

Elastic analysis in 3D anisotropic functionally graded solids by the MLPG

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Summary

Functionally graded materials are multi-phase materials with the phase volume fractions varying gradually in space, in a pre-determined profile. This results in continuously graded mechanical properties at the (macroscopic) structural scale. Often, these spatial gradients in material behaviour render FGMs as superior to conventional composites. FGMs possess some advantages over conventional composites because of their continuously graded structures and properties. Due to the high mathematical complexity of the initial-boundary value problems, analytical approaches for elastic analyses of FGMs are restricted to simple geometries and boundary conditions. The elastic analysis in FGM demands an accurate and efficient numerical method.

In spite of the great success of the finite and boundary element methods as effective numerical tools for the solution of boundary value problems on complex domains, there is still a growing interest in development of new advanced methods. Many meshless formulations are becoming to be popular due to their high adaptivity and a low cost to prepare input data for numerical analysis. The meshless local Petrov-Galerkin (MLPG) method is the fundamental base for derivations of many meshless formulations, since trial and test functions are chosen from different functional spaces. In this paper, the Heaviside step function is used as the test function. It yields a pure boundary integral formulation on local sub-domains for anisotropic elastostatics, while in elastodynamics an additional domain integral of inertia terms is involved. Small spherical subdomains surrounding the nodes are randomly distributed over the analyzed domain. The spatial variation of the displacement is approximated by the moving least-square (MLS) scheme. After performing the spatial integrations, one obtains the system ordinary differential equations for certain nodal unknowns. That system is solved numerically by the Houbolt finite difference scheme as a time stepping method. Alternatively, the Laplace transform is applied to eliminate the time variable. Then, the local integral equations are derived for Laplace transforms. Several quasi-static boundary value problems have to be solved for various values of the Laplace transform parameter. The Stehfest inversion method is applied to obtain the time-dependent solutions. The integral equations have a very simple nonsingular form. Moreover, both the boundary and domain integrations can be easily carried out on spherical

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sub-domains. The boundary conditions on the global boundary are satisfied by collocation of the MLS-approximation expressions for the displacements at boundary nodal points. To demonstrate the accuracy of the present method several numerical examples with simple and/or more complex geometry are considered for static and dynamic cases.