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Piezoelectric four-node geometrically exact solid-shell element with seven displacement degrees of freedom

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Introduction

In recent years, a considerable work has been carried out on 3D continuum-based finite elements that can handle analyses of thin piezoelectric laminated composite shells satisfactorily. These elements are typically defined by two layers of nodes at the bottom and top surfaces of the shell with three displacement degrees of freedom per node and known as piezoelectric *isoparametric* solid-shell elements [1,2]. In the isoparametric solid-shell element formulation, initial and deformed geometry are equally interpolated allowing one to describe rigid-body motions precisely. Still, the isoparametric solid-shell element formulation is computationally inefficient because stresses and strains are analyzed in the global and local orthogonal Cartesian coordinate systems, although the normalized element coordinates represent already curvilinear convected coordinates.

An alternative way is to develop the piezoelectric *geometrically exact* solid-shell element based on the curvilinear reference surface coordinates that finds its point of departure in a paper [3]. The term "geometrically exact" reflects the fact that reference surface geometry is described by analytically given functions. This geometrically exact piezoelectric solid-shell element formulation is based on the strain-displacement relationships of the first-order 6-parameter equivalent single-layer (ESL) theory written in curvilinear convected coordinates. It is remarkable that these strain-displacement relationships precisely represent all rigid-body shell motions and no assumptions except for standard Timoshenko-Mindlin kinematics are required to derive them. For this purpose, displacement vectors of the bottom and top surfaces of the shell are introduced but resolved, in contrast with the piezoelectric isoparametric solid-shell element formulation, in the reference surface frame.

Problem Formulation

Here, the more general study on the basis of the first-order 7-parameter ESL theory of piezoelectric shells is considered. As unknowns six displacements of the outer surfaces of the shell and an additional transverse displacement of the midsurface are chosen. Such choice of displacements gives the possibility to derive strain-displacement relationships, which are invariant under rigid-body shell motions. It should be mentioned that in some works developing the isoparametric solid-shell element formulation, displacement vectors of the face and middle surfaces are also utilized. But in the proposed 7-parameter shell theory selecting as

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unknowns the displacements of the outer and middle surfaces has a principally another mechanical sense and allows one to obtain the non-linear strain-displacement relationships with aforementioned attractive properties.

It is known that the 6-parameter piezoelectric shell theory on the basis of the complete 3D constitutive equations is deficient because thickness locking occurs [1-3]. This is due to the fact that the linear displacement field in the thickness direction results in a constant transverse normal strain, which in turn causes artificial stiffening of the shell element in the case of non-vanishing Poisson's ratios. In order to circumvent a locking phenomenon, the 3D constitutive equations have to be modified. However, the use of complete 3D constitutive laws within the shell analysis is of great importance for engineering applications. Thus, the first-order 7-parameter piezoelectric shell model is best suited for this purpose because such a model is optimal with respect to a number of degrees of freedom employed.

It is assumed that the electric potential is linear through the thickness of the piezoelectric layer and all displacement and electric potential degrees of freedom are coupled via constitutive equations. This allows us to formulate the efficient *curved* four-node solid-shell element for the analysis of thin piezoelectric laminated shells. To avoid shear and membrane locking and have no spurious zero energy modes, the assumed strain and stress resultant fields are invoked. As a result the piezoelectric geometrically exact solid-shell element developed does not contain any spurious zero energy modes and its stiffness matrix possesses six zero eigenvalues. It is remarkable that all elemental matrices require only direct substitutions and they are evaluated by using the 3D analytical integration. So, our geometrically exact element is very economical compared to conventional isoparametric finite elements because it additionally allows using extremely coarse meshes. Taking into account that electric signals generated by sensors are fed into microprocessors to activate a system of piezoelectric actuators in real time, a developed code is robust and very promising.

For the actuator-embedded shell analysis when only a prescribed input voltage is applied, the governing finite element equations can be expressed as

$$\mathbf{K}_{\mathrm{M}}\mathbf{U} = -\mathbf{K}_{\mathrm{ME}}\mathbf{\Phi},$$

where $\mathbf{U}$ and $\mathbf{\Phi}$ are the displacement and electric potential vectors of the element; $\mathbf{K}_{\mathrm{M}}$ and $\mathbf{K}_{\mathrm{ME}}$ are the stiffness and piezoelectric stiffness matrices. These equations have to be solved to assess a response of the coupled electromechanical system.

The performance of the proposed piezoelectric geometrically exact solid-shell element is evaluated with several problems extracted from the literature and authors' example as well, namely:

A bimorph cantilever beam consisting of two identical PVDF layers polarized in opposite directions parallel to the thickness direction. The electrical loading case is assumed to be a unit voltage applied across the beam thickness.

A simply supported plate composed of six graphite/epoxy layers with a staking sequence $[0/90/0]_S$ and PZT actuators attached to the outer planes. The plate is subjected initially to uniform pressure and then to a constant voltage. This problem deals with the shape control of a composite plate by piezoelectric actuators.

A cantilever laminated cylindrical shell made of six graphite/epoxy layers with a ply orientation $[30/30/0]_S$ and PZT actuators bonded to the bottom and top surfaces. Both piezoelectric layers are loaded statically by the same electric potential.

A cantilever laminated hyperbolic shell with six graphite/epoxy layers of different ply orientations and PVDF actuators attached to the outer surfaces.

All results are in a good agreement with those derived by using the existing piezoelectric isoparametric solid-shell elements [1,2] and geometrically exact solid-shell element [3] based on the 6-parameter shell theory.

Acknowledgement

The support of this work by Russian Ministry of Education and Science under Grant No 2.1.1/660 is gratefully acknowledged.

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