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On the shear influence on the free vibration behavior of magneto-electro-elastic beam

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Summary

A magneto-electro-elastic Timoshenko beam model is presented and employed to study the effect of the shear strain on the free vibration behavior of the beam. Once the differential governing equation for Timoshenko magneto-electro-elastic beam is derived, the Euler-Bernoulli model is obtained by letting be zero some of the governing equation coefficients. Results for the Timoshenko and Euler-Bernoulli beam are presented in comparison with two-dimensional finite element computation.

Introduction

Magneto-electro-elastic composites have recently emerged as a new and interesting materials for sensors and actuators applications, mainly due to their inherent and unique capability to convert energy among three different forms: magnetic, electric and mechanical. Magneto-electro-elastic composites are made up by piezoelectric and piezomagnetic phases that, combined together, give rise to a *smarter* composite, either in particulate or laminate form [1]. The obtained composites present both the electro-mechanical and magneto-mechanical coupling, characteristic of the piezoelectric and piezomagnetic phase respectively. Moreover, magneto-electro-elastic materials are also characterized by a strong coupling between the magnetic and the electric fields that stems as product property of the constituents electro-elastic and magneto-elastic coupling. The magneto-electric coupling is absent in the original phases and is a peculiar characteristic of the whole composite. Magneto-electro-elastic composites are exploited for the construction of magnetic field probes, electric packaging, hydrophones, medical ultrasonic imaging, sensors and actuators [2]. Because of their inherent multidisciplinary nature, a variety of studies are needed to obtain useful and valuable insights into their typical behaviour. Many research activities have been focused on chemical or technological features [3 - 5]. On the other hand, many aspects related to analytical or numerical modelling of magneto-electro-elastic media have been investigated [6-11]. In the present paper a model for free vibration of magneto-electro-elastic beam is presented. The model relies upon the Timoshenko's beam

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theory [12], which is extended to the present problem by considering the magneto-electro-mechanical constitutive relationships and the assumptions that no density charge and no density current act on the analyzed domain. Comparisons between the proposed Timoshenko magneto-electro-elastic beam model and the simplified Euler-Bernoulli [12] beam theory are also presented to highlight the influence of the shear effect on the magneto-electro-elastic beam free vibration behavior.

Basic assumption and governing equation

In order to write the magneto-electro-elastic beam governing equation, let L be the beam length and h the beam thickness. Moreover let us assume the electric and magnetic poling are directed along the thickness direction, namely the $y-axis$, and that the electric and magnetic field components transverse to the poling direction, i.e. along the $x-axis$, are negligible [13]. With the aim of describing the electric and magnetic beam characteristics, it is assumed that no external current density is present in the beam. Thus, both the electric and magnetic fields can be written in terms of an electric and a magnetic scalar potential functions, φ and ψ respectively, by virtue of the gradient relationships [8]. Under the aforementioned hypotheses and by assuming monoaxial stress state, the beam magneto-electro-elastic constitutive relationships read as

$$\begin{array}{lll} \sigma_{xx} = c\,\gamma_{xx} - e\,E_y - d\,H_y & D_x = e_{14}\,\gamma_{xy} & B_x = d_{14}\,\gamma_{xy} \\ \sigma_{xy} = c_{44}\,\gamma_{xy} & D_y = e\,\gamma_{xx} + \varepsilon\,E_y + \eta\,H_y & B_y = d\,\gamma_{xx} + \eta\,E_y + \mu\,H_y \end{array} \tag{1}$$

being σ_{ij} and γ_{ij} the stress and strain components, D_i and B_i the electric displacements and magnetic induction components while E_y and H_y are the electric and magnetic field components. In Eq.(1) c represents the elastic coefficient, ε and μ are the dielectric constant and the magnetic permeability of the material while e and d are representative of the piezoelctric and piezomagnetic coupling, respectively. The kinematical model is given by the displacement components u and v which read as

$$u = -y\,\vartheta(x,t), v = v(x,t)\,. \tag{2}$$

where ϑ is the cross sectional rotation. Starting from the Gauss' laws for electrostatic and magnetostatic problem in absence of electric charge density, and in view of the constitutive relationships and the previously mentioned assumptions, it follows that

$$\begin{aligned} \varphi(x,y,t) &= \left[\frac{\eta\,(d_{14}+d) - \mu\,(e_{14}+e)}{\varepsilon\mu - \eta^2}\frac{\partial\vartheta}{\partial x} + \frac{\mu\,e_{14} - \eta\,d_{14}}{\varepsilon\mu - \eta^2}\frac{\partial^2 v}{\partial x^2}\right]\frac{y^2}{2} + a_1 y + a_2 \\ \psi(x,y,t) &= \left[\frac{\eta\,(e_{14}+e) - \varepsilon\,(d_{14}+d)}{\varepsilon\mu - \eta^2}\frac{\partial\vartheta}{\partial x} + \frac{\varepsilon\,d_{14} - \eta\,e_{14}}{\varepsilon\mu - \eta^2}\frac{\partial^2 v}{\partial x^2}\right]\frac{y^2}{2} + a_3 y + a_4 \end{aligned} \tag{3}$$

Thus, by imposing the magneto-electric boundary conditions on the bottom and top surfaces of the beam, the integration constants a_i are set and both the potential functions distribution along the thickness directions become known.

