\theta_0'(0)$	
Pr	Ref.[34]	present	Errors
0.3	0.2148	0.2207	0.0059
0.6	0.2770	0.2777	0.0007
0.72	0.2955	0.2960	0.0005
1.0	0.3321	0.3323	0.0002
2.0	0.4223	0.4227	0.0004
3.0	0.4850	0.4856	0.0006
6.0	0.6133	0.6145	0.0012
10.0	0.7281	0.7304	0.0013
30.0	1.0517	1.0602	0.0085
60.0	1.3255	1.3459	0.0204
100.0	1.5718	1.6106	0.0388

A second-order accurate finite difference method is used to obtain solutions of these equations. Comparing $f_0''(0)$ and $f_1''(0)$ to results of [5] at various values of β showed a good agreement and these values are listed in Table 1. Also, computed values of $\theta_0'(0)$ at various values of Pr for flat-plate flow consist with [34], and are listed in Table 2. These tables indicated that the present results are correct, and the

Table 3: Ec vs. $\theta'(0)$, $\theta_0'(0)$ and $\theta_1'(0)$ (E=0.001, β=0, Pr=1, X=0.5, ξ=2)

Ec	$f_0''(0)$	$g_0'(0)$	$\theta_0'(0)$	$f_1''(0)$	$g_1'(0)$	$\theta_1'(0)$	$g'(0)$	$\theta'(0)$
0.000	0.4711	-0.4711	0.3331	1.9998	-39.6988	28.0695	-0.5110	0.3613
0.001	0.4711	-0.4711	0.3331	1.9998	-39.6988	28.0713	-0.5108	0.3612
0.005	0.4711	-0.4701	0.3324	1.9998	-39.7094	28.0788	-0.5099	0.3605
0.010	0.4711	-0.4690	0.3316	1.9998	-39.7221	28.0877	-0.5087	0.3597
0.020	0.4711	-0.4666	0.3299	1.9998	-39.7473	28.1056	-0.5064	0.3580
0.030	0.4711	-0.4642	0.3283	1.9998	-39.7727	28.1235	-0.5040	0.3564
0.040	0.4711	-0.4619	0.3266	1.9998	-39.7979	28.1414	-0.5017	0.3547
0.050	0.4711	-0.4595	0.3249	1.9998	-39.8232	28.1593	-0.4993	0.3531
0.060	0.4711	-0.4572	0.3233	1.9998	-39.8484	28.1771	-0.4970	0.3514
0.070	0.4711	-0.4548	0.3216	1.9998	-39.8737	28.1950	-0.4947	0.3498
0.080	0.4711	-0.4524	0.3199	1.9998	-39.8991	28.2129	-0.4923	0.3481
0.090	0.4711	-0.4501	0.3183	1.9998	-39.9244	28.2308	-0.4900	0.3465
0.100	0.4711	-0.4477	0.3166	1.9998	-39.9497	28.2487	-0.4877	0.3448

numerical method used is adequate. From table 3 shows the important factor Ec calculation results by this study, it is a novel to the others related studies. Hsiao et al. [36-39] and Vajravelu [40] were also using analytical and numerical solutions to solve the related problems. So that, some numerical technique methods will be applied to the same area in the future.

Figures 6, 7 and 8 show the conjugate fin temperature distributions and Figure 2,3,4 and 5 are their corresponding heat transfer coefficients . The results obtained from the present researchers' computation for different Ncc values and different elastic coefficients by centered finite difference methods. The calculations require about more than three iterations for convergence and provide a good solution to the problem.

In addition to the above results, several points require discussion. One consideration is the two kinds of flow fields selected to satisfy the boundary conditions for the fin, constituting a generalization for dealing with convection-conduction problems. Most other studies have used a simplified single flow field and selected the adiabatic boundary condition only for the convenience of analysis. A second consideration is the numerical method for the more accurate fourth-order Runge-Kutta method may be good in one by one shooting case, but may not be suit to the many unknowns initial guess conjugate problems. A third consideration is the accuracy depending on the numerical calculation results. So, selecting an appropriate numerical method is very important. At the presents study, the second-order Centered-difference method is selected as an example. A fourth consideration is a comparison with other related works. The paper has compared with exist solutions and list some of them on the tables (1), (2). All the comparisons seem to be having a good agreement.

