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A weakened weak (W2) formulation for certified solutions with bounds, real-time computation and inverse analysis of biomechanics problems

G. R. Liu[1,2]

Summary

This paper introduces first a weakened weakform (W2) using a generalized gradient smoothing technique for an unified formulation of a wide class of compatible and incompatible displacement methods including settings of the finite element methods (FEM) and meshfree methods of special properties including the upper bound properties. A G space is first defined to include discontinuous functions allowing the use of much more types of methods/techniques to create shape functions for numerical models; Properties and a set of important inequalities for G spaces are then proven in theory and analyzed in detail. We prove that the numerical methods developed based on the W2 formulation will be spatially stable, and convergent to exact solutions. We then present examples of some of the possible W2 models, and show the major properties of these models: 1) it is variationally consistent in a conventional sense, if the solution is sought in a H space (compatible cases); 2) it passes the standard patch test when the solution is sought in a G space with discontinuous functions (incompatible cases); 3) the stiffness of the discretized model is reduced compared to the FEM model and even the exact model, allowing us to obtain upper bound solutions with respect to both the FEM and the exact solutions; 4) the W2 models are less sensitive to the quality of the mesh, and triangular meshes can be used without any accuracy problems. These properties and theories have been confirmed numerically via examples solved using a number of W2 models including compatible and incompatible cases.

An NS-PIM model is then used to establish a real-time computation procedure based on the reduced basis approximation. The real-time computation model is then used to inversely identify the interface property of a dental implant system.

keywords: numerical methods, meshfree methods, fem, real-time computation, solution bound, inverse analaysis

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[1]Centre for Advanced Computations in Engineering Science, Department of Mechanical Engineering, National University of Singapore, 9 Engineering Drive 1, Singapore 117576. http://www.nus.edu.sg/ACES/

[2]Singapore-MIT Alliance (SMA), E4-04-10, 4 Engineering Drive 3, Singapore, 117576

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