Once the electro-magnetic variables distribution in the thickness direction is known, the shear force T and bending moment M can be written as follows

$$T = \int_{-\frac{h}{2}}^{\frac{h}{2}} \sigma_{xy} dy = S\left(\frac{\partial v}{\partial x} - \vartheta\right) \tag{4}$$

$$M = \int_{-\frac{h}{2}}^{\frac{h}{2}} \sigma_{xx} y dy = K_\vartheta \frac{\partial \vartheta}{\partial x} + K_v \frac{\partial^2 v}{\partial x^2} \tag{5}$$

where S is the bimorph shear stiffness, K_ϑ is the magneto-electro-elastic equivalent bending stiffness of the bimorph beam while K_v is an additional bending stiffness related to the second derivative of the transverse displacement that is present when piezoelectric or piezomagnetic coupling exist. Beam stiffness coefficients in Eq(4) and (5) are computed by using the constitutive relationships to express the stress components and by taking into account the kinematical model and the gradient relationships along with the electric and magnetic scalar functions Eq(3).

By virtue of Eq.(4) and (5), the beam equilibrium equations, taking into account both the translational and rotational inertia, lead to the following free vibration magneto-electro-elastic governing equation

$$(K_\vartheta + K_v)\frac{\partial^4 v}{\partial x^4} + (\frac{\rho h K_\vartheta}{S} + \frac{\rho h^3}{12})\frac{\partial^4 v}{\partial x^2 \partial t^2} + \rho h \frac{\partial^2 v}{\partial t^2} + \frac{\rho^2 h^4}{12S}\frac{\partial^4 v}{\partial t^4} = 0 \tag{6}$$

From Eq.(6) it follows that the Euler-Bernoulli beam governing equation is obtained by letting the beam shear stiffness S tend to infinity.

Modal Analysis

The problem of free vibrations is specified by the homogeneous differential equation of motion Eq.(6) and the homogenous boundary conditions. Three distinct boundary conditions configurations are considered in the present work, namely the simply supported beam, the cantilever beam and the clamped configuration. The general solution to the homogenous equation of motion is considered in the form

$$v(x,t) = V_n(x)\sin\omega_n t \tag{7}$$

where ω_n is the generic natural frequencies and V_n is the corresponding mode shapes written as

$$V_n(x) = C_{in} e^{\lambda_{in} x}, \qquad (i = 1,2,3,4) \tag{8}$$

thus, by using Eq.(10) and (8) into Eq.(6) the following characteristic equation is obtained

$$(K_\vartheta + K_v)\lambda_{ni}^4 + \left(\frac{\rho h^3}{12} + \frac{\rho h K_\vartheta}{S}\right)\omega_n^2\lambda_{ni}^2 + \left(\rho h \omega_n^2 - \frac{\rho^2 h^4}{12S}\right)\omega_n^2 = 0 \qquad (9)$$

which links the eigenvalues λ_{ni} to the eigenfrequencies ω_n and allows to express the mode shape Eq.(8) as function of the eigenfrequency only. Eventually, by writing the cross sectional rotation as

$$\vartheta(x,t) = \Theta_n(x)\sin\omega_n t \qquad (10)$$

and by expressing its mode shape Θ_n as function of the transverse displacement mode shape V_n, by means of the equilibrium equation, as follows

$$\Theta_n = \frac{1 + \frac{\rho h K_\vartheta}{S^2}\omega_n^2}{1 - \frac{\rho h^3}{12S}\omega_n^2} V_n' + \frac{K_\vartheta + K_v}{S - \frac{\rho h^3}{12}\omega_n^2} V_n''' \qquad (11)$$

where prime denotes differentiation with respect to x, the boundary conditions can be superimposed allowing the computation of the beam natural frequencies and mode shapes.