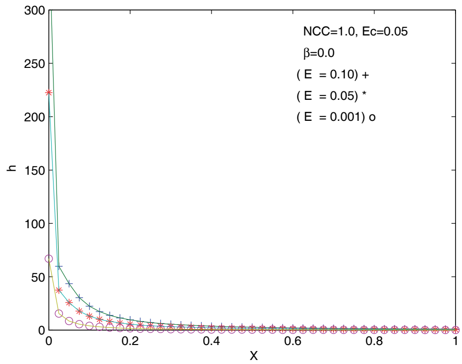

Figure 2: Local Convective heat transfer coefficient distributions

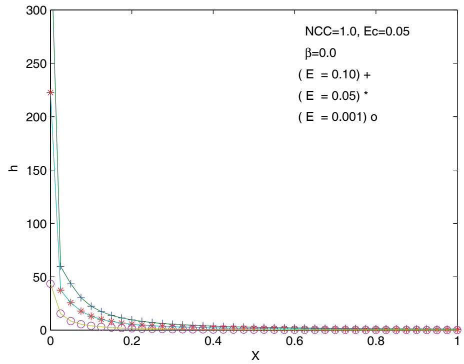

Figure 3: Local Convective heat transfer coefficient distributions

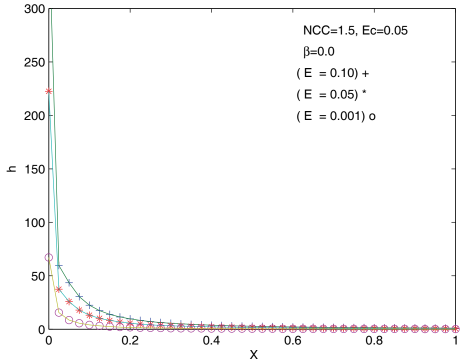

Figure 4: Local Convective heat transfer coefficient distributions

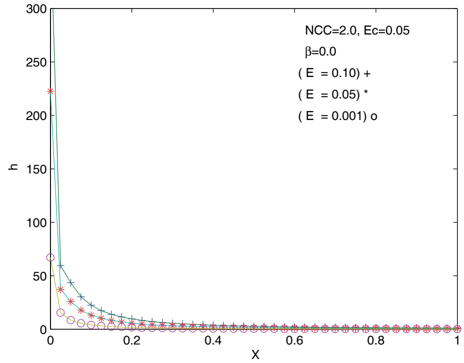

Figure 5: Local Convective heat transfer coefficient distributions

Conclusion

A steady two-dimensional forced convection of an incompressible second-grade fluid adjacent to a fin is studied. A similar solution was obtained and results indicate that the viscous dissipation force made an obviously influence to heat transfer performance. The variation of the magnitude of dimensionless wall shear stress important factor $f_0''(0)$ depends on relative quantities of E, Ec, $f_0'(0)$ and $f_1'(0)$. Dimensionless heat transfer important factor $\theta'(0)$ (according to Eq. (37)) increases with increasing values of Pr and/or $\theta'(0)$ is also increases with increasing E. In the present study, it has been introduced into analyses of a conjugate heat transfer problem of conduction in a solid fin and a forced convection in flow. The present conjugate problem is a hybrid system of the ordinary convective problem with a constant wall temperature. A local heat transfer coefficient is obtained from numerical solutions. Numerical results in the present study indicate that elastic effect