Natural frequencies

Some results are presented to show the effectiveness of the model and the influence of the shear force on the free vibration behavior of a magneto-electro-elastic beam. In the first application the eigenfrequencies of a cantilever magneto-electro-elastic beam are calculated and compared to those computed by using a two-dimensional finite element model, see [11]. The beam length is $L = 0.3\ m$ while its thickness is $h = 0.02\ m$. Material properties of a multiphase magneto-electro-elastic composite are taken from [6] and are arranged in light of the monoaxial stress state assumption. The reduced material constants are listed in Tab.(1) and the composite volume density is $\rho = 5550\ \frac{kg}{m^3}$.

c $[GPa]$	$c_{44}\,[GPa]$	e $\left[\frac{C}{m^2}\right]$	$e_{14}\left[\frac{C}{m^2}\right]$	d $\left[\frac{N}{Am}\right]$	$d_{14}\left[\frac{N}{Am}\right]$	$\eta\left[\frac{nNs}{VC}\right]$	$\varepsilon\left[\frac{nF}{m}\right]$	$\mu\left[\frac{Ns^2}{C^2}\right]$
120.6	45	6.5	0	32.6	180	-8.9	8.85	7.54

Table 1: Magneto-electro-elastic material constants.

The natural frequencies of the cantilever beam, considered as an Euler-Bernoulli (f_{EB}) or a Timoshenko (f_T) beam are reported in Tab.(2) where the percentage discrepancies with respect to the FEM analysis (f_{FEM}) are also highlighted. It appears from data shown in Tab.(2) that the effectiveness of the simplified Euler-Bernoulli beam model decreases as the natural frequency value increases reaching a percentage discrepancy of about 20% with respect to the finite element two-dimensional computation. By taking into account the shear force effect, by mean of the Timoshenko beam theory, the natural frequencies percentage discrepancy with respect to the FEM analyses, even though it increases with the frequency, is always negligible. Moreover it is seen that by using the Euler-Bernoulli model the seventh flexural mode is not captured.

Mode	f_T	f_{EB}	f_{FEM}[11]	$100\frac{f_T - f_{FEM}}{f_{FEM}}$	$100\frac{f_{EB} - f_{FEM}}{f_{FEM}}$
1	169	170	169	0	0.591
2	1043	1064	1043	0	2.013
3	2831	2974	2835	-0.141	4.903
4	5323	5817	5337	-0.262	8.993
5	8385	9594	8423	-0.451	13.902
6	11887	14300	11967	-0.668	19.495
7	15720	–	15867	-0.926	–

Table 2: Natural frequencies [Hz] for the flexural mode of the cantilever magneto-electro-elastic beam

The simply supported and clamped configurations are also studied for the same beam. Tab.(3) lists the natural frequencies of the simply-supported beam showing the lack of accuracy for the high-frequency mode and that the fifth mode of vibration is lost if the shear effect is not taken into account.

Mode	f_T	f_{EB}	f_{FEM}[11]	$100\frac{f_T - f_{FEM}}{f_{FEM}}$	$100\frac{f_{EB} - f_{FEM}}{f_{FEM}}$
1	475	478	474	0.210	0.843
2	1861	1915	1858	0.161	3.067
3	4049	4308	4045	0.0988	6.501
4	6900	7660	6896	0.0580	11.07
5	10272	–	10274	-0.0194	–
6	14039	11969	14056	-0.120	-14.848
7	18100	17235	18150	-0.275	-5.0413

Table 3: Natural frequencies [Hz] for the flexural mode of the simply-supported magneto-electro-elastic beam

The same behavior is observed for the clamped beam, whose natural frequencies for flexural modes are reported in Tab.(4). In this case both the fifth and seventh modes are not captured by using the Euler-Bernoulli approximation and the forth frequency of vibration presents a percentage difference with respect to the FEM result of about 23%. Even in this last case, the Timoshenko beam analytical solution has revealed to be effective and reliable in modeling flexural vibration of magneto-electro-elastic beam showing a percentage discrepancy with the numerical solution that is always acceptable.

Mode	f_T	f_{EB}	f_{FEM}[11]	$100\frac{f_T - f_{FEM}}{f_{FEM}}$	$100\frac{f_{EB} - f_{FEM}}{f_{FEM}}$
1	1053	1085	1054	-0.094	2.941
2	2800	2991	2805	-0.178	6.631
3	5250	5864	5266	-0.303	11.356
4	11675	14482	11758	-0.705	23.167
5	15423	–	15572	-0.956	–
6	19416	20227	19659	-1.236	2.889
7	23593	–	23430	0.695	–

Table 4: Natural frequencies [Hz] for the flexural mode of the clamped magneto-electro-elastic beam

Conclusions

A magneto-electro-elastic beam model based upon the Timoshenko beam theory to take into account shear force has been presented. Assumptions made on the electric and magnetic fields have allowed to include the piezoelectric, piezomagnetic and electromagnetic coupling in the beam equivalent bending stiffness coefficients. Free vibration analyses carried out by using the proposed model have shown that the effect of shear deformation is relevant in computing the natural frequencies and mode shapes of magneto-electro-elastic beam.

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