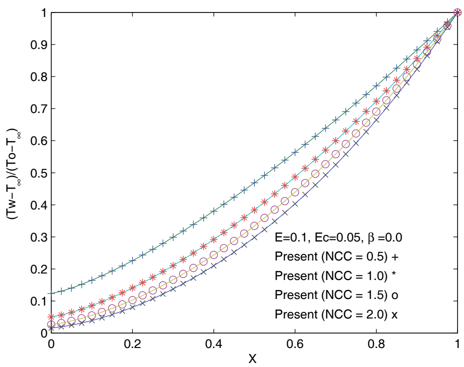

Figure 6: Conjugate fin temperature distributions for E=0.1; Ec=0.05; β=0.015

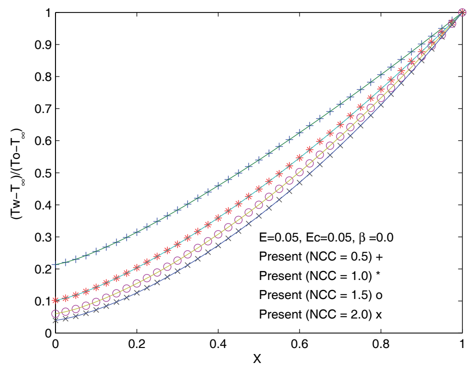

Figure 7: Conjugate fin temperature distributions for E=0.05; Ec=0.05; β=0.015

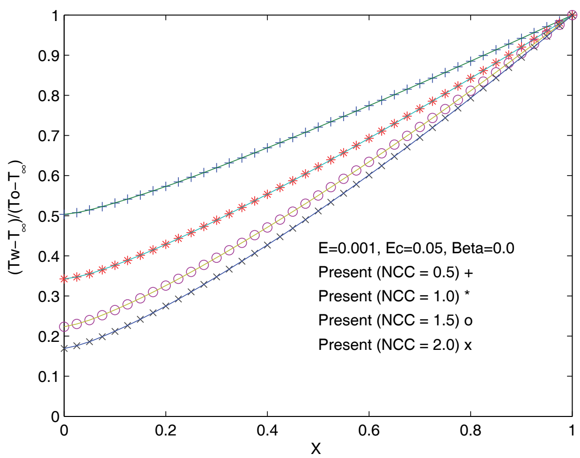

Figure 8: Conjugate fin temperature distributions for E=0.001; Ec=0.05; β=0.015

E in the flow can increase the local heat transfer coefficient and enhance the heat transfer of a fin. Also, a better heat transfer is obtained with a larger Ncc, E, Ec and a larger Pr.

References

1. J.P. Hartnett, Viscoelastic fluids: a new challenge in heat transfer, Transactions of ASME, 296-303, (1992).

2. A.C. Srivatsava, The flow of a non-Newtonian liquid near a stagnation point, Z. Angew. Math. Phys. 9, 80-84 (1958).

3. G. Rajeswari and S.L. Rathna, Flow of a particular class of non-Newtonian visco-elastic and visco-inelastic fluids near a stagnation point, Z. Angew.

Math. Phys. 13, 43-57 (1962).

4. S.P. Mishra and T.C. Panda, Effect of injection on the flow of second order fluid in the inlet region of a channel, Acta Mech. 32, 11-17 (1979).
5. K.R. Rajagopal, A.S. Gupta and T.Y. Na, A note on the Falkner-Skan flows of a non-Newtonian fluid, Int. J. Non-linear Mech. 18, 313-320 (1983).
6. M. Massoudi and M. Ramezan, Effect of injection or suction on the Falkner-Skan flows of second grade fluids, Int. J. Non-linear Mech. 24, 221-227 (1989).
7. Hsu C.E. Maneschy and M. Massoudi, Heat Transfer Analysis of A Non-Newtonian Fluid Past A Porous Plate, Int. J. Non-linear Mech. 28,131-143(1993).
8. K. R. Rajagopal, Boundary Layers in Non-linear Fluids, Pitman Monographs and Surveys in Pure and Applied Mathematics, 209-218, (1995).
9. H. T. Lin and L. K. Lin, Similarity solutions for laminar forced convection heat transfer from wedges to fluids of any Prandtl number, Int. J. Heat Mass Transfer 30, 1111-1118(1987)
10. Wen-Shing Yu and Hsiao-Tsung Lin, Conjugate problems of conduction and free convection on vertical and horizontal flat-plates, Int. J Heat Mass Transfer, Vol.36, No. 5, 1303-1313 (1993).
11. E. M. Sparrow and M. k. Chyu, Conjugate forced convection-conduction analysis of heat transfer in a flat-plate fin, ASME J. Heat Transfer, 104, 204-206(1982).
12. B. Sunden, The effect of Prandtl number on conjugate heat transfer from rectangular fins, Int. Commun. Heat Mass Transfer, 12, 225-232 (1985)
13. A. V. Luikov, T. L. Perelman and V. B. Rvskin, On determination of heat transfer coefficient in simultaneous conductive and convective heat transfer processes, 3^{rd} Int. Heat transfer Conf, Chicago (1966).
14. A. V. Luikov. V. A. Aleksashenko and A. A. Aleksashenko, Analytical methods of solution of conjugated problems in convective heat transfer, Int. J Heat Mass Transfer 14, 1047–1056 (1971).
15. A. V. Luikov, "Heat and Mass Transfer, Handbook, Energiya, Moscow (1972)
16. A. V. Luikov, Conjugate convective heat transfer problems, Int. J Heat Mass Transfer 17, 257-265 (I974)
17. Dibakar Rakshit and C. Balaji, Evaluation of candidate approaches in the study of conjugate convection from a fin array, International Communica-

tions in Heat and Mass Transfer, Volume 32, Issues 3-4, February 2005, Pages 529-538.

18. E. Velayati and M. Yaghoubi, Numerical study of convective heat transfer from an array of parallel bluff plates, International Journal of Heat and Fluid Flow, Volume 26, Issue 1, February 2005, Pages 80-91
19. S.V. Naidu, V. Dharma Rao, P.K. Sarma and T. Subrahmanyam, Performance of a Circular Fin in a Cylindrical Porous Enclosure, International Communications in Heat and Mass Transfer, Volume 31, Issue 8, November 2004, Pages 1209-1218
20. Bourhan Tashtoush, Magnetic and buoyancy effects on melting from a vertical plate embedded in saturated porous media, Energy Conversion and Management 46 (2005) 2566-2577.
21. Saleh M. Al-Harbi, Numerical study of natural convection heat transfer with variable viscosity and thermal radiation from a cone and wedge in porous media, Applied Mathematics and Computation 170 (2005) 64-75.
22. P.S. Datti, K.V. Prasad, M. Subhas Abel, Ambuja Joshi, MHD visco-elastic fluid flow over a non-isothermal stretching sheet, International Journal of Engineering Science 42 (2004) 935-946.
23. B. Sunden, Conjugate mixed convection heat transfer from a vertical rectangular fin, Int. Commun. Heat Mass Transfer, 10, 267-276 (1983).
24. B. Sunoen, A numerical investigation of coupled conduction-mixed convection for rectangular fins, Proc. 3rd Int. Conf Numer. Meth. in Laminar and Turbulent Flow, Aug. 1983, Seattle, pp. 809-819. Pineridge Press, Swansea (1983).
25. M.J. Huang and C. K. Chen, Vertical circular fin with conjugated forced convection-conduction flow, ASME J. Heat Transfer, 106, 658-661(1984)
26. M.J. Huang, C. K. Chen and W. Cleaver, Vertical circular fin with conjugated natural convection conduction flow, ASME J. Heat Transfer, 107, 242-245 (1985).
27. V. K. Garg and K. Velusamy, Heat transfer characteristics for a flat-plate fin, ASME J. Heat Transfer, 108, 224-226 (1986).
28. K. Velusamy and V. K. Garg, Heat transfer from a cylindrical fin in combined free-forced flow, Int. J. Heat Fluid Flow, 9(2), 233-240(1988).
29. Curtis F. Gerald, Applied Numerical Analysis, 2nd Ed., pp. 81-87.(1978).
30. R. S. Rivlin and J. L. Ericksen, Stress deformation relation for isotropic materials, J. Rat. Mech. Analysis, 4, 323-425(1955).

31. K. R. Rajagopal, T.Y. Na, and A. S. Gupta, Flow of a Visco-elastic fluid over a fin, Rheologica Acta, 23, 213-215(1984).
32. E.M. Sparrow and D.K. Hennecke, Temperature depression at the base of a fin, ASME Journal of Heat Transfer, 92, pp. 204-206(1970).
33. Cebeci Bradshaw, Physical and Computational Aspects of Convective Heat Transfer}, (1984).
34. Frank M. White, Viscous Fluid Flow, McGraw-Hill, 2nd Ed., pp.249 (1991).
35. Cheng-Hsing Hsu, Kai-Long Hsiao and Jyh-Tong Teng , Conjugate heat transfer of a triangular fin in the flow of a second-grade fluid, International Journal of Heat and Fluid Flow, Volume 19, Issue 3, June 1998, Pages 282-294
36. K.-L. Hsiao and G.-B. Chen, Conjugate Heat Transfer of Mixed Convection for Viscoelastic Fluid Past A Stretching Sheet, Mathematical Problems in Engineering, Volume 2007, Article ID 17058, 21 pages doi:10.1155/2007/17058.
37. K.-L. Hsiao, Conjugate Heat Transfer of Magnetic Mixed Convection with Radiative and Viscous Dissipation Effects for Second Grade Viscoelastic Fluid past a Stretching Sheet, Applied Thermal Engineering, 27/11-12 pp. 1895-1903 (2007).
38. K.-L. Hsiao, C.-H. Hsu, Conjugate heat transfer of mixed convection for visco-elastic fluid past a triangular fin, Nonlinear Analysis: Real World Applications (2007), doi:10.1016/j.nonrwa.2007.08.018
39. K-L. Hsiao, C-H. Hsu, Conjugate heat transfer of mixed convection for viscoelastic fluid past a Horizontal flat-plate fin, Applied Thermal Engineering (2008), doi: 10.1016/j.applthermaleng.2008.01.027
40. K. Vajravelu, Convection Heat Transfer at a Stretching Sheet with Suction and Blowing, J. of Mathematical Analysis and Application, 188, 1002-1011(1994).

Nomenclature

A	proportional constant.
$\mathbf{A}_1$, $\mathbf{A}_2$	kinematic tensors
$A(x)$	the flat plate fin area at section of x.
b	body force[Nm^{-3}].
B	proportional constant.
E	elastic parameter.
Ec	Eckert number.
f_0	zero-order dimensionless stream function.
f_0'	zero-order dimensionless velocity.

f_0''	zero-order dimensionless velocity gradient.
f_1	first-order dimensionless stream function.
f_1'	first-order dimensionless velocity.
f_1''	first-order dimensionless velocity gradient.
f	dimensionless stream function.
f'	dimensionless velocity.
f''	dimensionless velocity gradient.
g_0	zero-order dimensionless temperature funtion.
g_1	first-order dimensionless temperature function.
g_x	gravitational acceleration in the x-direction.
h	local heat transfer coefficient [$\mathrm{Wm^{-2}K^{-1}}$].
$\hat{h}$	dimensionless local heat transfer coefficient
$\mathbf{I}$	unit vector
k	thermal conductivity of the fluid [$\mathrm{Wm^{-1}K^{-1}}$].
k_1	visco-elastic parameter.
k_f	conductivity of the fin. [$\mathrm{Wm^{-1}K^{-1}}$].
L	characteristic length or fin length. [m]
N_{cc}	conduction-convection parameter
Nu	Nusselt number.
P, P^*	pressure. [$\mathrm{Nm^{-2}}$].
Pr	Prandtl number.
q	local heat transfer rate at the fin [W].
q_w	heat transfer rate at the wall surface.
Re_x	Reynolds number. $(u_\infty x/\nu)$
Re_L	Reynolds number. $(u_\infty L/\nu)$
t	fin half thickness. [m]
$\mathbf{T}$	stress tensor.
T	temperature. [K]
T_w	constant wall surface temperature. [K]
T_e	flow temperature at the outer edge of the boundary layer. [K]
T_f	fin temperature. [K]
T_∞	constant ambient fluid temperature. [K]
T_0	fin base temperature [K]
U^*	characteristic velocity.
u_e	edge velocity [$\mathrm{ms^{-1}}$]
u, v	velocity components in the x and y directions, respectively. [$\mathrm{ms^{-1}}$]
U, V	dimensionless horizontal and vertical flow velocities.
$\mathbf{V}$	velocity vector.
$\mathbf{X}$	dimensionless coordinate, (x/L)
x, y	horizontal and vertical coordinates.

X, **Y**	dimensionless horizontal and vertical coordinates.
V	velocity vector.
x, y	horizontal and vertical coordinate.

Greek symbols

α_1, α_2	first and second normal stress coefficients.
β	the thermal expansion coefficient.
β	shape factor
μ	dynamic viscosity. [$\mathrm{kgs^{-1}m^{-1}}$]
ν	kinematic viscosity. [$\mathrm{m^2s^{-1}}$]
ρ	density of the fluid. [$\mathrm{kgm^{-3}}$]
η	similar parameter.
ξ	dimensionless local parameter.
Φ	potential function.
Ψ	stream function.
θ	dimensionless temperature.
$\theta(0)$	dimensionless temperature at the wall for heat convection.
$\theta_f(0)$	dimensionless temperature at the wall for heat conduction.
$\theta_0(0)$	zero-order part of $\theta(0)$.
$\theta_1(0)$	first-order part of $\theta(0)$.
$\theta'(0)$	temperature gradient at the wall $\theta'(0) = -\left(\frac{m+1}{2}\right)^{1/2} \frac{\partial\theta}{\partial\eta}\Big\|_{\eta=0}$.
$\theta_0'(0)$	zero-order part of $\theta'(0)$, $\theta_0'(0) = -\left(\frac{m+1}{2}\right)^{1/2} \frac{\partial g_0}{\partial\eta}\Big\|_{\eta=0}$.
$\theta_1'(0)$	first-order part of $\theta'(0)$, $\theta_1'(0) = -\left(\frac{m+1}{2}\right)^{1/2} \frac{\partial g_1}{\partial\eta}\Big\|_{\eta=0}$.